Design of Digital PID Controller for Voltage Mode Control of DC-DC Converters

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Abstract— **DC/DC converters are massively used for switch-mode regulated power supply, renewable energy conversion systems and electrical drives. Conventionally analog methods were popular for control of these converters. This paper elucidates a digital controller using digital filter architecture, which supports fixed-point algorithm. Digital controller application to DC/DC converters has always been considered because of their superiority over analog controller. In digital controller, the control strategy can be altered or reprogrammed without the need of significant hardware changes. The digital controller improves response of DC-DC converter by varying loop-gain, cross-over frequency and phase margin. Closed loop digital control of buck and boost converter is presented and the results are obtained for varying operating conditions and verified using MATLAB/Simulink.**

Keywords— Boost; Buck; DC/DC Converters; Digital PID; PID Controller.

I. Introduction

DC/DC converters supply a fine and adjustable DC voltage to a variable load from unregulated supply. These converters generally have applications like television, battery chargers, receivers, computers & medical instruments, communication devices, armed force equipment etc. [1]. Buck & Boost converters are two fundamental types of DC/DC converters according to change in output level from given input level. All other topologies are derivatives of either buck or boost or a combination of these two, as they will either increases or decreases the input voltage [2].

Conventionally, analog controllers have been used to get a controlled and desired value of output from DC/DC converters. An analog control system (ACS) functions in genuine time with high bandwidth and conceptually infinite resolution [3]. Digital control has grown as a preferred solution for DC/DC switching converters from previous few years. The price of digital ICs is decreasing steadily so, the cost-prohibitive aspect of digital control technology has vanished [4]. Digital control of DC/DC converters is superior over analog control because advanced control algorithms like adaptive control and non-linear control can be easily executed by utilizing digital control [2].

Direct digital compensator development approach without constructing discrete model of converter is proposed in [5]. The response of a system is mainly resolute by initial samples of the compensator and this perception is used to fit a digital PID template to attain required response [6].

Digital implementation of controllers is most suitable to explore the control techniques which targets better dynamic response. Time optimal response is also a remarkable field of interest for converter application. Time optimal response results a sequence of control actions which takes minimum time against external disturbances. So, for DC/DC converters it is a sequence of precisely timed pulses.

State space averaging (SSA) method is used to calculate small signal transfer function (SSTF) of DC/DC converters from which ideal close-loop response is produced by substituting designing parameters. The open-loop response in discrete time is then obtained by transformation methods [7]. An averaged continuous-time model, considers switching duty cycle as input and describes the system's slow dynamics to avoid difficulties faced due to hybrid nature of system [8]. A mathematical depiction derived using SSA method for Buck-converter in continuous conduction mode (CCM) is conferred in [8]. A digitally controlled DC/DC converter is comprised of power-stage, analog to digital converter (ADC), digital controller and digital PWM [9].

For a digitally controlled converter, derivation of exact SSTF in discrete time is presented in [10]. This model relies on well-known approach for discrete-time modelling and the standard transform, considering modulation effect, sampling and delays produced in control loop. Digital implementation with existing resources like resolution of A/D and D/A (PWM) blocks and the computation time is a matter of attention [11].

In digital domain, all systems are contemplated as digital filters as they modify the wave-shape of signal and thus the harmonic amplitude. So, any design in digital domain can be constructed by digital filters. The discrete system model is built using only z^{-1} blocks and gain blocks [12].

The poles and zeros of boost converter are dependent on the switching duty cycle, so bode plots can exhibit significant variation. Therefore, PID controller may be unable to respond adroitly to significant fluctuations in operating points [13]. Online frequency measurement from bode plot is used as tuning algorithm for determination of digital controller parameters for DC/DC converters [14].

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The objective of adaptive control or auto-tuning of controller parameters according to system's dynamic behavior can be achieved by programmability feature of digital controllers. Digital controllers are able to precisely adjust switching frequency, to implement algorithms as well as reconfigure the power stage using power switch segmentation.

The analog TPS40k controller is compared with UCD921x digital controller for buck switch-mode converters [15]. The potency of digital controller for converters is established and algorithm is executed on TMS320F2812 DSP in reference [16]. A comparative study of performance of fuzzy and PID controllers for high output voltage boost converter is demonstrated and implemented on DSP [17].

Presented in this paper a complete process of digital controller design using digital filter technique that enables re-programmability. It can be implemented on low-cost fixed-point processors for fast and effective response with less memory and processor requirements.

II. MODELLING OF DC/DC CONVERTERS.

Linear compensators for converters are designed using a mathematical model of system. To obtain a specific performance objective, an accurate model is essential which can be derived by using SSA. This section presents open-loop control-to-output transfer function of uncompensated DC/DC converters.

A. Buck Converter

SSTF of buck converter is of 2nd order and it can be derived by SSA as given by (1). Fig. 1 illustrates schematic of buck converter for which SSTF is evaluated.

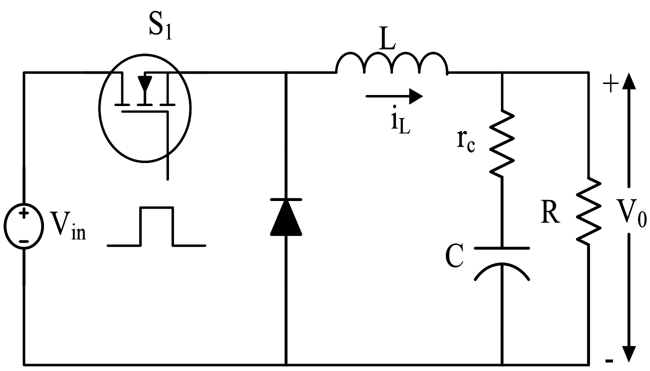

Fig.1. Buck converter schematic.

A zero in left half of s-plane is introduced by series equivalent resistance (ESR) of output capacitor that plays a vital role in defining the characteristics of converter.

$$\frac{\hat{v}_0(s)}{\hat{d}(s)} = \frac{V_{in}(1+sR_cC)}{1+s\left(R_cC+\left(\frac{RR_l}{R+R_l}\right)C+\frac{L}{R+R_l}\right)+s^2LC\left(\frac{R+R_C}{R+R_l}\right)} \quad (1)$$

Here, V_{in} is input source voltage, V_o is output load voltage, D is duty ratio, C is output capacitor, L is inductor and R is resistance of load. R_l and R_c are series equivalent resistances of inductor 'L' and capacitor 'C'. Assuming $R_l = 0$ in (1), the simplified equation is given as (2).

$$\frac{\hat{v}_0(s)}{\hat{d}(s)} = \frac{V_{in}(1+sR_cC)}{1+s\left(R_cC+\frac{L}{R}\right)+s^2LC\left(\frac{R+R_C}{R}\right)} \quad (2)$$

Here, $\hat{v}_0(s)$ and $\hat{d}(s)$ are small signal perturbations of output voltage and switching duty cycle [2]. Buck converter design parameters and values of components are displayed in TABLE I. Control-to-output transfer function as shown in (2) is used for development of controller for converter.

TABLE I. BUCK CONVERTER DESIGN PARAMETERS AND VALUES

Parameter	Nomenclature	Value
Capacitor	C	35.8µF
Inductor	L	34.8µH
Load Resistor	R	13.9Ω
ESR of Capacitor	R_C	69.64mΩ
Input DC voltage	V_{in}	24V

By feeding values of converter parameters from TABLE I in (2), the transfer function of converter can be evaluated. Fig. 2 illustrates frequency response (FR) characteristics of buck converter without controller.

$$\frac{\hat{v}_0(s)}{\hat{d}(s)} = \frac{5.98\times10^{-5}s+24}{1+s(4.99\times10^{-6})+s^2(1252\times10^{-12})} \quad (3)$$

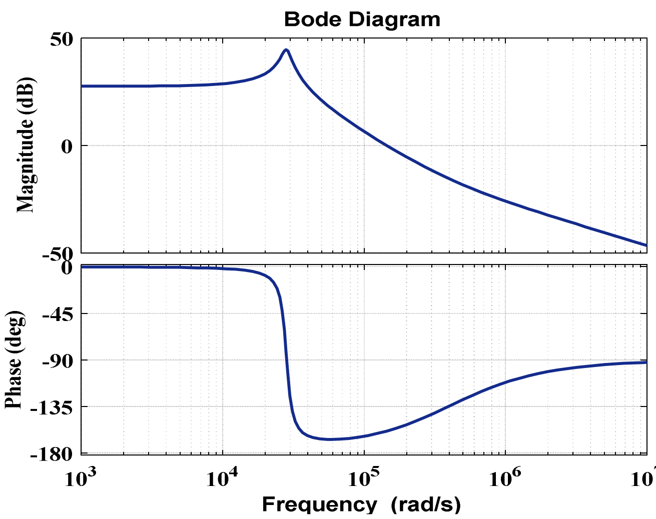

Fig.2. Uncompensated buck converter magnitude and phase response.

The output of open-loop buck converter is independent of load but it relies on input voltage directly, so load regulation is fine. Variation in input voltage causes proportional changes in output. So, line regulation is required to be improved and for this, closed-loop control is used.

B. Boost Converter

The SSTF of boost converter is derived similarly as developed for buck converter by SSA. Fig. 3 reveals schematic circuit diagram of boost converter for derivation of SSTF.

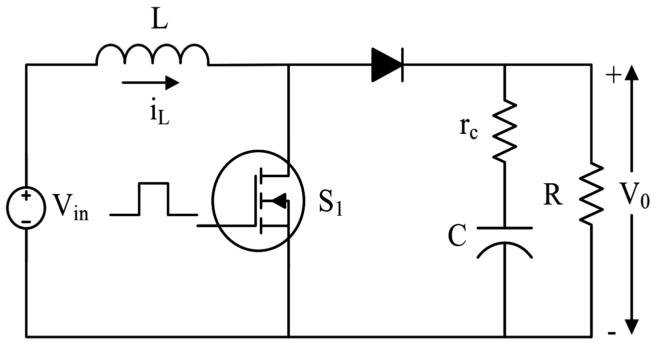

Fig.3. Boost converter.

The SSA of boost converter gives a 2nd order transfer function with two zeros. One is in left half side and other in right half side of s- plane.

$$\frac{\hat{v}_0(s)}{\hat{d}(s)} = \frac{V_o}{D_o L_e C} \frac{(1-sL_e R)(s^2 R_c C + R_c / R + 1)}{s^2 + sb_1 + b_2} \tag{4}$$

$$\text{Where, } L_e = \frac{L}{(1-D_o)^2} \text{ and } D_o = 1 - D$$

$$b_1 = \frac{(R_l / D_o^2) + (R_c / D_o)}{L_e} + \frac{1}{RC} \tag{5}$$

$$b_2 = \frac{(R_l / D_o^2) + (R_c / D_o)}{RL_e C} + \frac{1}{L_e C} \tag{6}$$

The expression of SSTF for boost converter can be simplified by substituting the expressions of b_1, b_2, L_e and taking R_l=0. Boost converter input-output parameters and values are displayed in TABLE II.

TABLE II. BOOST CONVERTER DESIGN PARAMETERS AND VALUES

Parameter	Nomenclature	Value
Capacitor	C	48.6μF
Inductor	L	6.075μH
Load Resistor	R	12Ω
ESR of Capacitor	R_C	16.66mΩ
Input DC voltage	V_{in}	5V

The values of parameters from TABLE II are substituted in (4) which gives final transfer function as given by (8). Fig. 4 shows magnitude and phase response of boost converter without controller.

$$\frac{\hat{v}_0(s)}{\hat{d}(s)} = 1.693\times10^{10} \times \frac{(1-420\times10^{-6}s)(809.676\times10^{-9}s+1.00138)}{s^2+s(1147.58)+(5.898\times10^8)} \tag{7}$$

$$\frac{\hat{v}_0(s)}{\hat{d}(s)} = \frac{(-5.756s^2 - 7.1\times10^6 s + 1.695\times10^{10})}{(s^2 + 1147.58s + 5.898\times10^8)} \tag{8}$$

Similar to buck converter, the output of boost converter doesn't depend on load resistance in open-loop, but it relies on input voltage so, the load-regulation is good. Variation in input directly affects output voltage. So, line-regulation needs to be improved.

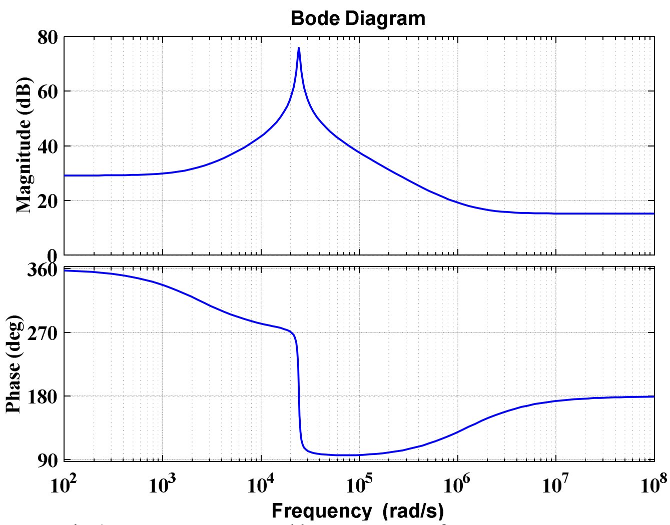

Fig.4. Uncompensated boost converter frequency response.

III. DIGITAL CONTROLLER DESIGN

In general, following two methodologies are utilized to design a digital controller for DC/DC converters:

1) *Digital re-design*
2) *Direct digital-design.*

In case of digital redesign, the controller is developed in continuous domain and then discretized to obtain digital form. In case of direct digital domain design, the system in continuous time domain is first discretized and then the controller is directly designed in discrete domain [18].

As we are much accustomed with frequency response (FR), it is convenient to design a controller using FR in s-domain. An analog controller is designed using FR characteristics and then mapped to discrete-time domain to obtain digital controller.

A target crossover-frequency 'f_c' and target phase-margin 'ϕ_m' is assumed before compensator design. The uncompensated loop gain at target crossover-frequency is evaluated from FR and then required compensation in phase is calculated. The controller is designed as per required compensation. A digital controller compensated system should have following characteristics:

1) Higher loop gain at low frequencies so that, steady state error get minimized and increase rejection of disturbances due to variation of voltage and current.

2) Higher crossover frequency (less than switching frequency) for quick transient response of converter.

3) The phase margin should be positive and sufficient to ensure stability of system.

The digital domain representation of a controller can be realized from digital filter configuration which enables programmable design of controller.

$$G_{con}(s) = K_p + \frac{K_i}{s} + s.K_d \tag{9}$$

Equation (9) represents analog PID controller in continuous time. A digital PID controller also follows (9), except the multiplication, integration and differentiation are performed numerically in digital computers. The response of system using digital controller is similar to that of analog controller, if numerical calculation is accurate.

Discrete time controller transfer function can be evaluated from analog PID controller by one of the methods like forward difference, backward difference, bilinear transformation etc. Here we used backward difference mapping method.

$$s = \frac{z-1}{z} \tag{10}$$

$$G_{con}(z) = K_p + \frac{K_i.z}{(z-1)} + \frac{K_d.(z-1)}{z} \tag{11}$$

$$G_{con}(z) = \frac{z^2(K_p + K_i + K_d) - z(K_p + 2K_d) + K_d}{z(z-1)} \tag{12}$$

$$G_{con}(z) = \frac{(K_p + K_i + K_d) - z^{-1}(K_p + 2K_d) + z^{-2}K_d}{(1-z^{-1})} \tag{13}$$

Discrete IIR filter transfer function with two zero and one pole is represented by (14)

$$G_{con}(z) = \frac{b_0 + z^{-1}.b_1 + z^{-2}.b_2}{(1-z^{-1})} \tag{14}$$

Comparing (13) and (14) gives values of IIR filter coefficients in terms of fundamental PID controller coefficients.

$$\begin{aligned} b_0 &= K_p + K_i + K_d \\ b_1 &= -(K_p + 2K_d) \\ b_2 &= K_d \end{aligned} \tag{15}$$

A digital filter can be represented using several structures such as: parallel, cascaded, direct form etc. Here direct form realization of an infinite impulse response (IIR) type digital filter is used to exhibit required poles and zeros in digital PID controller.

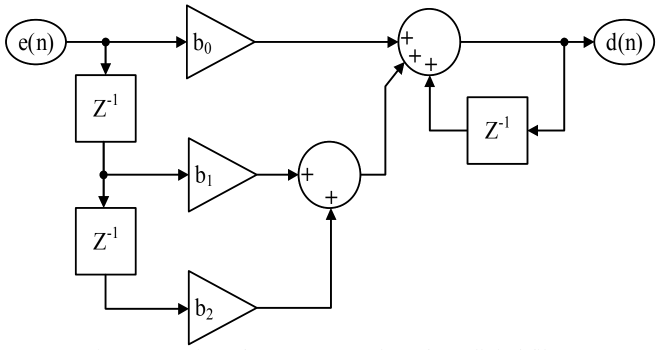

Fig.5. Direct form representation of IIR digital filter.

The digital PID controller realization from IIR digital filter using direct form is represented in Fig. 5, have two zeros required for compensation and a pole at origin for minimization of steady state error in system.

IV. Digital Control of DC/DC Converters

The output of DC/DC converters is a function of input dc voltage, switching duty-cycle and load current as depicted in (2) & (4). A constant output voltage is desirable during disturbances like abrupt load variation, sudden change in input dc voltage or load current. Thus, a negative feedback control is employed in DC/DC converters so that the duty ratio automatically adjusts to maintain a constant output dc voltage. Fig. 6 shows complete block schematic representation of digital control for converters.

Two components are essential while transiting from analog to digital domain. These components are: analog-digital (A/D) converter and sample & hold circuit.

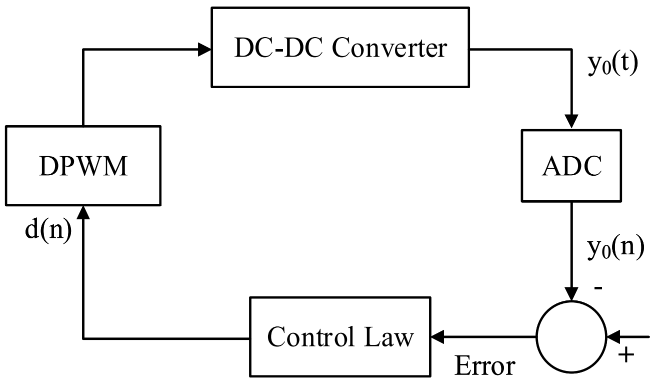

Fig.6. Block representation of digitally controlled DC-DC converter.

Digital PID controller substantially improves loop-gain, crossover frequency and phase-margin according to target values decide for given DC/DC converter. Fig. 7 shows close-loop scheme for Buck-converter with digital PID compensator. A digital pulse width modulator (DPWM) is used with digital controllers and it differs from analog PWM as it uses digital carrier for generation of PWM signal for switch. DPWM uses counter based or delay line based generation of sawtooth carrier wave.

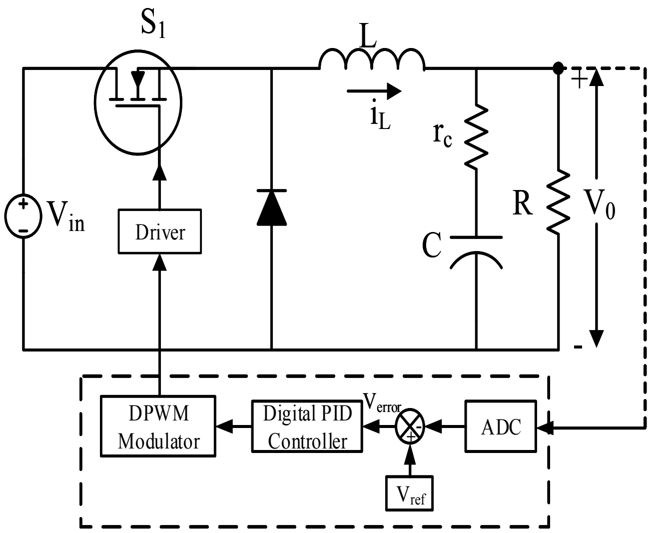

Fig.7. DC-DC buck converter with digital control.

V. SIMULATION RESULTS

The digital controller demonstrated here is applied to two fundamental DC/DC converters. A 24V to 12V buck converter and 5V to 12V boost-converter prototypes are considered and simulated using MATLAB/Simulink. The converters are subjected to practical conditions such as input voltage and load variations and it is observed that the output voltage regulates itself for an extensive range of input voltage and load resistance.

Fig. 8 and Fig. 9 shows output voltage reference tracking response of Buck and Boost converter respectively. The reference is changed at regular time interval of 40 msec and it is observed that the target value is achieved with very small settling time. Fig. 10 and Fig. 11 shows response of Buck and Boost converters for varying input voltage respectively. Fig. 12 and Fig. 13 shows response of Buck and Boost converters for varying load resistance values respectively.

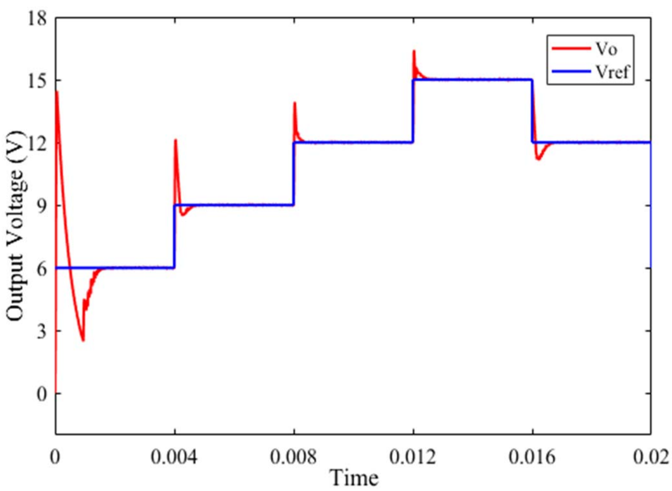

Fig.8. Reference Tracking of Buck converter.

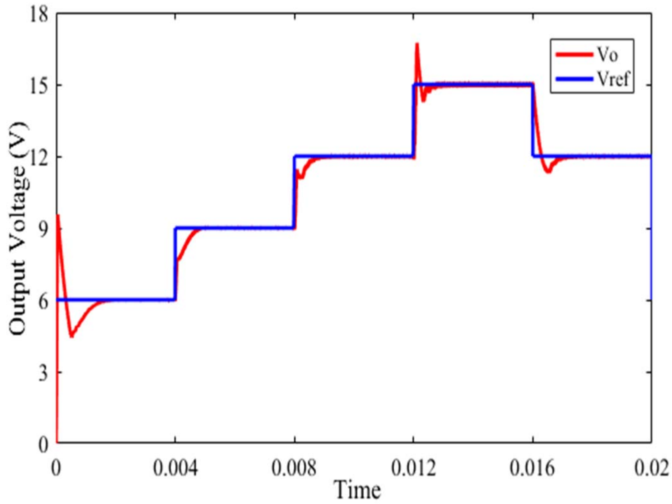

Fig.9. Reference Tracking of Boost converter.

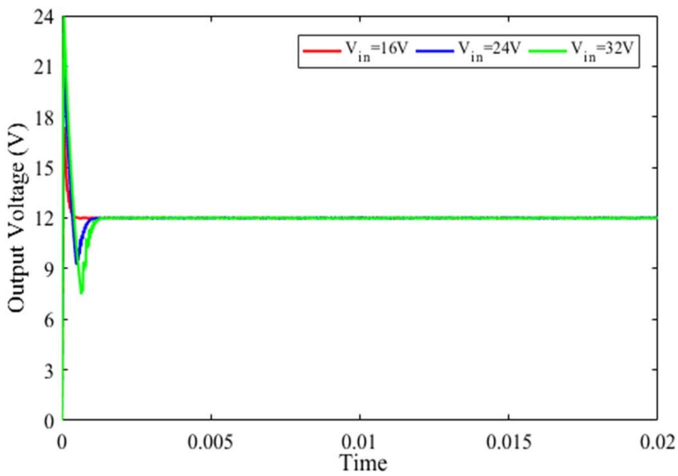

Fig.10. Response of Buck converter for varying input voltage.

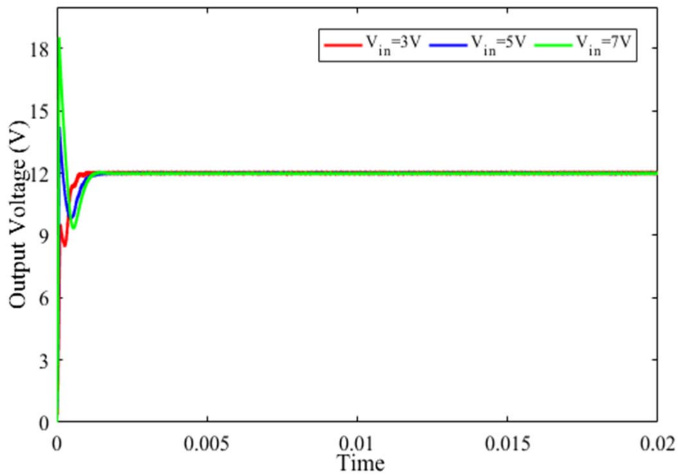

Fig.11. Response of Boost converter for varying input voltage.

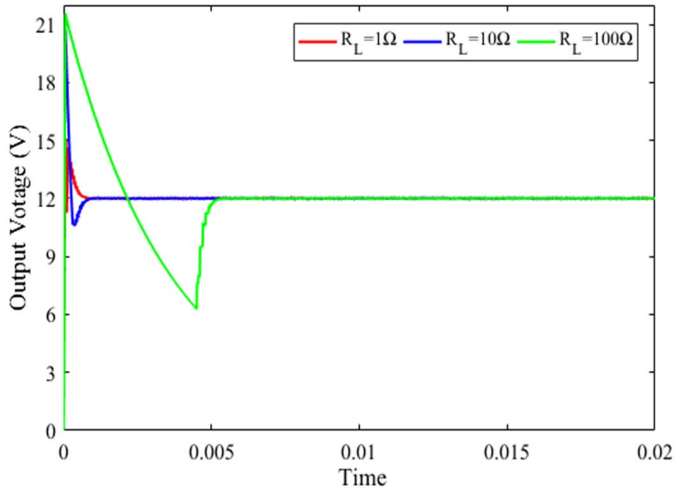

Fig.12. Response of Buck converter for varying load resistance value.

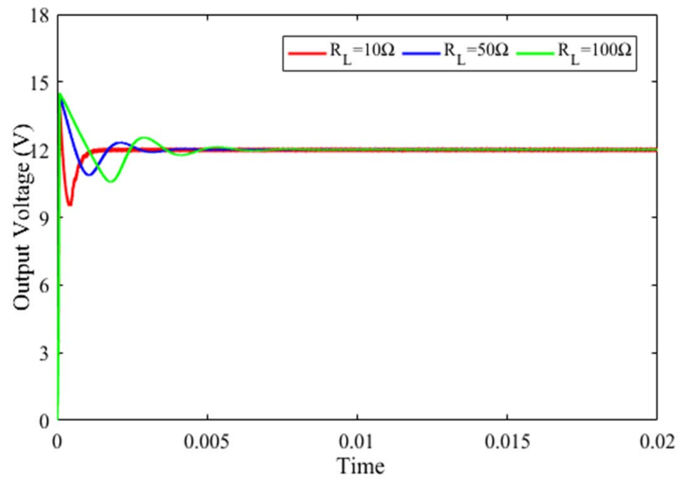

Fig.13. Response of Boost converter for varying load resistance value.

VI. Conclusion

This paper has described digital PID controller design for DC/DC converters using re-design approach. To obtain desired characteristics in terms of loop gain, crossover-frequency and phase-margin, digital filter direct form based PID controller is designed. The developed controller is mapped from continuous domain to discrete domain by using backward Eular method. The controller takes less memory size and less time to execute due to reduced complexity of architecture. MATLAB/Simulink based simulation results are obtained for practical operating conditions, which show that the digital control effectively improves the performance of DC/DC converters under disturbances. The output voltage ripple is within permissible limit and the settling time is of the order of 2-5 milliseconds. The fixed-point implementation of controller relies on the target platform and the digital controller proposed in this paper supports fixed-point algorithm thus can be implement on low-cost processors.

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