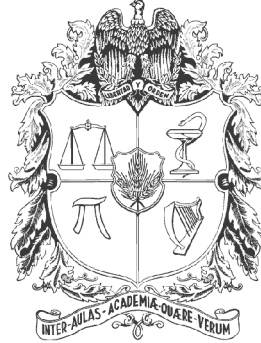


# 2011-II: Transmission Lines and Antennas



Javier Leonardo Araque Quijano  
Of: 453 – 204  
Ext. 14083  
jlaraqueq@unal.edu.co

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## Reduced Maxwell's Equations

- Assuming invariance of the medium along the  $z$  axis and a wave solution in that direction, the homogeneous (no sources) Maxwell's equations can be simplified by separating fields and the del operator into transverse (i.e. Restricted to the  $xy$  plane) and longitudinal/axial components (along  $z$ ):

$$\begin{aligned}\mathbf{E}(x, y, z) &= [\mathbf{e}_t(x, y) + \hat{\mathbf{z}}e_z(x, y)] e^{-j\beta z} \\ \mathbf{H}(x, y, z) &= [\mathbf{h}_t(x, y) + \hat{\mathbf{z}}h_z(x, y)] e^{-j\beta z}\end{aligned}$$

$$\nabla = \hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{y}} \frac{\partial}{\partial y} + \hat{\mathbf{z}} \frac{\partial}{\partial z} = \nabla_t - j\beta \hat{\mathbf{z}}$$

$$\begin{aligned}\nabla_t \times \mathbf{e}_t &= -j\omega\mu_0 \hat{\mathbf{z}}h_z \\ -\hat{\mathbf{z}} \times \nabla_t e_z - j\beta \hat{\mathbf{z}} \times \mathbf{e}_t &= -j\omega\mu_0 \mathbf{h}_t \\ \nabla_t \times \mathbf{h}_t &= j\omega\epsilon_0 \hat{\mathbf{z}}e_z \\ \hat{\mathbf{z}} \times \nabla_t h_z + j\beta \hat{\mathbf{z}} \times \mathbf{h}_t &= -j\omega\epsilon_0 \mathbf{e}_t \\ \nabla_t \cdot \mathbf{h}_t &= j\beta h_z \\ \nabla_t \cdot \mathbf{e}_t &= j\beta e_z\end{aligned}$$

## TE waves (H-waves)

- Also called H waves, these are obtained by setting  $e_z = 0$  in the reduced equations:

From Helmholtz equation:

$$\begin{aligned}\nabla_t^2 h_z + k_c^2 h_z &= 0 \\ \nabla_t^2 \mathbf{h}_t + k_c^2 \mathbf{h}_t &= 0 \\ k_c^2 &\equiv k_0^2 - \beta^2\end{aligned}$$

Curl of 3<sup>rd</sup> eq. at left, using then 5<sup>th</sup> at left and 2<sup>nd</sup> above:

$$\mathbf{h}_t = -\frac{j\beta}{k_c^2} \nabla_t h_z$$

$\hat{\mathbf{z}} \times$  2<sup>nd</sup> eq at left:

$$\mathbf{e}_t = -Z_0 \frac{k_0}{\beta} \hat{\mathbf{z}} \times \mathbf{h}_t = -Z_{TE} \hat{\mathbf{z}} \times \mathbf{h}_t$$

$$Z_{TE} \equiv Z_H \equiv Z_0 \frac{k_0}{\beta} > Z_0$$

$$\begin{aligned}\nabla_t \times \mathbf{e}_t &= -j\omega\mu_0 \hat{\mathbf{z}} h_z \\ -j\beta \hat{\mathbf{z}} \times \mathbf{e}_t &= -j\omega\mu_0 \mathbf{h}_t \\ \nabla_t \times \mathbf{h}_t &= 0 \\ \hat{\mathbf{z}} \times \nabla_t h_z + j\beta \hat{\mathbf{z}} \times \mathbf{h}_t &= -j\omega\epsilon_0 \mathbf{e}_t \\ \nabla_t \cdot \mathbf{h}_t &= j\beta h_z \\ \nabla_t \cdot \mathbf{e}_t &= 0\end{aligned}$$

## TM waves (E-waves)

- Solutions are obtained by setting  $h_z = 0$ , and are also called E waves. Previous solution can be reused via the duality transformations:

$$E \rightarrow H, H \rightarrow -E, \mu \rightarrow \epsilon, \epsilon \rightarrow \mu$$

$$\begin{aligned}\nabla_t^2 e_z + k_c^2 e_z &= 0 \\ \mathbf{e}_t &= -\frac{j\beta}{k_c^2} \nabla_t e_z \\ \mathbf{h}_t = \frac{1}{Z_0} \frac{k_0}{\beta} \hat{\mathbf{z}} \times \mathbf{e}_t &= \frac{1}{Z_{TM}} \hat{\mathbf{z}} \times \mathbf{e}_t \\ Z_{TM} \equiv Z_E &\equiv Z_0 \frac{\beta}{k_0} \leq Z_0\end{aligned}$$

## *Topics in guided propagation*

- Power flow
- Conductor loss
- Dielectric loss
- Inhomogeneous filling

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## *Loss in Transmission Lines and Waveguides*

- Perturbation techniques can be applied to low loss transmission media such as TLs and waveguides:
  - Solve for the modal fields neglecting loss
  - Insert loss as a correction factor in the propagation constant (attenuation constant).
  - Various loss mechanisms are included additively in the attenuation constant

$$\begin{aligned}\alpha &= \alpha_c + \alpha_d \\ \alpha_d &= \frac{k^2}{2\beta} \tan \delta \\ \alpha_c &= \frac{R_s Z_{\dagger}}{2} \frac{\int h_{tan}^2 dl}{\iint e_t^2 ds} \\ R_s &= \frac{1}{\sigma \delta_s}\end{aligned}$$

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