2012-1: Transmission Lines and Antennas

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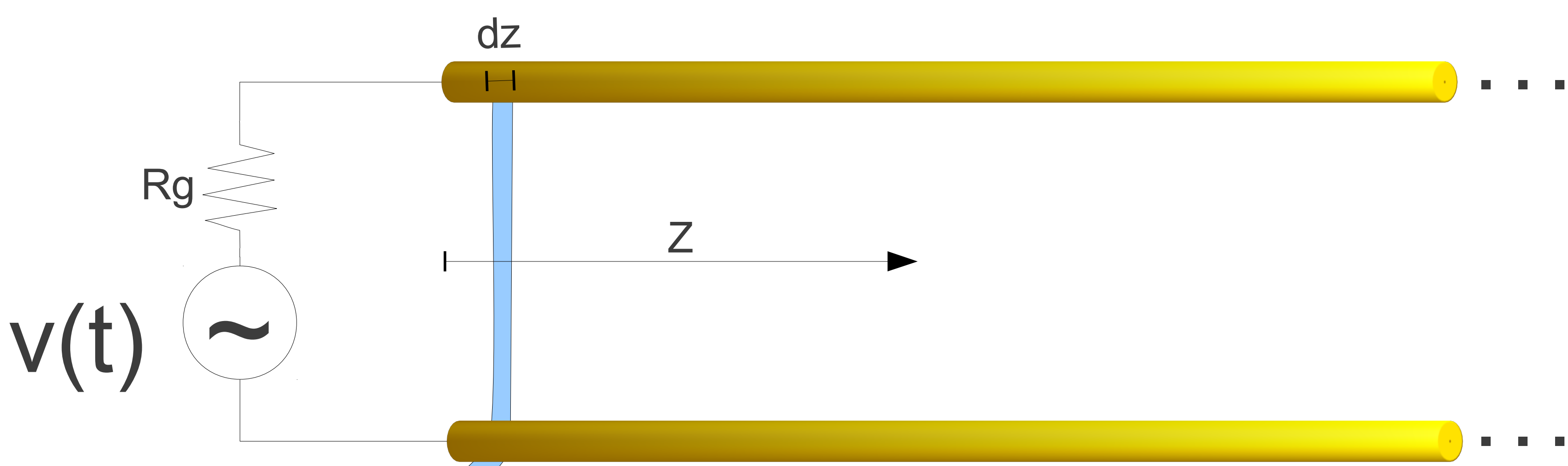
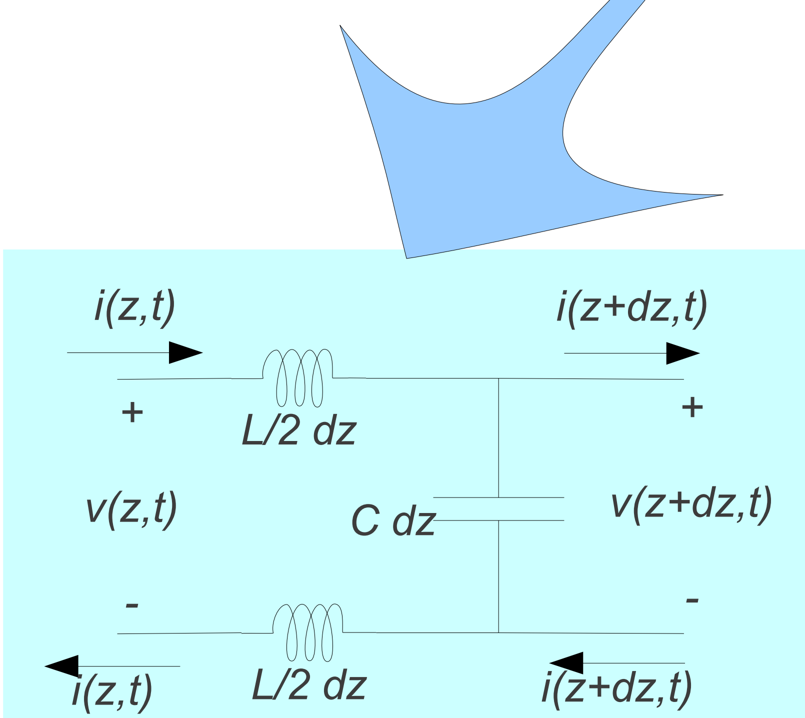
Transmission Lines





$$
\begin{aligned}
-i(z,t) + i(z+dz,t) + C\,dz\,\frac{dv(z+dz,t)}{dt} &= 0 \\
v(z,t) - v(z+dz,t) - 2\frac{L}{2}dz\,\frac{di(z+dz,t)}{dt} &= 0
\end{aligned}
$$

$$dz \to 0$$

Telegrapher's equations

$$
\boxed{
\begin{aligned}
\frac{\partial i(z,t)}{\partial z} + C\frac{\partial v(z,t)}{\partial t} &= 0 \\
\frac{\partial v(z,t)}{\partial z} + L\frac{\partial i(z,t)}{\partial t} &= 0
\end{aligned}
}
$$

Waves in Transmission Lines

- Partial derivation of equations above (one wrt t, other wrt z followed by substitution of the element with crossed differentials) results in identical (wave) equations for v(z,t) and i(z,t):

$$\begin{aligned} \frac{\partial^2 v(z,t)}{\partial z^2} - LC\frac{\partial^2 v(z,t)}{\partial t^2} &= 0 \\ \frac{\partial^2 i(z,t)}{\partial z^2} - LC\frac{\partial^2 i(z,t)}{\partial t^2} &= 0 \end{aligned}$$

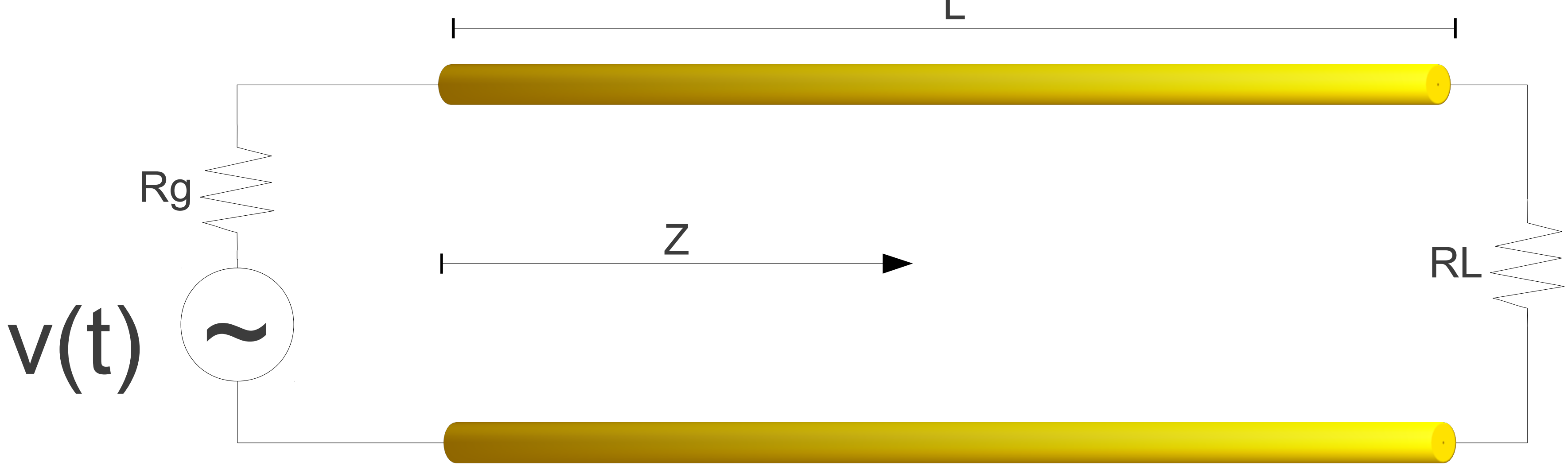
Units (s/m)², the inverse of a square velocity

- General solutions have the form:

$$\begin{aligned} v(z,t) &= V^+ f^+\left(t-\frac{z}{\mathrm{v}}\right) + V^- f^-\left(t+\frac{z}{\mathrm{v}}\right) \\ i(z,t) &= \frac{V^+}{Z_c} f^+\left(t-\frac{z}{\mathrm{v}}\right) - \frac{V^-}{Z_c} f^-\left(t+\frac{z}{\mathrm{v}}\right) \end{aligned}$$

$$Z_c = \sqrt{\frac{L}{C}} \qquad \mathrm{v} = \frac{1}{\sqrt{LC}}$$

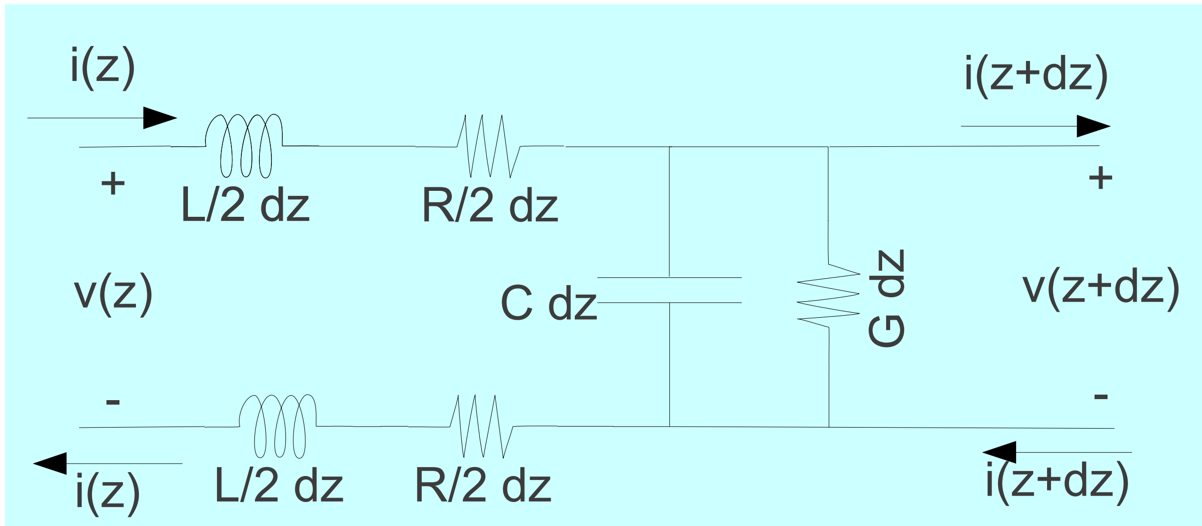
Terminated Transmission Lines (Time Domain)



In general both forward and backward waves are required to satisfy boundary conditions (voltage/current ratio at lumped loads). Whenever Zc and the terminating load are different, a reflected wave is generated, the reflection coefficient is:

$$\Gamma_L = \frac{R_L - Z_c}{R_L + Z_c} \qquad \Gamma_g = \frac{R_g - Z_c}{R_g + Z_c}$$

Lossy Transmission Lines (Frequency Domain)



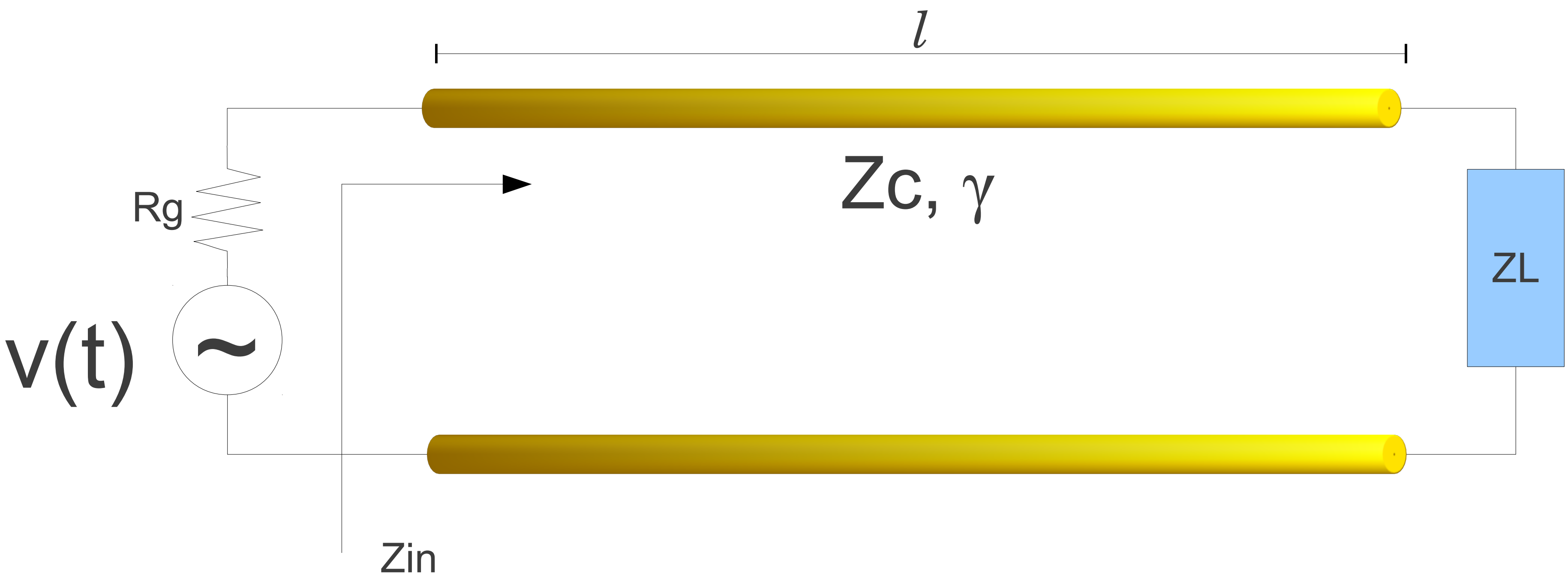
$$
\begin{aligned}
v(z) &= V^+ e^{-\gamma z} + V^- e^{\gamma z} \\
i(z) &= \frac{V^+}{Z_c} e^{-\gamma z} - \frac{V^-}{Z_c} e^{\gamma z}
\end{aligned}
$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Generic lossy lines are:
Dispersive = *phase velocity depends on frequency*
Distorting = *Attenuation constant depends on frequency*

Terminated Lossy Line (Frequency Domain)



$$Z_{in} = Z_C \frac{Z_L + Z_C \tanh(\gamma l)}{Z_C + Z_L \tanh(\gamma l)}$$

Important parameters

- Wavelength
- Phase velocity
- Power flow
- Terminated lines, load matching
- Reflection coefficient
- Standing Wave Ratio (SWR)
- Attenuation constant (perturbation technique)