



## Transmission Lines and Antennas (2016503)

### Exercises on Antennas

November 17, 2012

1. Derive the normalization condition for gain (valid for any antenna):  $\int G d\Omega = 4\pi\eta$ .

2. An antenna has a gain pattern approximated as:

$$g(\theta, \phi) = \begin{cases} G_0 & |\theta - \pi/2| \leq \theta_w/2, |\phi - \phi_0| \leq \phi_w/2 \\ 0 & \text{otherwise} \end{cases}$$

and a ohmic efficiency  $\eta$ .

- (a) Derive an expression for the maximum gain  $G_0$  as a function of the beam widths  $\theta_w$  and  $\phi_w$  by enforcing the normalization condition above.
- (b) Compute  $G_0$  in linear units and dB for  $\theta_w = \phi_w = 10^\circ$  and  $\eta = -2\text{dB}$ .
3. A GSM circular cell with a maximum diameter of 17 km is covered by three identical antennas working at 1800MHz with a pattern approximately described by the expression in exercise 2 with  $\theta_w = 12^\circ$ ,  $\phi_w = 120^\circ$ ,  $\eta = -0.5\text{dB}$  and  $\phi_0 = 0, 120^\circ$  and  $240^\circ$ .

Each antenna in the base station has an input power  $P_{in} = 15\text{W}$ . Considering as receiver a handset equipped with an antenna having  $G = -1\text{dB}$  and impedance  $50\Omega$  and a maximum polarization mismatch of  $-3\text{dB}$ , compute the minimum open terminal voltage at the receiving antenna (i.e. voltage amplitude of the equivalent generator considering that Friis equation assumes perfect receiver matching).

4. Compute by direct integration the radiation pattern of a half-wavelength dipole lying along  $\hat{z}$  and centered at the origin. Assume that the z-directed current is given by  $I(z) = I_0 \cos(kz)$ ,  $|z| < \lambda/4$ .
5. It is required to design an antenna for broadcasting in horizontal polarization that has almost omni-directional coverage in the horizontal plane and directive pattern in the vertical plane.
- (a) Design a broadside array with minimum complexity (i.e. minimum number of elements) having  $\text{FNBW} \leq 15^\circ$  and avoiding grating lobes at

850MHz. Use a computer to plot the array factor in a relevant plane to check conformance to specs.

- (b) Consider that the elementary radiator is a half-wavelength dipole whose pattern can be approximated by the following expression (valid only when dipole lies along  $\hat{z}$ ):

$$E_\theta = \frac{V_0}{r} \sin^3 \theta$$

- i. Draw a schematic of the whole array indicating clearly the horizontal plane and locations and orientations of elements.
- ii. Find the E and H planes of the overall antenna.
- iii. Compute the expressions for the field on these planes and use a computer to plot them.
6. (a) Design a rectangular patch antenna at 1850MHz having  $R_{in} = 50\Omega$  using inset feed, i.e. compute L, W and l. Substrate parameters are  $h = 1/20$  inch,  $\epsilon_r = 2.33$ .
- (b) Compute the frequency response of  $|S_{1,1}|_{\text{dB}}$  by using the transmission line model with reference impedance  $R_0 = R_{in}$  (patch input impedance at resonance) in a band  $\pm 10\%$  around the resonant frequency above. What is the 10dB-bandwidth (in percent)?
- (c) Compute and plot the resulting pattern when the patch designed is used as basic element in the array of the previous exercise.

## References

- [1] D. M. Pozar, *Microwave Engineering*. John Wiley & sons, 1998.
- [2] R. E. Collin, *Foundations of Microwave Engineering*. IEEE press, 2001.