Table 1 DISCRETE DISTRIBUTIONS

parametric family of distributions	Discrete density functions $f(\cdot)$	Parameter space	$ \text{Mean} \\ \mu = \mathscr{E}[X] $
Discrete uniform	$f(x) = \frac{1}{N} I_{\{1,, N\}}(x)$	$N=1,2,\ldots$	$\frac{N+1}{2}$
Bernoulli	$f(x) = p^{x}q^{1-x}I_{\{0, 1\}}(x)$	$0 \le p \le 1$ $(q = 1 - p)$	p
Binomial	$f(x) = \binom{n}{x} p^{x} q^{n-x} I_{\{0, 1,, n\}}(x)$	$0 \le p \le 1$ n = 1, 2, 3, (q = 1 - p)	np
Hypergeometric	$f(x) = \frac{\binom{K}{x} \binom{M-K}{n-x}}{\binom{M}{n}} I_{\{0,1,\ldots,n\}}(x)$	M = 1, 2, K = 0, 1,, M n = 1, 2,, M	$n\frac{K}{M}$
Poisson	$f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} I_{(0, 1,)}(x)$	$\lambda > 0$	λ
Geometric	$f(x) = pq^{x}I_{(0, 1, \ldots)}(x)$	0 $(q = 1 - p)$	$\frac{q}{p}$
Negative binomial	$f(x) = {r + x - 1 \choose x} p^{r} q^{x} I_{(0, 1,)}(x)$	$ 0  0 \\ (q = 1 - p) $	$\frac{rq}{p}$

Variance $\sigma^2 = \mathscr{E}[(X - \mu)^2]$	Moments $\mu_r' = \mathscr{E}[X^r]$ or $\mu_r = \mathscr{E}[(X - \mu)^r]$ and/or cumulants $\kappa_r$	Moment generating function & [e'x]
$\frac{N^2-1}{12}$	$\mu_3'=\frac{N(N+1)^2}{4}$	$\sum_{j=1}^{N} \frac{1}{N} e^{jt}$
	$\mu_4' = \frac{(N+1)(2N+1)(3N^2+3N-1)}{30}$	
Pq	$\mu_r' = p$ for all $r$	$q+pe^t$
npq	$\mu_3 = npq(q - p)  \mu_4 = 3n^2p^2q^2 + npq(1 - 6pq)$	$(q+pe^t)^n$
$n\frac{K}{M}\frac{M-K}{M}\frac{M-n}{M-1}$	$\mathscr{E}[X(X-1)\cdots(X-r+1)] = r! \frac{\binom{K}{r}\binom{n}{r}}{\binom{M}{r}}$	not useful
λ	$\kappa_r = \lambda$ for $r = 1, 2,$ $\mu_3 = \lambda$ $\mu_4 = \lambda + 3\lambda^2$	$\exp[\lambda(e^i-1)]$
$\frac{q}{p^2}$	$\mu_3 = \frac{q+q^2}{p^2}$	$\frac{p}{1-qe^t}$
	$\mu_4 = \frac{q + 7q^2 + q^3}{p^4}$	
<u>rq</u> p <sup>2</sup>	$\mu_3 = \frac{r(q+q^2)}{p^3}$	$\left(\frac{p}{1-qe^i}\right)^r$
	$\mu_4 = \frac{r[q + (3r + 4)q^2 + q^3]}{p^4}$	

Table 2 CONTINUOUS DISTRIBUTIONS

Name of parametric family of distributions	Cumulative distribution function $F(\cdot)$ or probability density function $f(\cdot)$	Parameter space	Mean $\mu = \mathscr{E}[X]$
Uniform or rectangular	$f(x) = \frac{1}{b-a} I_{[a,b]}(x)$	$-\infty < a < b < \infty$	$\frac{a+b}{2}$
Normal	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp[-(x-\mu)^2/2\dot{\sigma}^2]$	$-\infty < \mu < \infty$ $\sigma > 0$	μ
Exponential	$f(x) = \lambda e^{-\lambda x} I_{(0,\infty)}(x)$	$\lambda > 0$	$\frac{1}{\lambda}$
Gamma	$f(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x} I_{(0,\infty)}(x)$	$ \lambda > 0 \\ r > 0 $	$\frac{r}{\lambda}$
Beta	$f(x) = \frac{1}{B(a, b)} x^{a-1} (1-x)^{b-1} I_{(0,1)}(x)$	a > 0 b > 0	$\frac{a}{a+b}$
Cauchy	$f(x) = \frac{1}{\pi\beta\{1 + [(x-\alpha)/\beta]^2\}}$	$-\infty < \alpha < \infty$ $\beta > 0$	Does not exist
Lognormal	$f(x) = \frac{1}{x\sqrt{2\pi}\sigma} \exp[-(\log_e x - \mu)^2/2\sigma^2] I_{(0,\infty)}(x)$	$-\infty < \mu < \infty$ $\sigma > 0$	$\exp[\mu + \frac{1}{2}\sigma^2]$
Double exponential	$f(x) = \frac{1}{2\beta} \exp\left(-\frac{ x-\alpha }{\beta}\right)$	$-\infty < \alpha < \infty$ $\beta > 0$	α

Variance $\sigma^2 = \mathscr{E}[(X - \mu)^2]$	Moments $\mu'_r = \mathscr{E}[X^r]$ or $\mu_r = \mathscr{E}[(X - \mu)^r]$ and/or cumulants $\kappa_r$	Moment generating function $\mathscr{E}[e^{tx}]$
$\frac{(b-a)^2}{12}$	$\mu_r = 0$ for $r$ odd $\mu_r = \frac{(b-a)^r}{2^r(r+1)}$ for $r$ even	$\frac{e^{bt}-e^{at}}{(b-a)t}$
$\sigma^2$	$\mu_r = 0$ , $r \text{ odd}$ ; $\mu_r = \frac{r!}{(r/2)!} \frac{\sigma^r}{2^{r/2}}$ , $r \text{ even}$ ; $\kappa_r = 0$ , $r > 2$	$\exp[\mu t + \frac{1}{2} \sigma^2 t^2]$
$\frac{1}{\lambda^2}$	$\mu_r' = \frac{\Gamma(r+1)}{\lambda'}$	$\frac{\lambda}{\lambda - t}  \text{for } t < \lambda$
$\frac{r}{\lambda^2}$	$\mu_j' = \frac{\Gamma(r+j)}{\lambda^j \Gamma(r)}$	$\left(\frac{\lambda}{\lambda-t}\right)^r$ for $t<\lambda$
$\frac{ab}{(a+b+1)(a+b)^2}$	$\mu_r = \frac{B(r+a,b)}{B(a,b)}$	not useful
Does not exist	Do not exist	Characteristic function is $e^{i\alpha t - \beta t }$
$\frac{-\exp[2\mu + 2\sigma^2]}{-\exp[2\mu + \sigma^2]}$	$\mu_r' = \exp[r\mu + \frac{1}{2} r^2 \sigma^2]$	not useful
2β <sup>2</sup>	$\mu_r = 0$ for $r$ odd; $\mu_r = r! \beta^r$ for $r$ even	$\frac{e^{zt}}{1-(\beta t)^2}$

(continued)

Table 2 CONTINUOUS DISTRIBUTIONS (continued)

Cumulative distribution function $F(\cdot)$ or probability density function $f(\cdot)$	Parameter space	Mean $\mu = \mathscr{E}[X]$
$f(x) = abx^{b-1} \exp[-ax^b] I_{(0,\infty)}(x)$	$a > 0 \\ b > 0$	$a^{-1/b}\Gamma(1+b^{-1})$
$F(x) = [1 + e^{-(x-\alpha)/\beta}]^{-1}$	$-\infty < \alpha < \infty$ $\beta > 0$	α
$f(x) = \frac{\theta x_0^{\theta}}{x^{\theta+1}} I_{(x_0, \infty)}(x)$	$x_0 > 0$ $\theta > 0$	$\frac{\theta x_0}{\theta - 1}$
		for $\theta > 1$
$F(x) = \exp\left(-e^{-(x-\alpha)/\beta}\right)$	$\frac{-\infty < \alpha < \infty}{\beta > 0}$	$\alpha + \beta \gamma$ , $\gamma \approx .577216$
$f(x) = \frac{\Gamma[(k+1)/2]}{\Gamma(k/2)} \frac{1}{\sqrt{k\pi}} \frac{1}{(1+x^2/k)^{(k+1)/2}}$	k>0	$\mu = 0$ for $k > 1$
$f(x) = \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} \left(\frac{m}{n}\right)^{m/2}$	$m, n=1, 2, \ldots$	$\frac{n}{n-2}$
$\times \frac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}} I_{(0,\infty)}(x)$		for $n > 2$
$f(x) = \frac{1}{\Gamma(k/2)} \left(\frac{1}{2}\right)^{k/2} x^{k/2-1} e^{-(1/2)x} I_{(0,\infty)}(x)$	$k=1,2,\ldots$	k
	or probability density function $f(\cdot)$ $f(x) = abx^{b-1} \exp[-ax^b] I_{(0,\infty)}(x)$ $F(x) = [1 + e^{-(x-\alpha)/\beta}]^{-1}$ $f(x) = \frac{\theta x_0^{\theta}}{x^{\theta+1}} I_{(x_0,\infty)}(x)$ $F(x) = \exp(-e^{-(x-\alpha)/\beta})$ $f(x) = \frac{\Gamma[(k+1)/2]}{\Gamma(k/2)} \frac{1}{\sqrt{k\pi}} \frac{1}{(1+x^2/k)^{(k+1)/2}}$ $f(x) = \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} \left(\frac{m}{n}\right)^{m/2}$ $\times \frac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}} I_{(0,\infty)}(x)$	or probability density function $f(\cdot)$ space $f(x) = abx^{b-1} \exp[-ax^b] I_{(0,\infty)}(x) \qquad a > 0 \\ b > 0$ $F(x) = [1 + e^{-(x-\alpha)/\beta}]^{-1} \qquad -\infty < \alpha < \infty \\ \beta > 0$ $f(x) = \frac{\theta x_0^{\theta}}{x^{\theta+1}} I_{(x_0,\infty)}(x) \qquad x_0 > 0 \\ \theta > 0$ $F(x) = \exp(-e^{-(x-\alpha)/\beta}) \qquad -\infty < \alpha < \infty \\ \beta > 0$ $f(x) = \frac{\Gamma[(k+1)/2]}{\Gamma(k/2)} \frac{1}{\sqrt{k\pi}} \frac{1}{(1+x^2/k)^{(k+1)/2}} \qquad k > 0$ $f(x) = \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} \left(\frac{m}{n}\right)^{m/2} \qquad m, n = 1, 2, \dots$

Variance $\sigma^2 = \mathscr{E}[(X - \mu)^2]$	Moments $\mu'_r = \mathscr{E}[X^r]$ or $\mu_r = \mathscr{E}[(X - \mu)^r]$ and/or cumulants $\kappa_r$	Moment generating function &[e <sup>tX</sup> ]	_
$a^{-2/b}[\Gamma(1+2b^{-1}) - \Gamma^2(1+b^{-1})]$	$\mu_r' = a^{-r/b} \Gamma \left( 1 + \frac{r}{b} \right)$	$\mathscr{E}[X^t] = a^{-t/b}\Gamma\Big(1 + \frac{t}{b}\Big)$	
$\frac{\beta^2\pi^2}{3}$		$e^{\alpha t}\pi\beta t \csc(\pi\beta t)$	•
$\frac{\theta x_0^2}{(\theta-1)^2(\theta-2)}$	$\mu_r' = \frac{\theta x_0'}{\theta - r}  \text{for } \theta > r$	does not exist	•
for $ heta > 2$			
$\frac{\pi^2\beta^2}{6}$	$\kappa_r = (-\beta)^r \psi^{(r-1)}(1)$ for $r \ge 2$ , where $\psi(\cdot)$ is digamma function	$e^{\alpha t}\Gamma(1-\beta t)$ for $t<1/\beta$	•
	$\mu_r = 0$ for $k > r$ and $r$ odd		•
$\frac{k}{k-2}$	$\mu_r = \frac{k^{r/2}B((r+1)/2, (k-r)/2)}{B(\frac{1}{2}, k/2)}$	does not exist	ć
for $k > 2$	for $k > r$ and $r$ even		
$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$	$\mu_r' = \left(\frac{n}{m}\right)^r \frac{\Gamma(m/2+r)\Gamma(n/2-r)}{\Gamma(m/2)\Gamma(n/2)}$	does not exist	•
for $n > 4$	for $r < \frac{n}{2}$		
2 <i>k</i>	$\mu_j' = \frac{2^J \Gamma(k/2 + j)}{\Gamma(k/2)}$	$\left(\frac{1}{1-2t}\right)^{k/2}$	•
		for $t < 1/2$	

Variance $\sigma^2 = \mathscr{E}[(X - \mu)^2]$	Moments $\mu'_r = \mathscr{E}[X^r]$ or $\mu_r = \mathscr{E}[(X - \mu)^r]$ and/or cumulants $\kappa_r$	Moment generating function &[e'x]
$\frac{N^2-1}{12}$	$\mu_3' = \frac{N(N+1)^2}{4}$	$\sum_{j=1}^{N} \frac{1}{N} e^{jt}$
	$\mu_4' = \frac{(N+1)(2N+1)(3N^2+3N-1)}{30}$	
pq	$\mu_r' = p$ for all $r$	$q + pe^t$
npq	$\mu_{3} = npq(q - p) \mu_{4} = 3n^{2}p^{2}q^{2} + npq(1 - 6pq)$	$(q+pe^t)^n$
$n\frac{K}{M}\frac{M-K}{M}\frac{M-n}{M-1}$	$\mathscr{E}[X(X-1)\cdots(X-r+1)] = r! \frac{\binom{K}{r}\binom{n}{r}}{\binom{M}{r}}$	not useful
λ	$\kappa_r = \lambda$ for $r = 1, 2,$ $\mu_3 = \lambda$ $\mu_4 = \lambda + 3\lambda^2$	$\exp[\lambda(e^t-1)]$
$\frac{q}{p^2}$	$\mu_3 = \frac{q+q^2}{p^2}$	$\frac{p}{1-qe^t}$
	$\mu_4 = \frac{q + 7q^2 + q^3}{p^4}$	
$\frac{rq}{p^2}$	$\mu_3 = \frac{r(q+q^2)}{p^3}$	$\left(\frac{p}{1-qe^t}\right)^r$
	$\mu_4 = \frac{r[q + (3r + 4)q^2 + q^3]}{p^4}$	