SPICE Modeling of Analog Circuits Containing Constant Phase Elements

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***Abstract*—Simple models of constant phase elements (CPEs) for DC, AC, and transient analyses are proposed and implemented in SPICE-compatible simulation programs. The models are based on the operator impedances of the CPEs, which are rewritten to the Laplace-domain formulas of the corresponding behavioral controlled sources in SPICE. The controlled sources are designed so that their respective frequency and impulse characteristics take into account the specific limitations of the convolutional method of transient analysis used in SPICE when evaluating the responses of Laplace sources. Examples of the simulation tasks focusing on supply and communication systems highlight the advantages and weaknesses of the models.**

Keywords-SPICE, constant phase element, fractional order, transient analysis, transfer function, frequency response, impulse response, stability, Mittag-Leffler function

I. INTRODUCTION

Fractional calculus plays an important role in modeling real-world processes of living and non-living nature. A collection of real-world applications in science and engineering is presented in [1], covering physics, control systems, signal and image processing, mechanics and dynamics systems, biology, environmental science, materials, economics, and multidisciplinary fields. In electrical engineering and related fields, we register the development of applications particularly in communication systems (Fractional-order filters, oscillators, PLL (Phase Lock Loop), etc.), in control systems, and in the multidisciplinary symbiosis of electrical engineering and especially biochemistry and medicine [2].

The key element for the analog implementation of systems with fractional-order dynamics is the so-called CPE (Constant Phase Element) [3], whose impedance is proportional to the expression $s^{-\alpha}$. For α = 1 or –1, the CPE changes into a classical capacitor or inductor, characterized by a frequency-independent phase shift of $-\pi/2$ rads or $+\pi/2$ rads between voltage and current. Most commonly, a CPE is considered in the form of a fractional capacitor with the order α between 0 and 1, exhibiting a phase shift of $-\alpha\pi/2$ rads. It is implemented either by approximating RC or LC transverse structures [4, 5] or by a standard silicon process using fractal geometry [6]. Other possible methods are also known, exploiting the fractal properties of the metal-insulator-liquid interface or the properties of some natural tissues [7]. If the CPE is available as an existing two-terminal device, then it can be used to construct analog filters and oscillators by replacing the conventional capacitors. The resulting circuit will then exhibit the properties of a non-integer-order system [8], [9]. In a similar way, the CPE can be used to construct a fractional-order integrator, which is the basis for implementing control and other systems with non-integer dynamics.

For modeling and computer simulation of CPE applications, it should be taken into account that SPICE [10] and similar simulation programs provide mathematical models of the CPE only for α = +1, 0, and –1, i.e. for the inductor, resistor, and capacitor. While a CPE model can be created by modeling an approximation circuit, large-scale ladder circuit structures are required to ensure sufficient approximation accuracy. Little attention has been paid to this issue in the current literature, especially in terms of analyzing the relationship between the type of approximation circuit and the accuracy and speed of the analysis, the transient analysis in particular.

Another option is to use directly the CPE impedance model based on the controlled Laplace-type SPICE sources [11]. In this way, the CPEs and also complex linear circuits with fractional dynamics can in principle be modeled if their transfer functions $K(s)$ are known. The AC analysis is robust since it consists only in applying the well-known substitution $s = j\omega$, where ω is the circular frequency of the excitation signal, to the transfer function. The DC analysis is also straightforward since the model is reduced for the case $s = 0$. However, issues can arise in the transient analysis, which is implemented for circuits containing Laplace sources by the convolution of the impulse response and the signal driving the controlled source [11]. The impulse response is computed using the inverse fast Fourier transform (IFFT) algorithm from the transfer function. There are the following problems or constraints associated with this approach:

1. The system modeled via the controlled source should be stable. Otherwise, the samples of the impulse response cannot be correctly determined from the samples of the frequency response.

2. The frequency response resulting from the transfer function should exhibit significant attenuation in the upper frequency band of the IFFT window in order to rule out the presence of a Dirac impulse or its derivatives in the impulse response, and to treat the impulse response as causal.

3. The accuracy of the calculations can be affected by the discretization of the time axis and by numerical errors in the calculation of the impulse response of the inverse FFT.

4. The number of IFFT samples is limited by memory and computational speed requirements.

5. Calculating the responses of partial Laplace sources by the convolution method generally slows down simulations and is a potential source of additional numerical problems.

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A consequence of Constraint 1 is the impossibility of a plausible simulation of unstable processes in the circuit if this circuit is modeled via its s-domain transfer function.

A consequence of Constraint 2 is that there are fundamental problems in simulating wideband circuits with highpass and band-reject frequency responses and circuits with elements whose characteristics fade "slowly" with increasing frequency.

Constraints 3 and 4 can be eliminated by determining the impulse responses by other means than via the IFFT. For example, the simulation program Simetrix [12] offers, in addition to the classical IFFT method, an analytical method (starting from the s-domain formula of the transfer function, it tries to find an analytical expression for the impulse response), and the Stehfest method for computing the inverse Laplace transform. Unfortunately, this is not possible in conventional SPICE-like simulation programs.

Constraint 5 must be accepted as a fact.

The frequency window for the IFFT is determined in different ways in different programs. In general, however, the fundamental spectral resolution (Fmin) is derived from the inverse of the maximum simulation time (Tstop). In PSpice, the maximum frequency (Fmax) is determined by an algorithm (Fmax depends on the error criterion RELTOL the step ceiling Tmax, and it can be subsequently modified depending on the steepness of the frequency response). In LTSpice, the window attributes depend on the specified parameters of the controlled sources (on the number of FFT points, the time window length, and the mtol parameter). In HSPICE, both Fmin and Fmax can be specified for the controlled sources. In Micro-Cap, the algorithm is similar to PSpice, but it can be modified by specifying manually the controlled source parameters Number_Of_Datapoints and Tmax_For_Convolution.

Different programs deal with Constraint 2 in different, often undocumented, ways. In PSpice, while analyzing a wideband circuit, the algorithm described in [11] is activated to determine the upper frequency Fmax using the RELTOL and Tstop parameters. The IFFT then usually leads to a non-causal impulse response, and the user then has the option to shift it in time by the DELAY parameter. However, this option cannot be used for our purposes, because such a shift makes the results of the transient analysis significantly distorted.

Also worthy of attention is the way the simulation programs deal with s-domain transfer functions that tend to infinity for $s = 0$ (origin of the frequency axis, DC mode). This is the case of Laplace impedances of the CPE elements (see Section II). Then the LTSpice does not perform the transient analysis at all and reports "The Laplacian is singular at DC". Programs like PSpice and Micro-Cap handle such simulation task without this issue. Considering their easy availability (unlike the HSPICE for example), we focus on PSpice (specifically OrCAD PSpice A/D Demo version 9.1 and OrCAD PSpice A/D Lite version 17.2-2016) and Micro-Cap (version 12.2.0.5) for testing the CPE models in this work.

The paper shows how to implement the CPE model in SPICE simulators in order to eliminate Constraints 1 and 2 as much as possible. In Section II, following this introduction, a summary of CPE impedance models is presented. In Section III, the idea of their modeling using Laplace sources in order to eliminate Constraints 1 and 2 is explained. Section IV demonstrates how to use these models to simulate the CPE excitation from an external source and the step responses of fractional-order active filters employing the CPEs.

II. Constant Phase Elements

The constant phase element (CPE) is often referred to as a fractional-order capacitor with the impedance

$$Z_C = \frac{1}{s^\alpha C_F}\ , 0 < \alpha \le 1, \tag{1}$$

where C_F [F $s^{\alpha-1}$] is the fractional pseudo-capacitance.

Similarly, however, it is possible to define a fractional-order inductor with the impedance

$$Z_L = s^\alpha L_F\ , 0 < \alpha \le 1 \tag{2}$$

where L_F [H $s^{\alpha-1}$] is the fractional pseudo-inductance.

The impedance model (1) is also used to describe a generalized CPE with the parameter α in the range $-1 \le \alpha \le 1$, which is able to model the known elements L, R, C as the limiting cases for $\alpha = -1$, 0 and +1. For $\alpha = 0.5$ we obtain the Warburg element for modeling diffusion processes [13].

For $s = j\omega$, the frequency dependences of the impedances (1) and (2) are in the forms

$$Z_C = \frac{1}{\omega^\alpha C_F} e^{-j\frac{\pi}{2}\alpha}, Z_L = \omega^\alpha L_F e^{+j\frac{\pi}{2}\alpha},\ 0 < \alpha \le 1, \tag{3}$$

which confirm that in the harmonic steady state these elements exhibit a constant frequency-independent phase shift between voltage and current, which is determined by the parameter α.

It follows from (1) that the following relationships between the voltage v_C and the current i_C of fractional-order capacitor apply

$$v_C(t) = \frac{1}{C_F} I^{(\alpha)}\{i_C(t)\}, i_C(t) = C_F D^{(\alpha)}\{v_C(t)\} \tag{4}$$

where $I^{(\alpha)}$ and $D^{(\alpha)}$ are the operators of integration and differentiation of order α. The operator of non-integer differentiation is the inverse of the operator of non-integer integration, the latter being defined for $\alpha \in$ R+ by the α-order Riemann-Liouville integral [14]

$$I^{(\alpha)}\{f(t)\} = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\xi)^{\alpha-1} f(\xi)\, d\xi \tag{5}$$

where Γ ( ) is the gamma function.

A similar procedure can be used to obtain the relationships between the voltage v_L and the current i_L of the fractional-order inductor from (2):

$$v_L(t) = L_F D^{(\alpha)}\{i_L(t)\}, i_L(t) = \frac{1}{L_F} I^{(\alpha)}\{v_L(t)\} \tag{6}$$

The task is to build the CPE models (4), (6), utilizing the capabilities provided by standard SPICE programs.

III. SPICE Models of the CPE

The relationships (4) between voltage and current of the CPE (1) offer the voltage calculation by integrating the current

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or, alternatively, the current calculation by differentiating the voltage. For well-known reasons, the integration is preferred. For the transient analysis based on Laplace sources, this means starting from the impedance description (1) of the fractional-order capacitor, finding the impulse response by the inverse Laplace transform (1), and calculating the voltage response by its convolution with the driving current.

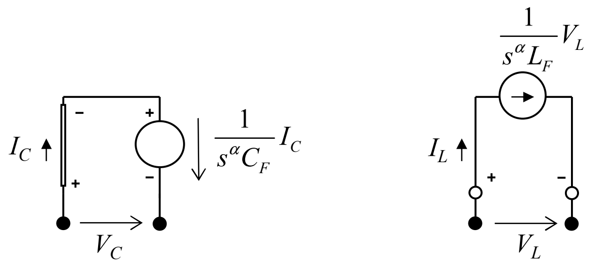
Similarly, for the fractional-order inductor (2) it will be more appropriate to start from the admittance description (inverse to (2)), since, according to (6), the current response of the CPE is determined by the integration of the voltage response.

It can be easily seen that the above strategy is consistent with Constraints 1 and 2 in Section I on the impulse response. We will show this on the example of a fractional capacitor.

Eq. (5) can be viewed as a convolution of the signal $f(t)$ driving a fractional-order integrator and its impulse response $g(t)$, whose Laplace transform is the operator impedance (1). Then the first equation (4) can be rewritten in the form

$$v_C(t) = g(t) \otimes i_C(t),\ \ g(t) = \frac{t^{\alpha-1}}{C_F \Gamma(\alpha)} \tag{7}$$

The element will be modeled in SPICE by a *current-controlled* Laplace *voltage source* as shown in Fig. 1(a). Similarly, the SPICE model of the fractional-order inductor is implemented by a *voltage-controlled* Laplace *current source* as shown in Fig. 1 (b).



(a) (b)

Fig. 1. SPICE model of the fractional-order (a) capacitor, (b) inductor.

The corresponding codes of models from Fig. 1, which can be used uniformly for both PSpice and Micro-Cap simulations, are shown in Table I.

TABLE I. SPICE MODELS OF CPES.

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************************************
* CPEC: Fractional-Order Capacitor
* CPEL: Fractional-Order Inductor
* For Transient, AC and DC analyses
* C – pseudo-capacitance, L – pseudo-inductance
* 0 < alpha <=1 - fractional order
*
.subckt CPEC 1 2 params: C=1 alpha=0.9
Ecpec 1 2 Laplace {i(Ecpec)} = {1/(C*s**alpha)}
.ends CPEC
*
.subckt CPEL 1 2 params: L=1 alpha=0.9
Gcpel 1 2 Laplace {v(1,2)} = {1/(L*s**alpha)}
.ends CPEL
************************************
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The transfer functions of the controlled sources lead to frequency responses with a monotonic decay with a steepness of 20α db per decade. Thus, it can be assumed that for low values of α a larger bandwidth (larger Fmax) should be chosen for the IFFT window than for the integer-order version (α = 1), but the Fmax increase has its limits (growth in the number of IFFT points and computational complexity). Furthermore, it is clear that the CPE models themselves are stable, so it can be expected that the transient analysis of applications built from them could proceed correctly even when these applications exhibit unstable behavior (see Constraint 1). We can also expect a correct behavior of the CPEC model for α > 1, because then its impedance shows a steep decrease with increasing frequency.

IV. EXAMPLES OF USING THE CPE MODELS

The CPE models were tested in Micro-Cap and PSpice in a variety of applications. Where possible, the outputs of the simulation programs were compared either with the analytical solution or with the outputs from Matlab, where the simulation problem was solved using the formulas of analyzed waveforms derived via the Mittag-Leffler functions. For demonstrations in this Section, the first from the dual CPEC and CPEL models was chosen since the vast majority of contemporary applications use just the fractional-order capacitors.

The first example concerns the charging and discharging process of the fractional capacitor, which can be considered as the simplest model of a supercapacitor [15]. The accuracy of the SPICE simulations is tested using an analytical function describing the charging and discharging curves. The second problem demonstrates the capabilities of a SPICE analysis of the transient response of communication filters composed of CPEs.

A. CPE Initialization via Current Excitation

In the context of studying the charging and discharging processes in supercapacitors and batteries, it is shown in [16] that the self-discharge curve of CPEs over time depends not only on the initial voltage value but also on the entire charging history. A schematic of the experiment is shown in Fig. 2. At time t = 0, the CPE with the impedance (1) and zero initial voltage is charged from a constant current source I_0. At time t_0, when the voltage on the CPE reaches the value V_0, the current is suddenly cut down to zero. It results in a spontaneous discharge of the CPE from the voltage V_0 gradually to zero. In [16], it is shown that the shape of the discharge curve depends not only on the voltage V_0 at time t_0 but also on the time t_0, and thus on how long and in what way the CPE has been charged to the voltage V_0.

Based on equations (4) and (5), the analytical formula of the CPE voltage can be derived [16]

$$v_C(t) = \frac{I_0}{C_F \Gamma(\alpha+1)} \left[t^\alpha 1(t) - (t-t_0)^\alpha 1(t-t_0) \right] \tag{8}$$

where 1( ) is the Heaviside unity step function.

The experiments described in [16] can be modeled using the CPEC model from Table I in SPICE. Figure 3 summarizes the results of transient analyses in Micro-Cap 12 and PSpice A/D Lite 17.2. For comparison, ideal waveforms generated from analytical relation (8) are imported into the Figure.

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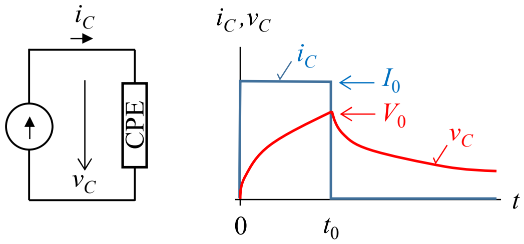

Fig. 2. Charging the capacitive CPE from a current source.

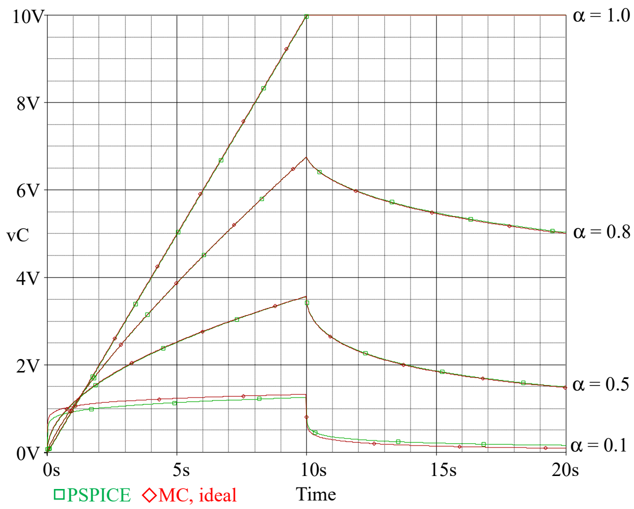

Fig. 3. Transient analysis computed in PSpice, Micro-Cap (MC) and analytically (ideal): charging and discharging processes for the parameters C_F = 1 $Fs^{\alpha-1}$, t_0 = 10 s, I_0 = 1 A, Tstop = 20 s, Tmax = 20 ms, RELTOL = 1×10^{-3}.

The accuracy of the results was quantified by the relative error of the computed voltage V_0 at time t_0 = 10 s. In general, the error increases if the parameter α decreases towards zero. For example, for α = 0.8, 0.5, and 0.1, Micro-Cap calculates the V_0 with an error of about 0.04%, 0.17%, and 1.13%, respectively, while PSpice generates an error of about 0.03%, 0.13%, and 5%. For α = 0.5 and 0.1, PSpice reports problems with the causality of the impulse response (the response is of 0.64% and 9.49% non-causal). It is thus clear that for α below ca 0.5, the Micro-Cap provides more relevant results than PSpice does.

B. Fractional-Order Sallen-Key Filter

Figure 4 shows an active fractional-order highpass filter, assembled from a pair of capacitive CPEs. Simulating the behavior of this application over time is a challenging test of the impedance models of CPEs with Laplace sources for two reasons:

1. The idealized model of the filter exhibits theoretically an infinite bandwidth (see Constraint 2).

2. Certain configurations of the orders α_1, α_2 of CPEs lead to unstable behavior. It is desirable to reveal this instability by transient analysis (see Constraint 1).

When using a pair of identical CPEs in the filter, i.e., $\alpha_1 = \alpha_2 = \alpha$ and $C_1 = C_2 = C$, then the transfer function of the filter will be

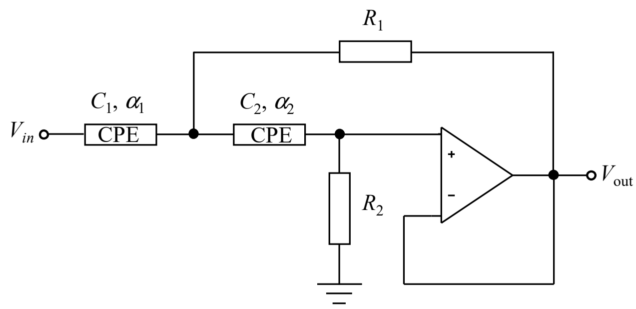

Fig. 4. Fractional-order active highpass filter. Adopted from the classical 2nd-order Sallen-Key structure. Parameters: $\alpha_1 = \alpha_2 = \alpha = 0.95$, $C_1 = C_2 = C$ = 22 $nFs^{\alpha-1}$, R_1 = 1 kΩ , R_2 = 120 kΩ.

$$K(s) = \frac{V_{out}}{V_{in}} = \frac{s^{2\alpha}}{s^{2\alpha} + s^{\alpha}\frac{\omega_0}{Q} + \omega_0^2}, \tag{9}$$

where the pseudo-characteristic frequency ω_0 and the pseudo-quality factor Q are given by

$$\omega_0 = \frac{1}{C\sqrt{R_1 R_2}}, Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}. \tag{10}$$

The cutoff frequency ω_{0F} of the asymptotic frequency characteristics of fractional filter will be [17]

$$\omega_{0F} = \omega_0^{1/\alpha}. \tag{11}$$

It can be shown [17] that the filter will be stable for $0 < \alpha < \alpha_{max}$, where

$$\alpha_{max} = 2(1 - \frac{1}{\pi}\tan^{-1}\sqrt{4Q^2 - 1}). \tag{12}$$

For the parameters given in the caption of Fig. 4, $f_{0F} = \omega_{0F}/(2\pi) \approx 1.02$ kHz, $Q \approx 5.48$, and $\alpha_{max} \approx 1.06$. The filter is therefore stable.

The correctness of the results of the transient circuit analysis in SPICE can be assessed by knowing the analytical formula for the transient response $h(t)$, which can be derived using the procedure described in [17] and [18]:

$$h(t) = \frac{1}{b}\mathrm{Im}\left\{t^{-\alpha}E_{\alpha,1-\alpha}(ct^{\alpha})\right\}, \tag{13}$$

where

$$c = a + jb, a = -\frac{\omega_0}{2Q}, b = \omega_0\sqrt{1 - \frac{1}{4Q^2}}, \tag{14}$$

$E_{\alpha,\beta}(\ )$ is a two-parametric Mittag-Leffler function [19] and Im{ } denotes the imaginary part of the argument.

The results of the AC analysis in PSpice with the CPEC model from Table I are shown in Fig. 5. The curves are in perfect agreement with the Micro-Cap outputs. A phase shift of about 170.87° was measured at low frequencies, and the gain increases with frequency with a steepness of about 38 db/decade, which corresponds to $\alpha \approx 0.95$.

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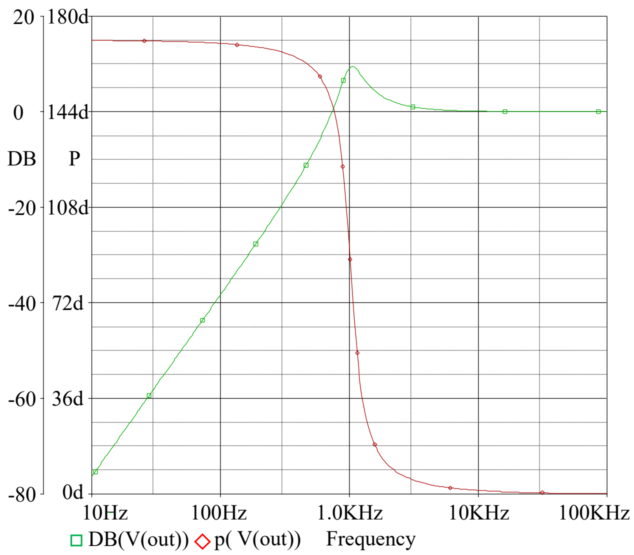

Fig. 5. Amplitude and phase frequency responses of the filter from Fig. 4.

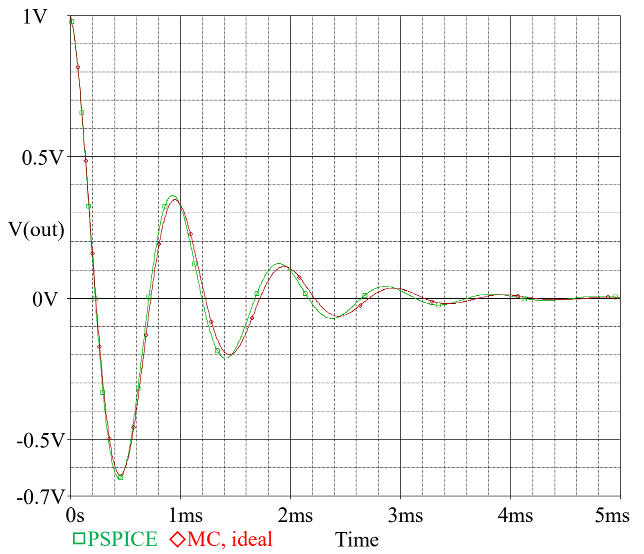

Fig. 6. Step responses computed in PSpice, Micro-Cap (MC), and Matlab (ideal,via Mittag-Leffler functions). Tstop = 5 ms, Tmax = 10 µs, RELTOL = 1×10^{-4}.

In Fig. 6, the step responses computed in PSpice and Micro-Cap are compared with those imported from Matlab. In Matlab, the analytical relation (13) was evaluated using the ml_deriv function by R. Garrappa for accurate computation of the Mittag-Leffler functions and their derivatives [20, 18]. The corresponding response (denoted as "ideal" in Fig. 6) quite coincides with the response obtained from Micro-Cap (MC). The response from PSpice is computed slightly less accurately than in Micro-Cap. Some correction can be achieved by reducing the RELTOL and Tmax parameters, but at the cost of a drastic increase in simulation time.

It follows from (12) that the filter becomes unstable when the critical value of the parameter $\alpha_{max} \approx 1.06$ is exceeded. The results of the simulation of the corresponding step response are summarized in Fig. 7. The experiments in this unstable mode show that both Micro-Cap and PSpice detect the threshold value α_{max} with a high accuracy as a cutoff between the transient response decaying or diverging over time. However, for the unstable response, the accumulated numerical errors associated with the IFFT algorithm and the convolution method of response calculation come into play. This can be clearly seen by comparing the ideal response with the response in Micro-Cap and especially in PSpice. The PSpice program reports a 1.9% non-causality of the impulse response for both CPEs. Attempting a transient analysis of the entire filter using a single Laplace source with the transfer function (9) then leads to a completely erroneous result where the impulse response of the filter exhibits a 99.1% non-causality.

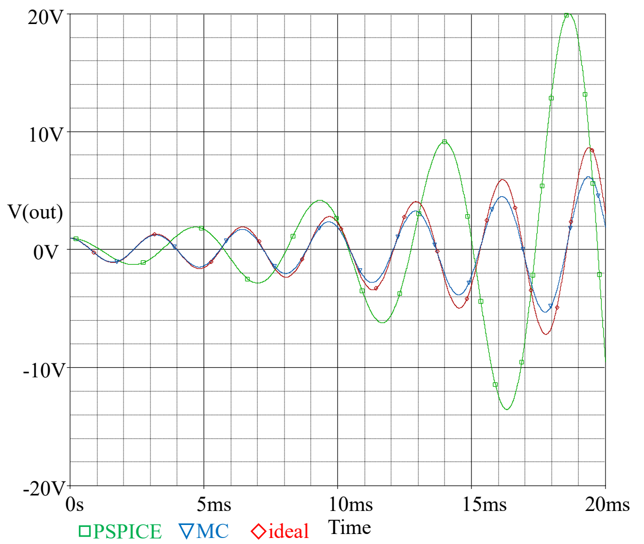

Fig. 7. Step responses computed in PSpice, Micro-Cap (MC), and Matlab (ideal,via Mittag-Leffler functions) for unstable filter with α = 1.1, Tstop = 20 ms, Tmax=10 µs, RELTOL = 1×10^{-5}.

V. Conclusions

The CPE model described is simple and easy to implement in any SPICE-compatible simulation program. It enables a convenient and robust AC analysis of a wide variety of applications involving the CPE components. As far as the transient analysis is concerned, the essential limitations of the convolution method used in SPICE must be taken into account. It turns out that the respective internal algorithms of this method in the available simulation programs behave differently in extreme situations. This is particularly evident when modeling wideband and unstable systems. Among the simulators tested, the Micro-Cap achieved the best results.

From the above it follows that there is a need to search for other alternative and more robust methods for the transient analysis of circuits employing CPEs in the SPICE environment. In the light of these findings, it appears useful to pursue the transient analysis while using analytical preprocessing, which provides relevant outputs unencumbered by the numerical errors of the convolutional method. However, such procedures must be implemented outside the SPICE environment [21].

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