HW 11.

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I. 1. $p \to q$
2. $p \to r$
Prove: $p \to q \wedge r$
3. $\sim p \vee q$ 1, cond.
4. $\sim p \vee r$ 2, cond.
5. $(\sim p \vee q) \wedge (\sim p \vee r)$ 3, 4, conj.
6. $\sim p \vee (q \wedge r)$ 5, distributive
7. $p \to (q \wedge r)$ 6, cond.

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1. $p \to q \vee r$
2. $p \to q \vee \bar{r}$
Prove $p \to q$
3. $\sim p \vee (q \vee r)$ 1, cond
4. $\sim p \vee (q \vee \bar{r})$ 2, cond
5. $(\sim p \vee (q \vee r)) \wedge (\sim p \vee (q \vee \bar{r}))$ 3, 4, conj.
6. $\sim p \vee ((q \vee r) \wedge (q \vee \bar{r}))$ 5, dist.
7. $\sim p \vee (q \vee (r \wedge \bar{r}))$ 6, dist.
8. $\sim p \vee (q \vee F)$ 7, negation
9. $\sim p \vee q$ 8, identity
10. $p \to q$ 9, cond.

II.

p	q	r	$q \wedge r$	$p \to (q \wedge r)$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
1	0	0	0	0
0	1	1	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

III. 1. $(p \wedge q \wedge r) \to (p \vee q)$

p	q	r	$p \wedge q \wedge r$	$p \vee q$	all
0	0	0	0	0	1
0	0	1	0	0	1
0	1	0	0	1	1
1	0	0	0	1	1
0	1	1	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	1	1

TAUTOLOGY

2.

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	all
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
1	0	0	0	1	0	0	1
0	1	1	1	0	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

TAUTOLOGY

3.

p	q	$p \to q$	$(p \to q) \to p$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

SATISFIABLE

4. $(p \vee q \vee r) \wedge (\bar{p} \vee \bar{q} \vee \bar{r}) \wedge (p \vee \bar{q}) \wedge (q \vee \bar{r}) \vee (r \vee \bar{p})$

p	q	r
*	*	1
*	0	*
0	1	0

0 1 0 ← does not satisfy

SATISFIABLE

IV. $p \vee (q \wedge \overline{(r \wedge (s \to t))})$

$\overline{r \wedge (\bar{s} \vee t)}$

$\bar{r} \vee \overline{\bar{s} \vee t}$

$\bar{r} \vee (s \wedge \bar{t})$

$p \vee (q \wedge (\bar{r} \vee (s \wedge \bar{t}))$

$p \vee ((q \wedge \bar{r}) \vee (q \wedge (s \wedge \bar{t})))$

$q \vee ((q \wedge \bar{r}) \vee (q \wedge s \wedge \bar{t}))$

$\boxed{q \vee (q \wedge \bar{r}) \vee (q \wedge s \wedge \bar{t})}$

V. Yes. If q is true, the formula is already true. I used a short certificate b/c I only needed to check the very first part of the formula to find an answer that satisfies the formula.