# 2012-III: Transmission Lines and Antennas


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# Reduced Maxwell's Equations

- Assuming invariance of the medium along the z axis and a wave solution in that direction, the homogeneous (no sources) Maxwell's equations can be simplified by separating fields and the del operator into transverse (i.e. Restricted to the xy plane) and longitudinal/axial components (along z):

$$\begin{aligned} \mathbf{E}(x,y,z) &= \left[\mathbf{e_t}(x,y) + \hat{\mathbf{z}} e_z(x,y)\right] e^{-j\beta z} \\ \mathbf{H}(x,y,z) &= \left[\mathbf{h_t}(x,y) + \hat{\mathbf{z}} h_z(x,y)\right] e^{-j\beta z} \end{aligned}$$

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z} = \nabla_t - j\beta\hat{\mathbf{z}}$$

$$\begin{aligned} \nabla_t \times \mathbf{e_t} &= -j\omega\mu_0 \hat{\mathbf{z}} h_z \\ -\hat{\mathbf{z}} \times \nabla_t e_z - j\beta\hat{\mathbf{z}} \times \mathbf{e_t} &= -j\omega\mu_0 \mathbf{h_t} \\ \nabla_t \times \mathbf{h_t} &= j\omega\epsilon_0 \hat{\mathbf{z}} e_z \\ \hat{\mathbf{z}} \times \nabla_t h_z + j\beta\hat{\mathbf{z}} \times \mathbf{h_t} &= -j\omega\epsilon_0 \mathbf{e_t} \\ \nabla_t \cdot \mathbf{h_t} &= j\beta h_z \\ \nabla_t \cdot \mathbf{e_t} &= j\beta e_z \end{aligned}$$

# TEM modes

$$
\begin{aligned}
\nabla_t \times \mathbf{e_t} &= 0 \\
\beta\hat{\mathbf{z}} \times \mathbf{e_t} &= \omega\mu_0\mathbf{h_t} \\
\nabla_t \times \mathbf{h_t} &= 0 \\
\beta\hat{\mathbf{z}} \times \mathbf{h_t} &= -\omega\epsilon_0\mathbf{e_t} \\
\nabla_t \cdot \mathbf{h_t} &= 0 \\
\nabla_t \cdot \mathbf{e_t} &= 0
\end{aligned}
$$

# TE waves (H-waves)

- Also called H waves, these are obtained by setting $e_z = 0$ in the reduced equations:

$$
\begin{aligned}
\nabla_t \times \mathbf{e_t} &= -j\omega\mu_0\hat{\mathbf{z}}h_z \\
-j\beta\hat{\mathbf{z}} \times \mathbf{e_t} &= -j\omega\mu_0\mathbf{h_t} \\
\nabla_t \times \mathbf{h_t} &= 0 \\
\hat{\mathbf{z}} \times \nabla_t h_z + j\beta\hat{\mathbf{z}} \times \mathbf{h_t} &= -j\omega\epsilon_0\mathbf{e_t} \\
\nabla_t \cdot \mathbf{h_t} &= j\beta h_z \\
\nabla_t \cdot \mathbf{e_t} &= 0
\end{aligned}
$$

From Helmholtz equation:

$$
\begin{aligned}
\nabla_t^2 h_z + k_c^2 h_z &= 0 \\
\nabla_t^2 \mathbf{h_t} + k_c^2 \mathbf{h_t} &= 0 \\
k_c^2 &\equiv k_0^2 - \beta^2
\end{aligned}
$$

Curl of 3rd eq. at left, using then 5th at left and 2nd above:

$$
\mathbf{h_t} = -\frac{j\beta}{k_c^2}\nabla_t h_z
$$

$\hat{\mathbf{z}}$ x 2nd eq at left:

$$
\mathbf{e_t} = -Z_0\frac{k_0}{\beta}\hat{\mathbf{z}} \times \mathbf{h_t} = -Z_{TE}\hat{\mathbf{z}} \times \mathbf{h_t}
$$

$$
Z_{TE} \equiv Z_H \equiv Z_0\frac{k_0}{\beta} > Z_0
$$

# *TM waves (E-waves)*

- Solutions are obtained by setting hz = 0, and are also called E waves. Previous solution can be reused via the duality transformations:

E → H, H → -E, μ → ε, ε → μ

$$
\begin{aligned}
\nabla_t^2 e_z + k_c^2 e_z &= 0 \\
\mathbf{e_t} &= -\frac{j\beta}{k_c^2}\nabla_t e_z \\
\mathbf{h_t} = \frac{1}{Z_0}\frac{k_0}{\beta}\hat{\mathbf{z}} \times \mathbf{e_t} &= \frac{1}{Z_{TM}}\hat{\mathbf{z}} \times \mathbf{e_t} \\
Z_{TM} \equiv Z_E &\equiv Z_0\frac{\beta}{k_0} \leq Z_0
\end{aligned}
$$

# *Computation of RLGC model parameters from Modal Fields (TEM lines)*

$$
\begin{aligned}
L &= \frac{\mu}{|I_0|^2}\int_S \mathbf{H}\cdot\mathbf{H}^*\mathrm{ds} \quad (\mathrm{H/m}) \\
C &= \frac{\epsilon}{|V_0|^2}\int_S \mathbf{E}\cdot\mathbf{E}^*\mathrm{ds} \quad (\mathrm{F/m}) \\
R &= \frac{R_s}{|I_0|^2}\int_{C_1+C_2} \mathbf{H}\cdot\mathbf{H}^*\mathrm{dl} \quad (\Omega/\mathrm{m}) \\
G &= \frac{\omega\epsilon''}{|V_0|^2}\int_S \mathbf{E}\cdot\mathbf{E}^*\mathrm{ds} \quad (\mathrm{S/m})
\end{aligned}
$$

# *Topics in guided propagation*

- Power flow
- Conductor loss
- Dielectric loss
- Inhomogeneous filling

# *Loss in Transmission Lines and Waveguides*

- Perturbation techniques can be applied to low loss transmission media such as TLs and waveguides:
  - Solve for the modal fields neglecting loss
  - Insert loss as a correction factor in the propagation constant (attenuation constant).
  - Various loss mechanisms are included additively in the attenuation constant

$$\alpha = \alpha_c + \alpha_d$$

$$\alpha_d = \frac{k^2}{2\beta}\tan\delta$$

$$\alpha_c = \frac{R_s Z_\dagger}{2}\frac{\int h_{tan}^2 dl}{\iint e_t^2 ds}$$

$$R_s = \frac{1}{\sigma\delta_s}$$