# Bandersnatch VRF-AD Specification

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Abstract

This specification delineates the framework for a Verifiable Random Function with Additional Data (VRF-AD), a cryptographic construct that augments a standard VRF by incorporating auxiliary information into its signature. We're going to first provide a specification to extend IETF's ECVRF as outlined in RFC-9381 [1], then we describe a variant of the Pedersen VRF originally introduced by BCHSV23 [2], which serves as a fundamental component for implementing anonymized ring signatures as further elaborated by VG24 [3]. This specification provides detailed insights into the usage of these primitives with Bandersnatch, an elliptic curve constructed over the BLS12-381 scalar field specified in MSZ21 [4].

# 1. Preliminaries

## 1.1. Common definitions

- F: Scalar field with prime order.
- **G**: Bandersnatch curve with order |**F**|.
- $\Sigma^k$ : Octets strings with length  $k \in \mathbb{N}$  (\* for arbitrary length).
- $G \in \mathbf{G}$ : Prime subgroup generator point.
- $x \in \mathbf{F}$ : Secret key scalar.
- $Y \leftarrow x \cdot G \in \mathbf{G}$ : Public key point.
- $i \in \Sigma^*$ : VRF input data.
- $I \in \mathbf{G}$ : VRF input point.
- $O \in \mathbf{G}$ : VRF output point.
- $o \in \Sigma^k$ : VRF output hash.

# 1.2. VRF Input

An arbitrary length octet-string provided by the user to generate some unbiasable verifiable random output.

# 1.3. VRF Input Point

A point in **G** generated from VRF input octet-string using the *Elligator 2 hash-to-curve* algorithm as described by section 6.8.2 of RFC-9380 [5].

$$I \leftarrow \mathtt{hash\_to\_curve}(i)$$

# 1.4. VRF Output Point

A point generated from VRF input point and secret key scalar.

$$O \leftarrow x \cdot I$$

# 1.4. VRF Output

A fixed-length octet string produced from the VRF output point using the *output-to-hash* procedure, which is *proof-to-hash* method described in Section 5.2 of RFC-9381, but with a specific focus on the output point component (referred to as *Gamma* in RFC-9381) extracted from the output point proof bundle.

$$o \leftarrow \mathtt{output\_to\_hash}(O)$$

## 1.5 Additional Data

An arbitrary length octet-string provided by the user to be signed together with the generated VRF output. This data doesn't influence the produced VRF output.

## 1.6. VRF-AD

A verifiable random function with additional data (VRF-AD) can be described by three functions:

- $prove(x, i, ad) \mapsto \Pi$
- $verify(y, i, ad, \Pi) \mapsto (\top | \bot)$
- $output(\Pi) \mapsto o$

Where:

•  $y \leftarrow \texttt{serialize\_compressed}(Y)$ 

- $\Pi \leftarrow \mathtt{encode\_compressed}((O, \pi))$
- $\pi$ : specific scheme proof

# 1.7. Challenge Procedure

Challenge construction mostly follows the procedure given in section 5.4.3 of RFC-9381 [1] with some tweaks to add additional data.

# Input:

- $\bar{P} \in \mathbf{G}^n$ : Sequence of n points.
- $ad \in \Sigma^*$ : Additional data octet-string.

# Output:

•  $c \in \mathbf{F}$ : Challenge scalar.

# Steps:

- 1.  $str_0 \leftarrow \texttt{suite\_string} \parallel 0x02$
- 2.  $str_i \leftarrow str_{i-1} \parallel point\_to\_string(P_{i-1}), i = 1 \dots n$
- 3.  $h \leftarrow \text{hash}(str_n \parallel ad \parallel 0x00)$
- $4. \ c \leftarrow \texttt{string\_to\_int}(h_{0...cLen-1})$

With point\_to\_string, string\_to\_int and hash as defined in section 2.1.

# 2. IETF VRF

Based on IETF RFC-9381 which is extended with the capability to sign additional user data (ad).

## 2.1. Configuration

Configuration is given by following the "cipher suite" guidelines defined in section 5.5 of RFC-9381.

- suite\_string = "Bandersnatch\_SHA-512\_ELL2".
- The EC group **G** is the prime subgroup of the Bandersnatch elliptic curve, in Twisted Edwards form, with finite field and curve parameters as specified in MSZ21. For this group, fLen = qLen = 32 and cofactor = 4.
- The prime subgroup generator  $G \in \mathbf{G}$  is defined as follows:

 $G_y \! = \! 19188667384257783945677642223292697773471335439753913231509108946878080696678$ 

 $- \ \, {\rm Compressed:} \ \, _{\tt 0x664197ccb667315e6064e4ee81ad8c3586d5dcba508b7d150f3e12da9e666c2a}$ 

- cLen = 32.
- The public key generation primitive is  $pk = sk \cdot G$ , with sk the secret key scalar and G the group generator. In this cipher suite, the secret scalar x is equal to the secret key sk.
- encode\_to\_curve\_salt = "" (empty no salt)
- The ECVRF\_nonce\_generation function is specified in section 5.4.2.2 of RFC-9381.
- The int\_to\_string function encodes into the 32 octets little endian representation.
- The string\_to\_int function decodes from the 32 octets little endian representation eventually reducing modulo the prime field order.
- The point\_to\_string function converts a point in **G** to an octet-string using compressed form. The *y* coordinate is encoded using int\_to\_string function and the most significant bit of the last octet is used to keep track of *x* sign. This implies that ptLen = flen = 32.
- The string\_to\_point function converts an octet-string to a point on **G**. The string most significant bit is removed to recover the *x* coordinate as function of *y*, which is first decoded from the rest of the string using int\_to\_string procedure. This function MUST outputs "INVALID" if the octet-string does not decode to a point on the prime subgroup **G**.
- The hash function hash is SHA-512 as specified in RFC-6234 [6], with hLen = 64.
- The ECVRF\_encode\_to\_curve function uses *Elligator2* method as described in section 6.8.2 of RFC-9380 and in section 5.4.1.2 of RFC-9381, with parametrized with h2c\_suite\_ID\_string = "Bandersnatch\_XMD:SHA-512\_ELL2\_RO\_" and domain separation tag DST = "ECVRF\_" || h2c\_suite\_ID\_string || suite\_string.

## 2.2. Prove

#### Input:

•  $x \in \mathbf{F}$ : Secret key

•  $I \in \mathbf{G}$ : VRF input point

•  $ad \in \Sigma^*$ : Additional data octet-string.

## Output:

•  $O \in \mathbf{G}$ : VRF output point

•  $\pi \in (\mathbf{F}, \mathbf{F})$ : Schnorr-like proof

#### Steps:

- 1.  $O \leftarrow x \cdot I$
- 2.  $Y \leftarrow x \cdot G$
- 3.  $k \leftarrow \mathtt{nonce}(x, I)$
- 4.  $c \leftarrow \mathtt{challenge}(Y, I, O, k \cdot G, k \cdot I, ad)$
- 5.  $s \leftarrow k + c \cdot x$
- 6.  $\pi \leftarrow (c,s)$

## Externals:

- nonce: refer to section 5.4.2.2 of RFC-9381.
- challenge: refer to section 1.6 of this specification.

# 2.3. Verify

# Input:

- $Y \in \mathbf{G}$ : Public key
- $I \in \mathbf{G}$ : VRF input point
- $ad \in \Sigma^*$ : Additional data octet-string.
- $O \in \mathbf{G}$ : VRF output point
- $\pi \in (\mathbf{F}, \mathbf{F})$ : Schnorr-like proof

# Output:

•  $\theta \in \{\top, \bot\}$ :  $\top$  if proof is valid,  $\bot$  otherwise.

## Steps:

- 1.  $(c,s) \leftarrow \pi$
- 2.  $U \leftarrow s \cdot G c \cdot Y$
- 3.  $V \leftarrow s \cdot I c \cdot O$
- 4.  $c' \leftarrow \mathtt{challenge}(Y, I, O, U, V, ad)$
- 5.  $\theta \leftarrow \top$  if c = c' else  $\bot$

## Externals:

• challenge: as defined for Prove

# 3. Pedersen VRF

Pedersen VRF resembles IETF EC-VRF but replaces the public key with a Pedersen commitment to the secret key, which makes this VRF useful in anonymized ring proofs.

The scheme proves that the output has been generated with a secret key associated with a blinded public key (instead of the public key). The blinded public key is a cryptographic commitment to the public key, and it can be unblinded to prove that the output of the VRF corresponds to the public key of the signer.

This specification mostly follows the design proposed by BCHSV23 [2] in section 4 with some details about blinding base point value and challenge generation procedure.

# 3.1. Configuration

Pedersen VRF is configured for prime subgroup G of Bandersnatch elliptic curve E, in Twisted Edwards form, defined in MSZ21 [4] with *blinding base*  $B \in G$  defined as follows:

 $B_x = 6150229251051246713677296363717454238956877613358614224171740096471278798312$ 

 $\bullet \ \ Compressed{:}\ \ _{0xe93da06b869766b158d20b843ec648cc68e0b7ba2f7083acf0f154205d04e23e}$ 

For all the other configurable parameters and external functions we adhere as much as possible to the Bandersnatch cipher suite for IETF VRF described in section 2.1 of this specification.

#### 3.2. Prove

#### Input:

- $x \in \mathbf{F}$ : Secret key
- $b \in \mathbf{F}$ : Secret blinding factor
- $I \in \mathbf{G}$ : VRF input point
- $ad \in \Sigma^*$ : Additional data octet-string.

## **Output:**

- $O \in \mathbf{G}$ : VRF output point
- $\pi \in (\mathbf{G}, \mathbf{G}, \mathbf{G}, \mathbf{F}, \mathbf{F})$ : Pedersen proof

# Steps:

- 1.  $O \leftarrow x \cdot I$
- 2.  $k \leftarrow \mathtt{nonce}(x, I)$
- 3.  $k_b \leftarrow \mathtt{nonce}(b, I)$
- 4.  $\bar{Y} \leftarrow x \cdot G + b \cdot B$
- 5.  $R \leftarrow k \cdot G + k_b \cdot B$

- 6.  $O_k \leftarrow k \cdot I$
- 7.  $c \leftarrow \mathtt{challenge}(\bar{Y}, I, O, R, O_k, ad)$
- 8.  $s \leftarrow k + c \cdot x$
- 9.  $s_b \leftarrow k_b + c \cdot b$
- 10.  $\pi \leftarrow (\bar{Y}, R, O_k, s, s_b)$

# 3.3. Verify

## Input:

- $I \in \mathbf{G}$ : VRF input point
- $ad \in \Sigma^*$ : Additional data octet-string.
- $O \in \mathbf{G}$ : VRF output point
- $\pi \in (\mathbf{G}, \mathbf{G}, \mathbf{G}, \mathbf{F}, \mathbf{F})$ : Pedersen proof

## Output:

•  $\theta \in \{\top, \bot\}$ :  $\top$  if proof is valid,  $\bot$  otherwise.

## Steps:

- 1.  $(\bar{Y}, R, O_k, s, s_b) \leftarrow \pi$
- 2.  $c \leftarrow \mathtt{challenge}(\bar{Y}, I, O, R, O_k, ad)$
- 3.  $\theta_0 \leftarrow \top$  if  $O_k + c \cdot O = I \cdot s$  else  $\bot$
- 4.  $\theta_1 \leftarrow \top$  if  $R + c \cdot \bar{Y} = s \cdot G + s_b \cdot B$  else  $\bot$
- 5.  $\theta = \theta_0 \wedge \theta_1$

# 4. Ring VRF

Anonymized ring VRF based of [Pedersen VRF] and Ring Proof as proposed in VG24.

# 4.1. Configuration

Ring proof is configured to work together with Pedersen VRF as presented in this specification.

The following configuration should be applied to specialize VG24 in order to instance the concrete scheme.

# • Groups and Fields:

 $-\mathbb{G}_{\mathbb{F}}$ : BLS12-381 prime order subgroup.

- $\mathbb{F}$ : BLS12-381 scalar field.
- J: Bandersnatch curve defined over  $\mathbb{F}$ .

# • Polynomial Commitment Scheme

KZG with SRS derived from Zcash powers of tau ceremony.

## • Fiat-Shamir Transform

- ark-transcript.
- Begin with empty transcript and "ring-proof" label.
- Push R to the transcript after instancing.
- TODO: Specify the order and how parameters are added to the transcript as we progress the protocol.
- Accumulator seed point in Twisted Edwards form:

 $S_x = 37805570861274048643170021838972902516980894313648523898085159469000338764576$ 

- Compressed: 0x6e5574f9077fb76c885c36196a832dbadd64142d305be5487724967acf9595a0
- Padding point in Twisted Edwards form:

 $\square_x = 26287722405578650394504321825321286533153045350760430979437739593351290020913$ 

 $\square_y = 19058981610000167534379068105702216971787064146691007947119244515951752366738$ 

 $- \ \, {\rm Compressed:} \ \, _{\tt 0x92ca79e61dd90c1573a8693f199bf6e1e86835cc715cdcf93f5ef222560023aa}$ 

A point with unknown discrete logarithm derived using the ECVRF\_encode\_to\_curve function as described in IETF suite [Configuration] section with input the string: "ring-proof-pad".

• Polynomials domain  $(\langle \omega \rangle = \mathbb{D})$  generator:

 $\omega = 49307615728544765012166121802278658070711169839041683575071795236746050763237$ 

•  $|\mathbb{D}| = 2048$ 

## 4.2. Prove

#### Input:

- $x \in \mathbf{F}$ : Secret key
- $P \in ?$ : Ring prover
- $k \in \mathbb{N}_k$ : prover public key position within the ring
- $b \in \mathbf{F}$ : Secret blinding factor

- $I \in \mathbf{G}$ : VRF input point
- $ad \in \Sigma^*$ : Additional data octet-string.

## Output:

- $O \in \mathbf{G}$ : VRF output point
- $\pi_p \in (\mathbf{G}, \mathbf{G}, \mathbf{G}, \mathbf{F}, \mathbf{F})$ : Pedersen proof
- $\pi_r \in ((G_1)^4, (\mathbf{F})^7, G_1, \mathbf{F}, G_1, G_1)$ : Ring proof

## Steps:

- 1.  $(O, \pi_p) \leftarrow Pedersen.prove(x, b, k, I, ad)$
- 2.  $\pi_r \leftarrow Ring.prove(P, b)$

# 4.3. Verify

## Input:

- $V \in (G_1)^3$ : Ring verifier (pre-processed commitment).
- $I \in \mathbf{G}$ : VRF input point.
- $O \in G$ : VRF output point.
- $ad \in \Sigma^*$ : Additional data octet-string.
- $\pi_p \in (\mathbf{G}, \mathbf{G}, \mathbf{G}, \mathbf{F}, \mathbf{F})$ : Pedersen proof
- $\pi_r \in ((G_1)^4, (\mathbf{F})^7, G_1, \mathbf{F}, G_1, G_1)$ : Ring proof

## Output:

•  $\theta \in \{\top, \bot\}$ :  $\top$  if proof is valid,  $\bot$  otherwise.

## Steps:

- 1.  $\theta_0 = Pedersen.verify(I, ad, O, \pi_p)$
- 2.  $(\bar{Y}, R, O_k, s, s_b) \leftarrow \pi_p$
- 3.  $\theta_1 = Ring.verify(V, \pi_r, \bar{Y})$
- 4.  $\theta \leftarrow \theta_0 \wedge \theta_1$

# Appendix A

The test vectors in this section were generated using ark-ec-vrfs libraries revision d90e180.

## A.1. IETF VRF Test Vectors

## Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (O): VRF output point,
out (beta): VRF output octet string,
proof_c: Proof 'c' component,
proof_s: Proof 's' component,
```

#### Vector 1

-, -.

c5eaf38334836d4b10e05d2c1021959a917e08eaf4eb46a8c4c8d1bec04e2c00, e7aa5154103450f0a0525a36a441f827296ee489ef30ed8787cff8df1bef223f, fdeb377a4ffd7f95ebe48e5b43a88d069ce62188e49493500315ad55ee04d744 ...2b93c4c91d5475370e9380496f4bc0b838c2483bce4e133c6f18b0adbb9e4722, 439fd9495643314fa623f2581f4b3d7d6037394468084f4ad7d8031479d9d101, 828bedd2ad95380b11f67a05ea0a76f0c3fef2bee9f043f4dffdddde09f55c01,

#### Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$ 

-,

8c1d1425374f01d86b23bfeab770c60b58d2eeb9afc5900c8b8a918d09a6086b, 60f32f5ad3e9694b82ccc0a735edb2f940f757ab333cc5f7b0a41158b80f574f, 44f3728bc5ad550aeeb89f8db340b2fceffc946be3e2d8c5d99b47c1fce344b3..c7fcee223a9b29a64fe4a86a9994784bc165bb0fba03ca0a493f75bee89a0946, 8aa1c755a00a6a25bdecda197ee1b60a01e50787bd10aa976133f4c39179330e, 18c74ffd67e6abc658e2d05ecd3101ddc0c33623823f2395538cf8d39e654f12,

#### Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506, -, 0b8c, c5eaf38334836d4b10e05d2c1021959a917e08eaf4eb46a8c4c8d1bec04e2c00,

67a348e256d908eb695d15ee0d869efef2bcf9f0fea646e788f967abbc0464dd, edde0178045133eb03ef4d1ad8b978a56ee80ec4eab8830d6bc6c08003138841 ..6657d3c449d9398cc4385d1c8a2bb19bcf61ff086e5a6c477a0302ce270d1abf, aec4d1cf308cb4cb400190350e69f4fb309255aa738fff5a6ac4ced7538fce03, 54e5d38a76f309ce63ca82465160abd8d75b78805a0b499e60c26436de4a8e01,

#### Vector 4

 $\label{eq:b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,$ 

-,
672e8c7a8e6d3eca67df38f11d50f3d7dbb26fa8e27565a5424e6f8ac4555dcc,
4d3e0524fc59374f1fdad8e471c695469b45ecf69c1de85c6c1230e888dd4cbe,
36127f8aee7c61048984f0a208bf6d334db9dacbeeeef9ff2d17117e81232832
..1462eb3ef602f5911d77ab11f815eb4154ba95c934e414198ef000a61b4de31a,
b72598f235145a377911caa794ba85820173c4c49b7be3b05d847b2c753e0311,
e8e34ad3131388a88eb7f80bd874f3421c378d4ad45911c4bc16e4cdc17b5716,

#### Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, \\ decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, \\ 42616e646572736e6174636820766563746f72, \\ \end{aligned}$ 

4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7, 9508104b820469687488d83f729288d9f70fc0523318beff44a47da10d490b3c, 4ee61f3c000544aa48c565e143e05c6501a623bdbf02a0a408b97433660b4907 ..715f75890cc0e45cdd7116e3da15b15c3c637782e8e05d05c0d5895e5fe583d1, ad6af59b4b84f18187c694ef374687d13517cb53508ff9dafa37d0c759e9601c, 4c1269d9d161dabd082fc606af979eca7f6c3ab68e78261dc6fb9fbbb98c9704,

#### Vector 6

da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72, 1f42.

4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7, 9508104b820469687488d83f729288d9f70fc0523318beff44a47da10d490b3c, 4ee61f3c000544aa48c565e143e05c6501a623bdbf02a0a408b97433660b4907 ..715f75890cc0e45cdd7116e3da15b15c3c637782e8e05d05c0d5895e5fe583d1, 4fa53519bd9d17acae4d1021416557d11b84dd4670b563770c14eb98161eaa08, 0f7f9bee9077427f547e69b919cf8d63823c14b20085fd9516768e0f5e3d3f0e,

#### Vector 7

35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500, b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623, 42616e646572736e6174636820766563746f72,

 $4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7,\\ 6d1dd583bea262323c7dc9e94e57a472e09874e435719010eeafae503c433f16,\\ 09106f062ac07846f3f841f64765527b333575143483855d633f99ccc2e8e306\\ ...e6239ff79a1272cff931e8d0ac6c390328486329118ad40a18b85184da1837ff,\\ 6dbeeab9648505fa6a95de52d611acfbb2febacc58cdc7d0ca45abd8c952ef12,\\ ce7f4a2354a6c3f97aee6cc60c6aa4c4430b12ed0f0ef304b326c776618d7609,\\ \end{aligned}$ 

## A.2. Pedersen VRF Test Vectors

#### Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
blinding: Blinding factor,
proof_pk_com (Y^-): Public key commitment,
proof_r: Proof 'R' component,
proof_ok: Proof 'O_k' component,
proof_s: Proof 's' component,
proof_sb: Proof 's_b' component
```

## Vector 1

-, -, -

c5eaf38334836d4b10e05d2c1021959a917e08eaf4eb46a8c4c8d1bec04e2c00, e7aa5154103450f0a0525a36a441f827296ee489ef30ed8787cff8df1bef223f, fdeb377a4ffd7f95ebe48e5b43a88d069ce62188e49493500315ad55ee04d744 ..2b93c4c91d5475370e9380496f4bc0b838c2483bce4e133c6f18b0adbb9e4722, 01371ac62e04d1faaadbebaa686aaf122143e2cda23aacbaa4796d206779a501, 3b21abd58807bb6d93797001adaacd7113ec320dcf32d1226494e18a57931fc4, 8123054bfdb6918e0aa25c3337e6509eea262282fd26853bf7cd6db234583f5e, ac57ce6a53a887fc59b6aa73d8ff0e718b49bd9407a627ae0e9b9e7c5d0d175b, 0d379b65fb1e6b2adcbf80618c08e31fd526f06c2defa159158f5de146104c0f,

e2ca83136143e0cac3f7ee863edd3879ed753b995b1ff8d58305d3b1f323630b,

#### Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$ 

\_

8c1d1425374f01d86b23bfeab770c60b58d2eeb9afc5900c8b8a918d09a6086b, 60f32f5ad3e9694b82ccc0a735edb2f940f757ab333cc5f7b0a41158b80f574f, 44f3728bc5ad550aeeb89f8db340b2fceffc946be3e2d8c5d99b47c1fce344b3 ..c7fcee223a9b29a64fe4a86a9994784bc165bb0fba03ca0a493f75bee89a0946, 99ff52abf49d67c4303ac4a8a00984d04c06388f5f836ebd37031f0e76245815, c1322e7a65b83996c25e37a84e36598333b0d417619242c0cb3d9d972edde848, 7a4363e0bf9cd18317287d681ab05704982b0088ce373f696dbdf3909a902b36, fc8770c209212640742d53e2f40e5c30fffae574f90fdc670ff11a1127586c03, 93f7c9d73eec05e500b758f645a2967e62b2206e57eff5f9b99bfc71812e620d, c864de36e0b428f6fb4ef470f94ec9601716cb26ad96f3359e4a1ec110794a0b,

#### Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506,

\_ ,

0b8c,

c5eaf38334836d4b10e05d2c1021959a917e08eaf4eb46a8c4c8d1bec04e2c00, 67a348e256d908eb695d15ee0d869efef2bcf9f0fea646e788f967abbc0464dd, edde0178045133eb03ef4d1ad8b978a56ee80ec4eab8830d6bc6c08003138841 ..6657d3c449d9398cc4385d1c8a2bb19bcf61ff086e5a6c477a0302ce270d1abf, e22ec3e4a2a4132237eb8a62bcc5ed864593cfde08e53b1632ecd3245761c808, 54c04f259f9e40ee086031d29960b12b6b6407e9de14985001c7265587941831, 9200b650a0c20b0ef73ccd7651ffc7af154e5e02879dc8666025c245aa547f01, 35f8dc0f744d1850513c46b6b4640716cbb4643da26cfe67f8c701486e0b4cae, 5faa89369589174f4202d6e53e8b4ef10a49b2ad8face60d7cb28bfc8f43bf0e, 017093ff8d22ba2f3852141365a1452fbb5ab8cf6f20cb04555e3163f8d88f13,

#### Vector 4

 $\label{eq:b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,$ 

-, -

 36127f8aee7c61048984f0a208bf6d334db9dacbeeeef9ff2d17117e81232832 ..1462eb3ef602f5911d77ab11f815eb4154ba95c934e414198ef000a61b4de31a, 755610da34cc224fbe60ce5e42add2ea6b272ef466aef18c13497363116d1c03, d26274e014ebfc19a9c1a951193858b972eae3360ed35635e89f1f9dbe432be5, 26202144ba4c4cb7ecde831c9e9662bec519493b29a098dd5803a8b4d261fc12, b9fa51c75d278d95f2ccace9609b28ec137b244c8b7d1523b16ed07c8e24b8e4, e42423127a2ca12d4f199287c8fa07784eacf9fc9b86a6bd56ee364cc352c009, 5371d6f9c76b560b4e42b9154a395bed60924d8de31284e926d06af382f5ad1b,

#### Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, \\ decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, \\ 42616e646572736e6174636820766563746f72, \\ \end{aligned}$ 

\_,

4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7, 9508104b820469687488d83f729288d9f70fc0523318beff44a47da10d490b3c, 4ee61f3c000544aa48c565e143e05c6501a623bdbf02a0a408b97433660b4907..715f75890cc0e45cdd7116e3da15b15c3c637782e8e05d05c0d5895e5fe583d1, fb0123dd6317dbd379afccded247f75b3c1c2e32b86eaa9d6c9d0eb5bef07919, a91807f0ee57d2344a8942808bf35c65b5bd4fde16752a98f3e3dc67be8c103d, dbc69ea4dd299deec2f29845e98a17700f07842cf2f6c5e9d88f388fa3a2831c, 311f94e886825c80a30fd44535be37218501bd072afcbc1298f8fba6c3e3c96d, 15f1562046078a7c0d152ef1b56bf3078c763089bf08790f10ff1cf3b9a5030b, f962cb2598032cf5b0a21b4c253514d75c91ace15acde9a3f716e40f70f06804,

#### Vector 6

da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72,

1f42,

 $4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7,\\ 9508104b820469687488d83f729288d9f70fc0523318beff44a47da10d490b3c,\\ 4ee61f3c000544aa48c565e143e05c6501a623bdbf02a0a408b97433660b4907\\ ..715f75890cc0e45cdd7116e3da15b15c3c637782e8e05d05c0d5895e5fe583d1,\\ 0752c5b639dffedf9a66ac111a765d3e9c4cfac9c8b26cc5af6d524967afdf0a,\\ d03caebf8577c1d2ed30a09708683195f11883411dc170e3ea9f09a2cbf86bab,\\ 8b16f0abb2873d6d56199280aeee9e02ce0274a9ca06a3194d6a72c25516ace8,\\ 311f94e886825c80a30fd44535be37218501bd072afcbc1298f8fba6c3e3c96d,\\ 9671cdae8b4cdeea640c24993ccf7e571fcfb3344d81d3cc6f36d03496777c1c,\\ 624e25cd6eccec59b09f0893ef9eab877b55c757b9e9c81260255145bffd9a0d,\\ \end{aligned}$ 

#### Vector 7

 $35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500,\\ b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623,\\ 42616e646572736e6174636820766563746f72,$ 

, 1f42,

4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7, 6d1dd583bea262323c7dc9e94e57a472e09874e435719010eeafae503c433f16, 09106f062ac07846f3f841f64765527b333575143483855d633f99ccc2e8e306 ..e6239ff79a1272cff931e8d0ac6c390328486329118ad40a18b85184da1837ff, 462ae9ad651e5caf11247b989fecb5f2b1729479c33b9133388d14fa35dbbd0c, 91f1ca92eeaa0b604faf3e4811c12b44991ea33cf582a529a4bc4429a3b6cc5a, b69946f270c46ccc59557bd40288a0a27607281da1892328fdb9da2dcb6c73cf, 5a02419120b814a5c81d67096aac728ee9bda5ddf9451cf554d871462a04831a, bd8c0c1e5e04577c8836e45fb64131d1275309fe28e1d4334b230e3aa639da1a, d93ccbd393ed88c8165b0a01aabe28c56a53b43e527e7927eeadff006dd22114,

# A.3. Ring VRF Test Vectors

KZG SRS parameters are derived from Zcash BLS12-381 powers of tau ceremony.

The evaluations for the ZK domain items, specifically the evaluations of the last three items in the evaluation domain  $\mathbb{D}$ , are set to 0 rather than being randomly generated.

## Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
blinding: Blinding factor,
proof_pk_com (Y^-): Pedersen proof public key commitment,
proof_r: Pedersen proof 'R' component,
proof_ok: Pedersen proof 'O_k' component,
proof_s: Pedersen proof 's' component,
proof_sb: Pedersen proof 's_b' component,
ring_pks: Ring public keys,
ring_pks_com: Ring public keys commitment,
ring_proof: Ring proof
```

#### Vector 1

3d6406500d4009fdf2604546093665911e753f2213570a29521fd88bc30ede18,

```
a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b,
c5eaf38334836d4b10e05d2c1021959a917e08eaf4eb46a8c4c8d1bec04e2c00,
e7aa5154103450f0a0525a36a441f827296ee489ef30ed8787cff8df1bef223f,
fdeb377a4ffd7f95ebe48e5b43a88d069ce62188e49493500315ad55ee04d744
 ..2b93c4c91d5475370e9380496f4bc0b838c2483bce4e133c6f18b0adbb9e4722,
01371ac62e04d1faaadbebaa686aaf122143e2cda23aacbaa4796d206779a501,
3b21abd58807bb6d93797001adaacd7113ec320dcf32d1226494e18a57931fc4,
8123054bfdb6918e0aa25c3337e6509eea262282fd26853bf7cd6db234583f5e,
ac57ce6a53a887fc59b6aa73d8ff0e718b49bd9407a627ae0e9b9e7c5d0d175b,
0d379b65fb1e6b2adcbf80618c08e31fd526f06c2defa159158f5de146104c0f,
e2ca83136143e0cac3f7ee863edd3879ed753b995b1ff8d58305d3b1f323630b,
7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313
 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471
..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5
..a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b
..4 fd11 f89 c2a1 aae fe856 bb1 c5d4 a1 fad73 f4de5 e41804 ca2 c17 ba26d6 e10050 ca2 c
..ad6fdeda0dde0a57c51d3226b87e3795e6474393772da46101fd597fbd456c1b
..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9,
afd34e92148ec643fbb578f0e14a1ca9369d3e96b821fcc811c745c320fe2264
 ..172545ca9b6b1d8a196734bc864e171484f45ba5b95d9be39f03214b59520af3
..137 ea80 e302730 a5 df 8 e4155003414 f6 dc f0523 d15 c6 ef5089806 e1 e8 e5782 beautiful filter for the contraction of the c
..92e630ae2b14e758ab0960e372172203f4c9a41777dadd529971d7ab9d23ab29
 ..fe0e9c85ec450505dde7f5ac038274cf,
98bc465cdf55ee0799bc25a80724d02bb2471cd7d065d9bd53a3a7e3416051f6
..e3686f7c6464c364b9f2b0f15750426a9107bd20fe94a01157764aab5f300d7e
{\tt ...2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd}
 ..93818c0c1c3bff5a793d026c604294d0bbd940ec5f1c652bb37dc47564d71dd1
..aa05aba41d1f0cb7f4442a88d9b533ba8e4788f711abdf7275be66d45d222dde
..988dedd0cb5b0d36b21ee64e5ef94e26017b674e387baf0f2d8bd04ac6faab05
..7510b4797248e0cb57e03db0199cd77373ee56adb7555928c391de794a07a613
..49021976970c858153505b20ac237bfca469d8b998fc928e9db39a94e2df1740
..ae0bad6f5d8656806ba24a2f9b89f7a4a9caef4e3ff01fec5982af8731433463
..62a0eb9bb2f6375496ff9388639c7ffeb0bcee33769616e4878fc2315a3ac351
 ..8a9da3c4f072e0a0b583436a58524f036c3a1eeca023598682f1132485d3a570
..88b63acd86c6c72288568db71ff15b7677bfe7218acdebb144a2bf261eb4f659
..80f830e77f37c4f8d11eac9321f302a089698f3c0079c41979d278e8432405fc
..14d80aad028f79b0c4c626e4d4ac4e643692a9adfdc9ba2685a6c47eef0af5c8
..f5d776083895e3e01f1f944cd7547542b7e64b870b1423857f6362533f7cd2a0
..1d231ffed60fe26169c28b28ace1a307fdc8d4b29f0b44659402d3d455d719d8
..96f83b7ee927f0652ca883e4cfa85a2f4f7bc60dda1b068092923076893db5bd
..477fa2d26173314d7512760521d6ec9f,
```

#### Vector 2

8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15, 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3, 0a, -, 8c1d1425374f01d86b23bfeab770c60b58d2eeb9afc5900c8b8a918d09a6086b, 60f32f5ad3e9694b82ccc0a735edb2f940f757ab333cc5f7b0a41158b80f574f. 44f3728bc5ad550aeeb89f8db340b2fceffc946be3e2d8c5d99b47c1fce344b3 ..c7fcee223a9b29a64fe4a86a9994784bc165bb0fba03ca0a493f75bee89a0946, 99ff52abf49d67c4303ac4a8a00984d04c06388f5f836ebd37031f0e76245815, c1322e7a65b83996c25e37a84e36598333b0d417619242c0cb3d9d972edde848, 7a4363e0bf9cd18317287d681ab05704982b0088ce373f696dbdf3909a902b36, fc8770c209212640742d53e2f40e5c30fffae574f90fdc670ff11a1127586c03, 93f7c9d73eec05e500b758f645a2967e62b2206e57eff5f9b99bfc71812e620d, c864de36e0b428f6fb4ef470f94ec9601716cb26ad96f3359e4a1ec110794a0b, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae4376accb086ae4376accb086ae4378accb086ae4378accb086ae4378accb086ae438accb086accb086accb086accb086a.. ad 6f ded a 0 dd e 0 a 57 c 51 d 3226 b 87 e 379 5 e 6474393772 da 46101 f d 597 f b d 456 c 1b d 500 f d..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 81ff2ae0324ba81dbc5f511fadd27d6fa23ff83d45a84ea96ed82f09ad73114a ..79349c978a86386c1a33c09f60c5362a99b73de3fe7f609d6f5f35736a6eb82c ..739943ad4a3d1fe3f1b589d5b173ad3351786b08e07a1369f82fee25b4a16001 ..fe0e9c85ec450505dde7f5ac038274cf, a57818b60d8fc54695a66b49a627b158a2f4141c696f0ac41b16831021e0ce56 ...04aaa76fab504c106e4a50621adcbeeb9107bd20fe94a01157764aab5f300d7e ..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd ..a5d0e0b9c8dfabd2a88713ea7448a6afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c035994f52a06a37045b7fe9bc886afed58c03596afed58c03596afed58c03596afed58c03596afed686afed58c03596afed686afed58c03596afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed686afed68666afed686afed686afed686afed686afed686afed686afed686afed686afed6866afed686afed..00939475c3beae30ee28aabdb0932bacb7967c476a0b2aaa9bc536fd18b487a6 ..5135ae128d4c6fe14dc98160c841a9183ac4a31adf99ad98c4f18368eed0733bac4a4a31adf99ad98c4f18368eed0733bac4a4a36eed0733bac4a4a36eed0733bac4a4a36eed0733bac4a4a36eed0733bac4a6eed0736eed0736eed0766eed0766eed0766eed0766eed0766eed076ee0076eed076eed076ee0076ee0076ee0076ee0076ee0076ee0076ee0076ee0..7b5126d767299e72a086d9cd9fda84ef8392425404173b80a430a9b320c6cf46 ..f203e0b7214333ab49b43bd68bef7db51fd7d55f3332122aa7e65dd990eb5c36 ..fdef18cfe2ef8624e1372d4ae51fb115572e4a67ada192739a8eddfee2f88c53 ..e9072a320d73c78176f8572b8021f5aa2bfe82834b546cd93295bf05d7b6b81a ..fa17e1d1a92e8a4e12672837f603e0b782235fee0b4f3f2673972730c14e224f ..0b6cd6e8a2e24358539a2cc242cf792d9b85cd784a6496192404c6ecc68ee370 ..b75f373ee9d9ba48a2de51d3b3f0a923a9385444eb6396f2ec220cefe3113bf2 ..08f2697fb1625da3c8d12e7ab8d405c8c05cc70074a7e2b76d73e9fc2e05e95b  ${\tt ...303920abf93139baaadba3911e3d2d63ae5335a8be8fb028df0052aded98f2d8}$ 

..1234fe608836b4896b2e080b9b9fa306be342e1aeb95368beb3099a97f2dd0c1 ..b10e54e38efb04c2b8977da7da8dfa801d6997de31337ea2c4ca2ddc77ad4356 ..6a614cc1742a24285ff9da590746aa6d,

#### Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506, 0b8c, c5eaf38334836d4b10e05d2c1021959a917e08eaf4eb46a8c4c8d1bec04e2c00, 67a348e256d908eb695d15ee0d869efef2bcf9f0fea646e788f967abbc0464dd, ..6657d3c449d9398cc4385d1c8a2bb19bcf61ff086e5a6c477a0302ce270d1abf, e22ec3e4a2a4132237eb8a62bcc5ed864593cfde08e53b1632ecd3245761c808, 54c04f259f9e40ee086031d29960b12b6b6407e9de14985001c7265587941831, 9200b650a0c20b0ef73ccd7651ffc7af154e5e02879dc8666025c245aa547f01, 35f8dc0f744d1850513c46b6b4640716cbb4643da26cfe67f8c701486e0b4cae. 5faa89369589174f4202d6e53e8b4ef10a49b2ad8face60d7cb28bfc8f43bf0e, 017093ff8d22ba2f3852141365a1452fbb5ab8cf6f20cb04555e3163f8d88f13, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c  ${\tt ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437}$ .. ad 6f ded a 0 dd e 0 a 57 c 51 d 3226 b 87 e 3795 e 6474393772 da 46101 f d 597 f b d 456 c 1b d 500 f d..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, b2c08aa40031c9f2a66353b9f3b4e30440704e46f277167fc04fdb7812953cc6 ..bcbca4beb8288a2585a26d786c313c8699b0dd6b2e008ea7aa3b00c273da21ba ..02bb6d51e39c8a2caf826bc2860d5b8e07b829647b1164c53ab307b9ca0c2bf5 ..fe0e9c85ec450505dde7f5ac038274cf, a28c6420603f4cd2efd457092ef74585f78eeae389e2ffabf58b9f9dd14ec4db ..9ffe14be02b7376f6ae7959e11ce1e559107bd20fe94a01157764aab5f300d7e ..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd ..a06a150e462634cd8e2a43f4a377bf503f37896a5ff3994e5dfe0d76d7080cab ..3c616ed081e9f13ba653904fce67a0e08476090b55153972ea2b7609d437cdee..cce2475bfd26aede213d5878a93ace5f99620bdabe5970d32feba4b425e600a8 ..f53b81887987ba0c9401bb8fd21d5219fc054674e8f6eea7824b5643e31d0d55 ..da7c33c6610cb183ed9fedca5321be353303da40b63bfc00719c8d253009520f ...67a8f11107b915e404a4b3066632c858314a3c2de105cf69d4ca584ae02c3e4a ${\tt ..60829ae05278cfcd8854e1eff8af8a3bda9d92797d404578011368e726052e22}$ 

 $..7c4e6f164ab9eb72050b7ab4845068ec48c4ef4933da4eb9f7bda8cbaee06d45\\..90d328064880d590e9cf3933d96a6c15d2f2cb1bb2d6247d522089587646f8de\\..386429e448d962155b5ebfcb4962e1ea3186608ce8a0928ae0eb90ebc38fac9e\\..0a78cf12d236efb73ecb943636ec911a8aed1c634ab079758eb1b68aaf765857\\..27dc23bc29c5e05797d7e1c0fc30a5eb00b0a323d8fe728072821dbf1c903394\\..932472e99a82bb70e69505a27ed9ace5862c1e247530904bef58d4ae28f9d8d8\\..2d61a5b7f8c92efeaaaccbe1ba0c9c97,$ 

#### Vector 4

b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,

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672e8c7a8e6d3eca67df38f11d50f3d7dbb26fa8e27565a5424e6f8ac4555dcc, 4d3e0524fc59374f1fdad8e471c695469b45ecf69c1de85c6c1230e888dd4cbe, 36127f8aee7c61048984f0a208bf6d334db9dacbeeeef9ff2d17117e81232832 ..1462eb3ef602f5911d77ab11f815eb4154ba95c934e414198ef000a61b4de31a. 755610da34cc224fbe60ce5e42add2ea6b272ef466aef18c13497363116d1c03, d26274e014ebfc19a9c1a951193858b972eae3360ed35635e89f1f9dbe432be5, 26202144ba4c4cb7ecde831c9e9662bec519493b29a098dd5803a8b4d261fc12, b9fa51c75d278d95f2ccace9609b28ec137b244c8b7d1523b16ed07c8e24b8e4, e42423127a2ca12d4f199287c8fa07784eacf9fc9b86a6bd56ee364cc352c009, 5371d6f9c76b560b4e42b9154a395bed60924d8de31284e926d06af382f5ad1b, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 .. ad 6f ded a 0 dd e 0 a 57 c 51 d 3226 b 87 e 3795 e 6474393772 da 46101 f d 597 f b d 456 c 1b d 500 f d..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 89804d665064e57f7de11fc3eeb50fcc7432972e344d14e9fe0f1c4a62ecb1a8 ..0bf0927ac19553787590c3a834feec1083c8f1104bb55779216dd5051aefe928 ...73b36f331309eaa429bb1bc14f8b31270964bdc061ae6c856e05b19117508112..fe0e9c85ec450505dde7f5ac038274cf.

..fe0e9c85ec450505dde7f5ac038274cf,

84c37dff677bda19d7ce202500196edeabb794b0e0970b52a76061d9fc9c396f
..5d6671db8da091886f4f894775b49a549107bd20fe94a01157764aab5f300d7e
..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd
..a8c53db306c4d0d30a509b2d3b48706279629cd604a3e220fdcd49c44876b6a0
..455419b418f22b18969cec9423e540dd89dd715839826e4aede1002bb651de89
..68c528cf5d2eb41f17e10494c89f3b475aedfbfd5b5de126f7f22c2fe83e2da0
..0dc1af96ea799fe2dcbd9f0985bc69dc6ce5d23c8c51075af7aa2062927f4d73
..b76d01e3ca5b67377c420afac1850d6e053114594e7680222b06c6d1ce239d13

 $..e69c933 daa0b6671896b92456a4856b9bf92dcca0329bd1743ee6dbf0912b347\\..7b1b03294a9b1553d7c79e6bc82448ee76f9dc8e9c869cca60346f0cc2700d20\\..0bec0aa5f9d4f1b91acc3d0e21456779b7a0abcb1a6e52e8126410131c63756c\\..2f161d49f441fdfaf719feeeedcd7b8c9f327eac0c279b42e251b71c104b280d\\..8d53a5dfd6be57c646fff62151f43372f768375472ea3bf5f23a23c040b4c26f\\..9460ee2b0f8b2c3946d8ce0909b80e7ee5608c1735ab06cc5bed2425fd823ce6\\..8938a4b576f245ad0fcab22a6609a9ebfc5c3716d67148dd570f64f2da5cde1f\\..801c0d421eb4f978026ff58aa8cb03118bd50badc10a9ac76c27579d7b074bff\\..cdfe592547df5db7b7cf36de8b7ac43ea2329196d2daef4faf01cafaf016f927\\..a376a1bcb5a8961c8135b8df70b98d34b6d32eade381a55595a3ba9a324d596b\\..5ea2a1f01565430d7ba3965ad07450c0,$ 

#### Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72,$ 

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4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7, 9508104b820469687488d83f729288d9f70fc0523318beff44a47da10d490b3c, 4 ee 61 f 3 c 000544 aa 48 c 565 e 143 e 05 c 6501 a 623 b db f 02 a 0a 408 b 97433660 b 4907 b 100 c 100..715f75890cc0e45cdd7116e3da15b15c3c637782e8e05d05c0d5895e5fe583d1, fb0123dd6317dbd379afccded247f75b3c1c2e32b86eaa9d6c9d0eb5bef07919, a91807f0ee57d2344a8942808bf35c65b5bd4fde16752a98f3e3dc67be8c103d, dbc69ea4dd299deec2f29845e98a17700f07842cf2f6c5e9d88f388fa3a2831c, 311f94e886825c80a30fd44535be37218501bd072afcbc1298f8fba6c3e3c96d, 15f1562046078a7c0d152ef1b56bf3078c763089bf08790f10ff1cf3b9a5030b, f962cb2598032cf5b0a21b4c253514d75c91ace15acde9a3f716e40f70f06804, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 .. decb 0151 cbeb 49 f76 f10 419 ab 6a 962 42 bdc 87 baac 8a 474 e5161123 de 430 4ac 2916 factorial decomposition of the control of the con..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 ..ad6fdeda0dde0a57c51d3226b87e3795e6474393772da46101fd597fbd456c1b ..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 8a73624024659145d99046f7c25332c0427a1a419b90c9c31f11dcb43bfa5dea ..23003968a3487ea5b5de2f9f4372727ac137ab04fec24c03a4ebd77055851650 ..fe0e9c85ec450505dde7f5ac038274cf,

a7185a7a63812926137b53a4776569fe2323e84689e9e2523e03d3c61beb0427
..7bde2c4a2a5e6acecbbfe1c09f16f9899107bd20fe94a01157764aab5f300d7e
..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd
..b2c7a1bdeee0b96e5636a6bc0e91bf3a2309e9a396c1a079f1be004fb8f01c58

 $..a559c750c46b100666e10f841924c3aaafcd90d79db59ea96d9f519cd23e85e1\\..498b18f2a1b5fde389db657178e77d55c946cfcc485343d2e14ed37fb5d39e7b\\..bbccea9103664414fe5eb0b4224721d790674a15b386d31f1d6b4ed8ccc74536\\..305372529273c12476382842eb14722352276ab05e1cdab867fe43b6da8d1149\\..41f5dd46bda9b29681e6210fbfba89c09e123ae6aa788845d4f87dbc42228b66\\..39a51dc61bc79ff3e53da217679648be2d6d4e4a0277d05afa98dd7378672337\\..3c3d8d4932f6258f174793f9597b9ef888e0e8bcb37ce0f79669411163c51570\\..e6ffd7afb89a3d8f43e1e9fdfce7c583c9a6bb32414c6787d83ad3d46812a43e\\..bf758c58417d169b0a8e7f08befd8beea50699e786f54037f8859c362a7b4a17\\..96c40e76ce0c05c0920f0d092c78123470cfc80e13198bca67c003bf554642fb\\..1cfecc2cc5c6a418a628e2d16152c8dadd9e069e3c3ee9f6715766760d4cdfab\\..d545348c67f22e576d6ab90184843e49989ff102693018ca1256aa1d20d73416\\..b6a2cf93aeaf2a985b109aef311e94f1b2b9951a341496a3584d386fe998b9ea\\..b014b5d452cf9489672089388ff5927e37593773ea18e316fd2bd0a8b0566f09\\..f7d36173103dc41ceb82e4dc9fcfc5f9,$ 

#### Vector 6

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, \\ decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, \\ 42616e646572736e6174636820766563746f72, \\ \end{aligned}$ 

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4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7, 9508104b820469687488d83f729288d9f70fc0523318beff44a47da10d490b3c, 4ee61f3c000544aa48c565e143e05c6501a623bdbf02a0a408b97433660b4907 ..715f75890cc0e45cdd7116e3da15b15c3c637782e8e05d05c0d5895e5fe583d1, 0752c5b639dffedf9a66ac111a765d3e9c4cfac9c8b26cc5af6d524967afdf0a, d03caebf8577c1d2ed30a09708683195f11883411dc170e3ea9f09a2cbf86bab, 8b16f0abb2873d6d56199280aeee9e02ce0274a9ca06a3194d6a72c25516ace8, 311f94e886825c80a30fd44535be37218501bd072afcbc1298f8fba6c3e3c96d, 9671cdae8b4cdeea640c24993ccf7e571fcfb3344d81d3cc6f36d03496777c1c, 624e25cd6eccec59b09f0893ef9eab877b55c757b9e9c81260255145bffd9a0d, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae4376accb086ae4376accb086ae4378accb086ae4378accb086ae4378accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086accb086accb086ae436accb086.. ad 6f de da 0 dd e 0 a 57 c 51 d 322 6 b 87 e 379 5 e 6474 393772 da 46101 f d 597 f b d 456 c 1 b d 500 f d 500..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 8a73624024659145d99046f7c25332c0427a1a419b90c9c31f11dcb43bfa5dea ..85fe0551e6e344c777470a6a75665195980c655ed0b3fe459512a901981313ce ..23003968a3487ea5b5de2f9f4372727ac137ab04fec24c03a4ebd77055851650 ..fe0e9c85ec450505dde7f5ac038274cf,

a8455660f642a887ca7bced683e7c5315c6ebff1d7d047ca43f5b5c7b34c244a ...3902f6ca62346b638ed58e4aa5b2c1c29107bd20fe94a01157764aab5f300d7e..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd ..85a6e99a7824dc5f71bbe12cd251236e62139b0fd1f2f17e979f098520fb37c4 ..cee0af5e1fa9f328d351bbe59044eb3f8156b5f8f7ee79d502b573f752c9d913 ..fdb27c803162ca2b0711a1544679c8c40247a02d5c555d5c1f70cc6c98e82adb ...77224cedf52fe9093b6dea787a5f04c67344bae1bc8a73ec67839de08202a95f ..3156b0923bfb4de17fc2fea219af465b11f1285c32008b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a88d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca59a86d7b085e23fa2064b9ca564b9c..95f99f59c457e2ecdffb9dc7fc3c8c169bc1195e70f5e7c49c56329297d0c133 ..9411db06e43c66b2fd1cbe3807d1225f8eb9ca6a5b19653271a51a737224204d ..55e9e06cb7eb1748f7770d9cdcda4b0030d84c0213b76ad5fbc9c36da4c8e21e ..daa6e8dc51fa80dfb938ec531e5ef248867c9d02f3604cbd8172a80a241feb49 ..61ae6a954dc81e266cff5a651b3127700848ae83e88246034d18a55f2e9eba26 ..b2cb13d93cb704fff63665b87bd0ccd595c8561ae10a4484af3a070a20537df6 ..79737410b648dd6c187cdc0277690b882497d53af3a8fcab1243a0bab5a49d69 ..22a4e3ae917b19959a085708d26cd90898763d698a75332d1aa817534af67420 ...f58c4126d7710ff7075d9a78bf4eaaf9ec30a9d4562f6db0f14ae82cf384179b ..8bd41390d2340377998f0ab8fb1547ebd4b45b36110a582db3e0d19ecb8cac6a ..8d35ba072f8176100cdde64c53a14b53,

#### Vector 7

 $35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500,\\ b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623,\\ 42616e646572736e6174636820766563746f72,$ 

-, 1f42,

4315192d2ce9e52ceb449a6b4da7f7e6636e53592c7f5e236763e21e9bac24c7, 6d1dd583bea262323c7dc9e94e57a472e09874e435719010eeafae503c433f16, 09106f062ac07846f3f841f64765527b333575143483855d633f99ccc2e8e306 ..e6239ff79a1272cff931e8d0ac6c390328486329118ad40a18b85184da1837ff, 462ae9ad651e5caf11247b989fecb5f2b1729479c33b9133388d14fa35dbbd0c, 91f1ca92eeaa0b604faf3e4811c12b44991ea33cf582a529a4bc4429a3b6cc5a, b69946f270c46ccc59557bd40288a0a27607281da1892328fdb9da2dcb6c73cf, 5a02419120b814a5c81d67096aac728ee9bda5ddf9451cf554d871462a04831a, bd8c0c1e5e04577c8836e45fb64131d1275309fe28e1d4334b230e3aa639da1a. d93ccbd393ed88c8165b0a01aabe28c56a53b43e527e7927eeadff006dd22114, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313  $\verb|..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471|\\$ ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 .. ad 6f ded a 0 dd e 0 a 57 c 51 d 3226 b 87 e 3795 e 6474393772 da 46101 f d 597 f b d 456 c 1b d 597 f b d 45..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, a359e70e307799b111ad89b162b4260fb4a96ebf4232e8b02d396033498d4216

```
{\tt ...} 305 ed9 cff 3584 a 4 c68 f 03 a b 3 df 87243 a 80 b f c 965633 ef c 23 c 82 c a 064 a f e 105 b a 200 f c 100 f c 10
```

- ..acccbf23e47b543d16c3c4466a83242a77acc16f79b8710051b5e97c85319cf3
- ..fe0e9c85ec450505dde7f5ac038274cf,

88a7fc8a8ae7d295bdd26553b06d298c7d7fdb3f08746aba8e3312d78254a201

- ..2fcba2178cb80851890a656d89550d0bebf60cca8c23575011d2f37cdc06dcdd
- ..807677ad0f02386ef4e6eb99d230aca4c9a73ee3f68397a65c98d6377bf32e1c
- ..e36ae082eafb81393786ab07f7a461798e0d87a09cf3d6ea801fc22d65c907dc
- ..b30ffb60128a307105cdd5db58f93967ec1c0497eba4db9dac64eb2dfe43baeb
- $\dots 30 cbe 5d 2184 c998 b9070 dcda 0c8170286 d793 d583982 adb 77f 01f 2ddd 295 a161 dcda 1962 dc$
- ..61399bcc4b0505757b49074173aca578544d676a44d37fc01321feec35e97437
- ${\tt ...6e2080c28a9bfd963566febe2aac12fe51326001b67abbbdc2d67a24c7605b56}$
- $\verb|..b4c12eb37e41fbace1e45bf37750057ca1bea272955a82bd2b9930d1a4c2f03d| \\$
- ..7dc19572954c84f2175706be08a9e7b12445f50bb4bcd6ed77c3d8f3356cc034
- ..6bc0a6359ca9384239bf09b160a5845710e688c4dbc7cb7e27494f0746818221
- ..f6b6e7af92a58c68f29415fdea5f75ab3e45b1138a7ecde2ff6eac44772f100f
- ..a4e6f4b97c4a401e919dbfd1386253a1dde71a3d6bbf24de9a01e1adca917d84
- ..e3f10888680e2b736f51a1336d7f6099000777bd3a6de5636654012b5873d109
- ..esi10000000ezb/30131a1330d/10099000///bd3a0de3030034012b30/3d103
- .. 4994887 e 59f7 a 6565999 b 922 cafac 10782 ba 51 d 1290 e 344 d d 3 e e 84 a 324 ba 241 d a 241 ba 241 d a 241 ba 241 d a 241 ba 241 ba 241 d a 241 ba 241 ba
- $\verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a32c6fb85bb7164878f2de12c9cece5ef7b92860a55fb25ccb65839a66b1| \verb|..b8c9a3c6cb65ac6cb$
- ..a73d4975f06823422cbe4aac82eef13e88eb6f6fd363bb19436f34b40825438d
- ..c55d3812bb0485d4ae4f9cc67af7074d,

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