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Computation of Transmission Line Parameters from Complex Reflection and Transmission Coefficients

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Abstract

This document provides formulas to compute the propagation constant and characteristic impedance of a transmission line given its physical length and the complex reflection and transmission parameters S_{11} and S_{21} relative to a known reference impedance, as measured by a Vector Network Analyzer (VNA). These formulas are valuable as they do not require that the unknown characteristic impedance bear any specific relation to the reference impedance of the measurement system nor that the line have a specific line length (e.g. $\lambda/4$). The formulas apply to measurements at one frequency sample; therefore, they apply equally well to dispersive and non-dispersive lines.

Formulation

Consider the situation in Fig. 1, where a transmission line under test is connected between two identical transmission lines with characteristic impedance Z_{ref}, assumed infinite in extent; this is representative of a measurement setup using a 2-port VNA to measure S_{11} and S_{21} parameters [1][Sec. 4.3] [2][Sec. 4.7], which require a matched generator at port 1 (left of line 1) and a matched termination at port 2 (right of line 2). This condition implies that in line one we have both incident and reflected waves V_1^+ and V_1^- (from the generator and the discontinuity respectively), while in line two we have only the transmitted wave, as there is no generator nor reflection at that side.

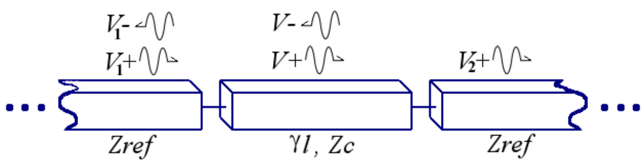

Figure 1: Schematic representation of a measurement setup using a 2-port VNA.

continuity of voltage and current at the interfaces results in these equations:

$$\begin{aligned}
V_1^+ + V_1^- &= V^+ + V^- \\
\frac{V_1^+ - V_1^-}{Z_{ref}} &= \frac{V^+ - V^-}{Z_c} \\
V^+e^{-\gamma\ell} + V^-e^{\gamma\ell} &= V_2^+ \\
\frac{V^+e^{-\gamma\ell} - V^-e^{\gamma\ell}}{Z_c} &= \frac{V_2^+}{Z_{ref}}
\end{aligned} \tag{1}$$

For the situation in Fig. 1, where wave amplitudes in lines 1 and 2 are computed with the phase reference at the corresponding interface, scattering parameters are given by:

$$S_{11} = \frac{V_1^-}{V_1^+} \qquad S_{21} = \frac{V_2^+}{V_1^+} \tag{2}$$

Manipulation of the non-linear system of equations (1)-(2) gives the unknown line parameters γ and Z_c in terms of the scattering parameters S_{11}, S_{21} and the characteristic impedance of the measurement system Z_{ref}:

$$\begin{aligned} e^{-\gamma l} &= \mathrm{S}_{21}\left(\frac{A}{2}+\frac{1}{2}\right) - \frac{(\mathrm{S}_{11}-1)\,(\mathrm{S}_{11}+\mathrm{S}_{11}\,A-A+1)}{2\,\mathrm{S}_{21}} \\ Z_c &= Z_{ref}A \end{aligned} \tag{3}$$

where the common term A is given by:

$$A = \sqrt{-\frac{{\mathrm{S}_{11}}^2+2\,\mathrm{S}_{11}-{\mathrm{S}_{21}}^2+1}{-{\mathrm{S}_{11}}^2+2\,\mathrm{S}_{11}+{\mathrm{S}_{21}}^2-1}} \tag{4}$$

In computing the propagation constant γ from the solution above, one must be careful since the function $\ln(x)$ is multivalued in the complex field, i.e. given a solution $\gamma = \alpha + j\beta$, the quantity $\gamma + j2\pi n$ is also a solution (for n integer). This in principle results in several possible solutions for β, which is simply a statment of the indistinguishability of the angles θ and $\theta + 2\pi n$. The correct value is of course only one, as the wave must undergo a specific number of oscillations in traversing the line; it can be obtained from measurements over a finite bandwidth and knowledge of the dispersion relation of the line, e.g. for TEM lines $\beta = 2\pi f\sqrt{\mu_r\epsilon_r}/c$, meaning that $\beta(f)$ is a line that crosses the horizontal axis at $f = 0$, hence exluding all the above possible solutions except one.

References

[1] D. M. Pozar, *Microwave Engineering*, 3rd ed. John Wiley & sons, 2005.

[2] R. E. Collin, *Foundations of Microwave Engineering.* IEEE press, 2001.