- A circuit with two pairs of accessible terminals, each satisfying the port condition, is called a two-port circuit. Such circuits are described by pairs of equations involving the four port variables.
- There are six distinct sets of two-port parameters: impedance or z-parameter, admittance or y-parameter, hybrid, inverse hybrid, transmission, and inverse transmission. They each result by specific choice of two of the four port variables to act as independent variables in the two-port equations.
- The transmission matrices of two-ports in cascade multiply, the admittance matrices of two-ports in parallel add, and the impedance matrices of two-ports in series add.
- Placing an ideal transformer in either port of a two-port guarantees that the port condition is satisfied.

PROBLEMS

16.1. Find the port variables i_1, i_2, v_1, v_2.

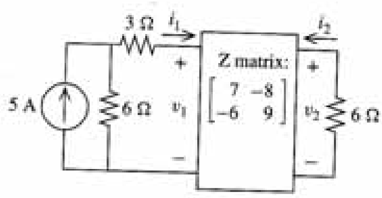

FIGURE P16.1

16.2. Find the transmission and inverse transmission parameters.

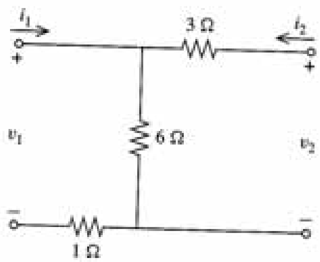

FIGURE P16.2

16.3. The y-parameters for a certain two-port are

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} 1+\frac{1}{s} & -\frac{2}{s} \\ -\frac{2}{s} & 3+\frac{3}{s} \end{bmatrix}$$

Find the hybrid parameters by using Table 16.2.

16.4. Find the y-parameters.

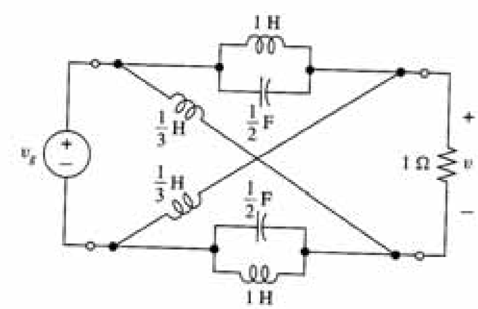

FIGURE P16.4

16.5. Show that the hybrid parameters may be obtained from the z-parameters by

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{12} = -\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$$

$$\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

16.6. Find the h- and g-parameters.

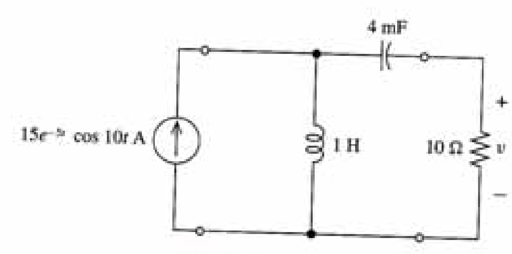

FIGURE P16.6

7. Show that the transmission parameters, **A**, **B**, **C**, and where

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

given by

$$\mathbf{A} = \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}}, \qquad \mathbf{B} = \frac{\mathbf{\Delta}_z}{\mathbf{z}_{21}}$$

$$\mathbf{C} = \frac{1}{\mathbf{z}_{21}}, \qquad \mathbf{D} = \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}}$$

8. Find the transmission parameters of the two-port twork in Fig. P16.6.

9. Find the y-parameters of the network shown. Ternate the output port with a 1-Ω resistor, and find the sulting network function $\mathbf{V}_2/\mathbf{V}_1$.

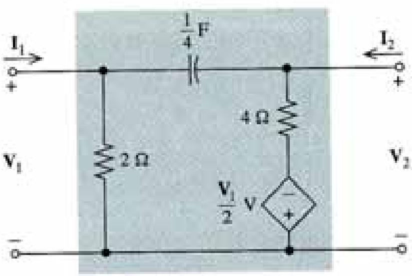

FIGURE P16.9

.10. Let $\mathbf{h}_{11} = 1$ kΩ, $\mathbf{h}_{12} = 10^{-4}$, $\mathbf{h}_{21} = 100$, and $\mathbf{h}_{22} = 10^{-4}$ S in Fig. 16.11(b) and find the network function $\mathbf{V}_2/\mathbf{V}_1$ if port 2 is open-circuited.

.11. Find the z-parameters for this two-port circuit.

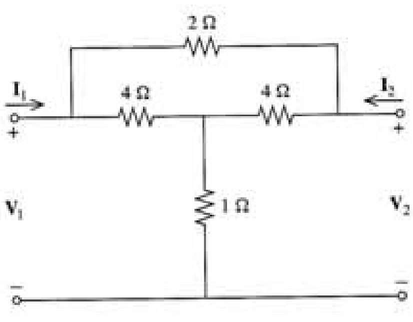

FIGURE P16.11

6.12. Find the z-parameters for this two-port circuit.

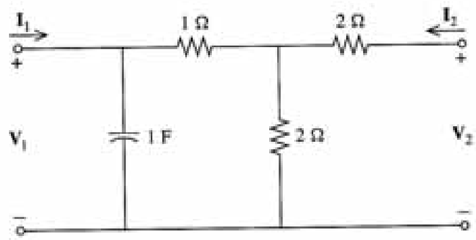

FIGURE P16.12

16.13. A circuit has admittance description

$$\begin{bmatrix} \mathbf{I}_1(s) \\ \mathbf{I}_2(s) \end{bmatrix} = \begin{bmatrix} s+1 & -s \\ -s & s+2+\frac{1}{s} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1(s) \\ \mathbf{V}_2(s) \end{bmatrix}$$

Find the **s**-domain port variables $\mathbf{I}_1(s)$, $\mathbf{I}_2(s)$, $\mathbf{V}_1(s)$, $\mathbf{V}_2(s)$ if $\mathbf{V}_1(s) = 1/s$ and the output port is terminated by a parallel RLC circuit with $R = 2$ Ω, $L = 4$ H, and $C = 1$ F.

16.14. Find the z-parameters for this two-port. Repeat for the n-parameters (do not use Table 16.2).

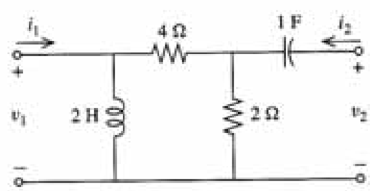

FIGURE P16.14

16.15. Find the inverse hybrid and inverse transmission parameters for the circuit of Problem 16.14. (Do not use Table 16.2.)

16.16. Find all six of the two-port parameter descriptions for the circuit of Problem 16.11, computing the g-parameter descriptions from two experiments and the rest from Table 16.2.

16.17. Find the z-parameters, and find the impedance looking into the input port with the output port open-circuited $\mathbf{V}_1/\mathbf{I}_1|_{\mathbf{I}_2=0}$ and the impedance looking into the output port with the input open-circuited $\mathbf{V}_2/\mathbf{I}_2|_{i_1=0}$.

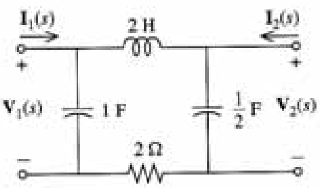

FIGURE P16.17

16.18. Determine the transmission parameters, and the transfer function $\mathbf{I}_1/\mathbf{I}_2|_{\mathbf{V}_1=0}$.

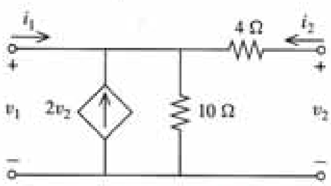

FIGURE P16.18

16.19. Determine a two-port's open-circuit voltage gain transfer function, defined as $\mathbf{H}(s) = \mathbf{V}_2(s)/\mathbf{V}_1(s)|_{I_2(s)=0}$, in terms of its z-parameters $\mathbf{z}_{11}(s)$, $\mathbf{z}_{12}(s)$, $\mathbf{z}_{21}(s)$, $\mathbf{z}_{22}(s)$. Repeat for the hybrid parameters.

16.20. Find the transconductance

$$\mathbf{H}(s) = \mathbf{I}_2(s)/\mathbf{V}_1(s)|_{\mathbf{I}_1(s)=0}$$

for a general two-port in terms of its inverse hybrid parameters and in terms of its transmission parameters.

16.21. Sketch a circuit for which the hybrid parameters do not exist, but all other parameters exist. Compute the values of all other 5 two-port descriptions for your circuit.

16.22. The symmetrical lattice is terminated in 1 Ω and $\mathbf{Z}_a$ and $\mathbf{Z}_b$ are as shown. Find the voltage ratio transfer function.

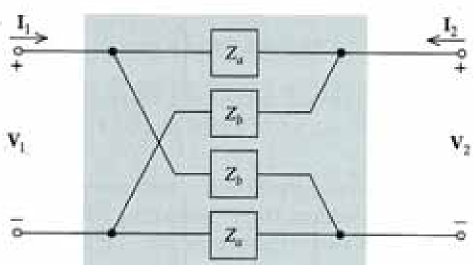

FIGURE P16.22

16.23. A certain two-port has y-parameter descriptions

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} s+1 & -s \\ -1 & s+2 \end{bmatrix}$$

Sketch a three-terminal model which includes no controlled current sources.

16.24. Find a model which has no internal loops for the circuit of a Problem 16.12.

16.25. Find a model which has no internal nodes (nodes not accessible at the input or output ports) for the circuit of Problem 16.11.

16.26. Show that the given circuit is equivalent to the general two-port network. Note how it differs from the equivalent circuit of Fig. 16.13.

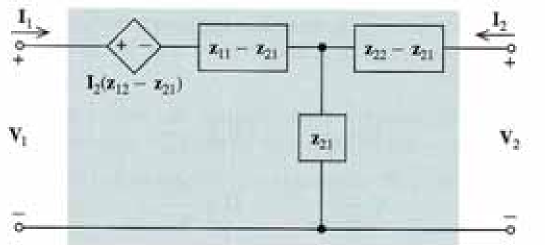

FIGURE P16.26

16.27. Show that the given circuits are equivalent to the general two-port network. Note how the two circuits differ.

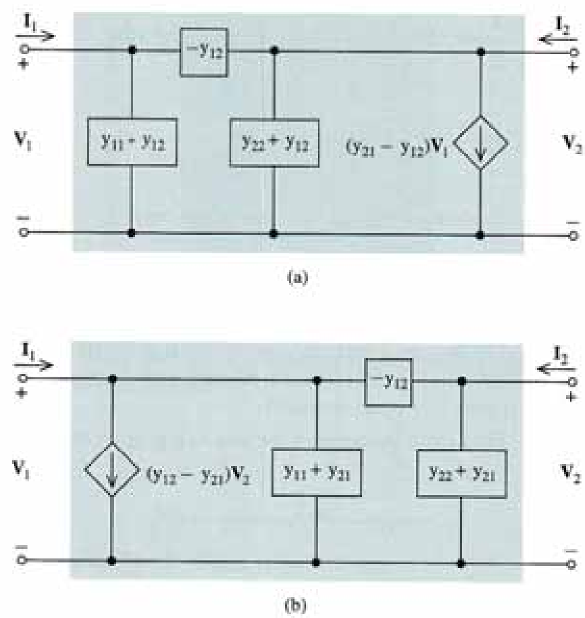

FIGURE P16.27

16.28. A circuit with inverse hybrid description

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = \frac{1}{(2s+3)} \begin{bmatrix} 1 & -(s+1) \\ (s+1) & s^2+7s+8 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

is terminated by a 3-Ω load resistor across the output port. What is its admittance $Y(j\omega)$ at $\omega = 10$ rad/s?

16.29. Find the hybrid parameters of the two-port consisting of two copies of the circuit of Problem 16.11 in parallel.

16.30. Find the y-parameters of the two-port consisting of two copies of the circuit of Problem 16.11 in cascade.

16.31. A circuit has transmission matrix

$$\begin{bmatrix} \mathbf{V}_1(s) \\ \mathbf{I}_1(s) \end{bmatrix} = \begin{bmatrix} 12 & 1 \\ -3 & 2s \end{bmatrix} \begin{bmatrix} \mathbf{V}_2(s) \\ -\mathbf{I}_2(s) \end{bmatrix}$$

Find the four port variables $\mathbf{V}_1(s)$, $\mathbf{I}_1(s)$, $\mathbf{V}_2(s)$, $\mathbf{I}_2(s)$ if both ports are terminated with copies of the one-port shown, aligned so that terminal a is connected to the plus ends of both ports.

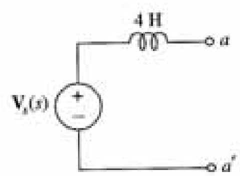

FIGURE P16.31

16.32. Find the z-parameters of three copies of the circuit of Problem 16.18 in cascade.

16.33. Find the g-parameters of three copies of the circuit of Problem 16.18 in parallel.

16.34. Find the inverse transmission parameters of three copies of the circuit of Problem 16.18 in series.

16.35. Find the y-parameter description.

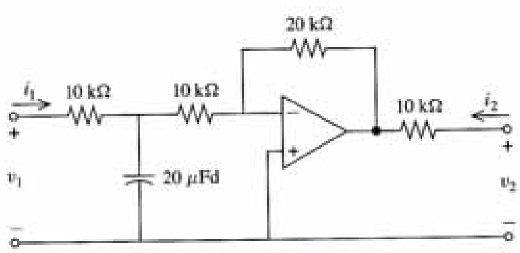

FIGURE P16.35

SPICE Problems

16.36. Find the z-parameters $\mathbf{z}_{11}(j\omega)$, $\mathbf{z}_{12}(j\omega)$, $\mathbf{z}_{21}(j\omega)$, and $\mathbf{z}_{22}(j\omega)$ at $\omega = 1$ and $\omega = 100$ rad/s. Use the ideal voltage amplifier model for the op amp with open loop gain 10^5.

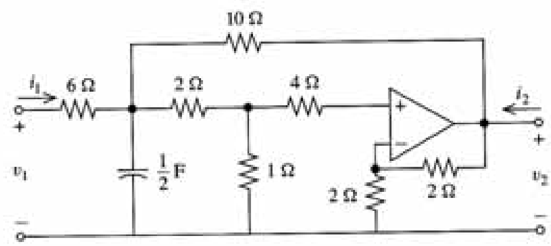

FIGURE P16.36

16.37. Determine the y-parameters $\mathbf{y}_{11}(j\omega)$, $\mathbf{y}_{12}(j\omega)$, $\mathbf{y}_{21}(j\omega)$, $\mathbf{y}_{22}(j\omega)$ at $\omega = 377$ rad/s if 10 copies of the circuit of Problem 16.39 are put in cascade. Solve using SPICE and subcircuits .SUBCKT.

16.38. Find the ac steady-state value of $v_2(t)$ using SPICE.

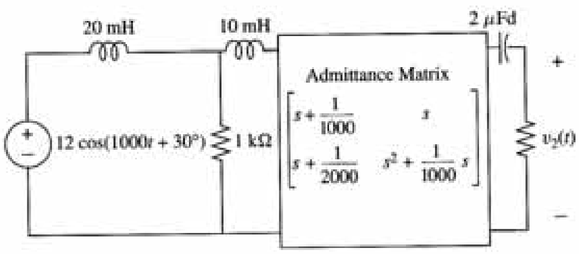

FIGURE P16.38

More Challenging Problems

16.39. Find the y-parameters for this circuit. Repeat for the transmission parameters (do not use Table 16.2).

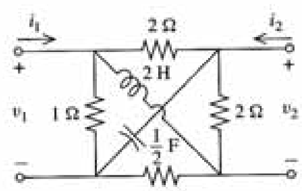

FIGURE P16.39

16.40. Find the hybrid parameters for this two-port with a–a' the input port and b–b' the output port. Then find $v_{aa'} = v_1$, $v_{bb'} = v_2$ in terms of $\mathbf{I}_g(s)$.

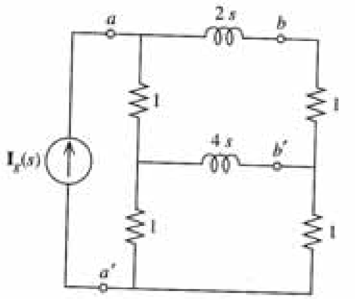

FIGURE P16.40

16.41. Find the h-matrix for the two-port.

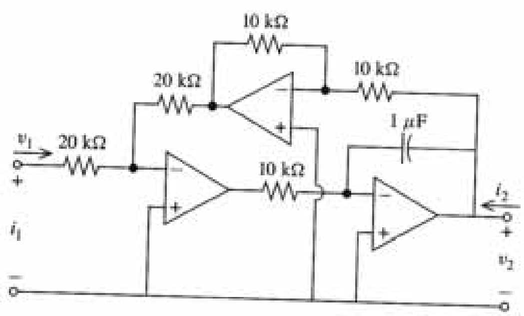

FIGURE P16.41

16.42. Show that for the circuit of Problem 16.4, in terms of the transmission parameters we have

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{\mathbf{A} + \mathbf{B}}$$

16.43. The circuit of Problem 16.22 (no termination) is a *lattice* with *series arms* both equal to $\mathbf{Z}_a$ and *cross arms* both equal to $\mathbf{Z}_b$. It is called a *symmetrical* lattice because the series arms are equal and the cross arms are equal. Show that the z- and y-parameters are given by

$$\mathbf{z}_{11} = \mathbf{z}_{22} = \frac{1}{2}(\mathbf{Z}_b + \mathbf{Z}_a)$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \frac{1}{2}(\mathbf{Z}_b - \mathbf{Z}_a)$$

and

$$\mathbf{y}_{11} = \mathbf{y}_{22} = \frac{1}{2}(\mathbf{Y}_b + \mathbf{Y}_a)$$

$$\mathbf{y}_{12} = \mathbf{y}_{21} = \frac{1}{2}(\mathbf{Y}_b - \mathbf{Y}_a)$$

where $\mathbf{Y}_a = 1/\mathbf{Z}_a$ and $\mathbf{Y}_b = 1/\mathbf{Z}_b$.

16.44. Find the hybrid and inverse hybrid descriptions for this circuit.

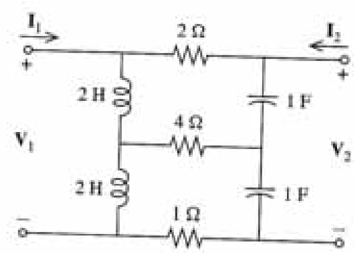

FIGURE P16.44

16.45. Use superposition to show that if a two-port contains independent sources, its two-port equations will have an extra source term. For instance, the impedance description is

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{21} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{1s} \\ \mathbf{V}_{2s} \end{bmatrix}$$

How would the new term $\begin{bmatrix} \mathbf{V}_{1s} \\ \mathbf{V}_{2s} \end{bmatrix}$ be found?

14.3. $\frac{-j\omega^3}{(4-3\omega^2)+j2\omega}$

14.5. $\sqrt{\frac{\omega^2+1}{\omega^6+2\omega^4-3\omega^2+1}}$, $\tan^{-1}\omega - \tan^{-1}\left(\frac{\omega-\omega^3}{1-2\omega^2}\right)$

14.7. $\sqrt{\frac{\omega^2+1}{\omega^2+2}}$, $\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$

14.9. $\frac{1}{s^2-1}$, $\frac{-1}{\omega^2+1}$, $-\frac{4}{5}\cos 2t$ V, no.

14.13. (a) 9 dB; (b) $\frac{9}{2}$ dB; (c) 50 dB; (d) 18 dB; (e) -72 dB; (f) 612 dB; (g) 2 dB; (h) -18 dB

14.15. (a) .05, (b) 20; (c) $5\sqrt{2}$ (d) $.01\sqrt{5}$; (e) $.005\sqrt{5}$; (f) $\sqrt{10}\times 10^{-20}$

14.17. (a) 121.62; (b) .254; (c) 7.08; (d) 3.16×10^8, (e) .0216, (f) 1.0001

14.19. $\frac{2(s-1/2)}{(s+1)^3}$, triple real pole with break frequency 1 rad/s, zero with break frequency 1/2 rad/s.

14.21. $\frac{\frac{1}{4}s}{s+3/4}$, 3/4 rad/s

14.25. 18 dB, -38 dB, -38 dB, -20 dB, -20 dB

14.27. $\omega_1 = 25.1$ rad/s, $\omega_2 = 631$ rad/s, $\omega_3 = 15.8$ krad/s, $\omega_4 = 398$krad/s, $\frac{(5\times 10^{-7})(s+1)^2(s+\omega_2)^2(s+\omega_4)^2}{s(s+\omega_1)^2(s+\omega_3)^2}$

14.29. $\frac{1/6}{(s+25/3)}$

14.31. $\frac{\frac{1}{4}s^2}{(s+1.47)(s^2+.279s+.34)}$

14.39. Series: 1 H, 400 pF, 12.5 Ω; parallel: 1 H, 400 pF, 2 MΩ

14.41. $\frac{A_0W_0}{jw+\frac{A_0\omega_0R_A}{R_A+R_F}}$, $\frac{A_0\omega_0R_A}{R_A+R_F}$ rad/s

14.61. $\frac{-1}{s(s+1)(s+3)}$

CHAPTER 15

15.1. $(\frac{3}{2}t+1)\times 10^{-4}$ W, $\frac{3}{2}\times 10^{-4}$ W, $(3t+\frac{7}{2})\times 10^{-4}$ W, $(\frac{3}{2}t+4)\times 10^{-4}$ W

15.3. $\frac{3}{2}$ H

15.5. $0.9t+1.05$ W, $0.45t^2+1.05t$ J

15.7. 2 H, 0 H

15.9. $4(e^{-t}-e^{-3t})$ A

15.11. $\left[-4e^{-(t/5)}+75\cos(4t-87.1^\circ)\right]$ mA

15.13. $\frac{5\sqrt{2}}{2}\sin(2t+135^\circ)$ V

15.15. 1250 W, 0.6 lagging

15.17. (a) $2e^{-2t}$ V; (b) $\frac{2\sqrt{2}}{3}\cos(8t-45^\circ)$ V

15.19. $-2e^{-t}$ V

15.21. $35.9\cos(t+152^\circ)$ A

15.23. $2\angle 76.4^\circ$, $1\angle 22.6^\circ$, $.343\angle -59.0^\circ$

15.25. $35.9\cos(t+152^\circ)$ A

15.27. -2

15.29. $\sqrt{2}e^{-t}\cos(t+45^\circ)-e^{-t}$ A

15.31. (a) $V_2=-8V_1$, $I_2=\frac{1}{8}I_1$, $v_2=-8v_1$, $i_2=\frac{1}{8}i_1$; (b) $V_2=-\frac{3}{8}V_1$, $I_2=\frac{8}{3}I_1$, $v_2=-\frac{3}{8}v_1$, $i_2=\frac{8}{3}i_1$

15.33. $4(1-e^{-(6/19)t})u(t)$ V

15.35. $2.67\angle -47.7^\circ$

15.37. $v_1 = 50\cos 377t$ V, $i_1 = 0.2\cos(377t-89.6^\circ)$ A, $v_2 = 0.2\cos(377t-89.3^\circ)$ V, $i_2 = .033\cos(377t+90.7^\circ)$ A

15.39. .761 H

15.41. $\frac{t_{21}}{t_{11}t_{22}-t_{12}t_{21}}$, $\frac{g_{11}}{g_{12}}$

15.43. $w \geq \frac{1}{2}L_1I_1^2+\frac{1}{2}L_2I_1^2-\sqrt{L_1L_2}I_1I_2 = \frac{1}{2}(\sqrt{L_1}I_1+\sqrt{L_2}I_2)^2 \geq 0$

15.45. $1.27\angle -158^\circ$ A

CHAPTER 16

16.1. 2.34 A, .938 A, 8.91 V, -5.62 V

16.3. $\begin{bmatrix} \frac{s}{s+1} & \frac{2}{s+1} \\ 3 & \frac{2s^2+6s-1}{s(s+1)} \end{bmatrix}$

16.5. Solve z-parameter equations for $\mathbf{I}_1$ and $\mathbf{V}_2$.

16.7. Solve z-parameter equations for $\mathbf{V}_1$ and $\mathbf{I}_2$.

16.9. $\frac{1}{4}\begin{bmatrix} s+2 & -s \\ 2-4s & s+1 \end{bmatrix}$, $\frac{2s-1}{2s+10}$

16.11. $\begin{bmatrix} 3.4 & 2.6 \\ 2.6 & 3.4 \end{bmatrix}$

16.13. $\mathbf{V}_1 = \frac{1}{s}$, $\mathbf{V}_2 = \frac{4s(8s^2+10s+5)}{(4s^2+2s+1)^2}$, $\mathbf{I}_1 = \frac{s+1}{s} - \frac{4s^2(8s^2+10s+5)}{(4s^2+2s+1)^2}$, $\mathbf{I}_2 = \frac{-(8s^2+10s+5)}{4s^2+2s+1}$

16.15. $\begin{bmatrix} \frac{s+3}{6s} & -\frac{1}{3} \\ +\frac{1}{3} & \frac{4s+3}{3s} \end{bmatrix}$, $\begin{bmatrix} \frac{(2s+3)(s+1)}{2s^2} & -\frac{(4s+3)}{s} \\ -\frac{(s+3)}{2s} & 3 \end{bmatrix}$

16.17. $\frac{1}{s(2s^2+2s+3)}\begin{bmatrix} 2(s^2+s+1) & 2 \\ 2 & 2(2s^2+2s+1) \end{bmatrix}$, $\frac{2(s^2+s+1)}{s(2s^2+2s+3)}$, $\frac{2(2^2+2s+1)}{s(2s^2+2s+3)}$

16.19. $\frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}$, $\frac{-\mathbf{h}_{21}}{\mathbf{h}_{11}\mathbf{h}_{22}-\mathbf{h}_{12}\mathbf{h}_{21}}$

16.23. Fig. 16.30 with $\mathbf{z}_{11}-\mathbf{z}_{12} = \frac{2}{s^2+2s+2}$, $\mathbf{z}_{22}-\mathbf{z}_{12} = \frac{1}{s^2+2s+2}$, $\mathbf{z}_{22}-\mathbf{z}_{21} = \frac{s}{s^2 2s+2}$, $\mathbf{z}_{12} = s$

16.25. Symmetric π-circuit with 6 Ω across each port, connected by 1.85 Ω.

16.27. (a) Apply KCL at input, output ports, (b) apply KCL at input, output ports.

16.29. $\begin{bmatrix} .706 & .765 \\ 1.00 & .588 \end{bmatrix}$

16.33. $\begin{bmatrix} -5.70 & -9.00 \\ 1.00 & 1.33 \end{bmatrix}$

16.35. $10^{-4}\begin{bmatrix} \frac{s+5}{s+10} & 0 \\ \frac{10}{s+10} & 1 \end{bmatrix}$

16.37. $\begin{bmatrix} 2.52 & 1.64\times 10^7 \\ -1.58\times 10^{-7} & 1.26 \end{bmatrix}$

16.43. Nodal analysis with negative terminal of v_1 as reference yields $\mathbf{y}_{11} = \mathbf{y}_{12}$ and $\mathbf{y}_{12} = \mathbf{y}_{21}$. Inverting yields the z-parameters.