2012-1: Transmission Lines and Antennas

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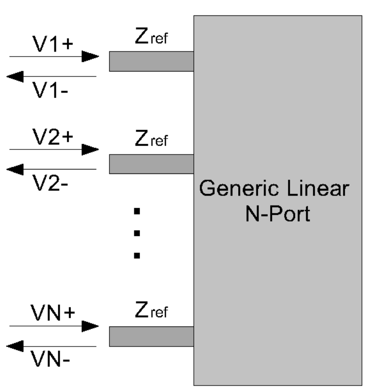
Scattering Matrix

The Scattering Matrix (S-Matrix) is the preferred representation at microwave frequencies as voltage/currents are not well defined in all cases as required for usual matrix representations (Z, Y, etc.))



$$\begin{bmatrix} v_1^- \\ \cdots \\ v_N^- \end{bmatrix} = \begin{bmatrix} S_{1,1} & \cdots & S_{1,N} \\ \cdots & & \cdots \\ S_{N,1} & \cdots & S_{N,N} \end{bmatrix} \begin{bmatrix} v_1^+ \\ \cdots \\ v_N^+ \end{bmatrix}$$

$$\mathbf{v}^- = \mathbf{S}\mathbf{v}^+$$

The Sm,n element is obtained as follows:

$$S_{m,n} = v_m^-/v_n^+ \text{ when } v_p^+ = 0 \quad \forall p \neq n$$

i.e. Feed only nth port, terminate all other ports in a matched load.

Properties of the S-Matrix

$$\mathbf{S} = \mathbf{S}^T \text{ for reciprocal networks}$$

$$\sum_{k=1}^{N} |S_{k,n}|^2 \leq 1 \quad \forall n \text{ for passive networks}$$

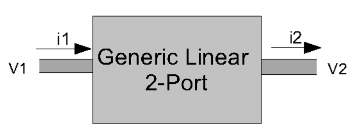
i.e. norm of column vectors is always ≤ 1
(equality holds for lossless networks)

$$\mathbf{S} = (\mathbf{Z}/Z_0 + \mathbf{I})^{-1} (\mathbf{Z}/Z_0 - \mathbf{I}) \qquad \mathbf{Z} = Z_0 (\mathbf{I} + \mathbf{S}) (\mathbf{I} - \mathbf{S})^{-1}$$
$$\mathbf{S} = (\mathbf{I} + \mathbf{Y}/Y_0)^{-1} (\mathbf{I} - \mathbf{Y}/Y_0) \qquad \mathbf{Y} = Y_0 (\mathbf{I} - \mathbf{S}) (\mathbf{I} + \mathbf{S})^{-1}$$

where $\mathbf{Z}$ and $\mathbf{Y}$ are the impedance and admittance matrices, $\mathbf{I}$ is the identity matrix and Z_0 and Y_0 are the port reference impedance/admittance.

The ABCD matrix

- 2-ports are probably the most common device.
- These are usually cascaded.
- ABCD matrix of a cascade of 2-ports is the matrix product of the individual ABCD matrices.



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

A is the ratio between an applied voltage v_1 and the resulting open-circuit voltage v_2 (i.e. $i_2 = 0$)
B is the ratio between an applied voltage v_1 and the resulting short-circuit current i_2 (i.e. $v_2 = 0$)
C is the ratio between an applied current i_1 and the resulting open-circuit voltage v_2 (i.e. $i_2 = 0$)
D is the ratio between an applied current i_1 and the resulting short-circuit current i_2 (i.e. $v_2 = 0$)

Properties of the ABCD matrix

$$\begin{bmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{bmatrix} = \frac{\begin{bmatrix} A + B/Z_0 - CZ_0 - D & 2\,(AD - BC) \\ 2 & -A + B/Z_0 - CZ_0 + D \end{bmatrix}}{A + B/Z_0 + CZ_0 + D}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{\begin{bmatrix} (1+S_{1,1})(1-S_{2,2}) + S_{1,2}S_{2,1} & Z_0\{(1+S_{1,1})(1+S_{2,2}) - S_{1,2}S_{2,1}\} \\ \{(1-S_{1,1})(1-S_{2,2}) - S_{1,2}S_{2,1}\}/Z_0 & (1-S_{1,1})(1+S_{2,2}) + S_{1,2}S_{2,1} \end{bmatrix}}{2S_{2,1}}$$

- For reciprocal networks, determinant of ABCD matrix AD-BC=1
- For symmetrical networks A = D = ±sqrt(1+BC)