

# Longitudinal\_Vehicle\_Model

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In this notebook, you will implement the forward longitudinal vehicle model. The model accepts throttle inputs and steps through the longitudinal dynamic equations. Once implemented, you will be given a set of inputs that drives over a small road slope to test your model.

The input to the model is a throttle percentage  $x_\theta \in [0, 1]$  which provides torque to the engine and subsequently accelerates the vehicle for forward motion.

The dynamic equations consist of many stages to convert throttle inputs to wheel speed (engine  $\rightarrow$  torque converter  $\rightarrow$  transmission  $\rightarrow$  wheel). These stages are bundled together in a single inertia term  $J_e$  which is used in the following combined engine dynamic equations.

$$J_e \dot{\omega}_e = T_e - (GR)(r_{eff} F_{load}) \quad (1)$$

$$m \ddot{x} = F_x - F_{load} \quad (2)$$

Where  $T_e$  is the engine torque,  $GR$  is the gear ratio,  $r_{eff}$  is the effective radius,  $m$  is the vehicle mass,  $x$  is the vehicle position,  $F_x$  is the tire force, and  $F_{load}$  is the total load force.

The engine torque is computed from the throttle input and the engine angular velocity  $\omega_e$  using a simplified quadratic model.

$$T_e = x_\theta(a_0 + a_1 \omega_e + a_2 \omega_e^2) \quad (3)$$

The load forces consist of aerodynamic drag  $F_{aero}$ , rolling friction  $R_x$ , and gravitational force  $F_g$  from an incline at angle  $\alpha$ . The aerodynamic drag is a quadratic model and the friction is a linear model.

$$F_{load} = F_{aero} + R_x + F_g \quad (4)$$

$$F_{aero} = \frac{1}{2} C_a \rho A \dot{x}^2 = c_a \dot{x}^2 \quad (5)$$

$$R_x = N(\hat{c}_{r,0} + \hat{c}_{r,1}|\dot{x}| + \hat{c}_{r,2}\dot{x}^2) \approx c_{r,1}\dot{x} \quad (6)$$

$$F_g = mg \sin \alpha \quad (7)$$

Note that the absolute value is ignored for friction since the model is used for only forward motion ( $\dot{x} \geq 0$ ).

The tire force is computed using the engine speed and wheel slip equations.

$$\omega_w = (GR)\omega_e \quad (8)$$

$$s = \frac{\omega_w r_e - \dot{x}}{\dot{x}} \quad (9)$$

$$F_x = \begin{cases} cs, & |s| < 1 \\ F_{max}, & \text{otherwise} \end{cases} \quad (10)$$

Where  $\omega_w$  is the wheel angular velocity and  $s$  is the slip ratio.

We setup the longitudinal model inside a Python class below. The vehicle begins with an initial velocity of 5 m/s and engine speed of 100 rad/s. All the relevant parameters are defined and like the bicycle model, a sampling time of 10ms is used for numerical integration.

```
In [ ]: import sys
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as mpimg

class Vehicle():
    def __init__(self):

        # =====
        # Parameters
        # =====

        #Throttle to engine torque
        self.a_0 = 400
        self.a_1 = 0.1
        self.a_2 = -0.0002

        # Gear ratio, effective radius, mass + inertia
        self.GR = 0.35
        self.r_e = 0.3
        self.J_e = 10
        self.m = 2000
        self.g = 9.81

        # Aerodynamic and friction coefficients
        self.c_a = 1.36
        self.c_r1 = 0.01

        # Tire force
        self.c = 10000
        self.F_max = 10000

        # State variables
        self.x = 0
        self.v = 5
```

```

self.a = 0
self.w_e = 100
self.w_e_dot = 0

self.sample_time = 0.01

def reset(self):
    # reset state variables
    self.x = 0
    self.v = 5
    self.a = 0
    self.w_e = 100
    self.w_e_dot = 0

```

Implement the combined engine dynamic equations along with the force equations in the cell below. The function *step* takes the throttle  $x_\theta$  and incline angle  $\alpha$  as inputs and performs numerical integration over one timestep to update the state variables. Hint: Integrate to find the current position, velocity, and engine speed first, then propagate those values into the set of equations.

```

In [ ]: class Vehicle(Vehicle):
        def step(self, throttle, alpha):
            # =====
            # Implement vehicle model here
            # =====
            pass

```

Using the model, you can send constant throttle inputs to the vehicle in the cell below. You will observe that the velocity converges to a fixed value based on the throttle input due to the aerodynamic drag and tire force limit. A similar velocity profile can be seen by setting a negative incline angle  $\alpha$ . In this case, gravity accelerates the vehicle to a terminal velocity where it is balanced by the drag force.

```

In [ ]: sample_time = 0.01
        time_end = 100
        model = Vehicle()

        t_data = np.arange(0,time_end,sample_time)
        v_data = np.zeros_like(t_data)

        # throttle percentage between 0 and 1
        throttle = 0.2

        # incline angle (in radians)
        alpha = 0

        for i in range(t_data.shape[0]):
            v_data[i] = model.v
            model.step(throttle, alpha)

```