Pérez Jacome David — Modelos Probabilísticos

1- Utiliza el algoritmo de Bayes Naive a partir de los sig datos

- Llueve : {nublado, temperatura baja}
- Llueve: {nublado, temperatura alta, humedad}
- Soleado {despejado, temperatura alta}

Determina la clase (soleado, llueve) a partir de las observación {despejado, temp. Alta}

$Y \begin{cases} 1 \text{ si llueve} \\ 0 \text{ si soleado} \end{cases}$

nublado, temp_baja, temp_alta, humedad, despejado
X_1, X_2, X_3, X_4, X_5

$X_i \begin{cases} 1 \text{ si esta presente} \\ 0 \text{ si no} \end{cases}$

Obs, {despejado, temp_alta}

Obs, (0, 0 1 0 1)

Soleado : Observaciones: Despejado, temp_alta

Llueve : Observaciones: Despejado, temp_alta

X_i	Soleado	Llueve
nublado	0	2/2
temp_baja	0	1/2
temp_alta	1/1	1/2
humedad	0	1/2
despejado	1/1	0

$p(\text{llueve}) = \frac{2}{3}$

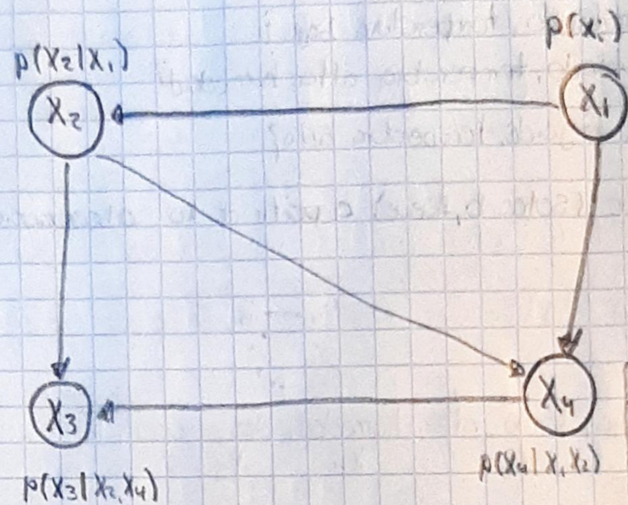
$P(\text{soleado}) = \frac{1}{3}$

$p(\text{Soleado, despejado, temp\_alta}) = \frac{1}{1} \frac{1}{1} \frac{1}{3} = \frac{1}{3} = 0.333$

$p(\text{Llueve, despejado, temp\_alta}) = 0 \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3} = 0.333$

2- A partir de la grafica bayesiana

X_1	T	F
T	0.4	0.6
F	0.2	0.8



T	F
.5	.5

X_2	X_4	T	F
T	T	0	1
T	F	1	0
F	T	0.5	0.5
F	F	0	1

X_1	X_2	T	F
T	T	0.5	0.5
T	F	0.2	0.8
F	T	1	0
F	F	0.4	0.6

Calcula la proba de las sig consultas:

- $P(X_1=T, X_2=T, X_3=T, X_4=T) = p(X_1=T) \cdot p(X_2=T|X_1=T) \cdot p(X_3=T|X_2=T, X_4=T) \cdot p(X_4=T|X_1=T, X_2=T)$

$$= (0.5)(0.4)(0)(0.5) = \boxed{0}$$

- $P(X_1=F, X_2=T, X_3=F, X_4=T) = p(X_1=F) \cdot p(X_2=T|X_1=T) \cdot p(X_3=F|X_2=T, X_4=T) \cdot p(X_4=T|X_1=F, X_2=T)$

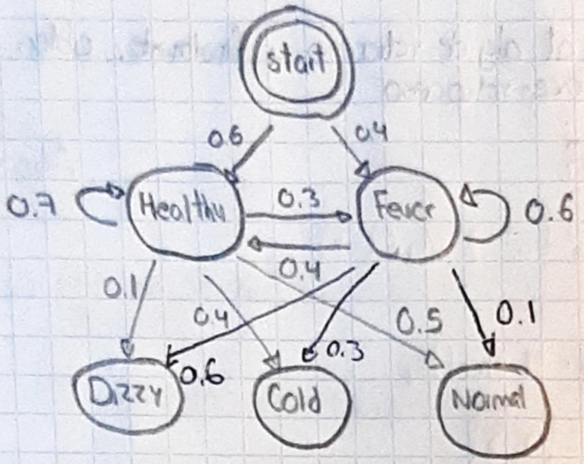
$$= (0.5)(0.4)(1)(1) = \boxed{0.2}$$

- $P(X_3=T, X_4=T) = (0.2)(0.5)(0.6)(0.5) + (0.4)(0.5)(0.8)(0.5) = \boxed{0.11}$

$= p(X_4=T|X_1=T\ X_2=T)\ p(X_3=T|X_2=T\ X_4=T)\ p(X_2=T|X_1=T)\ p(X_1=T) +$
$p(X_4=T|X_1=T\ X_2=F)\ p(X_3=T|X_2=F\ X_4=T)\ p(X_2=F|X_1=T)\ p(X_1=T) +$
$p(X_4=T|X_1=F\ X_2=T)\ p(X_3=T|X_2=T\ X_4=T)\ p(X_2=T|X_1=F)\ p(X_1=F) +$
$p(X_4=T|X_1=F\ X_2=F)\ p(X_3=T|X_2=F\ X_4=T)\ p(X_2=F|X_1=F)\ p(X_1=F)$

$=(0.5)(0)(0.4)(0.5) + (0.2)(0.5)(0.6)(0.5) + (1)(0)(0.6)(0.5) + (0.4)(0.5)(0.8)(0.5)$

3 Considera la Grafica Bayesiana



Obsevaciones : Dizzy, Cold y Normal

emisiones : Healthy y Fever

Construye maddo oculto de Markov de la grafica

$HMM = (S, \Sigma, A, B, \pi)$

$$\pi = \begin{array}{cc} \text{Healthy} & \text{Fever} \end{array}$$
$$\pi = [\,0.6 \quad 0.4\,]$$

$S = \{Healthy, Fever\}$

$\Sigma = \{Dizzy, Cold, Normal\}$

$$A = \begin{array}{c|cc} & \text{Healthy} & \text{Fever} \\ \hline \text{Healthy} & 0.7 & 0.4 \\ \text{Fever} & 0.3 & 0.6 \end{array}$$

$$B = \begin{array}{c|cc} & \text{Healthy} & \text{Fever} \\ \hline \text{Dizzy} & 0.1 & 0.6 \\ \text{Cold} & 0.4 & 0.3 \\ \text{Normal} & 0.5 & 0.1 \end{array}$$

4- Calcula la probabilidad de la cadena Normal Dizzy Cold

En base al alg. de avance, al alg de retroceso y finalmente, obten las etiquetas mas probables con el alg. de avance-retroceso

Avance

$x = \{$ Normal Dizzy Cold $\}$

$\alpha_i(t) = \sum_{j=1}^{N} P(w^t \mid S_j^t) P(S^t \mid S^{t-1}) \alpha_j(t-1)$

$\alpha_{Healthy}(1) = P(\text{Normal} \mid \text{Healthy}) \cdot \pi_{Healthy}$

$= 0.5 \cdot 0.6 = \boxed{0.3}$

$\alpha_{fever}(1) = P(\text{Normal} \mid \text{Fever}) \cdot \pi_{fever}$

$= 0.1 \cdot 0.4 = \boxed{0.04}$

$\alpha_{Healthy}(2) = P(\text{Dizzy} \mid \text{Healthy}) \cdot P(\text{Healthy} \mid \text{Healthy}) \cdot \alpha_{Healthy}(1)$
$+ P(\text{Dizzy} \mid \text{fever}) \cdot P(\text{Healthy} \mid \text{fever}) \cdot \alpha_{fever}(1) = (0.1)(0.7) \cdot (0.3) + (0.6)(0.4)(0.04)$

$= 0.0306$

$\alpha_{fever}(2) = P(\text{Dizzy} \mid \text{fever}) P(\text{fever} \mid \text{fever}) \alpha_{fever}(1) + P(\text{Dizzy} \mid \text{Healthy}) P(\text{fever} \mid \text{Hel}) \alpha_{Healthy}(1)$

$= (0.6)(0.6)(0.04) + (0.1)(0.3)(0.3) = 0.0234$

$\alpha_{Healthy}(3) = P(\text{Cold} \mid \text{Heal}) \cdot P(\text{Healthy} \mid \text{Healthy}) \cdot \alpha_{Healthy}(2) +$
$P(\text{Cold} \mid \text{fever}) P(\text{Healthy} \mid \text{fever}) \cdot \alpha_{fever}(2)$

$= (0.4)(0.7)(0.0306) + (0.3)(0.4)(0.0234) = \boxed{0.011376}$

$\alpha_{fever}(3) = P(\text{Cold} \mid \text{fever}) P(\text{fever} \mid \text{fever}) \alpha_{fever}(2) + P(\text{Cold} \mid \text{Healthy}) P(\text{fever} \mid \text{He})$
$\alpha_{Healthy}(2)$

$= (0.3)(0.6)(0.0234) + (0.4)(0.3)(0.0306) = \boxed{0.007884}$

$P(\text{Normal Dizzy Cold}) = 0.011376 + 0.007884 = \boxed{0.01926}$

Retroceso

$X = \{\text{Normal Dizzy Cold}\}$

$\beta_{healthy}(3) = 1$

$\beta_{fever}(3) = 1$

$\beta_{healthy}(2) = P(\text{Dizzy}|\text{Healthy})\, P(\text{Healthy}|\text{Healthy})\, \beta_{healthy}(3) + P(\text{Dizzy}|\text{fever})\, P(\text{Healthy}|\text{fever})\, P(\text{fever}(3))$

$= (0.1)(0.7)(1) + (0.6)(0.4)(1) = 0.31$

$\beta_{fever}(2) = P(\text{Dizzy}|\text{fever})\, P(\text{fever}|\text{fever})\, \beta_{fever}(3) + P(\text{Dizzy}|\text{Healt})\, P(\text{fever}|\text{Hel})\, \alpha_{Helt}(3)$

$= (0.6)(0.6)(1) + (0.1)(0.3)(1) = 0.39$

$\beta_{healthy}(1) = P(\text{Normal}|\text{Healthy})\, P(\text{Healthy}|\text{Healthy})\, \beta(\text{Healthy}(2)) + P(\text{Normal}|\text{fever})\, P(\text{Healthy}|\text{fever})\, P(\text{fever}(2)) = (0.5)(0.7)(0.31) + (0.1)(0.4)(0.39) = 0.1241$

$\beta_{fever}(1) = P(\text{Normal}|\text{fever})\, P(\text{fever}|\text{fever})\, \beta_{fever}(2) + P(\text{Normal}|\text{Healt})\, P(\text{fever}|\text{Healt})\, \beta_{Healt}(2)$

$= (0.1)(0.6)(0.39) + (0.5)(0.3)(0.31) = 0.0649$

$P(\text{Normal, Dizzy, Cold}) = (0.6)(0.1241) + (0.4)(0.0649) = 0.10042$

Avance-retroceso

$$\max \begin{cases} \alpha_{Helt}(1) \cdot \beta_{Helt}(1) = 0.3 \cdot 1 = 0.3 \\ \alpha_{fever}(1) \cdot \beta_{fever}(1) = 0.04 \cdot 1 = 0.04 \end{cases}$$

$$\max \begin{cases} \alpha_{Helt}(2) \cdot \beta_{Helt}(2) = (0.0306)(0.31) = 0.009486 \\ \alpha_{fever}(2) \cdot \beta_{fever}(2) = (0.0234)(0.39) = 0.009126 \end{cases}$$

$$\max \begin{cases} \alpha_{Helt}(3) \;\; \beta_{Helt}(3) = (0.011376)(1) = 0.011376 \\ \alpha_{fever}(3) \;\; \beta_{fever}(3) = (0.007884)(1) = 0.007884 \end{cases}$$