Bandersnatch VRF-AD Specification

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Abstract

This specification delineates the framework for a Verifiable Random Function with Additional Data (VRF-AD), a cryptographic construct that augments a standard VRF by incorporating auxiliary information into its signature. We're going to first provide a specification to extend IETF's ECVRF as outlined in RFC-9381 [1], then we describe a variant of the Pedersen VRF originally introduced by BCHSV23 [2], which serves as a fundamental component for implementing anonymized ring signatures as further elaborated by VG24 [3]. This specification provides detailed insights into the usage of these primitives with Bandersnatch, an elliptic curve constructed over the BLS12-381 scalar field specified in MSZ21 [4].

1. Preliminaries

Definition: A verifiable random function with additional data (VRF-AD) can be described with two functions:

- $Prove(sk, in, ad) \mapsto (out, \pi)$: from secret key sk, input in, and additional data ad returns a verifiable output out and proof π .
- $Verify(pk, in, ad, out, \pi) \mapsto (0|1)$: for public key pk, input in, additional data ad, output out and proof π returns either 1 on success or 0 on failure.

1.1. VRF Input

An arbitrary length octet-string provided by the user and used to generate some unbiasable verifiable random output.

1.2. VRF Input Point

A point in $\langle G \rangle$ generated from VRF input octet-string using the *Elligator 2 hash-to-curve* algorithm as described by section 6.8.2 of RFC-9380 [5].

1.3. VRF Output Point

A point in $\langle G \rangle$ generated from VRF input point as: $Output \leftarrow sk \cdot Input$.

1.4. VRF Output

A fixed length octet-string generated from VRF output point using the proof-to-hash procedure defined in section 5.2 of RFC-9381.

The first 32 bytes of the hash output are taken.

1.5 Additional Data

An arbitrary length octet-string provided by the user to be signed together with the generated VRF output. This data doesn't influence the produced VRF output.

1.6. Challenge Procedure

Challenge construction mostly follows the procedure given in section 5.4.3 of RFC-9381 [1] with some tweaks to add additional data.

Input:

- $\bar{P} \in \langle G \rangle^n$: Sequence of n points.
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

• $c \in \mathbb{Z}_r^*$: Challenge scalar.

Steps:

- 1. $str_0 \leftarrow \texttt{suite_string} \parallel 0x02$
- 2. $str_i \leftarrow str_{i-1} \parallel \texttt{point_to_string}(P_{i-1}), \ i = 1 \dots n$
- 3. $h \leftarrow \text{hash}(str_n \parallel ad \parallel 0x00)$
- 4. $c \leftarrow \texttt{string_to_int}(h_{0...cLen-1})$

With point_to_string, string_to_int and hash as defined in section 2.1.

2. IETF VRF

Based on IETF RFC-9381 which is extended with the capability to sign additional user data (ad).

2.1. Configuration

Configuration is given by following the "cipher suite" guidelines defined in section 5.5 of RFC-9381.

- suite string = "Bandersnatch SHA-512 ELL2".
- The EC group $\langle G \rangle$ is the prime subgroup of the Bandersnatch elliptic curve, in Twisted Edwards form, with finite field and curve parameters as specified in MSZ21. For this group, fLen = gLen = 32 and cofactor = 4.
- The prime subgroup generator $G \in \langle G \rangle$ is defined as follows:

- cLen = 32.
- The public key generation primitive is $pk = sk \cdot G$, with sk the secret key scalar and G the group generator. In this cipher suite, the secret scalar x is equal to the secret key sk.
- encode_to_curve_salt = pk_string (i.e. point_to_string(pk)).
- The ECVRF_nonce_generation function is specified in section 5.4.2.2 of RFC-9381.
- The int_to_string function encodes into the 32 bytes little endian representation.
- The string_to_int function decodes from the 32 bytes little endian representation eventually reducing modulo the prime field order.
- The point_to_string function converts a point in \(\G \) to an octet-string using compressed form. The y coordinate is encoded using int_to_string function and the most significant bit of the last octet is used to keep track of x sign. This implies that ptLen = flen = 32.
- The string_to_point function converts an octet-string to a point on $\langle G \rangle$. The string most significant bit is removed to recover the x coordinate as function of y, which is first decoded from the rest of the string using int_to_string procedure. This function MUST outputs "INVALID" if the octet-string does not decode to a point on the prime subgroup $\langle G \rangle$.
- The hash function hash is SHA-512 as specified in RFC-6234 [6], with hLen = 64.
- The ECVRF_encode_to_curve function uses Elligator2 method described in section 6.8.2 of RFC-9380 and is described in section 5.4.1.2 of RFC-9381, with h2c_suite_ID_string = "Bandersnatch_XMD:SHA-512_ELL2_RO_" and domain separation tag DST = "ECVRF_" || h2c_suite_ID_string || suite_string.

2.2. Prove

Input:

- $x \in \mathbb{Z}_r^*$: Secret key
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Schnorr-like proof

Steps:

- 1. $O \leftarrow x \cdot I$
- 2. $Y \leftarrow x \cdot G$
- 3. $k \leftarrow \mathtt{nonce}(x, I)$
- 4. $c \leftarrow \mathtt{challenge}(Y, I, O, k \cdot G, k \cdot I, ad)$
- 5. $s \leftarrow k + c \cdot x$
- 6. $\pi \leftarrow (c, s)$

Externals:

- nonce: refer to section 5.4.2.2 of RFC-9381.
- challenge: refer to section 1.6 of this specification.

2.3. Verify

Input:

- $Y \in \langle G \rangle$: Public key
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.
- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Schnorr-like proof

Output:

• $\theta \in \{\top, \bot\}$: \top if proof is valid, \bot otherwise.

Steps:

- 1. $(c,s) \leftarrow \pi$
- 2. $U \leftarrow s \cdot G c \cdot Y$

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3. V \leftarrow s \cdot I - c \cdot O
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4. $c' \leftarrow \mathtt{challenge}(Y, I, O, U, V, ad)$

5.
$$\theta \leftarrow \top$$
 if $c = c'$ else \bot

Externals:

• challenge: as defined for Prove

3. Pedersen VRF

Pedersen VRF resembles IETF EC-VRF but replaces the public key with a Pedersen commitment to the secret key, which makes this VRF useful in anonymized ring proofs.

The scheme proves that the output has been generated with a secret key associated with a blinded public key (instead of the public key). The blinded public key is a cryptographic commitment to the public key, and it can be unblinded to prove that the output of the VRF corresponds to the public key of the signer.

This specification mostly follows the design proposed by BCHSV23 [2] in section 4 with some details about blinding base point value and challenge generation procedure.

3.1. Configuration

Pedersen VRF is configured for prime subgroup $\langle G \rangle$ of Bandersnatch elliptic curve E, in Twisted Edwards form, defined in MSZ21 [4] with *blinding base* $B \in \langle G \rangle$ defined as follows:

 $B_y = 38436873314098705092845609371301773715650206984323659492499960072785679638442$

For all the other configurable parameters and external functions we adhere as much as possible to the Bandersnatch cipher suite for IETF VRF described in section 2.1 of this specification.

3.2. Prove

${\bf Input:}$

- $x \in \mathbb{Z}_r^*$: Secret key
- $b \in \mathbb{Z}_r^*$: Secret blinding factor
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof

Steps:

- 1. $O \leftarrow x \cdot I$
- 2. $k \leftarrow \mathtt{nonce}(x, I)$
- 3. $k_b \leftarrow \mathtt{nonce}(b, I)$
- 4. $\bar{Y} \leftarrow x \cdot G + b \cdot B$
- 5. $R \leftarrow k \cdot G + k_b \cdot B$
- 6. $O_k \leftarrow k \cdot I$
- 7. $c \leftarrow \mathtt{challenge}(\bar{Y}, I, O, R, O_k, ad)$
- 8. $s \leftarrow k + c \cdot x$
- 9. $s_b \leftarrow k_b + c \cdot b$
- 10. $\pi \leftarrow (\bar{Y}, R, O_k, s, s_b)$

3.3. Verify

Input:

- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.
- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof

Output:

• $\theta \in \{\top, \bot\}$: \top if proof is valid, \bot otherwise.

Steps:

- 1. $(\bar{Y}, R, O_k, s, s_b) \leftarrow \pi$
- 2. $c \leftarrow \mathtt{challenge}(\bar{Y}, I, O, R, O_k, ad)$
- 3. $\theta_0 \leftarrow \top$ if $O_k + c \cdot O = I \cdot s$ else \bot
- 4. $\theta_1 \leftarrow \top$ if $R + c \cdot \bar{Y} = s \cdot G + s_b \cdot B$ else \bot
- 5. $\theta = \theta_0 \wedge \theta_1$

4. Ring VRF

Anonymized ring VRF based of [Pedersen VRF] and Ring Proof as proposed in VG24.

4.1. Configuration

Ring proof is configured to work together with Pedersen VRF as presented in this specification.

The following configuration should be applied to specialize VG24 in order to instance the concrete scheme.

• Groups and Fields:

- $-\mathbb{G}_{\mathbb{H}}$: BLS12-381 prime order subgroup.
- F: BLS12-381 scalar field.
- J: Bandersnatch curve defined over \mathbb{F} .

• Polynomial Commitment Scheme

- KZG with SRS derived from Zcash powers of tau ceremony.

• Fiat-Shamir Transform

- merlin library implementation.
- Begin with empty transcript with empty label.
- Push R to the transcript after instancing.
- TODO: Specify the order and how parameters are added to the transcript as we progress the protocol.
- Accumulator seed point (Twisted Edwards form):

 $S_x = 3955725774225903122339172568337849452553276548604445833196164961773358506589$

• Padding point (Twisted Edwards form):

 $\square_x = 5259734940318236869621856335705224150406219599146660415951585879123115970561$

 $\square_y = 23297815351169973518610888463679675079080900957871871916328881498043316508082$

• Polynomials domain $(\langle \omega \rangle = \mathbb{D})$ generator:

 $\omega = 49307615728544765012166121802278658070711169839041683575071795236746050763237$

• $|\mathbb{D}| = 2048$

4.1.1. Short Weierstrass Form Requirement

The Ring-Proof scheme, as outlined in VG24, mandates that all points must be in Short Weierstrass form. Therefore, any point used in this scheme, whether derived from Twisted Edwards form or otherwise, must first be converted to Short Weierstrass form. This requirement applies to both user-related values, such as the ring points used by the ring public keys, and to configuration points like the accumulator and padding.

4.2. Prove

Input:

- $x \in \mathbb{Z}_r^*$: Secret key
- $P \in ?$: Ring prover
- $k \in \mathbb{N}_k$: prover public key position within the ring
- $b \in \mathbb{Z}_r^*$: Secret blinding factor
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

- $O \in \langle G \rangle$: VRF output point
- $\pi_p \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof
- $\pi_r \in ((G_1)^4, (\mathbb{Z}_r^*)^7, G_1, \mathbb{Z}_r^*, G_1, G_1)$: Ring proof

Steps:

- 1. $(O, \pi_p) \leftarrow Pedersen.prove(x, b, k, I, ad)$
- 2. $\pi_r \leftarrow Ring.prove(P, b)$

4.3. Verify

Input:

- $V \in (G_1)^3$: Ring verifier (pre-processed commitment).
- $I \in \langle G \rangle$: VRF input point.
- $O \in G$: VRF output point.
- $ad \in \Sigma^*$: Additional data octet-string.
- $\pi_p \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof
- $\pi_r \in ((G_1)^4, (\mathbb{Z}_r^*)^7, G_1, \mathbb{Z}_r^*, G_1, G_1)$: Ring proof

Output:

• $\theta \in \{\top, \bot\}$: \top if proof is valid, \bot otherwise.

Steps:

```
1. \theta_0 = Pedersen.verify(I, ad, O, \pi_p)

2. (\bar{Y}, R, O_k, s, s_b) \leftarrow \pi_p

3. \theta_1 = Ring.verify(V, \pi_r, \bar{Y})

4. \theta \leftarrow \theta_0 \wedge \theta_1
```

Appendix A

The test vectors in this section were generated using code provided at https://github.com/davxy/ark-ec-vrfs.

A.1. IETF VRF Test Vectors

Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
proof_c: Proof 'c' component,
proof_s: Proof 's' component,
```

Vector 1

-,

b923c55b4b7d8c28156c87e005c6d8385a6f26019eee3149aaeb7ee7ce284b38, 208d1eacbedbfb00708a7068c708a565c0bd41c8155010c52e55c6837fecfa52, 96b48404e1df9c738557ccbdfb5bc6f7b8fa3d281aa51742a5928e7a5d77cf5b ..4fc6ed61fc0f7e073dfc3ee8e06b1e5de55e93ecff8ad926cc99a08e8aa6a779, 106f39b9ba10c49df8dfeeea43f8ff02823110fcd8de3ce6110124d29f75881c, 49584112e665526173bfebb6f8949348b1accf72da122c77b501cd395464330c,

Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$

-,
d905aaf894a97094b1d707ea7685fbc4ac501fc01cef25586a9c36288c5c6302,
25c5ab15ce5d973bfec7b6dd428b5b5971958a056d10cc18d5e9ccd0ee4c7b86,
2ae6660f435f733482e4fb6a2c743288fc1d8a6b173b01f490929cd128514c51
..8112bed1659bb8eab1535e279f9b7349fa316ba6f7bd8baa4ae410141bb565d2,
ac8c53d06bb8c0946c479f1732e16800e810810fedda70f37b8a9c4f1016df11,
9a3d82d40e8600276b5fd92cd8d21287abbece6ee357ff5e086126cf912e3d0a,

Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506, -, 0b8c.

587f7c01731c52ce4e02405a9642bf39da4b62befa0a0811f00dd1710a975cc4, 002030eb901d08fe85873b46cd5a1bd2a2c9fbce4f15e9e39066c1fe91be1c1f, 5ca9dc5e02e908b5f1de31c85d30a064353420ab930a541db5f518eee07fb059 ..323df22d2ce82d36a5bac52aa322f08072cc0b9c555a5e4179e3c11a067de7a2, 2ae1f37e6427ec7f3b71e90b54eac7b0b21425760f46ca78908bc0fd2077ca16, 78c7f35f0b3e8edd83a08a36a70c263cd7dba1ab81a2d6ee60242b4af06f2d03,

Vector 4

b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,

, c1cde8432c5bf619b14a403d611140c117a52ba31004574238bd58bf8fc6181f, 5d5a673794b7a0003a1c36f299c4d61055e4b680bb3c2ccd8858dce89c6cd5d3, 0db282523110f629d8c9424afa66f4dfcb9e6dcea5f7891ab2ffc09eeb72a0ac ..11ac36841ec72644a5d24c1fa879872d3091c5e5b81940761f9f8f378f5013ae, 7eb5a8b661e9d93203d7f7aa4b597e695be7c139b457fa5e33a866f4a66f2f12, cde921089ee5ec8d2d940e75819a6347cd8f0ccd215b712f90b278ed186cbb03,

Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, \\ decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, \\ 42616e646572736e6174636820766563746f72, \\ \end{aligned}$

, 8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635, 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 4ddb0d1ebe4d7da9e2cca5c85e39b51166c969dfa30bbf69baafa22121b2000e, 2616dff1f59ff7e7bfc25fa0fea37a9c37e93cf1b88a5e73505a195138590c0c,

Vector 6

da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72, 1f42.

8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635, 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 087914abfd2a59a593384c538bb2f11480d4b196ae2a973ac33cb7dd2cc1541b, 9ad1cdabc97035a05d76c4f4e3c1826deafbc3e4d41df6bf66eaa21d1ba63018,

Vector 7

35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500, b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623, 42616e646572736e6174636820766563746f72, 1f42.

69dec7fe79f816d095b04cead45e856ff6c7e798f513e09291958e35a5590443, 9adeacd15eacdc651e4db1ea4c0917973eac2000479edf6132f3774601cc6902, ff5f6324ea18bbb4df92f7d6304bf27a0a44fa80fd40b985de8d43963a7e02c6 ..ef6f0947911604155c6fe40f68cc91c96ffd358275b58960554274498a70f144, 50a14bab81a42e118e8c167136db35b731a9194a250ae5e65452592742cbdb0e, a75b5327d1b921bb72e2e8c525c18d2fce661b365379ae9f1168c75d281d0100,

A.2. Pedersen VRF Test Vectors

Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
blinding: Blinding factor,
proof_pk_com (Y^-): Public key commitment,
proof_r: Proof 'R' component,
proof_ok: Proof 'O_k' component,
proof_s: Proof 's' component,
proof_sb: Proof 's_b' component
```

Vector 1

3d6406500d4009fdf2604546093665911e753f2213570a29521fd88bc30ede18,

 $\verb| a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b, -, \\$

b923c55b4b7d8c28156c87e005c6d8385a6f26019eee3149aaeb7ee7ce284b38, 208d1eacbedbfb00708a7068c708a565c0bd41c8155010c52e55c6837fecfa52, 96b48404e1df9c738557ccbdfb5bc6f7b8fa3d281aa51742a5928e7a5d77cf5b ..4fc6ed61fc0f7e073dfc3ee8e06b1e5de55e93ecff8ad926cc99a08e8aa6a779, a3f1a139943f3dc02c624505a5794dcc1a75651f60ca69081ebf9bdbd7458616, 2882f90320afdcf99680b8662efe846e2fd477cce00a47ac154f996c910b920a, 71d85bb1a0edcf4362ec8137cdef1a856096e4f9995cc3a4db1781d3e9c7b817, 647c218cec9610102b202bcf7d29bdbf91770c326f07586051fa40bee863b63e, cda38b375717fa7790c18c70dcfcd6ce8f19b13819f088b74688f21dd127c412, 9b52eff1cc2ab908070a1ba89059ae3f6823b43702c60272c5d5943cceb6ac0e,

Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$

-,

d905aaf894a97094b1d707ea7685fbc4ac501fc01cef25586a9c36288c5c6302, 25c5ab15ce5d973bfec7b6dd428b5b5971958a056d10cc18d5e9ccd0ee4c7b86, 2ae6660f435f733482e4fb6a2c743288fc1d8a6b173b01f490929cd128514c51 ..8112bed1659bb8eab1535e279f9b7349fa316ba6f7bd8baa4ae410141bb565d2, 85a94726bcaeaf2db516a6a532ec2450488e7d093374f54de0ba05d2a36bb00a, b28263558234202119a143c295a3fc5a35a6f830dd0c7018e3f33862d1986c1c, 4cb8186c3da92e9be0179f894cdc364aabe1a890340aee9fd886bed45f5017e7, 83a9519edb8ecc4f360eee599c6c1310019c4c3451ca42b4887328e347003bdf, 3e1b408e4ceb5a81e5b71527b01f541d5069438aaa279aa48c39bb7e34f24001, 1dc7b84f188a7fb5bf051464be19e54495f42bd723130992319bad7560023714,

Vector 3

 $6 db 187 202 f69 e627 e432296 ae 1 d0 f166 ae 6 ac 3 c1222585 b6 cea e80 ea 07670 b14, \\9 d97151298 a5339866 ddd 3539d16696 e19e6b68 ac 731562 c807 fe63 a1 ca 49506, \\$

, 0b8c,

587f7c01731c52ce4e02405a9642bf39da4b62befa0a0811f00dd1710a975cc4, 002030eb901d08fe85873b46cd5a1bd2a2c9fbce4f15e9e39066c1fe91be1c1f, 5ca9dc5e02e908b5f1de31c85d30a064353420ab930a541db5f518eee07fb059 ..323df22d2ce82d36a5bac52aa322f08072cc0b9c555a5e4179e3c11a067de7a2, cb3a17d3578d86e2f3b23bb47160327c391c808da28c6be53ed3189d22d78205, f99d09a38f1a1ead7d9503fd601e2d8a56c09eaeb5fb3130035803e04033b49a, de58f590cd204247192f5b49d86c81ddc691fd6b55561fb33ccbec24ecbc86db, d502f832afaddb7bb54e8c28cce458a2a9c3c6c230e4b85539913ec531de168b, 1dd33771a9bfdcf94e6e95fa43e4667adf3279d9c2b22e0877abeb5e99a9e01b,

7863bbac83653e1a48bc0e814e4792c6b2d884522f5556bbb1844c151dcdb700,

Vector 4

b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,

c1cde8432c5bf619b14a403d611140c117a52ba31004574238bd58bf8fc6181f,
5d5a673794b7a0003a1c36f299c4d61055e4b680bb3c2ccd8858dce89c6cd5d3,
0db282523110f629d8c9424afa66f4dfcb9e6dcea5f7891ab2ffc09eeb72a0ac
..11ac36841ec72644a5d24c1fa879872d3091c5e5b81940761f9f8f378f5013ae,
141a8a762dff63c7c05b26d022a8027c515e57f067b5546532296f0ca40a1909,
e926e6b3cbca7b66c42cfc603c4ef2dabc3f5e1276b20d2807f007e974675cb1,
29c56732de262411e71908326037f0f961776db2082bf3d88537265af6a57c92,
c59024c715d21f2a08fb0cd8cb24046558222c6753180853f9601d92186c5e3b,
b818a32590aeb6d79d24cdc6cacb6d5cdc58ccb7025b82be1c1ba2cd34c2f005,
e854b63f9c4e0aab3a051885498d42b5ec354e619491ee9ff239bd3fb486b509,

Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, \\ decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, \\ 42616e646572736e6174636820766563746f72, \\ \end{aligned}$

%af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635, 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 4749f32b7aa36158a4fdfb5bc7e63c40b62eb1d7c75036676e093571a3e9cb06, e159e5494957bb478c4a4d142cde10dadd73a038f8b198c4321dff1271ab61b4, 16a8409cc245978bf55279447d854adca637a58c8c7894a0972b190ad7314492, 3639790d6414b474aa1d53de4e7a896b4e6458c078867acd22200f00f20f280a, bbfd0996c8937c9aaabad9a254614b75c529f892fdfcfcfbe73486888545b610, 6bce65ffb002c6349213b720115ee1457214796c983618f32b4b79c8c559851b,

Vector 6

da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72, 1f42.

 $8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635,\\ 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c,\\ 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4\\ ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a,\\ 1f64d22282d00a58d17d4fe4dc6e8b9772109b6091e1684649c6084fc842391b,\\$

89e230c832f5c2ee1072d9d110151a2dafa4577d64b7fb0845855ae3d1c12fec, e3bd5e3a3f07efb256c989f22fcfe8494219dcd37b35419f5f10da68de09f125, 3639790d6414b474aa1d53de4e7a896b4e6458c078867acd22200f00f20f280a, ceff5ef2315be8be839b1f3c0314b72d976c2e14a2a27c2d1ce8465e90c98607, 0ea7abf79fc1bdebc8b9009cc5744358071c12e82a31565d35a8f91069b55c1b,

Vector 7

35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500, b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623, 42616e646572736e6174636820766563746f72, 1f42,

 $69 \\ dec7fe79f816d095b04cead45e856ff6c7e798f513e09291958e35a5590443, \\ 9 \\ adeacd15eacdc651e4db1ea4c0917973eac2000479edf6132f3774601cc6902, \\ ff5f6324ea18bbb4df92f7d6304bf27a0a44fa80fd40b985de8d43963a7e02c6 \\ ..ef6f0947911604155c6fe40f68cc91c96ffd358275b58960554274498a70f144, \\ ea1f922fce5e359d92e0fdcda53a1d2e6b791c7e7a8ffad915f3535c6175f115, \\ f674ad5f72661aa0c2bc5ca83aee9794c8b8bbc4017abcc00a11a23a0b558e68, \\ f77eaec55fe36b06f1d1f7eef7db24fdcce74c83fde19b1c322aca288e39948f, \\ b846dfbceb2a74fe102b3aec94e7b8460f5adcb609c407839ab6cb06d1e3bd38, \\ 35a41d1cb4d22b5c162d319b206db940b6fcef71bbe0c13a6376a89788292519, \\ c04b177f954d17e7c129ce8d55cb7f148b3957078c96e7229100dc50b7d62b02, \\ \end{aligned}$

A.3. Ring VRF Test Vectors

KZG SRS parameters are derived from Zcash BLS12-381 powers of tau ceremony.

The evaluations for the ZK domain items, specifically the evaluations of the last three items in the evaluation domain \mathbb{D} , are set to 0 rather than being randomly generated.

Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
blinding: Blinding factor,
proof_pk_com (Y^-): Pedersen proof public key commitment,
proof_r: Pedersen proof 'R' component,
proof_ok: Pedersen proof 'O_k' component,
proof_s: Pedersen proof 's' component,
proof_sb: Pedersen proof 's_b' component,
ring_pks: Ring public keys,
ring_pks_com: Ring public keys commitment,
```

ring_proof: Ring proof

Vector 1

3d6406500d4009fdf2604546093665911e753f2213570a29521fd88bc30ede18, a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b, b923c55b4b7d8c28156c87e005c6d8385a6f26019eee3149aaeb7ee7ce284b38, 208d1eacbedbfb00708a7068c708a565c0bd41c8155010c52e55c6837fecfa52, ..4fc6ed61fc0f7e073dfc3ee8e06b1e5de55e93ecff8ad926cc99a08e8aa6a779, a3f1a139943f3dc02c624505a5794dcc1a75651f60ca69081ebf9bdbd7458616, 2882f90320afdcf99680b8662efe846e2fd477cce00a47ac154f996c910b920a, 71d85bb1a0edcf4362ec8137cdef1a856096e4f9995cc3a4db1781d3e9c7b817, 647c218cec9610102b202bcf7d29bdbf91770c326f07586051fa40bee863b63e, cda38b375717fa7790c18c70dcfcd6ce8f19b13819f088b74688f21dd127c412, 9b52eff1cc2ab908070a1ba89059ae3f6823b43702c60272c5d5943cceb6ac0e, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 ..ad6fdeda0dde0a57c51d3226b87e3795e6474393772da46101fd597fbd456c1b..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 89e2e79b6178c12684ac3a6bf9437af3a69dcc529f0021ec40bb006506837ae1 ..82bf4b908e46733d3a23507791169fda8ea11b18665fe894ee9f0754c0c3fec7 ${\tt ..0c6b8d1444d9b604ce949cbf130642d89f72b6cb1f08e32a18cdbb00aadfdf1b}$..fe0e9c85ec450505dde7f5ac038274cf, 84251f616e4a04227cd2fcf59077db2a3f43575bdc4a60c21a09c6f4b98fecb7 ${\tt ..95104cb5533c6c088015846caa08ffe49107bd20fe94a01157764aab5f300d7e}$..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd ..8ed8d119f14c6f5029fbd58a337b351834e37e239cb56557779eb5dbd5934534 ..d417e99b8534393ae55084a9765233ad849bf0906e4c3601c2c63bb7c9b55e08 ..91203a8e8ce31ae910c75281f2961814e967f610d080dda67c683c170c66f8e9 ..98f8633aef4b1416e768447dc1761896d146e06d53108506fbcb95d414ee1c59 ..67093934 a fa 4217 d 96 c 6 c 5 d b 561 a f 7f 6f 14 e 6 e c 34 d 5502 f 8b 126 d b 63 c e 9e b 863 a factorial de la companya del companya de la companya de la companya del companya de la companya del companya de la companya de la companya de la companya de la companya del companya de la companya del companya de la companya de la companya de la companya de la..d1443a4af4ca6e9153a2b311bdfc68953d5da24e72144c453db8a622613e2e68..33b617d0cbb8d1c9dc40b9c1122caccbdfa5781c0391b77490ae766dd3169c66 ...37e9d5d76d4f702350e2ed9aba6f8d160d27f3675e2ccb8a26ab5f1075c17624..e08efd3f76c4c4dde783aba37cc93172efb5306a1aef0591d5b9a306f2e51334 $\dots 742 \\ fcc685 \\ e7e0958 \\ e324 \\ db86 \\ ebe4bf94777 \\ c339 \\ fb5e97b370 \\ c3429795 \\ afa940 \\ above 100 \\ above$..b0909da6d65c28199b1cabf958c3bb318c280e375c8ae8e2694143f59eda03ba ${\tt ..0c5bf14d8bbd7a9bd8a70f7db4ecc5e5f7612b6282f1f0875030f7248064e8711}$

..9f5c058d17369eb46a949ac02c97123a91c065f30e3d24d282202ee39de179cc ..11aede1b407a76de21fd7202c986e7dd2ce4b92a1ac5342a1f214348414cf624 ..b2e7243cc5e8a5d1d02465df72cb84e2c266cffdf6365d0b845e81c2fa859213 ..649db1f2995334a3561d730fe4d0f62f,

Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$

d905aaf894a97094b1d707ea7685fbc4ac501fc01cef25586a9c36288c5c6302, 25c5ab15ce5d973bfec7b6dd428b5b5971958a056d10cc18d5e9ccd0ee4c7b86, 2ae6660f435f733482e4fb6a2c743288fc1d8a6b173b01f490929cd128514c51 ..8112bed1659bb8eab1535e279f9b7349fa316ba6f7bd8baa4ae410141bb565d2, 85a94726bcaeaf2db516a6a532ec2450488e7d093374f54de0ba05d2a36bb00a, b28263558234202119a143c295a3fc5a35a6f830dd0c7018e3f33862d1986c1c, 4cb8186c3da92e9be0179f894cdc364aabe1a890340aee9fd886bed45f5017e7, 83a9519edb8ecc4f360eee599c6c1310019c4c3451ca42b4887328e347003bdf. 3e1b408e4ceb5a81e5b71527b01f541d5069438aaa279aa48c39bb7e34f24001, 1dc7b84f188a7fb5bf051464be19e54495f42bd723130992319bad7560023714. 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ${\tt ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437}$.. ad 6f ded a 0 dd e 0 a 57 c 51 d 3226 b 87 e 3795 e 6474393772 da 46101 f d 597 f b d 456 c 1b d 500 f d..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 894fd4149cce66e5f39f11c0de38825da7d07c52de1d8e74ed170c6b1a2feec7 ..bc158b35068bbcfa9455fd76f699c15cb5e9dfaba7a93cb264c07d9228e8c642 ..73e2d5febe689b4b6279f21b1b0b26ec956f6d6d3fd5650edc1e4f7bf8d1663b

 $88e15ab86bfd02c6bbc4a6fda0dae46f9efe6a96ae3701daed38d6750a2d5ac3\\..3b96f7dd7a34b78ad3eda732740784bd9107bd20fe94a01157764aab5f300d7e\\..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd\\..ae75edd956b647c773b4b59548c27ee717acecd6b3f8a36be3bfaea9841e0bf6\\..6a7c591d5c336a8aec25aa996f4670daa6101f1a3d5517d847e7406df60f049f\\..8435ff5f9e3b09789d10a6a3d8e13bb959212aa4e0cf0b957699349e225525a2\\..ac00259d4cea4c90e929942bcf0d28119fe7381d8e30c6f66d0fa82c70a6df2e\\..f84eda886a2c5989b152a947759be27a0b5744f79e6ea20c19c28542d24de230\\..928a8c0ed5f17fe06f626eda156341ba142d3f2b05d646b41c71a877f4453a68\\..364d5793f1607b9249c1aa275ba0dc9185d48976833bf0ad3855ff62dd159a67\\..7d73eabf04c6f305d43bad31fbf258b5396670ba7b049f28714ebe595cfec311\\..27d3dfad0501160eb879c2f4bbe23fb2fbadf78a9024c15870243a104652f81c$

..fe0e9c85ec450505dde7f5ac038274cf,

 $..8b075a21e0ed8af0b1fa718957088e6271a2914adb24e6a0e0aaa23b4c675c4f\\ ..a76e0537da88fc7e03caab26f766483a76018683ac0eab5089e1c4cf8d6f0236\\ ..626971b02d3c2575ec5d54696081cb46506db5c7575659a2e74eabe15e6ff21a\\ ..a3dccd1d7f3cc02fbf322c7504fa503c9199d570ec1696e8da7586ea50629819\\ ..bb91eeba62157b244e2f97c7b710e4460b27ca4bd468600db1008a61d1514d12\\ ..b51470335da7f5b4a663416092253498aa52a9d3223bdc4a69161dc0db587eb6\\ ..4456db6bad7d94d45695a41e3bef98dd,$

Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506, 0b8c, 587f7c01731c52ce4e02405a9642bf39da4b62befa0a0811f00dd1710a975cc4, 002030eb901d08fe85873b46cd5a1bd2a2c9fbce4f15e9e39066c1fe91be1c1f, 5 ca 9 dc 5 e 0 2 e 9 0 8 b 5 f 1 de 31 c 8 5 d 30 a 0 6 4 3 5 3 4 20 a b 9 30 a 5 4 1 d b 5 f 5 1 8 e e e 0 7 f b 0 5 9 a 5 d 30 a 5 4 1 d b 5 f 5 1 8 e e e 0 7 f b 0 5 9 a 5 d 30 a 5 4 1 d b 5 f 5 1 8 e e e 0 7 f b 0 5 9 a 5 d 30 a 5 4 1 d b 5 f 5 1 8 e e e 0 7 f b 0 5 9 a 5 d 30 a 5 4 1 d b 5 f 5 1 8 e e e 0 7 f b 0 5 9 a 5 d 30 a 5 d 30...323df22d2ce82d36a5bac52aa322f08072cc0b9c555a5e4179e3c11a067de7a2, cb3a17d3578d86e2f3b23bb47160327c391c808da28c6be53ed3189d22d78205. f99d09a38f1a1ead7d9503fd601e2d8a56c09eaeb5fb3130035803e04033b49a, de58f590cd204247192f5b49d86c81ddc691fd6b55561fb33ccbec24ecbc86db, d502f832afaddb7bb54e8c28cce458a2a9c3c6c230e4b85539913ec531de168b, 1dd33771a9bfdcf94e6e95fa43e4667adf3279d9c2b22e0877abeb5e99a9e01b, 7863bbac83653e1a48bc0e814e4792c6b2d884522f5556bbb1844c151dcdb700, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ${\tt ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5}$..9 d97151298 a5339866 ddd3539 d16696 e19 e6b68 ac731562 c807 fe63 a1 ca49506 feasible for the contraction of the contraction..4 fd11 f89 c2a1 aae fe856 bb1 c5d4 a1 fad73 f4de5 e41804 ca2 c17 ba26d6 e10050 ca2 c..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 .. ad 6f de da 0 dd e 0 a 57 c 51 d 322 6 b 87 e 379 5 e 6474 393772 da 46101 f d 597 f b d 456 c 1 b d 500 f d 500..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, a90130fa47aaf758299818bd119e7fecdddb62674541f78c5fa5371b9db62d0f ..8afd73d28225fb1ae60e8959c5f0e929b861ba122a1c8fa45fc9d2b8fb66666e ..f55fdcdfdae22addff823236613fb08b49a694b9f1ec38b72fc0a021857d3026 ...92e630ae2b14e758ab0960e372172203f4c9a41777dadd529971d7ab9d23ab29..fe0e9c85ec450505dde7f5ac038274cf, ae28c697167281871ae0b4d8eae5f7189e8b905bdfc652eea9f0a92f96eab819 ${\tt ..1439c44e5cf78f2824f9b19ed577f6b19107bd20fe94a01157764aab5f300d7e}$..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd ..90346560e90a53ec6ec7c27eb6d55a1f8efca883bd7f6e15d0211127173601df ..c3769a6dd1d43d907cbdff645caa98a9a42a5c1a713d7302ff9eae55b3ca8a79 ..671d06fdb4ce619d78b2fdfe543a5d224629ac3625349437b6519acbc95d70d0

..f7392d425a6f3e661a0f31bc44ad43909faaecb4dd59bc5682163a2d2f889a09
..a9aa852ed39ab26aacec48b809c0941e9d2e579e488937676678d388d14ecb36
..d5489fd64bbfcbbd835b084e3c842c16276482cf2fdd1c8470b0781b33ffd06e
..08d1096b39f8468e727fdf4b7f2176cdedc56a6439eff6891d253794231cd10b
..b71d50c3d33e1c07d2a1e8988483e1ce0aad0ee8ba51d177bb4778b36221c87f
..f27948b3a84118eac577df67a473d2bec0acd5aa6205c4fb54078a0a86c316fd
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..91ed863c7037144c196b7bc37379d1e3fd0ff5ebe2f68d6c46c3805b01d17f59
..92890f2ad8376ade62923147c1aba065cdcb8d32053c0335cc015f0dce8bc380
..064f7e70ed10c876f7c248b8afb98eab,

Vector 4

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Vector 5

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Vector 6

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Vector 7

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- ${\tt ...92e630ae2b14e758ab0960e372172203f4c9a41777dadd529971d7ab9d23ab29}$
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- ..5f266084fb8504d436c2fbfff9f6daa1,

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5.

6.

 $\hbox{Internet Engineering Task Force $\textit{Hashing to Elliptic Curves}$; RFC Editor, 2023; } \\$

Internet Engineering Task Force US Secure Hash Algorithms; RFC Editor, 2011;