Q6

1. $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i) = \frac{1}{N}\left(\sum_{i=1}^{N} a + b\sum_{i=1}^{N} x_i\right) = \frac{1}{N}(Na) + b\frac{1}{N}\sum_{i=1}^{N} x_i$
$= a + bm(X)$

2. $cov(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X)) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2$
$= s^2$

3. let $Z = a+bY$ and $z_i = a+by_i$
$m(Z) = m(a+bY) = a+bm(Y)$
$cov(X,Z) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(z_i - m(Z)) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - a+bm(Y)))$
$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\cdot b(y_i - m(Y)) = b\cdot\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y)))$
$= b\,cov(X,Y)$

4. let $u = a+bX$, $v = a+bY$
$u_i - m(U) = b(x_i - m(X))$ and $z_i - m(Z) = b(y_i - m(Y))$
$cov(U,Z) = \frac{1}{N}\sum_{i=1}^{N}(u_i - m(U))(z_i - m(Z))$
$= \frac{1}{N}\sum_{i=1}^{N}[b(x_i - m(X))][b(y_i - m(y))]$
$= b^2\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(y))\right]$
$= b^2 cov(X,Y)$

5. yes, the transformation $x \mapsto a+bx$ is strictly increasing so it preserves the order of the data
- median is $Q_{0.5}$ so $a+b\,med(X)$
$IQR(a+bX) = (a+bQ_{.75}(X)) - (a+bQ_{.25}(X)) = b(Q_{.75}(X) - Q_{.25}(X))$
$= b\,IQR(X)$

6. let $X = \{0,2\}$
$m(X) = \frac{0+2}{2} = 1 \Rightarrow (m(X))^2 = 1$
$X^2 = \{0^2, 2^2\} = \{0,4\} = m(X^2) = \frac{0+4}{2} = 2$
$m(X^2) = 2 \neq 1 = (m(X))^2$
let $X = \{0,4\}$
$m(X) = \frac{0+4}{2} = 2 \Rightarrow \sqrt{m(X)} = \sqrt{2}$
$\sqrt{X} = \{\sqrt{0}, \sqrt{4}\} = \{0,2\} \Rightarrow m(\sqrt{X}) = \frac{0+2}{2} = 1$
$m(\sqrt{X}) = 1 \neq \sqrt{2} = \sqrt{m(X)}$