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SPICE simulation of fractional order element without using Ladder circuit

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***Abstract*—Recent trend of research area got priority to use fractional order elements in different areas of science and engineering. So far multi-component fractional order elements (ladder fractor) are implemented using SPICE simulation as there is no tool box in SPICE for fractional elements implementation. If the ladder fractor is used, the circuit becomes complicated that may stop the circuit simulation. In this context, single component fractional order elements have been simulated for the first time using control voltage source (E) in SPICE for different practical circuits. Characteristics of the circuit using fractional inductor can be easily analysed as single component fractional inductor which has not yet fabricated. It has been shown that using this simulation method, fractional order circuits can be realized easily for the entire frequency range for different α values. For comparative studies single component fractional capacitors fabricated in IIT Kharagpur, have been used for circuit realization.**

***Index Terms*—fractional elements, SPICE simulation, filters, fractional order circuits**

I. Introduction

In recent time fractional order circuits and systems have gained enormous importance as it provides extra degree of freedom than the integer order counter part. Literature shows the application of fractional circuits almost in all the domains of electrical and electronic engineering ([1]- [12]). But researchers face two major roadblocks in practical implementation of fractional order circuits: (i) unavailability of a commercial fractional order element similar to resistor, capacitor, or inductor used in day to day work; and (ii) absence of fractional order element in the library of electronic design and automation (EDA) software. These two constrains force the researchers to use ladder based circuits to realize fractional order element for simulation and practical implementation purpose. The ladder based circuit is bulky and hence, not suitable for circuit design.

In this context this work aims to develop library/toolbox for SPICE ([13]- [18]) from where the user can choose fractional order element/s as per the requirement of their designed circuit/s. To start with we have identified three tasks:

1) To develop netlist based SPICE program to simulate fractional order element.
2) To develop the graphical user interface for the user
3) To ingrate the library/toolbox with the existing SPICE software

In this paper SPICE based program to realize single component type fractional order element has been reported. To validate the program, same circuit is implemented in hardware using the fabricated solid state fractional capacitor ([19]- [22]) and compared with the simulated result.

The paper is organized as follows: First, a brief description of the fractional order element is presented in section-II. Next SPICE programs developed to realize single component fractional elements are provided. Then the simulated results are validated with those obtained by experimental results. In the next section, realization of different filters considering whole ranges of the exponent value (α) have been presented. Examples of different SPICE simulation are given in the Section V.

II. Brief background of fractional order element (FOE)

The impedance of any circuit element can be represented as

$$Z_F(s) = \frac{1}{Fs^{\alpha}} \tag{1}$$

Let us call F as "fractance" and α is the exponent. Depending on the values of α, impedance of this device becomes resistor (for $\alpha = 0$), capacitor ($\alpha = 1$) and inductor (for $\alpha = -1$). If we exclude these integer values of α, then the device can be called a fractional order element. In frequency domain, the magnitude of the FOE impedance is $|Z_F| = 1/F\omega^{\alpha}$ $\mho$ and phase angle is $\theta = -90\alpha$ degree. If we generalize further the circuit element in four quadrants can be defined as shown in the Fig. 1a [23]. In this paper we will follow the terminology as given:

1) Acute angle capacitive fractional order element: AC-FOE
2) Obtuse angle capacitive fractional order element: OC-FOE
3) Acute angle inductive fractional order element: AI-FOE
4) Obtuse angle inductive fractional order element: OI-FOE

Fractional capacitor is represented in SPICE by using a controlled voltage source (E) or by a controlled current source

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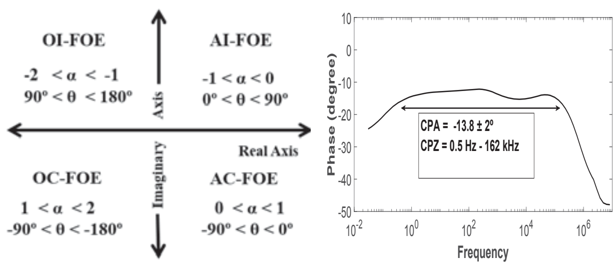
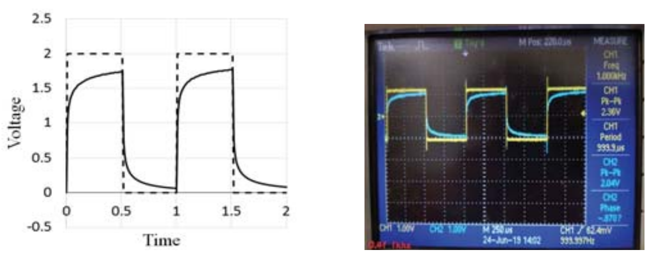
(G). Transfer function of a control voltage source in SPICE is $1/s$. To make s^{α}, parametric analysis has been done and the α value is varied as per the requirement; see Section V (.PARAM statement). F is a simple multiplicative term with s. Using this property the transfer function of a fractional capacitor has been implemented. For controlled current source, a high resistance should be added in parallel to the source to avoid floating errors as logically no current will be flowing through that path. A solid state single element fractional capacitor has been taken to measure phase response (Fig. 1b) using Novocontrol Alpha-A Frequency Analyzers. A constant phase is obtained from the requency 0.5 Hz - 162 kHz and we call it as CPA zone. A simple circuit connecting a resistor and AC-FOC in series is simulated in SPICE. For different α values, the behaviour of the circuit has been studied. Both SPICE results and practical results for $\alpha = 0.4$ and $F = 10\ \mu\mho s^{\alpha}$ are shown in Fig. 2.



(a) (b)

Fig. 1: (a) FOE in different impedance plane, (b) Phase response of single component FOC



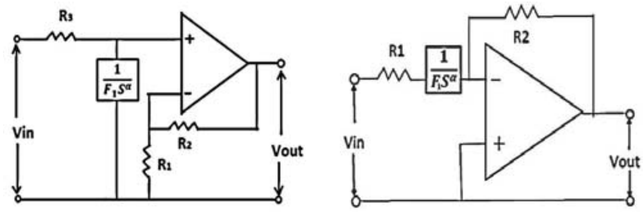
(a) (b)

Fig. 2: Voltage wave form (a) SPICE (b) experiment across $AC - FOE$ for resistance with $AC - FOE$ circuit ($f = 1\ kHz$, $\alpha = 0.4$, $F = 10\ \mu\mho s^{\alpha}$)

Fractional inductor is difficult to fabricate, mostly realized by GIC (Generalized Impedance Converter). Using SPICE theoretical behaviour of fractional inductor has been simulated, replacing classical inductor by a controlled voltage (E) or current source (G) which has transfer function $1/FS^{-\alpha}$. Programs of the SPICE modeling for fractional capacitor and inductor are given in the Section V.

III. Realization of different filters using SPICE

This section deals with realization of different filters in SPICE using fractional capacitors for the two cases; $\alpha<1$ and $\alpha>1$. All the filters are first implemented using classical capacitor and then replaced by fractional capacitor (using control sources) and the results are obtained by varying the value of α.



(a) (b)

Fig. 3: (a) Active Non-inverting Low-pass filter (b) Active High-pass filter

A. Low-pass and high-pass filter

To realize active lowpass filter, the circuit shown in Fig. 3a has been considered. The transfer function and the cut-off frequency are given by

$$H(s) = \frac{1 + \frac{R_2}{R_3}}{1 + R_3 F_1 s^{\alpha}} \tag{2}$$

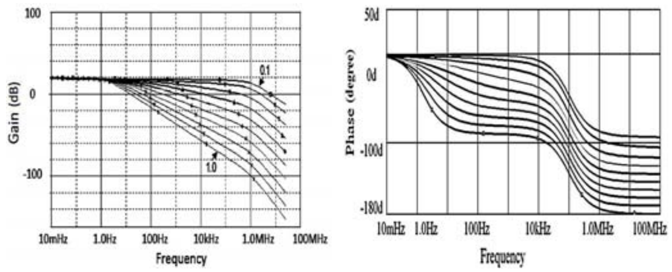
$f_c = \frac{1}{2\pi F_1 R_3}$ respectively. The circuit shown in Fig. 3b has been used to realize a high-pass filter response with the transfer function given by

$$H(s) = \frac{-\frac{R_2}{R_1}}{1 + R_1 F_1 s^{\alpha}} \tag{3}$$

and the cut-off frequency is $f_{cL} = \frac{1}{2\pi F_1 R_1}$. In SPICE for the fixed values of R and F (Table I), α is varied from 0.1 to 1.0 and 1.0 to 1.9. Simulated frequency responses of low pass filter are given by the Figs. 4 and 5 respectively and the frequency responses of the high pass filter are given by Figs. 6 and 7. Later part of this work will cover multiple fractional components.

TABLE I: Parameter values considered for implementing different filters

Filters	Resistor	Capacitor	Inductor
Low-pass	R_1=1k, R_2=8.6k, R_3=10 k	F_1=10$\mu\mho s^{\alpha}$	0
High-pass	R_1=1 k, R_2=47 k	F_1=10 $\mu\mho s^{\alpha}$	0
Band-pass	R_1=10 k, R_2=10 k	F_1=100 $n\mho s^{\beta}$ F_2=10 n^{α}	0
Band-pass with inductor	R_1=1 k R_F=10 k	C_1=10 μF	L_1=10 $m\mho s^{-\alpha}$
Notch 1	R_1=10k, R_2=10k R_3=5 k, R_4=2 k R_5=1 k	C_1=0.1μF C_2=0.1μF F_3=0.2$\mu\mho s^{\alpha 3}$	0 0 0
Notch 2	R_1=10k, R_2=10k R_3=5 k, R_4=2 k R_5=1 k	C_1=0.1μF F_2=0.1$\mu\mho s^{\alpha 2}$ C_3=0.2μF	0 0 0



(a) (b)

Fig. 4: Frequency response of the low pass filter for α = 0.1 to 1.0 (a) magnitude plot and (b) phase plot

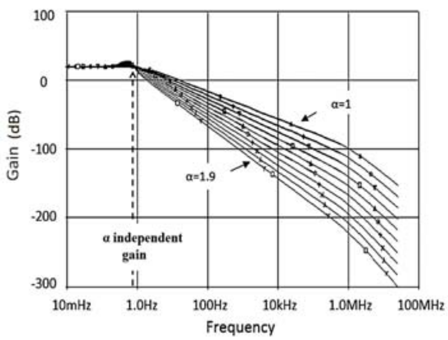


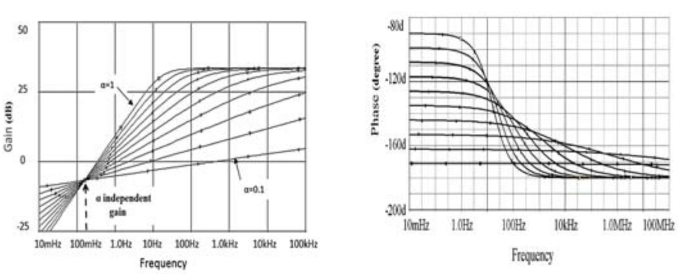
Fig. 5: Frequency response of the low pass filter for α = 1.0 to 1.9

B. Band-pass filter

To realize active band-pass filter the circuit shown in Fig. 8 has been considered for SPICE simulation with single component type AC-FOE. The transfer function is given as

$$H(s) = \frac{-\frac{R_2}{R_1}(R_1 F_1 s^{\beta})}{(1 + R_1 F_1 s^{\beta})(1 + R_2 F_2 s^{\alpha})} \tag{4}$$

with the cut-off frequencies $f_{cL} = \frac{1}{2\pi F_1 R_1}$ and $f_{cH} = \frac{1}{2\pi F_2 R_2}$. The low and high frequency stopband attenuations are independent of each other which is the advantage for fractional-order bandpass filters [25]. The parameter values have been considered as given Table I and both the capacitors are in fractional order during simulation. From the frequency



(a) (b)

Fig. 6: Frequency response of active high-pass filter for α = 0.1 to 1.0 (a) magnitude plot and (b) phase plot

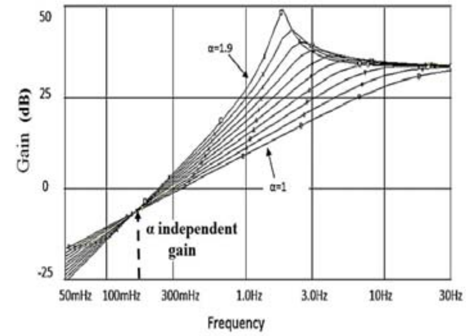


Fig. 7: Frequency response (magnitude plot) of high-pass filter for α = 1.0 to 1.9

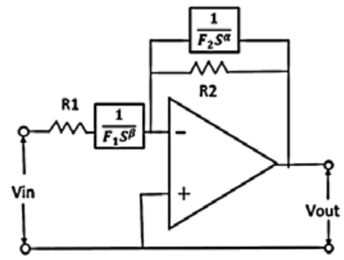
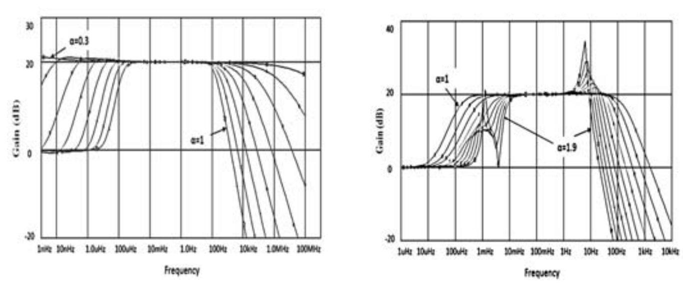


Fig. 8: Active band-pass filter

response (Fig. 9a) it is apparent that with the decrease in the α values within the range 1.0 to 0.1, both low pass and high pass roll-off become slower that results an increase in bandwidth. Also an increase in the α within 1.0 - 1.9, quick roll-off in both the region results an decrease in bandwidth and represented by the Fig. 9b. An interesting characteristic we can observe from this Figure, there is a spike in the lower roll off region from α = 1.5 to 1.9. Fig. 10 has been considered to realize band pass filter using fractional order inductor and the frequency response for the variation of exponents is given in Fig. 11 keeping the capacitor as integer one. The SPICE modeling is given in the Section V. From the Fig. 11 it is apparent that



(a) (b)

Fig. 9: (a) Frequency response (magnitude plot) of band-pass filter considering both the capacitor as fractional order (α = 0.1-1.0) (b) Frequency response (magnitude plot) of band-pass filter considering both the capacitor as fractional order (α = 1.0-1.9)

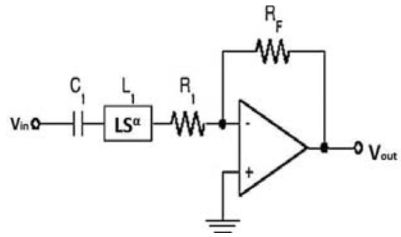


Fig. 10: Band-pass filter circuit using capacitor and inductor

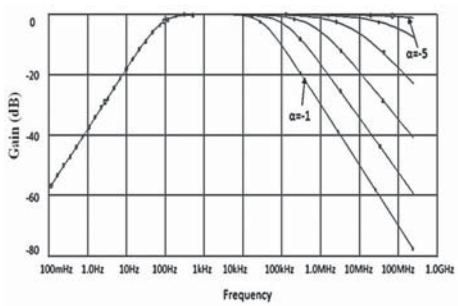


Fig. 11: Frequency response (magnitude plot) of band-pass filter considering fractional inductor keeping the capacitor as classical one

fractional inductor also maintain the attenuation characteristics as fractional capacitor.

C. Notch filter

Fig. 12 has been considered to implement notch filter. In

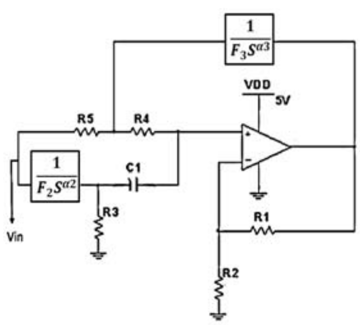
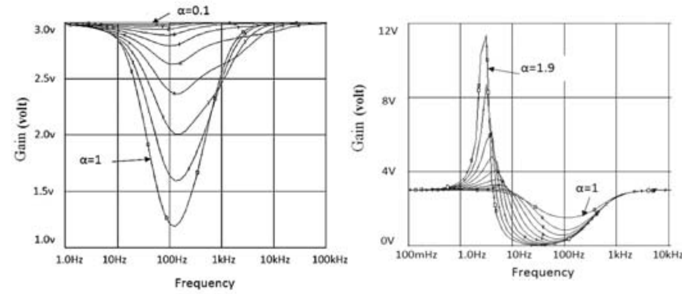


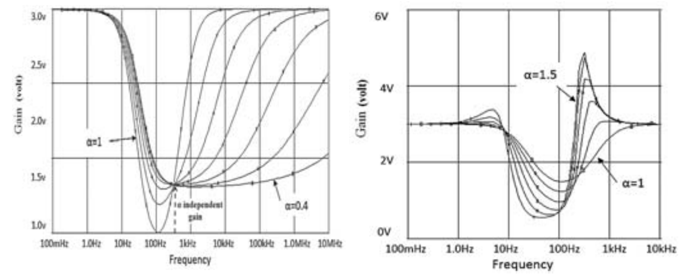
Fig. 12: Active Notch-filter

the first part of implementation, C_3 is replaced as fractional order keeping both C_1 and C_2 as classical capacitor and the frequency response for both the cases (α = 0.1 to 1.0 and α=1.0 to 1.9) are represented by Figs. 13a and 13b respectively. C_3 dominates the low frequency region and the attenuation of the low pass region is according to α values. In the 2nd part of implementation C_2 is fractional order which dominates the high frequency attenuation. Figs. 14a and 14b depict for both the cases (α = 0.1 to 1.0 and α = 1.0 to 1.9) high frequency attenuation is according to the α values. The frequency response of all the notch filters are presented in volt (instead of dB) to clearly observe the attenuation.



(a) (b)

Fig. 13: (a) Frequency response (magnitude plot) of notch-filter1 for α = 0.1 to 1 (b) Frequency response (magnitude plot) of notch-filter1 for α = 1 to 1.9



(a) (b)

Fig. 14: (a) Frequency response (magnitude plot) of notch-filter2 for α = 1 to 0.4 (b) Frequency response (magnitude plot) of notch-filter2 for α = 1 to 1.5

IV. Results and Discussion

The advantage of using SPICE program for simulation is that we can cover the whole frequency region if we use the fractional element as single component ($1/FS^{\alpha}$) which are not possible using ladder based fractional order element. Any circuit related to ladder has only a limited frequency zone of operation. So the SPICE program extends the scope of simulation using fractional order element. The peaking in the response for $\alpha > 1$ (Figs. 5, 7, 9b, 13b, 14b) is expected to increase due to the overall filter order approaches to the second-order filter. Another characteristics that we can observe (Figs. 5, 6, 7), at a particular frequency the gain of the filters are same. More elaborately, at that particular frequency, the system response is independent of fractional exponent. Practically fractional inductor can be realized by GIC and hence, researchers use ladder based FOE along with GIC circuit for a circuit simulation. Using SPICE program, single component fractional inductor has been implemented. For validation of the SPICE results, experimental data using single component fractional capacitor have been presented. Band-pass filter using inductor could not be presented as single component fractional inductor is not still available for research purpose but in simulation it is possible using SPICE.

For low pass filter, experimental results have been presented in Fig. 15 for α = 0.155 using fractional capacitor (single component). Both theoretical and practical results coincide within CPA range. It is expected as the fabricated single component type fractional capacitor has constant phase angle (CPA) from 0.5 Hz to 165 kHz, so beyond that range the results will deviate. For the entire frequency range, time domain response was also observed. Fig. 16 represents time domain response for 2 kHz square pulse, and as expected SPICE simulation and experimental results are in close match. So in the time domain also SPICE can give the desired output by using control source for fractional elements.

The experimental results for high pass filter are presented in Fig. 17 for α = 0.153 and f = 5 kHz. Within the CPA zone (0.5 Hz - 162 kHz) both the theoretical and practical results coincide. Also the time domain response (Fig. 18) has the same characteristics for both SPICE simulation and experimental results. Instead of SPICE, researchers use Matlab toolbox (FOMCON) for implementing fractional order system. But, in Matlab implementation the system response will be according to the transfer function only that may not match with the practical results as FOMCON implements reduced order model.

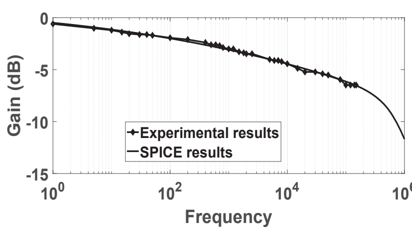
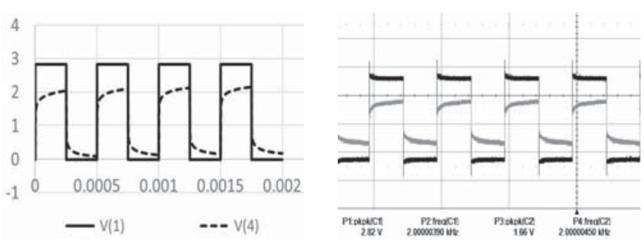


Fig. 15: SPICE and experimental results for α = 0.155, R_1= 9.65 k, R_2= 1.475 k, R_3= 0.985 k, F = 172 $\mu \mho s^{\alpha}$



(a) (b)

Fig. 16: Time domain response for 2 kHz square pulse (a) SPICE implementation (b) practical realization

V. SPICE PROGRAM

1) Implementation of active low-pass filter using fractional capacitor .PARAM alpha=1 C=100n

```
VS3 1 0 AC 1
R3 1 2 10k
*C1 2 0 100n
E1 2 0 LAPLACE {i(VS3)} = {1/((s^(alpha)*C))}
R1 3 0 1K
R2 3 4 9K
XOP 2 3 4 OPAMP1
.SUBCKT OPAMP1 1 2 6
RIN 1 2 10MEG
EGAIN 3 0 1 2 100K
RP1 3 4 1000
CP1 4 0 1.5915UF
EBUFFER 5 0 4 0 1
ROUT 5 6 10
.ENDS
.AC DEC 5 0.01u 20MEG
.PLOT AC V(1) V(3)
.PROBE
.END
```

2) Implementation of active band-pass filter using fractional inductor

```
vin 1 0 AC 1
.PARAM alpha3=-1 L=10m
R1 3 4 10k
*L1 2 3 10m *for classical inductor
E3 2 3 LAPLACE {i(vin)} = {L/s^(alpha3)}
C1 1 2 200n ic=0
Rf 4 5 10k
E 5 0 0 4 999k
.AC DEC 10 1n 100MEG
.PRINT AC VM(5) VP(5)
.PLOT AC VM(5) VP(5)
.PROBE
```

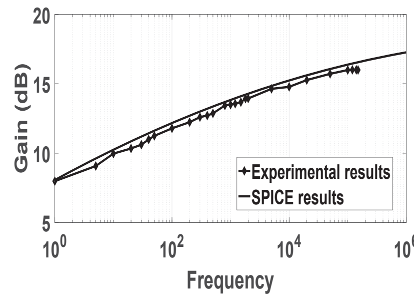
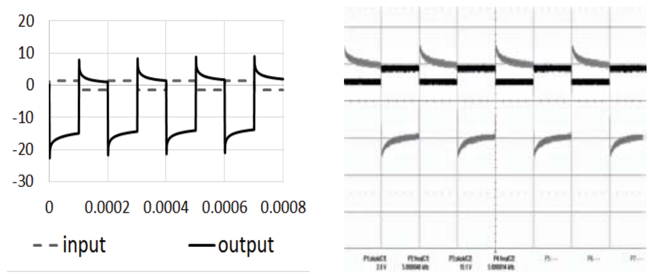


Fig. 17: SPICE and experimental results for α = 0.153, R_1 = 0.985 k, R_2 = 9.65 k, F = 265 $\mu \mho s^{\alpha}$



(a) (b)

Fig. 18: Time domain response of differentiator at 5 kHz square pulse (a) SPICE implementation (b) practical realization

.END

VI. Conclusion

In this work SPICE programs have been developed to realize single component type fractional order element. In all the above analysis, the advantage to use SPICE modeling is that only by changing the α values, the response of a system can be obtained for the entire range of frequencies. To validate the simulated results, practical experiment has been carried out and the results are compared with the simulated values (Figs. 15, 16, 17 and 18). From the above analysis, it is clear that, both the results coincide (frequency domain and time domain) within the CPA zone.

This implementation covers the realization of bandpass (Fig. 9) and band-reject (notch-filter) responses (Figs. 13 and 14) with order less than 2 and 3 respectively. In addition, the above given bandpass and band-reject filters show asymmetric bandpass characteristics with independent control of the stop-band attenuations. The manipulation in the order of the fractional elements can be done as per the system requirement using SPICE modeling for the entire range of α values.

Application of notch filter can be found in the reception of the transmitted signal (communication system), biomedical applications (rejection of noises from the biological signal at a particular frequency). To achieve the required Q-factor and cut-off frequency, both the fractance and exponent values of the fractional element can be tuned. This tuning of the response can be adjusted using SPICE implementation for entire frequency range and will help the circuit designer to implement practically. For analyzing filters researchers can also use fractional inductor (Fig. 11) and the system response for the inductor also can be tuned by changing α values to achieve the desired goal.

Both high pass and low pass fractional filter response imply that fractional filters can be applied where slow roll off is required. All the fractional filters can be used in audio and video synthesis and that can be investigated by further research.

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