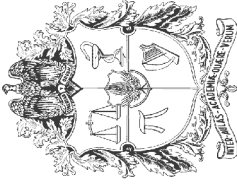


2012-1: Transmission Lines and Antennas



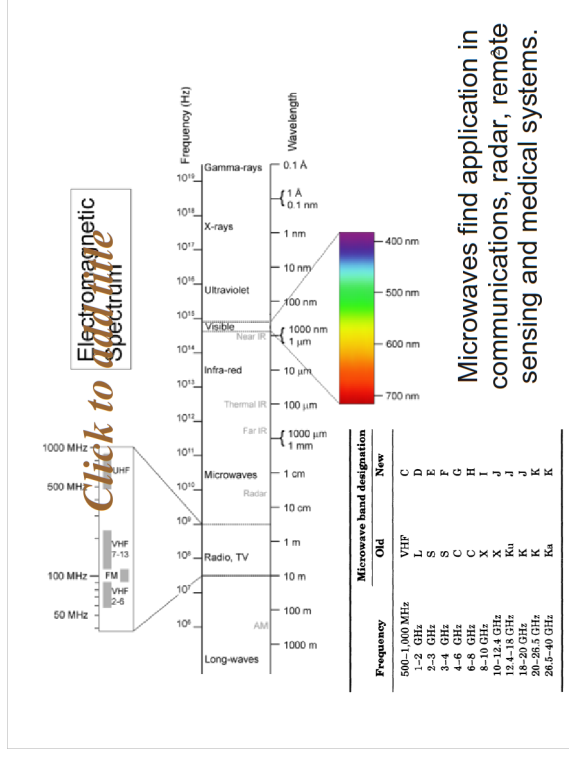
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Intro - phasors

Phasors provide a compact way to represent and operate with time-harmonic quantities

"In-phase" (I) "quadrature" (Q)

=complex n.

= "Phasor"

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Prerequisites

- Vector algebra
- Coordinate transformations
- Vector calculus
- Maxwell's equations
- Mathematical software (MATLAB, OCTAVE, SCILAB, etc)

Intro – Maxwell's equations

Description of electromagnetic phenomena at the microscopic level: linear dimensions and charge magnitudes are large compared to that of single atoms.

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -j\omega \mathbf{B} \\ \nabla \times \mathbf{H} &= j\omega \mathbf{D} + \mathbf{J} \end{aligned}$$

Frequency-domain Maxwell's equations (differential form)

Not independent: (3) and (4) suffice

$$\begin{aligned} \epsilon_0 &= 8.854 \times 10^{-12} \text{ F/m} \\ \mu_0 &= 4\pi \times 10^{-7} \text{ H/m} \end{aligned}$$

$$\begin{aligned} \mathbf{D} &= \epsilon \mathbf{E} \\ \mathbf{B} &= \mu \mathbf{H} \\ \nabla \cdot \mathbf{J} &= -j\omega \rho \end{aligned}$$

Constitutive relations and continuity equation

Intro – Maxwell's equations (2)

Presence of material media modify constitutive relations (isotropic case considered). Electric and magnetic susceptibilities affect fields.

$$\begin{aligned} \mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P}_e = \epsilon_0 (1 + \chi_e) \mathbf{E} = \epsilon \mathbf{E} \\ \epsilon &= \epsilon' - j\epsilon'' = \epsilon_0 (1 + \chi_e) \\ \mathbf{B} &= \mu_0 (1 + \chi_m) \mathbf{H} = \mu \mathbf{H} \\ \mu &= \mu' - j\mu'' = \mu_0 (1 + \chi_m) \end{aligned}$$

Conducting media

$$\mathbf{J} = \sigma \mathbf{E}$$

Dielectric media

Magnetic media

For dielectric conducting media Ampère-Maxwell equation becomes:

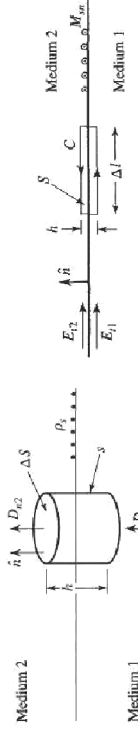
$$\begin{aligned} \nabla \times \mathbf{H} &= j\omega \mathbf{D} + \mathbf{J} \\ &= j\omega \epsilon \mathbf{E} + \sigma \mathbf{E} \\ &= j\omega \epsilon' \mathbf{E} + (\omega \epsilon'' + \sigma) \mathbf{E} \\ &= j\omega \left(\epsilon' - j\epsilon'' - j\frac{\sigma}{\omega} \right) \mathbf{E} \end{aligned}$$

$$\tan \delta = \frac{\omega \epsilon'' + \sigma}{\omega \epsilon'}$$

Loss is described by the loss tangent (frequency dependent)

Intro – field at interfaces

Integral form of Maxwell's equations can be used to obtain boundary conditions at media interfaces:



$$\begin{aligned} \hat{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) &= \rho_s \\ \hat{n} \cdot \mathbf{B}_2 &= \hat{n} \cdot \mathbf{B}_1 \\ \mathbf{E}_2 \times \hat{n} &= \mathbf{E}_1 \times \hat{n} \\ \hat{n} \times (\mathbf{H}_2 - \mathbf{H}_1) &= \mathbf{J}_s \end{aligned}$$

Dielectric Interfaces	$\hat{n} \cdot \mathbf{D}_2 = \hat{n} \cdot \mathbf{D}_1$	$\hat{n} \cdot \mathbf{D} = \rho_s$
	$\hat{n} \cdot \mathbf{B}_2 = \hat{n} \cdot \mathbf{B}_1$	$\hat{n} \cdot \mathbf{B} = 0$
	$\mathbf{E}_2 \times \hat{n} = \mathbf{E}_1 \times \hat{n}$	$\mathbf{E} \times \hat{n} = \mathbf{0}$
	$\hat{n} \times \mathbf{H}_2 = \hat{n} \times \mathbf{H}_1$	$\hat{n} \times \mathbf{H} = \mathbf{J}_s$

At PEC (perfect electric conductor) surfaces:

Intro – The wave equation

Consider Maxwell's curl equations in a source-free medium

$$\begin{aligned} \nabla \times \mathbf{E} &= -j\omega \mu \mathbf{H} \\ \nabla \times \mathbf{H} &= j\omega \epsilon \mathbf{E} \end{aligned}$$

Curl[(5)] → (6)

$$\nabla \times \nabla \times \mathbf{E} = -j\omega \mu \nabla \times \mathbf{H} = \omega^2 \mu \epsilon \mathbf{E}$$

Using identity:

$$\nabla \times \nabla \times \mathbf{A} = \nabla \nabla \cdot \mathbf{A} - \nabla^2 \mathbf{A}$$

$$\nabla^2 \mathbf{E} + \omega^2 \mu \epsilon \mathbf{E} = \mathbf{0}$$

This is the Helmholtz equation, that describes wave propagation in linear isotropic and source-free media. Note that an identical equation may be obtained for H.

Intro – Plane waves

- In order to solve for a particular case, assume that electric field is oriented along x and constant in the xy plane:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 \quad k = \omega \sqrt{\mu\epsilon}$$

Wave number (nm)

- This second-order linear scalar equation with constant coefficients has two independent solutions with arbitrary amplitude constants:

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz}$$

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Intro – Plane waves in lossless media

- Lossless media are characterized by a real wave number k. For real amplitude constants the time domain expression of electric field is:

$$\mathcal{E}_x(z, t) = \mathcal{E}^+ \cos(\omega t - kz) + \mathcal{E}^- \cos(\omega t + kz)$$

- Total field is the superposition of waves propagating along +z and -z. For one of these, a fixed-phase point (e.g. 0 radians) travels at a velocity called phase velocity:

$$v_p = \frac{dz}{dt} = \frac{d}{dt} \left(\frac{\omega t - \text{constant}}{k} \right) = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

In free space:

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.998 \times 10^8 \text{ m/s}$$

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Intro – plane waves in lossless media (2)

- Wavelength: distance between two successive minima for fixed t (can use maxima or any other phase reference)

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}$$

- Maxwell's curl equation (Faraday's law) gives magnetic field intensity H:

$$H_y = \frac{1}{\eta} (E^+ e^{-jkz} - E^- e^{jkz})$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

- Both E and H are perpendicular to the direction of propagation: TEM wave. E and H are related by the wave impedance η

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Intro – plane waves in lossy media

- Form of solution is the same as above with the difference that the wave number is complex.

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}$$

Conducting material



$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$$

$$\gamma = j\omega\sqrt{\mu\epsilon} = jk = j\omega\sqrt{\mu\epsilon'}(1 - j\tan\delta)$$



Lossy dielectric

- Time-domain form is modified accordingly:

$$e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$$



$$e^{-\alpha z} \cos(\omega t - \beta z)$$

Intro – plane waves in lossy media (2)

Magnetic vector can be computed as before

$$H_y = \frac{1}{\eta} (E^+ e^{-\gamma z} - E^- e^{\gamma z})$$

For a good conductor:

$$\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

Skin depth tells the distance at which wave amplitude has decreased to 37% (power to 13%)

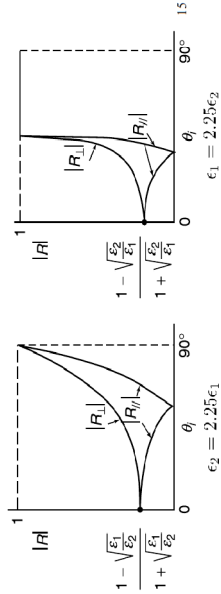
$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\eta = \frac{j\omega\mu}{\gamma} \simeq (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\frac{1}{\sigma\delta_s}$$

Wave impedance \rightarrow

Reflection/Transmission (Smooth Interface)

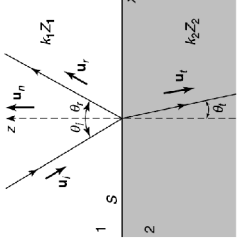
- Reflected wave amplitude depends on:
 - Incidence angle
 - Media impedance $Z = \sqrt{\frac{\mu}{\epsilon}}$
- Separate cases for analysis: TE (perpendicular) and TM (parallel)



Wave Interaction with Matter

Reflection/transmission: occurs in presence of large material discontinuities.

- For plane waves/interfaces, the Fresnel coefficients relate E_t , E_r with E_i .
- According to interface roughness, reflection may be specular or diffuse.



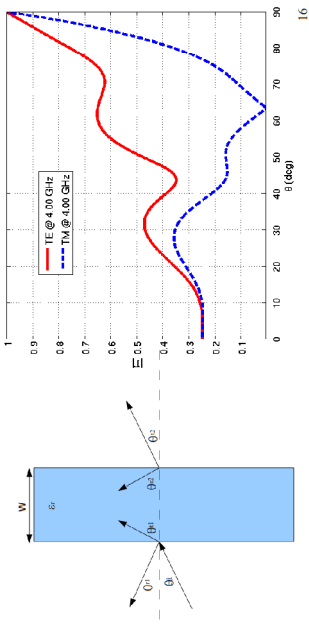
$$\theta_r = \theta_i$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$$

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Example

- Reflection from a lossy brick wall ($\epsilon_{psr}=4-j0.1$) with 30cm thickness @ 4GHz



Intro - Wave polarization

- General elliptical polarization
- Circular polarization
- Handedness

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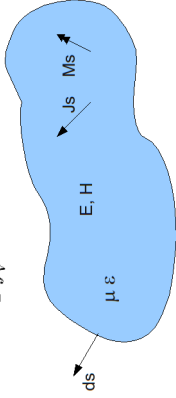
Poynting Theorem – Poynting Vector

$$-\frac{1}{2} \int_V (\mathbf{E} \cdot \mathbf{J}_s^* + \mathbf{H}^* \cdot \mathbf{M}_s) = \text{Sources}$$

$$\frac{1}{2} \oint_{\partial V} (\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s} + \text{Outwards power flow}$$

$$\frac{\sigma}{2} \int_V E^2 dv + \frac{\omega}{2} \int_V (\epsilon'' E^2 + \mu'' H^2) dv + \text{Loss}$$

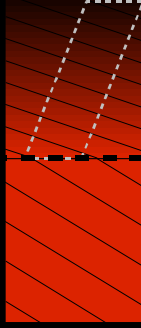
$$j \frac{\omega}{2} \int_V (\mu' H^2 - \epsilon' E^2) dv \text{ Reactive storage}$$



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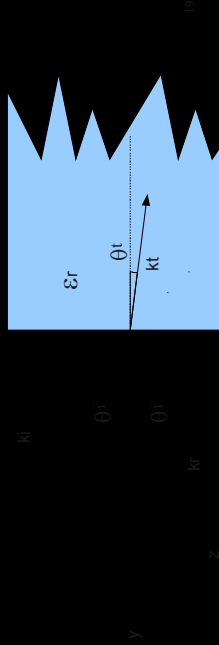
Solution

- Using Snell's law:
 - $\sin(\theta_2) = 0.1971 + j0.0066$
 - why is this complex? In medium 2, phase increase and attenuation must occur in different directions in order to satisfy the boundary conditions, see k later.
- Using 4th eq in slide 16 (note that $\cos^2 + \sin^2 = 1$ works also in the complex field):
 - $E_t = 0.7119 + j0.0159j$
 - This has only x component (TE wave)
 - $\mathbf{E} = x^0 E_t$
- Compute wave vector (complex):
 - $\mathbf{k} = \cos(\theta_2) \hat{y} \sin(\theta_2) + \hat{z} \cos(\theta_2)$
 - $\mathbf{k} = k_0 [-j0.34 + \hat{z}(1.7 - j5.9e-2)]$
 - $\text{re} \rightarrow \text{phase} / \text{m} \rightarrow \text{attenuation}$
- Compute real part of Poynting vector:
 - $\mathbf{S}_r = \text{re}(\mathbf{E} \times (-\mathbf{E}^* \times \mathbf{k}^*) / (\omega \mu))$
 - $\mathbf{S}_r = \text{re}(\mathbf{k}^*) |\mathbf{E}_t|^2 / (\omega \mu)$
 - $\mathbf{S}_r = [\hat{y} 0.34 + \hat{z} 1.7] |\mathbf{E}_t|^2 / (120\pi)$
- Integrate on a convenient surface, a slanted cylinder with 1m^2 cross section. Only integral on front cap (on interface) is non-zero: sides are tangent to \mathbf{S}_r , while attenuation makes it negligible on back cap.
 - $dP/dA = \langle \mathbf{S}_r \rangle \cdot \hat{\mathbf{z}} / 2 = 1.1 \text{ mW/m}^2$



Example

- A TE wave with amplitude 1V/m going through free-space impinges on a dielectric interface with $\theta_{\text{inc}} = 20^\circ$. The dielectric half space has $\epsilon_r = 3 - j0.2$. Compute the power lost per unit area.



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