Bandersnatch VRF-AD Specification

Davide Galassi Sey

Seyed Hosseini

11 Aug 2024 - Draft 14

Abstract

This specification delineates the framework for a Verifiable Random Function with Additional Data (VRF-AD), a cryptographic construct that augments a standard VRF by incorporating auxiliary information into its signature. We're going to first provide a specification to extend IETF's ECVRF as outlined in RFC-9381 [1], then we describe a variant of the Pedersen VRF originally introduced by BCHSV23 [2], which serves as a fundamental component for implementing anonymized ring signatures as further elaborated by Vasilyev [3]. This specification provides detailed insights into the usage of these primitives with Bandersnatch, an elliptic curve constructed over the BLS12-381 scalar field specified in MSZ21 [4].

1. Preliminaries

Definition: A verifiable random function with additional data (VRF-AD) can be described with two functions:

- $Prove(sk, in, ad) \mapsto (out, \pi)$: from secret key sk, input in, and additional data ad returns a verifiable output out and proof π .
- $Verify(pk, in, ad, out, \pi) \mapsto (0|1)$: for public key pk, input in, additional data ad, output out and proof π returns either 1 on success or 0 on failure.

1.1. VRF Input

An arbitrary length octet-string provided by the user and used to generate some unbiasable verifiable random output.

1.2. VRF Input Point

A point in $\langle G \rangle$ generated from VRF input octet-string using the *Elligator 2 hash-to-curve* algorithm as described by section 6.8.2 of RFC-9380 [5].

1.3. VRF Output Point

A point in $\langle G \rangle$ generated from VRF input point as: $Output \leftarrow sk \cdot Input$.

1.4. VRF Output

A fixed length octet-string generated from VRF output point using the proof-to-hash procedure defined in section 5.2 of RFC-9381.

The first 32 bytes of the hash output are taken.

1.5 Additional Data

An arbitrary length octet-string provided by the user to be signed together with the generated VRF output. This data doesn't influence the produced VRF output.

2. IETF VRF

Based on IETF RFC-9381 which is extended with the capability to sign additional user data (ad).

2.1. Configuration

Configuration is given by following the "cipher suite" guidelines defined in section 5.5 of RFC-9381.

- suite_string = "Bandersnatch_SHA-512_ELL2".
- The EC group $\langle G \rangle$ is the prime subgroup of the Bandersnatch elliptic curve, in Twisted Edwards form, with finite field and curve parameters as specified in MSZ21. For this group, ${\tt fLen} = {\tt qLen} = 32$ and ${\tt cofactor} = 4$.
- The prime subgroup generator $G \in \langle G \rangle$ is defined as follows:

 $G_y \! = \! 19188667384257783945677642223292697773471335439753913231509108946878080696678$

- cLen = 32.
- The public key generation primitive is $pk = sk \cdot G$, with sk the secret key scalar and G the group generator. In this cipher suite, the secret scalar x is equal to the secret key sk.
- encode_to_curve_salt = pk_string (i.e. point_to_string(pk)).
- The ECVRF_nonce_generation function is specified in section 5.4.2.2 of RFC-9381.

- The int_to_string function encodes into the 32 bytes little endian representation.
- The string_to_int function decodes from the 32 bytes little endian representation eventually reducing modulo the prime field order.
- The point_to_string function converts a point in $\langle G \rangle$ to an octet-string using compressed form. The y coordinate is encoded using int_to_string function and the most significant bit of the last octet is used to keep track of x sign. This implies that ptLen = flen = 32.
- The string_to_point function converts an octet-string to a point on $\langle G \rangle$. The string most significant bit is removed to recover the x coordinate as function of y, which is first decoded from the rest of the string using int_to_string procedure. This function MUST outputs "INVALID" if the octet-string does not decode to a point on the prime subgroup $\langle G \rangle$.
- The hash function hash is SHA-512 as specified in RFC-6234 [6], with hLen = 64.
- The ECVRF_encode_to_curve function uses Elligator2 method described in section 6.8.2 of RFC-9380 and is described in section 5.4.1.2 of RFC-9381, with h2c_suite_ID_string = "Bandersnatch_XMD:SHA-512_ELL2_RO_" and domain separation tag DST = "ECVRF_" || h2c_suite_ID_string || suite_string.

2.2. Prove

Input:

- $x \in \mathbb{Z}_r^*$: Secret key
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Schnorr-like proof

Steps:

- 1. $O \leftarrow x \cdot I$
- $2. \ Y \leftarrow x \cdot G$
- 3. $k \leftarrow nonce(x, I)$
- 4. $c \leftarrow challenge(Y, I, O, k \cdot G, k \cdot I, ad)$
- 5. $s \leftarrow k + c \cdot x$
- 6. $\pi \leftarrow (c, s)$

7. return (O, π)

Externals:

- nonce: refer to section 5.4.2.2 of RFC-9381.
- *challenge*: refer to section 5.4.3 of RFC-9381 and section 2.4 of this specification.

2.3. Verify

Input:

- $Y \in \langle G \rangle$: Public key
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.
- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Schnorr-like proof

Output:

• True if proof is valid, False otherwise

Steps:

- 1. $(c,s) \leftarrow \pi$
- 2. $U \leftarrow s \cdot G c \cdot Y$
- 3. $V \leftarrow s \cdot I c \cdot O$
- 4. $c' \leftarrow challenge(Y, I, O, U, V, ad)$
- 5. if $c \neq c'$ then return False
- 6. **return** True

Externals:

 \bullet challenge: as defined for Sign

2.4. Challenge

Challenge construction mostly follows the procedure given in section 5.4.3 of RFC-9381 [1] with some tweaks to add additional data.

Input:

- $Points \in \langle G \rangle^n$: Sequence of n points.
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

• $c \in \mathbb{Z}_r^*$: Challenge scalar.

Steps:

```
1. str = suite\_string \parallel 0x02

2. for each P in Points: str = str \parallel point\_to\_string(P)$

3. str = str \parallel ad \parallel 0x00

4. h = hash(str)

5. h_t = h[0] \parallel ... \parallel h[cLen - 1]

6. c = string\_to\_int(h_t)

7. return c
```

With point_to_string, string_to_int and hash as defined in section 2.1.

3. Pedersen VRF

Pedersen VRF resembles IETF EC-VRF but replaces the public key with a Pedersen commitment to the secret key, which makes this VRF useful in anonymized ring proofs.

The scheme proves that the output has been generated with a secret key associated with a blinded public key (instead of the public key). The blinded public key is a cryptographic commitment to the public key, and it can be unblinded to prove that the output of the VRF corresponds to the public key of the signer.

This specification mostly follows the design proposed by BCHSV23 [2] in section 4 with some details about blinding base point value and challenge generation procedure.

3.1. Configuration

Pedersen VRF is configured for prime subgroup $\langle G \rangle$ of Bandersnatch elliptic curve E, in Twisted Edwards form, defined in MSZ21 [4] with *blinding base* $B \in \langle G \rangle$ defined as follows:

For all the other configurable parameters and external functions we adhere as much as possible to the Bandersnatch cipher suite for IETF VRF described in section 2.1 of this specification.

3.2. Prove

Input:

- $x \in \mathbb{Z}_r^*$: Secret key
- $b \in \mathbb{Z}_r^*$: Secret blinding factor
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof

${\bf Steps:}$

- 1. $O \leftarrow x \cdot I$
- 2. $k \leftarrow nonce(x, I)$
- 3. $k_b \leftarrow nonce(b, I)$
- 4. $\bar{Y} \leftarrow x \cdot G + b \cdot B$
- 5. $R \leftarrow k \cdot G + k_b \cdot B$
- 6. $O_k \leftarrow k \cdot I$
- 7. $c \leftarrow challenge(\bar{Y}, I, O, R, O_k, ad)$
- 8. $s \leftarrow k + c \cdot x$
- 9. $s_b \leftarrow k_b + c \cdot b$
- 10. $\pi \leftarrow (\bar{Y}, R, O_k, s, s_b)$
- 11. return (O, π)

3.3. Verify

Input:

- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.
- $O \in \langle G \rangle$: VRF output point
- $\pi \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof

Output:

• True if proof is valid, False otherwise

Steps:

- 1. $(\bar{Y}, R, O_k, s, s_b) \leftarrow \pi$
- 2. $c \leftarrow challenge(\bar{Y}, I, O, R, O_k, ad)$
- 3. if $O_k + c \cdot O \neq I \cdot s$ then return False
- 4. if $R + c \cdot \bar{Y} \neq s \cdot G + s_b \cdot B$ then return False
- 5. **return** True

4. Ring VRF

Anonymized ring VRF based of [Pedersen VRF] and Ring Proof as proposed by Vasilyev.

4.1. Configuration

Ring proof is configured to work together with Pedersen VRF as presented by this specification.

The following configuration should be applied to the parameters left unspecified by the Vasilyev ring proof spec.

• Groups and Fields:

- G: BLS12-381 prime order subgroup.
- F: BLS12-381 scalar field.
- J: Bandersnatch curve defined over \mathbb{F} .

• Polynomial Commitment Scheme

- KZG with SRS derived from Zcash powers of tau ceremony.
- PCS.Commit \equiv KZG.Commit

• Fiat-Shamir Transform

- merlin library implementation.
- Begin with empty transcript with empty label.
- Push R to the transcript after creation
- TODO: Specify how parameters are added to the transcript as we progress through the protocol
- Accumulator seed point (Twisted Edwards form):

 $\mathbf{S}_x = 3955725774225903122339172568337849452553276548604445833196164961773358506589$

• Padding point (Twisted Edwards form):

 $\square_x = 5259734940318236869621856335705224150406219599146660415951585879123115970561$

 $\square_y = 23297815351169973518610888463679675079080900957871871916328881498043316508082$

• Polynomials domain $(\langle \omega \rangle = \mathbb{D})$ generator:

• $|\mathbb{D}| = 2048$

4.2. Prove

Input:

- $x \in \mathbb{Z}_r^*$: Secret key
- $P \in TODO$: Ring prover
- $b \in \mathbb{Z}_r^*$: Secret blinding factor
- $I \in \langle G \rangle$: VRF input point
- $ad \in \Sigma^*$: Additional data octet-string.

Output:

- $O \in \langle G \rangle$: VRF output point
- $\pi_p \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof
- $\pi_r \in ((G_1)^4, (\mathbb{Z}_r^*)^7, G_1, \mathbb{Z}_r^*, G_1, G_1)$: Ring proof

Steps:

- 1. $(O, \pi_p) \leftarrow Pedersen.prove(x, b, I, ad)$
- 2. $\pi_r \leftarrow Ring.prove(P, b)$
- 3. return (O, π_p, π_r)

4.3. Verify

Input:

- $V \in (G_1)^3$: Ring verifier.
- $I \in \langle G \rangle$: VRF input point.
- $O \in G$: VRF output point.
- $ad \in \Sigma^*$: Additional data octet-string.
- $\pi_p \in (\langle G \rangle, \langle G \rangle, \langle G \rangle, \mathbb{Z}_r^*, \mathbb{Z}_r^*)$: Pedersen proof

```
• \pi_r \in ((G_1)^4, (\mathbb{Z}_r^*)^7, G_1, \mathbb{Z}_r^*, G_1, G_1): Ring proof
```

Output:

• True if proof is valid, False otherwise

Steps:

- 1. $r_p = Pedersen.verify(I, ad, O, \pi_p)$
- 2. if $r_p \neq True$ return False
- 3. $(\bar{Y}, R, O_k, s, s_b) \leftarrow \pi_p$
- 4. $r_r = Ring.verify(V, \pi_r, \bar{Y})$
- 5. if $r_r \neq True$ return False
- 6. return True

Appendix A

The test vectors in this section were generated using code provided at https://github.com/davxy/ark-ec-vrfs.

A.1. IETF VRF Test Vectors

Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
proof_c: Proof 'c' component,
proof_s: Proof 's' component,
```

Vector 1

-, -

b923c55b4b7d8c28156c87e005c6d8385a6f26019eee3149aaeb7ee7ce284b38, 208d1eacbedbfb00708a7068c708a565c0bd41c8155010c52e55c6837fecfa52, 96b48404e1df9c738557ccbdfb5bc6f7b8fa3d281aa51742a5928e7a5d77cf5b ..4fc6ed61fc0f7e073dfc3ee8e06b1e5de55e93ecff8ad926cc99a08e8aa6a779, 106f39b9ba10c49df8dfeeea43f8ff02823110fcd8de3ce6110124d29f75881c, 49584112e665526173bfebb6f8949348b1accf72da122c77b501cd395464330c,

Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$

-,

d905aaf894a97094b1d707ea7685fbc4ac501fc01cef25586a9c36288c5c6302, 25c5ab15ce5d973bfec7b6dd428b5b5971958a056d10cc18d5e9ccd0ee4c7b86, 2ae6660f435f733482e4fb6a2c743288fc1d8a6b173b01f490929cd128514c51 ..8112bed1659bb8eab1535e279f9b7349fa316ba6f7bd8baa4ae410141bb565d2, ac8c53d06bb8c0946c479f1732e16800e810810fedda70f37b8a9c4f1016df11, 9a3d82d40e8600276b5fd92cd8d21287abbece6ee357ff5e086126cf912e3d0a,

Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506,

, 0b8c,

587f7c01731c52ce4e02405a9642bf39da4b62befa0a0811f00dd1710a975cc4, 002030eb901d08fe85873b46cd5a1bd2a2c9fbce4f15e9e39066c1fe91be1c1f, 5ca9dc5e02e908b5f1de31c85d30a064353420ab930a541db5f518eee07fb059 ..323df22d2ce82d36a5bac52aa322f08072cc0b9c555a5e4179e3c11a067de7a2, 2ae1f37e6427ec7f3b71e90b54eac7b0b21425760f46ca78908bc0fd2077ca16, 78c7f35f0b3e8edd83a08a36a70c263cd7dba1ab81a2d6ee60242b4af06f2d03,

Vector 4

b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,

-,

c1cde8432c5bf619b14a403d611140c117a52ba31004574238bd58bf8fc6181f, 5d5a673794b7a0003a1c36f299c4d61055e4b680bb3c2ccd8858dce89c6cd5d3, 0db282523110f629d8c9424afa66f4dfcb9e6dcea5f7891ab2ffc09eeb72a0ac ..11ac36841ec72644a5d24c1fa879872d3091c5e5b81940761f9f8f378f5013ae, 7eb5a8b661e9d93203d7f7aa4b597e695be7c139b457fa5e33a866f4a66f2f12, cde921089ee5ec8d2d940e75819a6347cd8f0ccd215b712f90b278ed186cbb03,

Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, \\ decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, \\ 42616e646572736e6174636820766563746f72, \\ \end{aligned}$

, 8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635, 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 4ddb0d1ebe4d7da9e2cca5c85e39b51166c969dfa30bbf69baafa22121b2000e, 2616dff1f59ff7e7bfc25fa0fea37a9c37e93cf1b88a5e73505a195138590c0c,

Vector 6

da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72,

 $8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635,\\ 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c,\\ 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4\\ ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a,\\ 087914abfd2a59a593384c538bb2f11480d4b196ae2a973ac33cb7dd2cc1541b,\\ 9ad1cdabc97035a05d76c4f4e3c1826deafbc3e4d41df6bf66eaa21d1ba63018,\\ \end{aligned}$

Vector 7

35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500, b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623, 42616e646572736e6174636820766563746f72, 1f42

69dec7fe79f816d095b04cead45e856ff6c7e798f513e09291958e35a5590443, 9adeacd15eacdc651e4db1ea4c0917973eac2000479edf6132f3774601cc6902, ff5f6324ea18bbb4df92f7d6304bf27a0a44fa80fd40b985de8d43963a7e02c6 ..ef6f0947911604155c6fe40f68cc91c96ffd358275b58960554274498a70f144, 50a14bab81a42e118e8c167136db35b731a9194a250ae5e65452592742cbdb0e, a75b5327d1b921bb72e2e8c525c18d2fce661b365379ae9f1168c75d281d0100,

A.2. Pedersen VRF Test Vectors

Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
blinding: Blinding factor,
proof_pk_com (Y^-): Public key commitment,
proof_r: Proof 'R' component,
proof_ok: Proof 'O_k' component,
proof_s: Proof 's' component,
```

```
proof_sb: Proof 's_b' component
```

Vector 1

3d6406500d4009fdf2604546093665911e753f2213570a29521fd88bc30ede18, a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b, -

-,

b923c55b4b7d8c28156c87e005c6d8385a6f26019eee3149aaeb7ee7ce284b38, 208d1eacbedbfb00708a7068c708a565c0bd41c8155010c52e55c6837fecfa52, 96b48404e1df9c738557ccbdfb5bc6f7b8fa3d281aa51742a5928e7a5d77cf5b ..4fc6ed61fc0f7e073dfc3ee8e06b1e5de55e93ecff8ad926cc99a08e8aa6a779, a3f1a139943f3dc02c624505a5794dcc1a75651f60ca69081ebf9bdbd7458616, 2882f90320afdcf99680b8662efe846e2fd477cce00a47ac154f996c910b920a, 71d85bb1a0edcf4362ec8137cdef1a856096e4f9995cc3a4db1781d3e9c7b817, 647c218cec9610102b202bcf7d29bdbf91770c326f07586051fa40bee863b63e, cda38b375717fa7790c18c70dcfcd6ce8f19b13819f088b74688f21dd127c412, 9b52eff1cc2ab908070a1ba89059ae3f6823b43702c60272c5d5943cceb6ac0e,

Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$

-,

 $\label{eq:d905aaf894a97094b1d707ea7685fbc4ac501fc01cef25586a9c36288c5c6302, 25c5ab15ce5d973bfec7b6dd428b5b5971958a056d10cc18d5e9ccd0ee4c7b86, 2ae6660f435f733482e4fb6a2c743288fc1d8a6b173b01f490929cd128514c51 ..8112bed1659bb8eab1535e279f9b7349fa316ba6f7bd8baa4ae410141bb565d2, 85a94726bcaeaf2db516a6a532ec2450488e7d093374f54de0ba05d2a36bb00a, b28263558234202119a143c295a3fc5a35a6f830dd0c7018e3f33862d1986c1c, 4cb8186c3da92e9be0179f894cdc364aabe1a890340aee9fd886bed45f5017e7, 83a9519edb8ecc4f360eee599c6c1310019c4c3451ca42b4887328e347003bdf, 3e1b408e4ceb5a81e5b71527b01f541d5069438aaa279aa48c39bb7e34f24001, 1dc7b84f188a7fb5bf051464be19e54495f42bd723130992319bad7560023714,$

Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506,

0b8c,

587f7c01731c52ce4e02405a9642bf39da4b62befa0a0811f00dd1710a975cc4, 002030eb901d08fe85873b46cd5a1bd2a2c9fbce4f15e9e39066c1fe91be1c1f, 5ca9dc5e02e908b5f1de31c85d30a064353420ab930a541db5f518eee07fb059 ..323df22d2ce82d36a5bac52aa322f08072cc0b9c555a5e4179e3c11a067de7a2, cb3a17d3578d86e2f3b23bb47160327c391c808da28c6be53ed3189d22d78205,

f99d09a38f1a1ead7d9503fd601e2d8a56c09eaeb5fb3130035803e04033b49a, de58f590cd204247192f5b49d86c81ddc691fd6b55561fb33ccbec24ecbc86db, d502f832afaddb7bb54e8c28cce458a2a9c3c6c230e4b85539913ec531de168b, 1dd33771a9bfdcf94e6e95fa43e4667adf3279d9c2b22e0877abeb5e99a9e01b, 7863bbac83653e1a48bc0e814e4792c6b2d884522f5556bbb1844c151dcdb700,

Vector 4

b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,

-, c1cde8432c5bf619b14a403d611140c117a52ba31004574238bd58bf8fc6181f, 5d5a673794b7a0003a1c36f299c4d61055e4b680bb3c2ccd8858dce89c6cd5d3, 0db282523110f629d8c9424afa66f4dfcb9e6dcea5f7891ab2ffc09eeb72a0ac ..11ac36841ec72644a5d24c1fa879872d3091c5e5b81940761f9f8f378f5013ae, 141a8a762dff63c7c05b26d022a8027c515e57f067b5546532296f0ca40a1909, e926e6b3cbca7b66c42cfc603c4ef2dabc3f5e1276b20d2807f007e974675cb1, 29c56732de262411e71908326037f0f961776db2082bf3d88537265af6a57c92, c59024c715d21f2a08fb0cd8cb24046558222c6753180853f9601d92186c5e3b, b818a32590aeb6d79d24cdc6cacb6d5cdc58ccb7025b82be1c1ba2cd34c2f005, e854b63f9c4e0aab3a051885498d42b5ec354e619491ee9ff239bd3fb486b509,

Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72,$

8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635, 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 4749f32b7aa36158a4fdfb5bc7e63c40b62eb1d7c75036676e093571a3e9cb06, e159e5494957bb478c4a4d142cde10dadd73a038f8b198c4321dff1271ab61b4, 16a8409cc245978bf55279447d854adca637a58c8c7894a0972b190ad7314492, 3639790d6414b474aa1d53de4e7a896b4e6458c078867acd22200f00f20f280a, bbfd0996c8937c9aaabad9a254614b75c529f892fdfcfcfbe73486888545b610, 6bce65ffb002c6349213b720115ee1457214796c983618f32b4b79c8c559851b,

Vector 6

da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72, 1f42, 8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635,

022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 1f64d22282d00a58d17d4fe4dc6e8b9772109b6091e1684649c6084fc842391b, 89e230c832f5c2ee1072d9d110151a2dafa4577d64b7fb0845855ae3d1c12fec, e3bd5e3a3f07efb256c989f22fcfe8494219dcd37b35419f5f10da68de09f125, 3639790d6414b474aa1d53de4e7a896b4e6458c078867acd22200f00f20f280a, ceff5ef2315be8be839b1f3c0314b72d976c2e14a2a27c2d1ce8465e90c98607, 0ea7abf79fc1bdebc8b9009cc5744358071c12e82a31565d35a8f91069b55c1b,

Vector 7

 $35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500,\\ b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623,\\ 42616e646572736e6174636820766563746f72,\\ 1f42.$

 $69 \\ dec7fe79f816d095b04cead45e856ff6c7e798f513e09291958e35a5590443, \\ 9 \\ adeacd15eacdc651e4db1ea4c0917973eac2000479edf6132f3774601cc6902, \\ ff5f6324ea18bbb4df92f7d6304bf27a0a44fa80fd40b985de8d43963a7e02c6 \\ ..ef6f0947911604155c6fe40f68cc91c96ffd358275b58960554274498a70f144, \\ ea1f922fce5e359d92e0fdcda53a1d2e6b791c7e7a8ffad915f3535c6175f115, \\ f674ad5f72661aa0c2bc5ca83aee9794c8b8bbc4017abcc00a11a23a0b558e68, \\ f77eaec55fe36b06f1d1f7eef7db24fdcce74c83fde19b1c322aca288e39948f, \\ b846dfbceb2a74fe102b3aec94e7b8460f5adcb609c407839ab6cb06d1e3bd38, \\ 35a41d1cb4d22b5c162d319b206db940b6fcef71bbe0c13a6376a89788292519, \\ c04b177f954d17e7c129ce8d55cb7f148b3957078c96e7229100dc50b7d62b02, \\ \end{aligned}$

A.3. Ring VRF Test Vectors

Generated using Zcash BLS12-381 URS.

Schema:

```
sk (x): Secret key,
pk (Y): Public key,
in (alpha): Input octet-string,
ad: Additional data octet-string,
h (I): VRF input point,
gamma (0): VRF output point,
out (beta): VRF output octet string,
blinding: Blinding factor,
proof_pk_com (Y^-): Pedersen proof public key commitment,
proof_r: Pedersen proof 'R' component,
proof_ok: Pedersen proof 'O_k' component,
proof_s: Pedersen proof 's' component,
proof_sb: Pedersen proof 's_b' component,
ring_pks: Ring public keys,
```

ring_pks_com: Ring public keys commitment,

ring_proof: Ring proof

Vector 1

3d6406500d4009fdf2604546093665911e753f2213570a29521fd88bc30ede18, a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b, b923c55b4b7d8c28156c87e005c6d8385a6f26019eee3149aaeb7ee7ce284b38, 208d1eacbedbfb00708a7068c708a565c0bd41c8155010c52e55c6837fecfa52, 96b48404e1df9c738557ccbdfb5bc6f7b8fa3d281aa51742a5928e7a5d77cf5b..4fc6ed61fc0f7e073dfc3ee8e06b1e5de55e93ecff8ad926cc99a08e8aa6a779, a3f1a139943f3dc02c624505a5794dcc1a75651f60ca69081ebf9bdbd7458616, 2882f90320afdcf99680b8662efe846e2fd477cce00a47ac154f996c910b920a, 71d85bb1a0edcf4362ec8137cdef1a856096e4f9995cc3a4db1781d3e9c7b817, 647c218cec9610102b202bcf7d29bdbf91770c326f07586051fa40bee863b63e, cda38b375717fa7790c18c70dcfcd6ce8f19b13819f088b74688f21dd127c412, 9b52eff1cc2ab908070a1ba89059ae3f6823b43702c60272c5d5943cceb6ac0e. 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..a1b1da71cc4682e159b7da23050d8b6261eb11a3247c89b07ef56ccd002fd38b ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 ..ad6fdeda0dde0a57c51d3226b87e3795e6474393772da46101fd597fbd456c1b..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 89e2e79b6178c12684ac3a6bf9437af3a69dcc529f0021ec40bb006506837ae1 ..82bf4b908e46733d3a23507791169fda8ea11b18665fe894ee9f0754c0c3fec7 ..0c6b8d1444d9b604ce949cbf130642d89f72b6cb1f08e32a18cdbb00aadfdf1b ..fe0e9c85ec450505dde7f5ac038274cf, ae5cf31262305cc0d061a88275a6afeed634e1c7566c79c720812529759a5845 ..810a0e88b88aa8ae60f937f1ca4f21da914ad8aba8c1092777dfb212803dfd38 ..9c0e3b1678c6e5e0bf02c3c8179537ca205bb3580e43016ccb584c0c08b1114a ..9756566a3bf9085c2669ccf6e6c6fb939a67624a147ebe76112bab427d22edc0 ..141fe2828bd0cda558a0b16cf94ad7e4afa58b5c4a914114f7d03572b527346c ..ef906d05c896b820c8ade0122fe727bd262ee283af9b30bb67e8b926f8fa041b ..a9789444ce27e82c0caecb12c94b4a0fcb1514ee4905b644b229b1f7f8734e63 ..42ee26310278b91edac541e2d20d28a826f3ee7182acf582e2f1103d90ee3a63 ${\tt ...69b4f91f800eff0bfc83fc9463cc100104de722c78a897f8a5bcac3fa555724d}$..3cc17b108fa299caea8a5e14f12bc3da02a51af9ee175c6aea68532d1d7e0c2c ..4a87507258bd695edab7ea8d6f86c47c84c29d7fd237590fae5a2df5280c8601..684042360445eaf6a6f6b0a38017ce2cf710004662e0a987d87929c15ee74742 ..e5e0c83147132513675439c879219b61283005541826556f1700964c3345b62d ..b39e9b2db283d075ef33af3a97344467351ab3b800c29a13c5a02289d0c15868

..e3811debfe757222ea63a0f35bfb91c251ac3e4c74e838af87949b00756a363d ..f0418975ad1b8ca3cc557c6d0896c944adeaf58838f2b8b1bfabf18f05fd4abd ..41a35135149da932e3617bc2ff49f021d9aebe34ca1d3e7fab1ace2b7cfb4fd1 ..b2f41416e104129b952a6b9d22d4bb0802668fbc5e2e9735c69b3f44eaff87a4 ..bcd6a9a71991feaaa7e7232bc18a4733,

Vector 2

 $8b9063872331dda4c3c282f7d813fb3c13e7339b7dc9635fdc764e32cc57cb15,\\ 5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3,\\ 0a,$

d905aaf894a97094b1d707ea7685fbc4ac501fc01cef25586a9c36288c5c6302, 25c5ab15ce5d973bfec7b6dd428b5b5971958a056d10cc18d5e9ccd0ee4c7b86, 2ae6660f435f733482e4fb6a2c743288fc1d8a6b173b01f490929cd128514c51 ..8112bed1659bb8eab1535e279f9b7349fa316ba6f7bd8baa4ae410141bb565d2, 85a94726bcaeaf2db516a6a532ec2450488e7d093374f54de0ba05d2a36bb00a, b28263558234202119a143c295a3fc5a35a6f830dd0c7018e3f33862d1986c1c, 4cb8186c3da92e9be0179f894cdc364aabe1a890340aee9fd886bed45f5017e7. 83a9519edb8ecc4f360eee599c6c1310019c4c3451ca42b4887328e347003bdf, 3e1b408e4ceb5a81e5b71527b01f541d5069438aaa279aa48c39bb7e34f24001, 1dc7b84f188a7fb5bf051464be19e54495f42bd723130992319bad7560023714, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..5ebfe047f421e1a3e1d9bbb163839812657bbb3e4ffe9856a725b2b405844cf3 ..4 fd11 f89 c2a1 aae fe856 bb1 c5d4 a1 fad73 f4de 5e41804 ca2c17 ba26d6 e10050 ca2c17 ba26..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae4376accb086ae4376accb086ae4378accb086ae4378accb086ae4378accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086ae436accb086accb086accb086ae436accb086accb0.. ad 6f ded a 0 dd e 0 a 57 c 51 d 3226 b 87 e 3795 e 6474393772 da 46101 f d 597 f b d 456 c 1b d 500 f d..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9 894fd4149cce66e5f39f11c0de38825da7d07c52de1d8e74ed170c6b1a2feec7 ..bc158b35068bbcfa9455fd76f699c15cb5e9dfaba7a93cb264c07d9228e8c642 ..92e630ae2b14e758ab0960e372172203f4c9a41777dadd529971d7ab9d23ab29

 $b8cba6c3a415bdb4104a228dc4a431325a8d61b15786a3d7f1450387dbf2b84d\\ ..f1568203e80a81c1293f6b0fbe4f7e6b914ad8aba8c1092777dfb212803dfd38\\ ..9c0e3b1678c6e5e0bf02c3c8179537ca205bb3580e43016ccb584c0c08b1114a\\ ..b758b758807789cc310a41ef5f43cffbab02c806d9cd83d3fe41879b6ee0bbf2\\ ..99179e16cf5f6f76346215dd0ea22525a65893a06327a22dd6c5164285768387\\ ..924e0ca971569a70bbd084fd9dbcc5c8f969fe4e3146a51e8441740b79d2a73d\\ ..5a2eea6ad577933b99821a94b5630a03b55f44cc7c189c76834676c1d5ce492c\\ ..e5ae09a8ecdc7830884a0125b92b38e71ee3b1142da83fd9431c4dac81fef152\\ ..322c5bc6ed916f61e619b10c26b6c05c9fa2ddf3ac6d667bacaa602324b27b25\\ ..cbb4f64138d23da3d647375bd60680b9cd0cc23ecd1d115ae7c4ae5581857143\\ ..e508623ef4d7c3d9a4242309c91ad643cdbd099a451f616ec192fc45e68baa45$

..fe0e9c85ec450505dde7f5ac038274cf,

..2c31f3b8eb737f70105deb3318fb56f10e56ba042106a3da3c7d743b461d383e
..80f6a56ae688c6e0bcbbfbf760b0067debd79fdc39c3effae076c9133cbb9a6e
..970219c73f080bba8e32bb467f687f124c8986bcd2413f31aa39247c86c6a03e
..18617ea068d6ff5231374b028d621935826b8ae29351a9c006f4ff56e19247a7
..b8fbd55fc31eb427854bad441e3da66096994659ffc40d227668cd2c527fafab
..c92e219e57a9cff6b73532ee8c76225e7aaa402141bc0e9013b5880ee43ba547
..ac4ed2c8deec88414533eca85dc1126386c93e1427999bee8beee14d14a39f06
..3ebaba896479110b7773742af2f24a84,

Vector 3

6db187202f69e627e432296ae1d0f166ae6ac3c1222585b6ceae80ea07670b14, 9d97151298a5339866ddd3539d16696e19e6b68ac731562c807fe63a1ca49506, 0b8c. 587f7c01731c52ce4e02405a9642bf39da4b62befa0a0811f00dd1710a975cc4, 002030eb901d08fe85873b46cd5a1bd2a2c9fbce4f15e9e39066c1fe91be1c1f, 5ca9dc5e02e908b5f1de31c85d30a064353420ab930a541db5f518eee07fb059 ..323df22d2ce82d36a5bac52aa322f08072cc0b9c555a5e4179e3c11a067de7a2. cb3a17d3578d86e2f3b23bb47160327c391c808da28c6be53ed3189d22d78205, f99d09a38f1a1ead7d9503fd601e2d8a56c09eaeb5fb3130035803e04033b49a, de58f590cd204247192f5b49d86c81ddc691fd6b55561fb33ccbec24ecbc86db, d502f832afaddb7bb54e8c28cce458a2a9c3c6c230e4b85539913ec531de168b, 1dd33771a9bfdcf94e6e95fa43e4667adf3279d9c2b22e0877abeb5e99a9e01b, 7863bbac83653e1a48bc0e814e4792c6b2d884522f5556bbb1844c151dcdb700, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..9 d97151298 a5339866 ddd3539 d16696 e19 e6b68 ac731562 c807 fe63 a1 ca49506 feasible for the contraction of the contraction..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 .. ad 6f de da 0 dd e 0 a 57 c 51 d 322 6 b 87 e 379 5 e 6474 393772 da 46101 f d 597 f b d 456 c 1b a 6474 a 64..3 f9 dc0 c4 f67 f207 974123830 c2 d66988 fb3 fb44 becbbba5a64143 f376 edc51 d9, and becbbba64143 f376 edc51 d9, and becbbba641443 f376 edc51 d9, and becbbba641443 f376 edc51 d9, and becbbba641444 f376 edc51 d9, and becbbba641444 f376 edc51a90130fa47aaf758299818bd119e7fecdddb62674541f78c5fa5371b9db62d0f ..8 afd 73d 28225fb 1ae 60e 8959c 5f0e 929b 861ba 122a 1c8fa 45fc 9d 2b8fb 66666e..f55fdcdfdae22addff823236613fb08b49a694b9f1ec38b72fc0a021857d3026 ..fe0e9c85ec450505dde7f5ac038274cf. a5b1c2890ca37ac292af4f1b736f60fdb463fd46dfc029ec1d490286cad8411d..bd87f54a692d702c3bf63ef31789e91d914ad8aba8c1092777dfb212803dfd38 ..9c0e3b1678c6e5e0bf02c3c8179537ca205bb3580e43016ccb584c0c08b1114a ..b4662ecc45202cdbb759a76aae448f7ee0e0ef12d6d4a49fe9c15a8d5fe432c6 ..2073680cb2744396ca33d7c44b90b1f3a6f836fb65712a521a33bacc26103c8b ..aa81afc9a8931a7b5f9e3ea7bc5d30fef146bdfdcab3d7b92ee2bc4a9a29af38 $\verb| ..cb68cb84820f0c4d9c4001b18ab21f80fb4cfec3d079a7d3bed13e32a996362e| \\$

 $..693983d2ff6be8002b0278718e4529759ab3e3a275428cdbe6788b92938ef400\\ ..ce55785c59dd2f1f4d510abaa80c3e6655100041da2726288f81ba7a733b856b\\ ..0f62cd45dcd49628e368b2d5fa1c5ca055fcd68a339779f3789d05cb28cf9909\\ ..fa545aa5e98cb68a56f247e387c3cdf5c62c52cb0886d25d77061e67f3d02a4a\\ ..409df07a6ce6e6d42e4ed84b95f2d24b31bbfdc4c10c1c6769e625f2158ff75e\\ ..81a4bb6e4aa3b77f9ba1009ac4337b4a40e1f32f36c9fe2739a908f0961d7842\\ ..420fec2b8ba6a74594873fa68609ba6f6174959ee58fd2823cb582120f5f1f06\\ ..e926e6a0e03ca3041ffa8e5b59c5e549ada411c6eaa4511bc779e4f9923f0fbd\\ ..f74511f723b015c45c77f9621a9280976aab33de095292f8846f9b3bd61b353e\\ ..a71f1d471a3dfc5734b21c2595af996b21946699d864af09663e061afd370aa2\\ ..d61e9b0a1a95f2e97fda089ce75df1d5,$

Vector 4

 $\label{eq:b56cc204f1b6c2323709012cb16c72f3021035ce935fbe69b600a88d842c7407, dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99, 73616d706c65,$

c1cde8432c5bf619b14a403d611140c117a52ba31004574238bd58bf8fc6181f. 5d5a673794b7a0003a1c36f299c4d61055e4b680bb3c2ccd8858dce89c6cd5d3, 0db282523110f629d8c9424afa66f4dfcb9e6dcea5f7891ab2ffc09eeb72a0ac ..11ac36841ec72644a5d24c1fa879872d3091c5e5b81940761f9f8f378f5013ae, 141a8a762dff63c7c05b26d022a8027c515e57f067b5546532296f0ca40a1909, e926e6b3cbca7b66c42cfc603c4ef2dabc3f5e1276b20d2807f007e974675cb1, 29c56732de262411e71908326037f0f961776db2082bf3d88537265af6a57c92, c59024c715d21f2a08fb0cd8cb24046558222c6753180853f9601d92186c5e3b, b818a32590aeb6d79d24cdc6cacb6d5cdc58ccb7025b82be1c1ba2cd34c2f005,e854b63f9c4e0aab3a051885498d42b5ec354e619491ee9ff239bd3fb486b509, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..dc2de7312c2850a9f6c103289c64fbd76e2ebd2fa8b5734708eb2c76c0fb2d99 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 .. ad 6f de da 0 dd e 0 a 57 c 51 d 322 6b 87 e 379 5e 6474 393772 da 46101 f d 597 f b d 456 c 1b a 100 f d 100 f d..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9. b62f3bf3e83646318894151bb51bb535a2539581773a01956f1874cb64e7a952 ..809d40be330de7d34bf01162adb2675e94c21ba7db9087beeb87d536cce326fb ${\tt ...20a5b816654432c73a772ede266d0d3bbae3f6aa0bcb31b5de62d33863a0098aa}$..fe0e9c85ec450505dde7f5ac038274cf, a6e4e910b16cd26a1e1c92b407e8616a521d5640c59129e44d3c5ed3248c9fa9 ..0eea775fff48c0f7596d56e321c0013f914ad8aba8c1092777dfb212803dfd38 ..9c0e3b1678c6e5e0bf02c3c8179537ca205bb3580e43016ccb584c0c08b1114a ..a1c341fe390f7ef3e45fa49d12fc655ba90805eafd0dee2442ef2618208692e4 ..e4931cbbf11ac60aa0384e965036023e85ff0e1317f4a1ce508a40f781596644

..a923d723c0eee4b6acc12e689cd74616ed6d5d38551e5eec82a3c4e6e462de4b
.b95d314c5883fbaa2b101576f5896af8eeb01ec35af191d94371107c79dc345f
.4823f4122a46d5eb06635b11f58e4a93cd59e91e5d7d0b30ce37b11b147f7c08
.661b89b1f0dec5b74602957c6325639cb0f4da1077442441d902799eaef9d92e
.c05d026c4282859350b82be6ee0ec3a484b9252de9522988f42144103a8c8453
.30b002c4f1da7e45b0d6e363141e3097284be9ac846113806e9d2d2123bdae43
.2bf1419d2b6ac8e83fff2bc501938fc9f39ccd8be1ae6c5520419236fc1c791a
.49219e97029ced5760d79eb5c6cb1facab6b83147eb86f6cddaa9990c9fe6f48
.8314bbfc225ed62f9cf14241a49ef9289ad537fa3f36c1fd1fde14ec79ae5eb9
.29ab90ad7d2ea5cfe4b680b8dfe42c408b547cbef65033aadf30aa369984d430
.73b2992b48e2a7e2a22da52f6872fc46afb4f8ac25e80bbd3deeb6e6bed84e9e
.b0d3a832987a1fc58c9739fb772cd89a9c352d8c055f7545735d9b38fb5b9857
.91c4f730c95137aeefb8fb591a95b0624984533b97ac182b390210312b2be0b8
.474d3e230efda6f36f276ea6e8712775,

Vector 5

 $\label{lem:da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72,$

8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635, 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 4749f32b7aa36158a4fdfb5bc7e63c40b62eb1d7c75036676e093571a3e9cb06, e159e5494957bb478c4a4d142cde10dadd73a038f8b198c4321dff1271ab61b4, 16a8409cc245978bf55279447d854adca637a58c8c7894a0972b190ad7314492, 3639790d6414b474aa1d53de4e7a896b4e6458c078867acd22200f00f20f280a, bbfd0996c8937c9aaabad9a254614b75c529f892fdfcfcfbe73486888545b610, 6bce65ffb002c6349213b720115ee1457214796c983618f32b4b79c8c559851b, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 ..ad6fdeda0dde0a57c51d3226b87e3795e6474393772da46101fd597fbd456c1b..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 9436b3535d5dcffd6f15628fb028095f5c0733d067222f8893bb106f2fdac0f6 ... 3 dfc f69a5715522c7318b9b311264ee5a2b499057db5d1211e6b9f4633ad433d..22dce5f20a95b8a8618b99539bb697791e02b1afcf6e2de8240d067396196b83 ...92e630ae2b14e758ab0960e372172203f4c9a41777dadd529971d7ab9d23ab29..fe0e9c85ec450505dde7f5ac038274cf,

8af467c31da3b5e558a885b12d7ef226df7a8061e67229ab2bf39271d1c5d292...e9ce57631c8e04d372168d669a39fc5d914ad8aba8c1092777dfb212803dfd38

..9c0e3b1678c6e5e0bf02c3c8179537ca205bb3580e43016ccb584c0c08b1114a ..b16ecd5427ea98b2ae4a7fd8492356cdc3cac256cbca4aa1bf816890794e2f78 ..dbebb9a0827679bd8e2e3d4d645d05fd86355b4350c56886912878aeae1626a8 ..b7dc38866c27ca62a3824e2286d125907639602f49b36bace5e98e85a0fb0a33 ..09560c6efc420851f634642ddafefcc3494e904ac78b68f6a8c4c66cc5c06305 ..fc39025d4b2ababf059975eb47ec3e64e93fabc0d11695e6e39cea883b83fc28 ...0e13c2df6ba448b21ee03dd938e4a3b1958231255033828a9f1f16fa95768f72..531 a e 8 df 20 f 04923 cc 20312641 b 4 a 3 b c 8 d 4 c 3 f 8524392 f 8 b a 004 a 02 e 4 a c 9354 d a 2012 f 8 b a 201..9cdff871a6cf080db416b89e414d7895e5cb5ebccb6ac8b8c93a215185a78c71 ..928cbfe0d635d6625bdde37d110bd1005da656a0b03cfe6372e8fc48f7f31f6c ..eff5c9a971012e038bf10f824bd8c8710da8b10854656e4cf48b32fe4a23de57 ..96d770aff3b9a6faac33ad5a9a88f6ed524f5e6faa46bd0375bd2d07211ea2eb ..6d8341fd2e67d5bad6e71ec09ff141df80f8b1507f427d972d2d2bd30e1f1595..6b7108371df92eafe7b1ccf7b384705d8abb439bffbeea9e9f0d7caa06ef99c0 ..e4e4b32776953d16c0c7cea0a6f544d48e4eba06d1d541c51a2ac72f80b905eb ..9742dbba4aff852f64aa519eb65aac2f35b71f94c9263ef3b39a7b815b46a5f6 ..d6d28374c8ebe9a6df5103f6e253a8e5,

Vector 6

da36359bf1bfd1694d3ed359e7340bd02a6a5e54827d94db1384df29f5bdd302, decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29, 42616e646572736e6174636820766563746f72, 1f42,

8af6936567d457e80f6715f403e20597c2ca58219974c3996a4e4414c3361635, 022abfa7670d5051a6a0e212467666abb955faafe7fe63446f50eb710383444c, 126296afb914aa1225dfdddfe3bfd185b488801810e18034330b1c07409ccdc4 ..f8deccfc30be219cb5186f80a523ae41720031ae39a78f18d3b14df8bb6d8e8a, 1f64d22282d00a58d17d4fe4dc6e8b9772109b6091e1684649c6084fc842391b, 89e230c832f5c2ee1072d9d110151a2dafa4577d64b7fb0845855ae3d1c12fec, e3bd5e3a3f07efb256c989f22fcfe8494219dcd37b35419f5f10da68de09f125, 3639790d6414b474aa1d53de4e7a896b4e6458c078867acd22200f00f20f280a, ceff5ef2315be8be839b1f3c0314b72d976c2e14a2a27c2d1ce8465e90c98607, Oea7abf79fc1bdebc8b9009cc5744358071c12e82a31565d35a8f91069b55c1b, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471 ..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..decb0151cbeb49f76f10419ab6a96242bdc87baac8a474e5161123de4304ac29 ..4 fd11 f89 c2a1 aae fe856 bb1 c5d4 a1 fad73 f4de5 e41804 ca2 c17 ba26d6 e10050 ca2 c..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 .. ad 6f ded a 0 dd e 0 a 57 c 51 d 3226 b 87 e 3795 e 6474393772 da 46101 f d 597 f b d 456 c 1b d 500 f d..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, 9436b3535d5dcffd6f15628fb028095f5c0733d067222f8893bb106f2fdac0f6 ..3dfcf69a5715522c7318b9b311264ee5a2b499057db5d1211e6b9f4633ad433d ${\tt ...22dce5f20a95b8a8618b99539bb697791e02b1afcf6e2de8240d067396196b83}$..92e630ae2b14e758ab0960e372172203f4c9a41777dadd529971d7ab9d23ab29

..fe0e9c85ec450505dde7f5ac038274cf, a12e0817ce74eb4ca03e2062e1470d92173eba8d25a931b34856bace49ca0798 ...199218268a5cb7eb627c3e7a4c9a37e5914ad8aba8c1092777dfb212803dfd38 ..9c0e3b1678c6e5e0bf02c3c8179537ca205bb3580e43016ccb584c0c08b1114a ..a68e0243c11f79b48e15a79e552271053cb87b511a0e07d66557b0e60ff8e8eb ..6b8c00630f6f1f885569fd4de3e0565e9260fcc2be2f62922728c9aa58bc1e57 ..5eb3de75aa871f738dc2e86d7cab09ec924ec3eeda5bf7e226104c93df965e46 ..5c62365cb0a9d995867bab19da3cecdec967be8db19881c58358c30b778d3b4c ..783af4e70a9e2bb3be8acfb929772e717c0c533585d96ee42f12d21c7deba934 ..5e07fdadb158ce200294089daaef01db40361366cb3677bacc07b656e2dd380a ..20de6c88a436bf991a97e7d783e39a71534500af00f59a4d8317735fb38d8628 ..7d79a69dadbb9a7c886c125ec3169e4d931ce0aeece7cb704d2191d587210106 ..9f5d02c97d44e713d8507b814c5cde061b361e4ec3d7022b2624f15fd0cbdd0f..33462f57672c9cf12d6d20aeb03c7da7f3f25d0ccb35f295094e034eaa11b154 ..8ca4d3ce10b52a176dcbf05f33a574a6e44f99715e8a2f7e8a27f530219fecc8 ..994b0e595b715fd4d41027c9eeaa4438523bebdd270c9d8bb4b50baab7710877 ..a410431d48e389228b9c8f4abdf6050fa4a3dd6f2204e275b0526c219636266a ..85c7005fed57d50bc53fb758af199ed53e4d4f4c8edff106a208960f52a6c18c ..a3ea8533a3e51719b2041c7769b2c6b070eeec65a24617b1168809358a97b073 ..3b852f96cd63e049e7e941b872ee9055,

Vector 7

35b877a25c394512292b82bdf8468e98eaf03c79c7fc9d53546dadc5fb75b500, b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623, 42616e646572736e6174636820766563746f72, 1f42.

69dec7fe79f816d095b04cead45e856ff6c7e798f513e09291958e35a5590443, 9adeacd15eacdc651e4db1ea4c0917973eac2000479edf6132f3774601cc6902, ff5f6324ea18bbb4df92f7d6304bf27a0a44fa80fd40b985de8d43963a7e02c6 ..ef6f0947911604155c6fe40f68cc91c96ffd358275b58960554274498a70f144, ea1f922fce5e359d92e0fdcda53a1d2e6b791c7e7a8ffad915f3535c6175f115, f674ad5f72661aa0c2bc5ca83aee9794c8b8bbc4017abcc00a11a23a0b558e68, f77eaec55fe36b06f1d1f7eef7db24fdcce74c83fde19b1c322aca288e39948f, b846dfbceb2a74fe102b3aec94e7b8460f5adcb609c407839ab6cb06d1e3bd38, 35a41d1cb4d22b5c162d319b206db940b6fcef71bbe0c13a6376a89788292519. c04b177f954d17e7c129ce8d55cb7f148b3957078c96e7229100dc50b7d62b02, 7b32d917d5aa771d493c47b0e096886827cd056c82dbdba19e60baa8b2c60313 ..d3b1bdb321123449c6e89d310bc6b7f654315eb471c84778353ce08b951ad471c6471c84778353ce08b951ad471c8471c84778353ce08b951ad471c8471c84778353ce08b951ad471c8471c84778353ce08b951ad471c847783550ce08b951ad471c847783550ce08b951ad471c847783550ce08b951ad471c84778350ce08b951ad471c84778350ce08b951ad471c84778350ce08b951ad471c84778350ce08b951ad471c84778350ce08b951ad471c84778350ce08b951ad471c8477850ce08b951ad471c8477850ce08b951ad471c8477850ce08b951ad471c8477850ce08b951ad471c8477850ce08b951ad471c84760ce08b951ad47160ce08b951ad470ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b951ad670ce08b950ce..561fdb0dcfb8bd443718b942f82fe717238cbcf8d12b8d22861c8a09a984a3c5 ..b0e1f208f9d6e5b310b92014ea7ef3011e649dab038804759f3766e01029d623 ..4fd11f89c2a1aaefe856bb1c5d4a1fad73f4de5e41804ca2c17ba26d6e10050c ..86d06ee2c70da6cf2da2a828d8a9d8ef755ad6e580e838359a10accb086ae437 .. ad 6f ded a 0 dd e 0 a 57 c 51 d 322 6 b 87 e 379 5 e 6474 393772 da 461 01 f d 597 f b d 456 c 1 b d 500 f d 500..3f9dc0c4f67f207974123830c2d66988fb3fb44becbbba5a64143f376edc51d9, b8d97722ccfc97a5cf2cc77aa0bbf5a146dca7762b98e2b6bf4b8e34e04e214b

```
{\tt ...28d838eb642749b18ec6b8a0d79d54a3acd644b13615f791f33d648026ed6e16}
```

- ..9bd516e3413b47ea35c9a8879bc1290d9fea32db7f127ecb33185d102875de50
- ..fe0e9c85ec450505dde7f5ac038274cf,

98d4f4aa83e7332f2598162283b79c2e09a499623a91dd95bea57ad90cd44bfa

- ..4bb53ee7c8296272ecfc13821eeab57b914ad8aba8c1092777dfb212803dfd38
- ..9c0e3b1678c6e5e0bf02c3c8179537ca205bb3580e43016ccb584c0c08b1114a
- ..83ac15b9d6ebb93fa8dae235893cc41ebd891e1a6dce2ec79ade16223becd3bb
- .. ec 2124 a 36 e 0 35 e 2f 0 ab f 3811 c 499 a f 3181 a b d c 9 c d 113 d a 1 a a 2b 8 a 43 c 89 d 7576 b a constant a
- ..0570fb7a273d3cde227429acbd034c761f97c0f681a38dc654f90e5c6c983cbd
- ${\tt ...0eda1561f831d3fbea4702dcdb4ff16bdc81aadc27338a1858df812ac179f82ac179f62ac179f62ac179f82ac179f62ac179f62ac179f62ac179f62ac179f62ac179f62ac179f62ac179f62ac179f62ac179f62ac179f62ac170f62ac170f664ac170f62ac170f62ac170f62ac170f62ac170f62ac170f62ac170f62ac170f62ac170f62ac170f62ac170f$
- .. 88 acc 5774 b 63 f 49 d 8 b c de 61578 f f 0 c 9 a 7 c b da 80 b 5 a 0 f 6 f b 5 a 80 a 56 f 4 c 40 3570 d a 60 b 5 a 60 f 6 f b 5 a 80 a 56 f 4 c 40 3570 d a 60 b 5 a 60 f 6 f b 5 a 80 a 56 f 4 c 40 3570 d a 60 b 5 a 60 f 6 f b 5 a 80 a 56 f 4 c 40 3570 d a 60 b 5 a 60 f 6 f b 5 a 80 a 56 f 4 c 40 3570 d a 60 b 5 a 60 b
- ${\tt ..41cb30835b46489c1d52507e13f639984d783608d6b54335384bd580c57ff10c}$
- ..4cc8c86403bcc8d86b67a51cf4a28a571363e17d42ca7f229672ed3a5891ee3a
- ..a5d0a3a5cf716c8c5fbce739123d2a0965388193ecadd3e9b9d20d390c19e841
- .. bf85b8b981 ae 23 a 36542 f 3724 e 13448 c 8f7f47 c d 55463 b 7e 450f4 e a 29c85d615
- ${\tt ...1d0cdef59645fd25a2c9ca74341b3d901e4f5a8a850454fb78da2f7c06769659}$
- ..8560291b44425e1fecce87f46113967c722675dac020e197ec36e70a6d3957a5
- ..d29317b23d507eccccf5c4c66c35ed83d29d7cb896109c1868739b7e7abe72d9
- ..d7d2e777f586efa88c85aed4273d8559984e66b683a66163ba0f30a9ed271322
- ..04745b711b63fc62319deb6eafd9f784121fa18a529f84422e368e1ffa0b1577
- ..95bb53f788e75be61241ca4506c96da94dc59ed23c788cb4b385106b9cfac0a5
- ..16c3be766abb7f3eb1bc3d87d6ff2335,

References

1.

Internet Engineering Task Force Verifiable Random Functions; RFC Editor, 2023;

2.

Burdges, J.; Ciobotaru, O.; Alper, H.K.; Stewart, A.; Vasilyev, S. Ring Verifiable Random Functions and Zero-Knowledge Continuations 2023.

3.

Vasilyev, S. Ring Proof Technical Specification 2024.

4.

Masson, S.; Sanso, A.; Zhang, Z. Bandersnatch: A Fast Elliptic Curve Built over the Bls12-381 Scalar Field 2021.

5.

Internet Engineering Task Force Hashing to Elliptic Curves; RFC Editor, 2023;

6.

Internet Engineering Task Force US Secure Hash Algorithms; RFC Editor, 2011;