Wave velocities and Dispersion

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TX lines as linear systems

$$F_i(\omega) \longrightarrow \boxed{Z(\omega)} \longrightarrow F_o(\omega)$$

General TX line transfer function:

$$Z(\omega) = e^{-\alpha\ell} e^{-j\beta\ell}$$

$$\beta = \sqrt{\omega^2 \mu\epsilon - k_c^2}$$

Complicated nonlinear function of freq.

In frequency domain we use product:

$$F_o(\omega) = Z(\omega) F_i(\omega)$$

If signal is narrowband, we can use a linearized version of beta (**note: this representation is exact for TEM lines**):

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega - \omega_0)$$

Signals

INPUT:

$$
\begin{aligned}
f_i(t) &= \cos(\omega_c t)\, f_m(t) \\
f_i(t) &= \Re\left[e^{j\omega_c t} f_m(t)\right] \\
F_i(\omega) &= F_m(\omega - \omega_c)
\end{aligned}
$$

Narrowband modulated signal

$$\omega_m \ll \omega_c$$

OUTPUT:

$$
\begin{aligned}
F_o(\omega) &= F_m(\omega - \omega_c)\, e^{-j\beta\ell} \\
F_o(\omega) &\approx F_m(\omega - \omega_c)\, e^{-j\beta_0\ell} e^{-j\beta_0'(\omega-\omega_0)\ell}
\end{aligned}
$$

Time domain:

$$f_o(t) \approx \frac{1}{2\pi}\Re\left[\int F_m(\omega - \omega_c)\, e^{-j\beta_0\ell} e^{-j\beta_0'(\omega-\omega_0)\ell} e^{j\omega t} d\omega\right]$$

$$\omega = \omega - \omega_c: \longrightarrow \quad f_o(t) \approx \frac{1}{2\pi}\Re\left[e^{j(\omega_c t - \beta_0\ell)} \int F_m(\omega)\, e^{j\omega\left(t-\beta_0'\ell\right)} d\omega\right]$$

$$\boxed{f_o(t) \approx \cos(\omega_c t - \beta_0\ell)\, f_m(t - \beta_0'\ell)}$$