SATFD

Lab. 2

March 7, 2024

1 Introduction

The exercise aims to introduce signal analysis, familiarize with concepts such as sampling frequency, Nyquist frequency, aliasing, zero padding, upsampling, and basic properties of the Fourier transform in a short time window, as well as to gain proficiency in using the Matlab/Python environment in time series analysis.

1.1 Signal generation

For further analysis in the exercise, we will use a signal with specified properties, particularly with specified: sampling frequency, duration, amplitude, frequency, and phase. Example: Sinusoidal signal $x(t) = Asin(2\pi ft)$ of length 1000 samples, amplitude 5, frequency 10Hz, and sampling frequency 1000Hz.

```
N=1000; A=5; f=10; fs=1000;
dt=1/fs; % okres probkowania
t=dt*(0:N-1); % wektor chwil probkowania
x=A*sin(2*pi*f*t); % sygnal
plot(t,x); grid; title("Sygnal x(t)"); xlabel("czas [s]");
```

1.2 Fourier Transform

The Fourier Transform is a fundamental tool for frequency analysis of signals:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$X(f)$ is the complex Fourier spectrum of the signal $x(t)$ and contains information about its frequency content (f - frequency [Hz], $x(t)$ - the signal under consideration).

Example: Calculate the coefficients of the complex Fourier series. Draw their real part, imaginary part, and modulus.

```
X=fft(x); % szybka dyskretna transformacja Fouriera
% wyznaczenie czestosci dla kazdego binu FFT
f=fs*(mod(((0:N-1)+floor(N/2)), N)-floor(N/2))/N;
figure(1)
subplot(311); plot(f,real(X)); grid; title("Real(X)"); xlabel("Hz");
subplot(312); plot(f,imag(X)); grid; title("Imag(X)"); xlabel("Hz");
subplot(313); plot(f,abs(X)); grid; title("Abs(X)"); xlabel("Hz");
figure(2)
plot(f(1:N/2+1),abs(X(1:N/2+1))/(N/2)); grid; title("Po wyskalowaniu");
```

1.3 Window functions

The frequency resolution of the spectrum is $fs = N$. If an intermediate frequency appears in the signal (always in a real signal), it will be blurred - "leak" to other bins. Therefore, a segment of the signal is cut out using a time window (multiplying the signal by a window function). Many different functions can be used as windows, for example Bartlett, Blackman, Hamming, Hann, Kaiser.

1.4 The Whittaker-Nyquist-Kotielnikov-Shannon theorem

The Whittaker-Nyquist-Kotielnikov-Shannon theorem states that the sampling frequency must be greater than twice the highest frequency component in the measured signal. Improper selection of the sampling frequency can lead to aliasing, which is an irreversible distortion of the signal.

2 Realization of the task

The suggested environment is MATLAB or Python.

Prepare a script that will illustrate the following issues:

1. Comparison of signals in the time domain and spectra (images in two columns):
   (a) Infinite signal (theoretical plot)
   (b) Rectangular window function (choose a short window covering at most a few periods of the signal)
   (c) Finite signal, i.e., the product of the infinite signal and the rectangular window function
   (d) Non-rectangular window function (choose a short window covering at most a few periods of the signal)
   (e) Smoothed finite signal using a non-rectangular window, i.e., the product of the infinite signal and the non-rectangular window function
2. Sampling theorem. Aliasing. Spectrum of a sampled signal below and above the Nyquist frequency. As a signal, you can use a musical fragment from Lab 1. You can also use your favorite song in wav format. The sampling frequency is stored in the wav file header and returned by the wavread function. It is best to start with sampling above the Nyquist frequency and then perform several decimations (removing every other sample), displaying the signal and spectrum plot each time.
3. Zero padding - filling the signal with zeros (up to 4x the length of the signal, for analysis, take either a theoretical signal or a musical fragment).
4. Upsampling - filling the signal with zeros between samples (up to 4x the length of the signal, for analysis, take either a theoretical signal or a musical fragment).

Content and form of the report: electronic form (Word, TeX, pdf, ...), the report should contain a description of the tasks performed and a complete set of used scripts.