2012-1: Transmission Lines and Antennas

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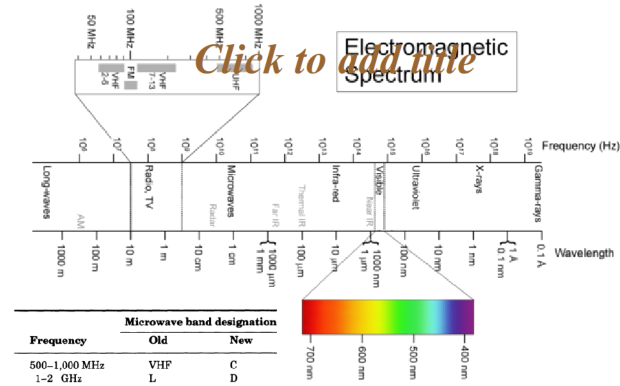
Electromagnetic Spectrum



Frequency	Microwave band designation Old	New
500–1,000 MHz	VHF	C
1–2 GHz	L	D
2–3 GHz	S	E
3–4 GHz	S	F
4–6 GHz	C	G
6–8 GHz	C	H
8–10 GHz	X	I
10–12.4 GHz	X	J
12.4–18 GHz	Ku	J
18–20 GHz	K	J
20–26.5 GHz	K	K
26.5–40 GHz	Ka	K

Microwaves find application in communications, radar, remote sensing and medical systems.

Prerequisites

- Vector algebra
- Coordinate transformations
- Vector calculus
- Maxwell's equations
- Mathematical software (MATLAB, OCTAVE, SCILAB, etc)

Intro - phasors

Phasors provide a compact way to represent and operate with time-harmonic quantities

"In-phase" (I) "quadrature" (Q)

=complex n.

="Phasor"

Intro – Maxwell's equations

Description of electromagnetic phenomena at the microscopic level: linear dimensions and charge magnitudes are large compared to that of single atoms.

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho && (1)\\ \nabla \cdot \mathbf{B} &= 0 && (2)\\ \nabla \times \mathbf{E} &= -j\omega\mathbf{B} && (3)\\ \nabla \times \mathbf{H} &= j\omega\mathbf{D} + \mathbf{J} && (4) \end{aligned}$$

Frequency-domain Maxwell's equations (differential form)

Not independent: (3) and (4) suffice

$$\begin{aligned} \mathbf{D} &= \epsilon\mathbf{E}\\ \mathbf{B} &= \mu\mathbf{H}\\ \nabla \cdot \mathbf{J} &= -j\omega\rho \end{aligned}$$

Constitutive relations and continuity equation

$$\begin{aligned} \epsilon_0 &= 8.854 \times 10{-}12\mathrm{F/m}\\ \mu_0 &= 4\pi \times 10^{-7}\mathrm{H/m} \end{aligned}$$

Intro – Maxwell's equations (2)

Presence of material media modify constitutive relations (isotropic case considered). Electric and magnetic susceptibilities affect fields.

$$\begin{aligned} \mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}_e = \epsilon_0(1+\chi_e)\,\mathbf{E} = \epsilon\mathbf{E}\\ \epsilon = \epsilon' - j\epsilon'' = \epsilon_0(1+\chi_e) \end{aligned}$$

Dielectric media

$$\begin{aligned} \mathbf{B} = \mu_0(1+\chi_m)\,\mathbf{H} = \mu\mathbf{H}\\ \mu = \mu' - j\mu'' = \mu_0(1+\chi_m) \end{aligned}$$

Magnetic media

Conducting media

$$\mathbf{J} = \sigma\mathbf{E}$$

For dielectric conducting media Ampère-Maxwell equation becomes:

$$\begin{aligned} \nabla \times \mathbf{H} &= j\omega\mathbf{D} + \mathbf{J}\\ &= j\omega\epsilon\mathbf{E} + \sigma\mathbf{E}\\ &= j\omega\epsilon'\mathbf{E} + (\omega\epsilon'' + \sigma)\,\mathbf{E}\\ &= j\omega\left(\epsilon' - j\epsilon'' - j\frac{\sigma}{\omega}\right)\mathbf{E} \end{aligned}$$

$$\tan\delta = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon'}$$

Loss is described by the loss tangent (frequency dependent)

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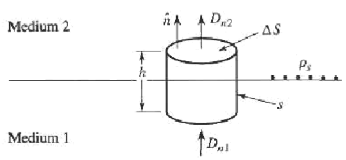
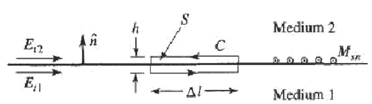
Intro – field at interfaces

Integral form of Maxwell's equations can be used to obtain boundary conditions at media interfaces:





$$\begin{aligned} \hat{\mathbf{n}} \cdot (\mathbf{D}_2 - \mathbf{D}_1) &= \rho_s\\ \hat{\mathbf{n}} \cdot \mathbf{B}_2 &= \hat{\mathbf{n}} \cdot \mathbf{B}_1 \end{aligned}$$

$$\begin{aligned} \mathbf{E}_2 \times \hat{\mathbf{n}} &= \mathbf{E}_1 \times \hat{\mathbf{n}}\\ \hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) &= \mathbf{J}_s \end{aligned}$$

Dielectric Interfaces

$$\begin{aligned} \hat{\mathbf{n}} \cdot \mathbf{D}_2 &= \hat{\mathbf{n}} \cdot \mathbf{D}_1\\ \hat{\mathbf{n}} \cdot \mathbf{B}_2 &= \hat{\mathbf{n}} \cdot \mathbf{B}_1\\ \mathbf{E}_2 \times \hat{\mathbf{n}} &= \mathbf{E}_1 \times \hat{\mathbf{n}}\\ \hat{\mathbf{n}} \times \mathbf{H}_2 &= \hat{\mathbf{n}} \times \mathbf{H}_1 \end{aligned}$$

At PEC (perfect electric conductor) surfaces:

$$\begin{aligned} \hat{\mathbf{n}} \cdot \mathbf{D} &= \rho_s\\ \hat{\mathbf{n}} \cdot \mathbf{B} &= 0\\ \mathbf{E} \times \hat{\mathbf{n}} &= \mathbf{0}\\ \hat{\mathbf{n}} \times \mathbf{H} &= \mathbf{J}_s \end{aligned}$$

Intro – The wave equation

Consider Maxwell's curl equations in a source-free medium

$$\begin{aligned} \nabla \times \mathbf{E} &= -j\omega\mu\mathbf{H} && (5)\\ \nabla \times \mathbf{H} &= j\omega\epsilon\mathbf{E} && (6) \end{aligned}$$

Curl[(5)] → (6)

$$\nabla \times \nabla \times \mathbf{E} = -j\omega\mu\nabla \times \mathbf{H} = \omega^2\mu\epsilon\mathbf{E}$$

Using identity: $\nabla \times \nabla \times \mathbf{A} = \nabla\nabla \cdot \mathbf{A} - \nabla^2\mathbf{A}$

$$\nabla^2\mathbf{E} + \omega^2\mu\epsilon\mathbf{E} = \mathbf{0}$$

This is the Helmholtz equation, that describes wave propagation in linear isotropic and source-free media. Note that an identical equation may be obtained for H.

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Intro – Plane waves

- In order to solve for a particular case, assume that electric field is oriented along x and constant in the xy plane:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 \qquad k = \omega\sqrt{\mu\epsilon}$$

Wave number (1/m)

- This second-order linear scalar equation with constant coefficients has two independent solutions with arbitrary amplitude constants:

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz}$$

Intro – Plane waves in lossless media

- Lossless media are characterized by a real wave number k. For real amplitude constants the time domain expression of electric field is:

$$\mathcal{E}_x(z,t) = \mathcal{E}^+ \cos(\omega t - kz) + \mathcal{E}^- \cos(\omega t + kz)$$

- Total field is the superposition of waves propagating along +z and -z. For one of these, a fixed-phase point (e.g. 0 radians) travels at a velocity called phase velocity:

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - constant}{k}\right) = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

In free space: $v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.998 \times 10^8 \mathrm{m/s}$

Intro – plane waves in lossless media (2)

- Wavelength: distance between two successive minima for fixed t (can use maxima or any other phase reference)

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}$$

- Maxwell's curl equation (Faraday's law) gives magnetic field intensity H:

$$H_y = \frac{1}{\eta}\left(E^+ e^{-jkz} - E^- e^{jkz}\right) \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

- Both E and H are perpendicular to the direction of propagation: TEM wave. E and H are related by the wave impedance η.

Intro – plane waves in lossy media

- Form of solution is the same as above with the difference that the wave number is complex.

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{\gamma z}$$

Conducting material → $\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$

$\gamma = j\omega\sqrt{\mu\epsilon} = jk = j\omega\sqrt{\mu\epsilon'(1 - j\tan\delta)}$ ← Lossy dielectric

- Time-domain form is modified accordingly:

$e^{-\gamma z} = e^{-\alpha z}e^{-j\beta z}$ → $e^{-\alpha z}\cos(\omega t - \beta z)$

Intro – plane waves in lossy media (2)

Magnetic vector can be computed as before

$$H_y = \frac{1}{\eta}\left(E^+ e^{-\gamma z} - E^- e^{\gamma z}\right)$$

For a good conductor:

$$\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

Skin depth tells the distance at which wave amplitude has decreased to 37% (power to 13%)

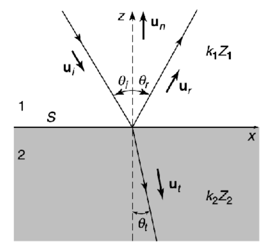
$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Wave impedance →

$$\eta = \frac{j\omega\mu}{\gamma} \simeq (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\frac{1}{\sigma\delta_s}$$

Wave Interaction with Matter

Reflection/transmission: occurs in presence of large material discontinuities.



- For plane waves/interfaces, the Fresnel coefficients relate Et, Er with Ei.
- According to interface roughness, reflection may be specular or diffuse.

$$\theta_r = \theta_i$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i$$

Reflection/Transmission (Smooth Interface)

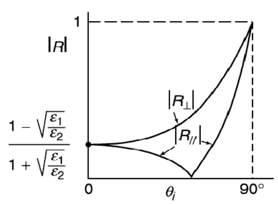
- Reflected wave amplitude depends on:
  - Incidence angle
  - Media impedance $Z = \sqrt{\frac{\mu}{\epsilon}}$
- Separate cases for analysis: TE (perpendicular) and TM (parallel)

$$\left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{Z_2 cos\theta_t - Z_1 cos\theta_i}{Z_2 cos\theta_t + Z_1 cos\theta_i}$$

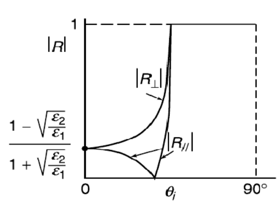
$$\left(\frac{E_t}{E_i}\right)_{\parallel} = \frac{2Z_2 cos\theta_i}{Z_1 cos\theta_i + Z_2 cos\theta_t}$$

$$\left(\frac{E_r}{E_i}\right)_{\perp} = \frac{Z_2 cos\theta_i - Z_1 cos\theta_t}{Z_2 cos\theta_i + Z_1 cos\theta_t}$$

$$\left(\frac{E_t}{E_i}\right)_{\perp} = \frac{2Z_2 cos\theta_i}{Z_2 cos\theta_i + Z_1 cos\theta_t}$$



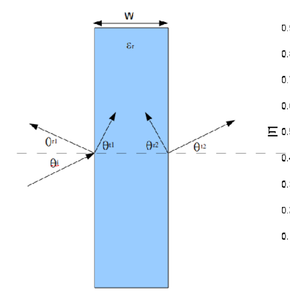
$\epsilon_2 = 2.25\epsilon_1$

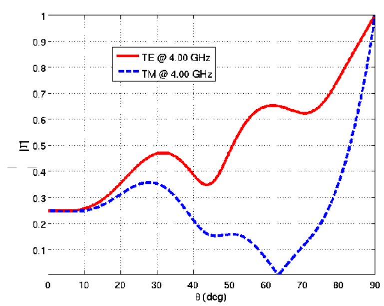


$\epsilon_1 = 2.25\epsilon_2$

Example

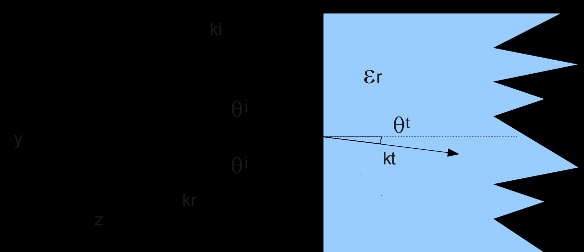
- Reflection from a lossy brick wall (epsr=4-j0.1) with 30cm thickness @ 4GHz





Example

- A TE wave with amplitude 1V/m going through free-space impinges on a dielectric interface with theta_i = 20°. The dielectric half space has epsilon_r = 3- j0.2. Compute the power lost per unit area.



Solution

- Using Snell's law:

  sin(theta_t) = 0.1971 + 0.0066i

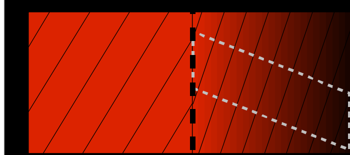
  why is this complex? In medium 2, phase increase and attenuation must occur in different directions in order to satisfy the boundary conditions, see **k** later.

- Using 4th eq in slide 15 (note that cos^2+sin^2=1 works also in the complex field):

  Et = 0.7119 + 0.0159i

  This has only x component (TE wave)

  E = **x^** Et



- Compute wave vector (complex):

  k = ωsqrt(με)(-**ŷ** sin(θt) + **ẑ**cos(θt))

  k = k0[-**ŷ** 0.34 + **ẑ** (1.7 – j5.9e-2)]

  re → phase / im → attenuation

- Compute real part of Poynting vector:

  Sr = re(**E** x (-**E*** x **k***) / (ωμ))

  Sr = re(**k***) |Et|^2/(ωμ)

  Sr=[-**ŷ** 0.34 + **ẑ**1.7]|Et|^2 /(120π)

- Integrate on a convenient surface, a slanted cylinder with 1m^2 cross section. Only integral on front cap (on interface) is non-zero: sides are tangent to **Sr**, while attenuation makes it negligible on back cap.

- dP/dA = <**Sr**| **ẑ**>/2 = 1.1mW/m^2

Intro - Wave polarization

- Polarization expresses the time behavior of the electric field vector at a fixed point in space. According to the curve described by the E arrow tip it may be:
  - Linear
  - Elliptical (general case), may be LH or RH
  - Circular, may be LH or RH
- A general E field in the frequency domain is expressed in terms of a complex vector:

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_r + j\mathbf{E}_i \\ \mathcal{E} &= \mathbf{E}_r \cos(\omega t) - \mathbf{E}_i \sin(\omega t)\end{aligned}$$

Poynting Theorem – Poynting Vector

$$-\frac{1}{2}\int_V (\mathbf{E}\cdot\mathbf{J_s}^* + \mathbf{H}^*\cdot\mathbf{M_s}) = \qquad \text{Sources}$$

$$\frac{1}{2}\oint_{\partial V} (\mathbf{E}\times\mathbf{H}^*)\cdot ds + \qquad \text{Outwards power flow}$$

$$\frac{\sigma}{2}\int_V E^2 dv + \frac{\omega}{2}\int_V (\epsilon'' E^2 + \mu'' H^2)\, dv + \qquad \text{Loss}$$

$$j\frac{\omega}{2}\int_V (\mu' H^2 - \epsilon' E^2) dv \qquad \text{Reactive storage}$$