Things to do:

Double check that the diagonalizable function that sage has built in works correctly. Look at eigenvalues of the counter-examples.

Check K matrix as well to see if there are any cospectral graphs. Likewise, look at adjacency matrices that are cospectral and see if their non backtracking B matrix and K matrix are cospectral.

Hunt for counter-examples to the potential theorem that the nonbacktracking matrix for non regular minimum 2 degree graphs have unique spectrums. (look at cospectral d-regular graphs whwere d>2)

read over new paper

Research Questions to research

Do equitable partitions maintain their property of having constant row sums for non backtracking graphs?
(An equitable partition is a partition in which the rows of the partition of the adjacency matrix add to the same value. In regards to the vertices, each vertex belongs to a subset of vertices, which has the same amount of edges connecting itself to a different subset of vertices)

Try and get a relationship
B[ ] = [ ]K but for ^~P [ ] = something[ ]

Or a P = [ ] * [ ] - ....
Like how we have B = ST - tau

Recurrence relation ^~P^k like ^~A^k (see polya's theorem)

Kemeny's constant for non-backtracking: Is K_nb <= K_e for all graphs with enough edges?

H(i,j) <= O(n^2) for non backtracking walks? H(i,j) = O(n^3) for normal random walks.

Does the cospectrality of the eigenvalues of the nonbacktracking random walks distinguish between two different graphs where d_min > 1? (Whenever there are degree one vertices, it contributes a 0 so that doesn't distiniguish them)

Holds on up to 7 vertices so far ^^^^

Things disproven:

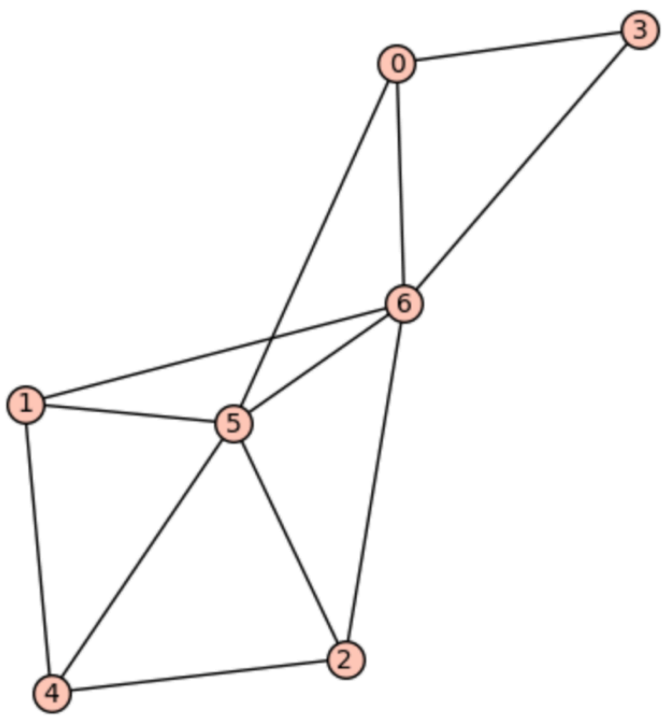
FALSE: If the non backtracking matrix converges to a stationary distribution, does that imply that the matrix is diagonalizable?

FALSE: Conjecture: The Ihara matrix is diagonalizable whenever G is connected and not a cycle and d_min >=2 (AKA whenever B is irreducible)

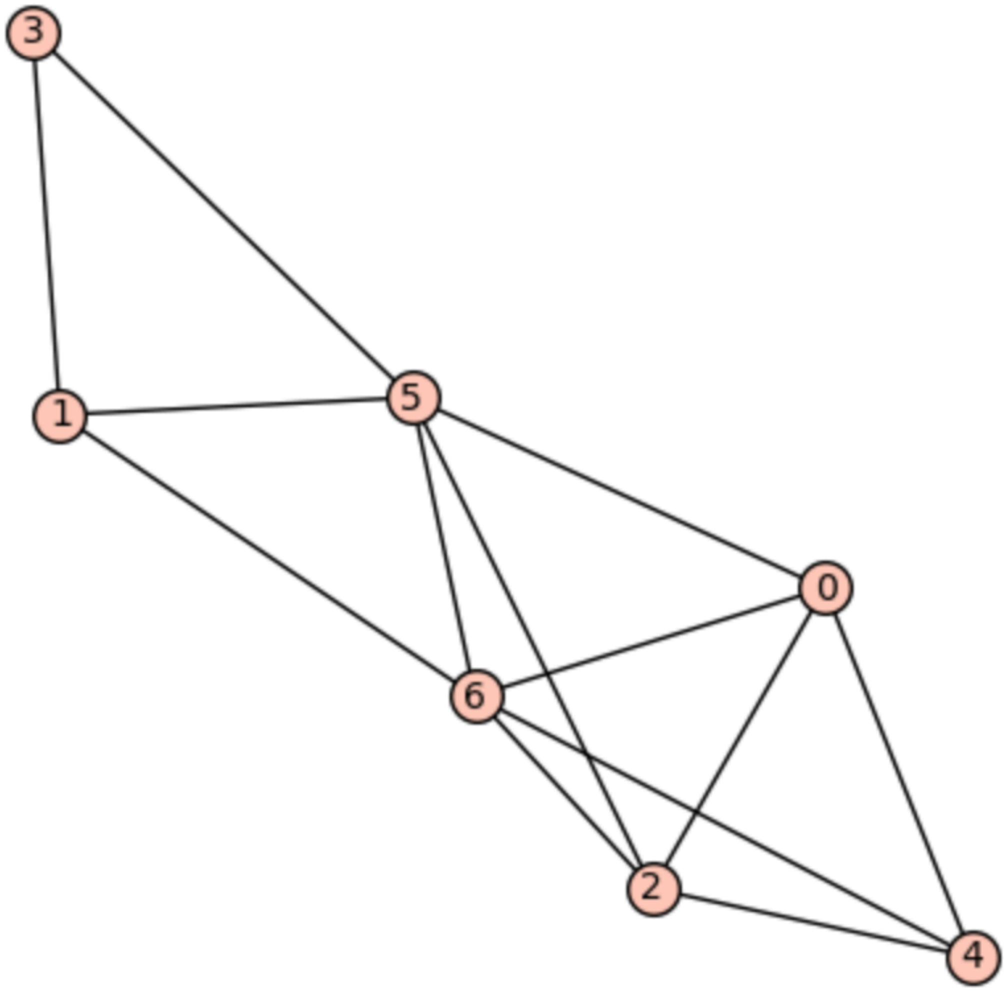
New Findings: there doesn't appear to be a one to two correspondence between eigenvalues of the adjacency matrix and eigenvalues of the ihara matrix, except for those which are connected, not a cycle, an d_min >=2

New Findings: There are two graphs on 7 vertices which are connected, not cycles, and have min d >= 2, for which their Ihara matrices are not diagonalizable:

```
g = {0:[3,6,5], 1:[4,5,6], 2:[4,5,6], 3:[0,6], 4:[1,2,5], 5:[0,1,2,4,6], 6:[0,3,5]}
sad_graph = Graph(g)
show(sad_graph)
k_sad = ihara_matrix(sad_graph)
print("Is diagonalizable: " + str(k_sad.is_diagonalizable()))
```




```
Is diagonalizable: False
```


3
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