```
EXE15-5.2
*
V1          1        0     DC        2
R1          1        2     4
L1          0        2     4      IC=0
L2          0        3     4      IC=-0.5
KL          1       L2     0.5
V2          4        3     DC       -1
L3          4        5     1      IC=-0.5
R2          5        0     2
.TRAN            .1       10     UIC
.PRINT     TRAN      I(V2)
.END
```

SUMMARY

Elements may interact electrically, through the wires that connect them, or magnetically, when the magnetic field spawned by one element threads the other. Mutual inductance causes the terminal current and voltage of one inductor to depend on those of another through their electromagnetic interaction.

- The voltage induced in a coupled coil consists of a term proportional to the derivative of its own current plus terms proportional to the derivatives of the currents flowing through each of its coupled coil partners.
- The dot convention requires positive coupling terms in the i–v laws if both inductive currents flow into, or both out of, the dotted ends.
- The coupling coefficient between two coupled coils is their mutual inductance divided by the geometric mean of their self-inductances.
- A transformer is a device for changing voltage, current, or impedance levels in a circuit. It consists of a pair of coupled coils on a common core.
- An ideal transformer steps up the voltage by the turns ratio and steps down the current by the negative inverse turns ratio.
- An ideal transformer dissipates no electrical power.

PROBLEMS

15.1. Suppose $L_1 = L_2 = 0.1$ H, $M = 0.05$ H, $N_1 = N_2 = 1000$ turns. Find the leakage fluxes ϕ_{L1}, ϕ_{L2} and the total fluxes ϕ_1 and ϕ_2 threading the inductors if $i_1 = 3t + 2$ A, $i_2 = 3$ A.

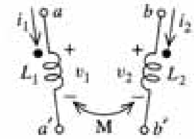

FIGURE P15.1

15.2. A pair of coupled coils have self-inductances $L_1 =$ 1 H, $L_2 = 5$ H. If $i_1(t) = 5\cos 2t$ A, $i_2(t) = 2\cos 2t$ A, what is the maximum amplitude A of the sinusoidal flux linkage $\lambda_1(t) = A\cos(2t+\phi)$ threading coil 1? The minimum amplitude A?

15.3. When a' and b' in the figure for Problem 15.1 are shorted, the inductance between a and b is 1 H, and when a' is shorted to b, the inductance between a and b' is 7 H. Find the mutual inductance M.

15.4. $L_1 = L_2 = 2$ H, $M = 1$ H and positive di_1/dt leads to negative v_2. Find i_1 and i_2 for $t > 0$.

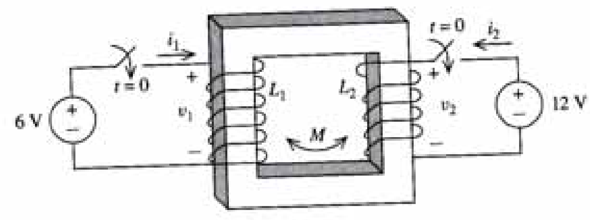

FIGURE P15.4

15.5. If $\omega(0) = 0$, determine the power $p(t)$ and energy $\omega(t)$ for the coupled coils of Problem 15.1 above, with the indicated currents.

15.6. Suppose $L_1 = 1$ H, $L_2 = 5$ H, $M = 2$ H in the figure for Problem 15.1. Find the peak instantaneous power in watts supplied to the inductors if $i_1(t) = 3\cos t$ A, $i_2 = \sin(t + 45°)$ A. Is this power dissipated in the form of heat? Explain.

15.7. When two identical coils are each made to simultaneously carry a dc current of 1 A, they each have flux linkage $i_1 = i_2 = 10$ Wb. When they are shielded from one another so no flux from one threads the other, the flux linkage rises to 12 Wb. Find M in both cases.

15.8. Write the i–v laws in the time and s domains. In both cases $i_1(0-) = 1$ A, $i_2(0-) = 0$.

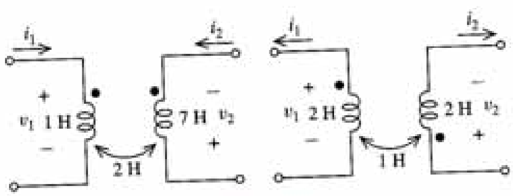

FIGURE P15.8

15.9. Find i for $t > 0$ if $i(0) = 0$ and $v(0) = 4$ V.

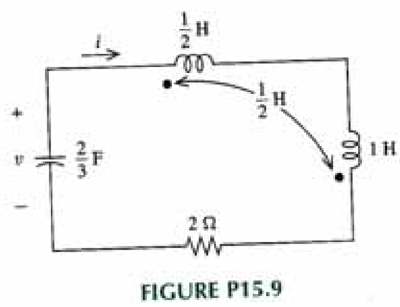

FIGURE P15.9

15.10. Determine $Z(s)$, the impedance looking into this circuit.

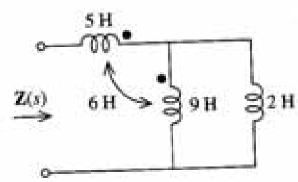

FIGURE P15.10

15.11. Find $i_2(t)$ in ac steady state.

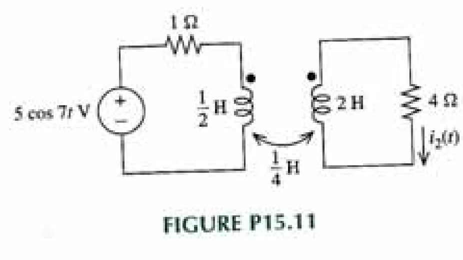

FIGURE P15.11

15.12. Find the ac steady state value of $v_R(t)$.

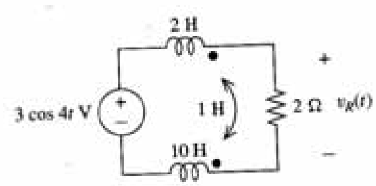

FIGURE P15.12

15.13. Find the steady-state value of v.

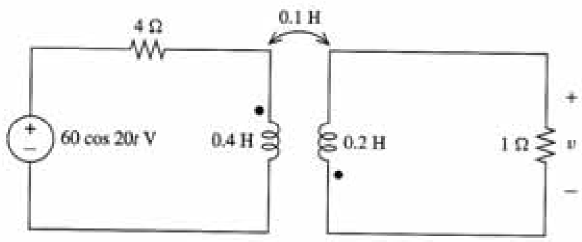

FIGURE P15.13

15.14. Find the steady-state currents i_1 and i_2.

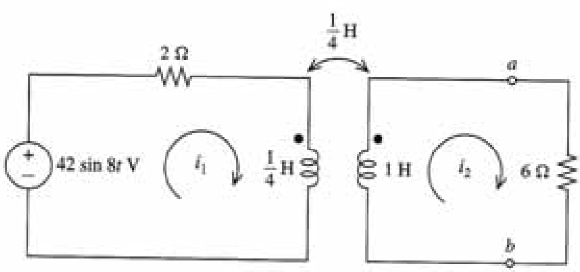

FIGURE P15.14

15.15. Find the power dissipated in the 1-Ω resistor in the primary circuit and the power factor seen by the source.

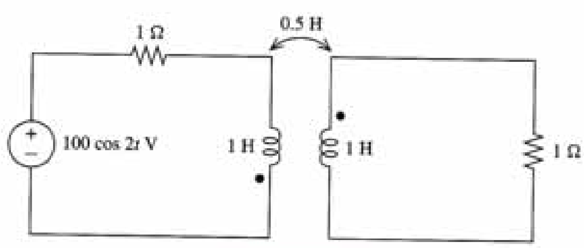

FIGURE P15.15

15.16. Find the steady-state current i_2.

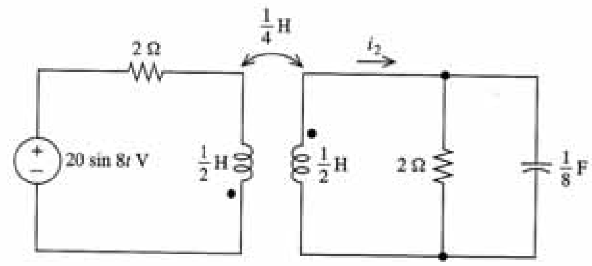

FIGURE P15.16

15.17. (a) Find v for $t > 0$ if $v_g = 4u(t)$ V and (b) find the steady-state value of v if $v_g = 4\cos 8t$ V and the output terminals are loaded with a resistor of 8 Ω.

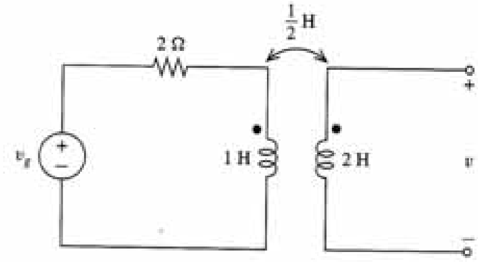

FIGURE P15.17

15.18. Find $\mathbf{I}_1$ and $\mathbf{I}_3$ using mesh analysis.

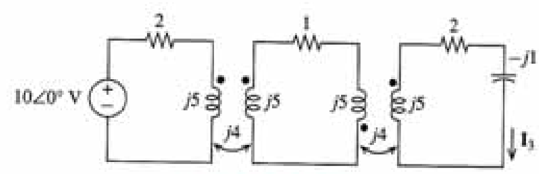

FIGURE P15.18

15.19. Find v for $t > 0$ if $i_g = 2u(t)$ A. Use mesh analysis and assume no initial stored energy at $t = 0$.

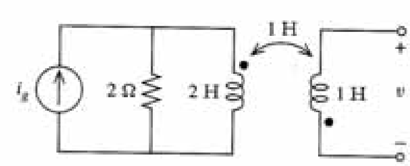

FIGURE P15.19

15.20. Repeat 15.19 using nodal analysis.

15.21. Find the ac steady state current $i_2(t)$ using mesh analysis.

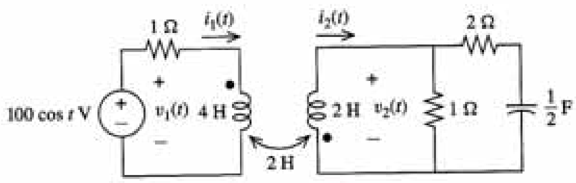

FIGURE P15.21

15.22. Find the voltage **V** across the source.

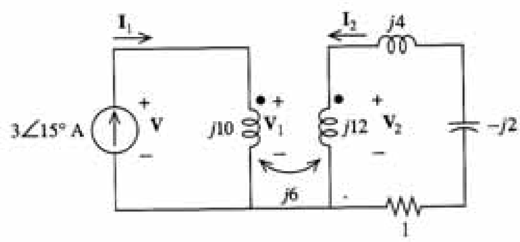

FIGURE P15.22

15.23. Determine the reflected impedance Z_R, current and voltage ratios $\mathbf{I}_2/\mathbf{I}_1$, $\mathbf{V}_2/\mathbf{V}_1$ for the circuit of Problem 15.21.

15.24. Determine the reflected impedance Z_R, current and voltage ratios $\mathbf{I}_2/\mathbf{I}_1$, $\mathbf{V}_2/\mathbf{V}_1$ for the circuit of Problem 15.22.

15.25. Solve 15.21 using the uncoupled coil model of Fig. 15.11 and nodal analysis.

15.26. Find $\mathbf{I}_L$.

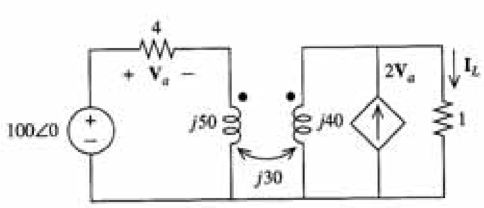

FIGURE P15.26

15.27. Find the transfer function

$$\mathbf{H}(s) = \mathbf{V}_2(s)/\mathbf{V}_1(s)|_{\mathbf{I}_2(s)=0}$$

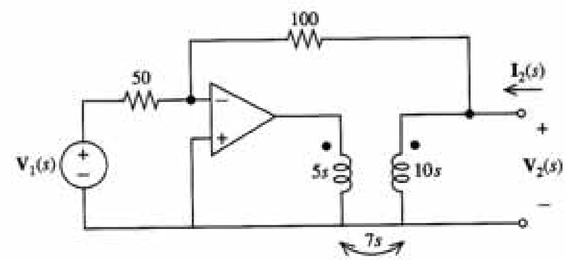

FIGURE P15.27

15.28. Find the steady-state voltage v using the equivalent circuit of Fig. 15.11(b) for the transformer.

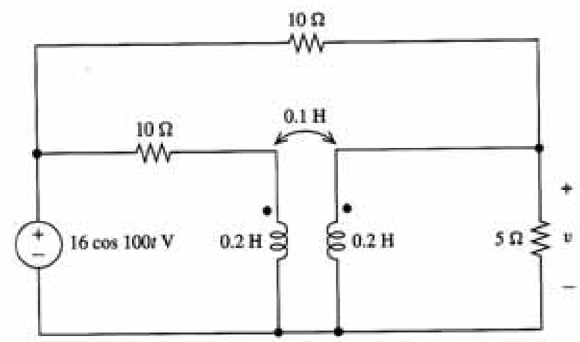

FIGURE P15.28

15.29. Find $i(t)$ for $t > 0$ if $i(0+) = 0$.

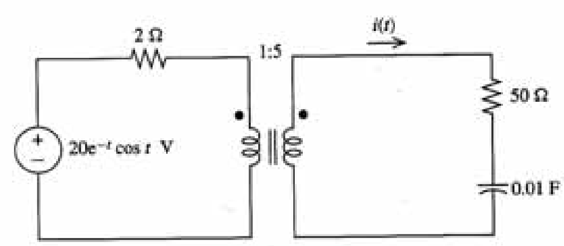

FIGURE P15.29

15.30. Find the average power delivered to the 8-Ω resistor.

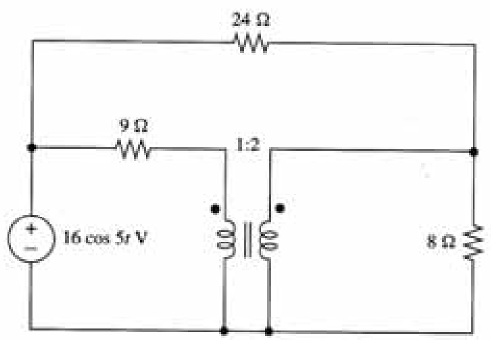

FIGURE P15.30

15.31. Write the i–v laws in the time and s domains. Do not assume initial currents are zero.

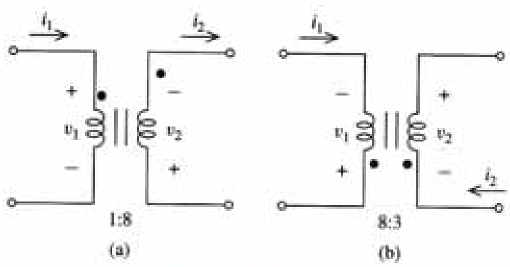

FIGURE P15.31

15.32. Find the ac steady state value of $v_2(t)$ for $v_1(t) = 50 \sin 3t$ V.

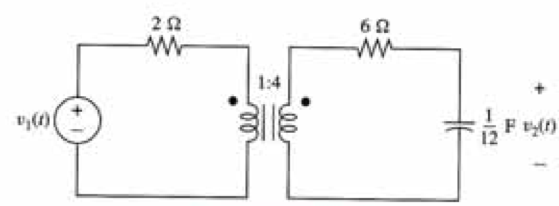

FIGURE P15.32

15.33. Find $v_2(t)$ in Problem 15.32 if all initial conditions at time $t = 0$ are zero and $v_1(t) = u(t)$.

15.34. If $\mathbf{I} = 2/\!\underline{0}$ A, what is the turns ratio n?

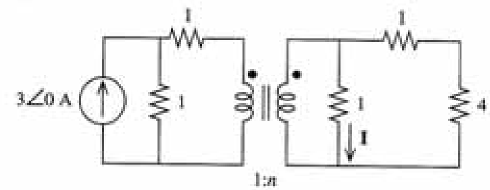

FIGURE P15.34

15.35. Find $\mathbf{I}_1$ in Problem 15.18 by reflecting impedances into primaries twice.

15.36. Find the real average power delivered by the source.

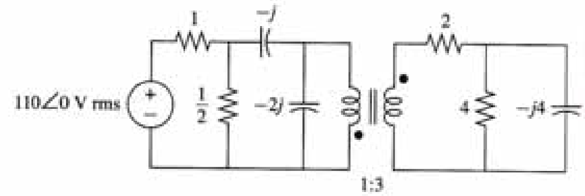

FIGURE P15.36

SPICE Problems

15.37. Use SPICE to determine the ac steady state values for the four port variables.

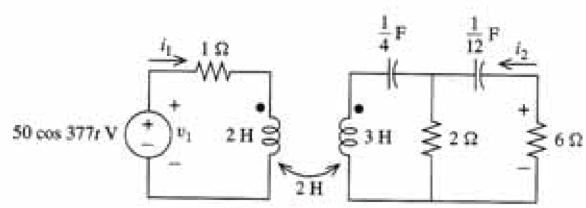

FIGURE P15.37

15.38. Repeat Problem 15.37 if the coupled coils are replaced by an ideal transformer with turn ratio $n = 20$.

15.39. Find the value for M at which the ac steady state amplitude of v_2 is reduced to $\frac{1}{10}$ of its value when the coils are unity coupled.

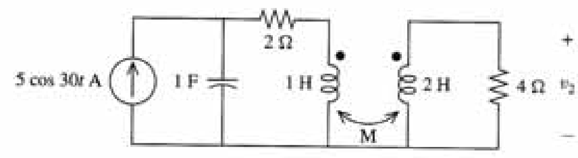

FIGURE P15.39

15.40. Plot i for $0 < t < 5$ s in the circuit Problem 15.9.

15.41. In the circuit of Problem 15.28, replace the 10-Ω resistor bridging the transformer with a series connection of a 10-Ω resistor and a 1000-mF capacitor. Use SPICE to plot the frequency response for the voltage v of the resulting circuit in the interval $1 < f < 50$ Hz.

More Challenging Problems

15.42. When b is shorted to b', the other inductor satisfies $v_1 = 4di_1/dt$. When a is shorted to a', the other inductor satisfies $v_2 = 2di_2/dt$. If $L_1 = 5$ H, find M and indicate whether the dot should be at b or b'.

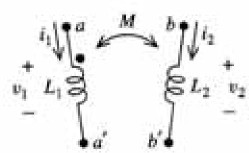

FIGURE P15.42

15.43. Show that a real transformer, with $0 \le k \le 1$, satisfies the passivity condition $\omega(t) \ge 0$ for all t.

15.44. Find $i_1(t)$ in ac steady state. The 4 H and 6 H coils are coupled with $M_a = 2$ H (filled dots). The 6 H and 5 H coils are also coupled with $M_b = 4$ H (open dots). The 4 H and 5 H coils are not coupled.

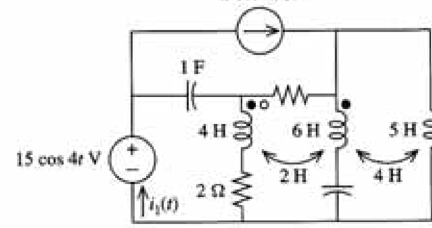

FIGURE P15.44

15.45. Find $\mathbf{I}_3$ in Problem 15.18 by reflecting sources into secondaries twice.

14.3. $\frac{-j\omega^3}{(4-3\omega^2)+j2\omega}$

14.5. $\sqrt{\frac{\omega^2+1}{\omega^6+2\omega^4-3\omega^2+1}}, \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega-\omega^3}{1-2\omega^2}\right)$

14.7. $\sqrt{\frac{\omega^2+1}{\omega^2+2}}, \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$

14.9. $\frac{1}{s^2-1}, \frac{-1}{\omega^2+1}, -\frac{4}{5}\cos 2t$ V, no.

14.13. (a) 9 dB; (b) $\frac{9}{2}$ dB; (c) 50 dB; (d) 18 dB; (e) −72 dB; (f) 612 dB; (g) 2 dB; (h) −18 dB

14.15. (a) .05, (b) 20; (c) $5\sqrt{2}$ (d) $.01\sqrt{5}$; (e) $.005\sqrt{5}$; (f) $\sqrt{10} \times 10^{-20}$

14.17. (a) 121.62; (b) .254; (c) 7.08; (d) 3.16×10^8, (e) .0216, (f) 1.0001

14.19. $\frac{2(s-1/2)}{(s+1)^3}$, triple real pole with break frequency 1 rad/s, zero with break frequency 1/2 rad/s.

14.21. $\frac{\frac{1}{4}s}{s+3/4}$, 3/4 rad/s

14.25. 18 dB, −38 dB, −38 dB, −20 dB, −20 dB

14.27. $\omega_1 = 25.1$ rad/s, $\omega_2 = 631$ rad/s, $\omega_3 = 15.8$ krad/s, $\omega_4 = 398$krad/s, $\frac{(5\times10^{-7})(s+1)^2(s+\omega_2)^2(s+\omega_4)^2}{s(s+\omega_1)^2(s+\omega_3)^2}$

14.29. $\frac{1/6}{(s+25/3)}$

14.31. $\frac{\frac{1}{3}s^2}{(s+1.47)(s^2+.279s+.34)}$

14.39. Series: 1 H, 400 pF, 12.5 Ω; parallel: 1 H, 400 pF, 2 MΩ

14.41. $\frac{A_0 W_0}{jw+\frac{A_0\omega_0 R_A}{R_A+R_F}}, \frac{A_0\omega_0 R_A}{R_A+R_F}$ rad/s

14.61. $\frac{-1}{s(s+1)(s+3)}$

CHAPTER 15

15.1. $(\frac{3}{2}t+1)\times10^{-4}$ W, $\frac{3}{2}\times10^{-4}$ W, $(3t+\frac{7}{2})\times10^{-4}$ W, $(\frac{3}{2}t+4)\times10^{-4}$ W

15.3. $\frac{3}{2}$ H

15.5. $0.9t + 1.05$ W, $0.45t^2 + 1.05t$ J

15.7. 2 H, 0 H

15.9. $4(e^{-t} - e^{-3t})$ A

15.11. $\left[-4e^{-(t/5)} + 75\cos(4t - 87.1^\circ)\right]$ mA

15.13. $\frac{5\sqrt{2}}{2}\sin(2t + 135^\circ)$ V

15.15. 1250 W, 0.6 lagging

15.17. (a) $2e^{-2t}$ V; (b) $\frac{2\sqrt{2}}{3}\cos(8t - 45^\circ)$ V

15.19. $-2e^{-t}$ V

15.21. $35.9\cos(t + 152^\circ)$ A

15.23. $2\angle 76.4^\circ$, $1\angle 22.6^\circ$, $.343\angle -59.0^\circ$

15.25. $35.9\cos(t + 152^\circ)$ A

15.27. −2

15.29. $\sqrt{2}e^{-t}\cos(t+45^\circ) - e^{-t}$ A

15.31. (a) $V_2 = -8V_1, I_2 = \frac{1}{8}I_1, v_2 = -8v_1, i_2 = \frac{1}{8}i_1$; (b) $V_2 = -\frac{3}{8}V_1, I_2 = \frac{8}{3}I_1, v_2 = -\frac{3}{8}v_1, i_2 = \frac{8}{3}i_1$

15.33. $4(1 - e^{-(6/19)t})u(t)$ V

15.35. $2.67\angle -47.7^\circ$

15.37. $v_1 = 50\cos 377t$ V, $i_1 = 0.2\cos(377t - 89.6^\circ)$ A, $v_2 = 0.2\cos(377t - 89.3^\circ)$ V, $i_2 = .033\cos(377t + 90.7^\circ)$ A

15.39. .761 H

15.41. $\frac{t_{21}}{t_{11}t_{22}-t_{12}t_{21}}, \frac{g_{11}}{g_{12}}$

15.43. $w \ge \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_1^2 - \sqrt{L_1L_2}I_1I_2 = \frac{1}{2}(\sqrt{L_1}I_1 + \sqrt{L_2}I_2)^2 \ge 0$

15.45. $1.27\angle -158^\circ$ A

CHAPTER 16

16.1. 2.34 A, .938 A, 8.91 V, −5.62 V

16.3. $\begin{bmatrix} \frac{s}{s+1} & \frac{2}{s+1} \\ 3 & \frac{2s^2+6s-1}{s(s+1)} \end{bmatrix}$

16.5. Solve z-parameter equations for $\mathbf{I}_1$ and $\mathbf{V}_2$.

16.7. Solve z-parameter equations for $\mathbf{V}_1$ and $\mathbf{I}_2$.

16.9. $\frac{1}{4}\begin{bmatrix} s+2 & -s \\ 2-4s & s+1 \end{bmatrix}, \frac{2s-1}{2s+10}$

16.11. $\begin{bmatrix} 3.4 & 2.6 \\ 2.6 & 3.4 \end{bmatrix}$

16.13. $\mathbf{V}_1 = \frac{1}{s}, \mathbf{V}_2 = \frac{4s(8s^2+10s+5)}{(4s^2+2s+1)^2}$, $\mathbf{I}_1 = \frac{s+1}{s} - \frac{4s^2(8s^2+10s+5)}{(4s^2+2s+1)^2}, \mathbf{I}_2 = \frac{-(8s^2+10s+5)}{4s^2+2s+1}$

16.15. $\begin{bmatrix} \frac{s+3}{6s} & -\frac{1}{3} \\ +\frac{1}{3} & \frac{4s+3}{3s} \end{bmatrix}, \begin{bmatrix} \frac{(2s+3)(s+1)}{2s^2} & -\frac{(4s+3)}{s} \\ -\frac{(s+3)}{2s} & 3 \end{bmatrix}$

16.17. $\frac{1}{s(2s^2+2s+3)}\begin{bmatrix} 2(s^2+s+1) & 2 \\ 2 & 2(2s^2+2s+1) \end{bmatrix}$, $\frac{2(s^2+s+1)}{s(2s^2+2s+3)}, \frac{2(2^2+2s+1)}{s(2s^2+2s+3)}$

16.19. $\frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \frac{-\mathbf{h}_{21}}{\mathbf{h}_{11}\mathbf{h}_{22}-\mathbf{h}_{12}\mathbf{h}_{21}}$

16.23. Fig. 16.30 with $\mathbf{z}_{11} - \mathbf{z}_{12} = \frac{2}{s^2+2s+2}, \mathbf{z}_{22} - \mathbf{z}_{12} = \frac{1}{s^2+2s+2}, \mathbf{z}_{22} - \mathbf{z}_{21} = \frac{s}{s^2 2s+2}, \mathbf{z}_{12} = s$

16.25. Symmetric π-circuit with 6 Ω across each port, connected by 1.85 Ω.

16.27. (a) Apply KCL at input, output ports, (b) apply KCL at input, output ports.

16.29. $\begin{bmatrix} .706 & .765 \\ 1.00 & .588 \end{bmatrix}$

16.33. $\begin{bmatrix} -5.70 & -9.00 \\ 1.00 & 1.33 \end{bmatrix}$

16.35. $10^{-4}\begin{bmatrix} \frac{s+5}{s+10} & 0 \\ \frac{10}{s+10} & 1 \end{bmatrix}$

16.37. $\begin{bmatrix} 2.52 & 1.64\times10^7 \\ -1.58\times10^{-7} & 1.26 \end{bmatrix}$

16.43. Nodal analysis with negative terminal of v_1 as reference yields $\mathbf{y}_{11} = \mathbf{y}_{12}$ and $\mathbf{y}_{12} = \mathbf{y}_{21}$. Inverting yields the z-parameters.