Homework Assignment 2

Numerical analysis for PDE's

Thomas Frachson - Davoud Saffar Shamshirgar

1.

2.

3. a. Considering the the following second order PDE

$$\mathfrak{L} = -(u_{xx} + 2u_{xy} + u_{yy}) = 0,$$

and the following schemes

$$\mathfrak{L}_{\Delta_1} = \frac{1}{h^2}(2u_{i+1,j} + 2u_{i,j+1} + 2u_{i-1,j} + 2u_{i,j-1} - 6u_{i,j} - u_{i+1,j-1} - u_{i-1,j+1}), \tag{1.1}$$

$$\mathfrak{L}_{\Delta_2} = \frac{1}{h^2}(u_{i+1,j+1} - 2u_{i,j} - u_{i-1,j-1}), \tag{1.2}$$

we can show that the schemes above are consistent. Note that the schemes have the following stencils. To show that these schemes are both consistent, we start by doing a Taylor expansion.

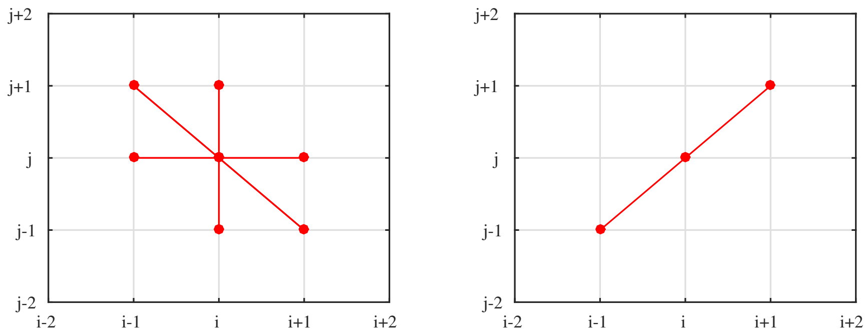

Figure 1.1 – Stencil for the scheme 1 (left) and scheme 2 (right).

$$u(x+ih, y+jh) = u(x,y) + h(iu_x + ju_y) + \frac{h^2}{2}(i^2 u_{xx} + ij u_{xy} + j^2 u_{yy}) + \mathcal{O}(h^3). \tag{1.3}$$

Using equation (1.3), we can write the schemes above as

$$\mathfrak{L}_{\Delta_1} - \mathfrak{L} = C_1 h^2 + \mathcal{O}(h^3), \tag{1.4}$$

$$\mathfrak{L}_{\Delta_2} - \mathfrak{L} = C_2 h^2 + \mathcal{O}(h^3). \tag{1.5}$$

From this, it is easy to verify that

$$\lim_{h\to 0} \mathfrak{L}_{\Delta_i} - \mathfrak{L} = 0, \quad i = 1,2.$$

This proves the consistency of both methods.