Phil/LPS 31 Introduction to Inductive Logic
Lecture 3

David Mwakima
dmwakima@uci.edu
Department of Logic and Philosophy of Science
University of California, Irvine

April 7th 2023

Topics

- Sentential Logic: Arguments and Inference
- Deductive Logic
- Validity
- Soundness

Sentential Logic: Arguments and Inference

- One feature of a natural language like English is that we use it to communicate with others; and especially to offer them reasons why they can or should accept whatever we are saying.

Sentential Logic: Arguments and Inference

- One feature of a natural language like English is that we use it to communicate with others; and especially to offer them reasons why they can or should accept whatever we are saying.
- The reasons we give, hoping to convince or persuade someone to accept a claim, are called premises. We hope that if someone believes the premises then they will infer or draw the conclusion, which we claim.

Sentential Logic: Arguments and Inference

- One feature of a natural language like English is that we use it to communicate with others; and especially to offer them reasons why they can or should accept whatever we are saying.
- The reasons we give, hoping to convince or persuade someone to accept a claim, are called premises. We hope that if someone believes the premises then they will infer or draw the conclusion, which we claim.
- This set of sentences consisting of (1) premises and (2) a conclusion which are connected by (3) inference is called an argument.

Sentential Logic: Arguments and Inference

- One feature of a natural language like English is that we use it to communicate with others; and especially to offer them reasons why they can or should accept whatever we are saying.
- The reasons we give, hoping to convince or persuade someone to accept a claim, are called premises. We hope that if someone believes the premises then they will infer or draw the conclusion, which we claim.
- This set of sentences consisting of (1) premises and (2) a conclusion which are connected by (3) inference is called an argument.
- We wish to use sentential logic to represent the structure of good arguments made in natural language.

From Sentential Logic to Deductive Logic

- The process of drawing or inferring a conclusion is called deduction if the premises entail the conclusion.

p	q	$(p \rightarrow q)$
1	1	1
1	0	0
0	1	1
0	0	1

From Sentential Logic to Deductive Logic

- The process of drawing or inferring a conclusion is called deduction if the premises entail the conclusion.
- To understand what the word "entail" means, we need the truth table for the truth function $(p \rightarrow q)$, which is read as "If p, then q". See Homework 2 for why $(p \rightarrow q)$ is truth-functionally equivalent to $(\neg p \vee q)$.

p	q	$(p \rightarrow q)$
1	1	1
1	0	0
0	1	1
0	0	1

From Sentential Logic to Deductive Logic

- We say that a formula F entails another formula G in sentential logic just in case $(F \rightarrow G)$ is a tautology.

p	q	$(p \rightarrow q)$
1	1	1
1	0	0
0	1	1
0	0	1

From Sentential Logic to Deductive Logic

- We say that a formula F entails another formula G in sentential logic just in case $(F \rightarrow G)$ is a tautology.
- From the truth table for $(p \rightarrow q)$, we see that $(F \rightarrow G)$ is false just in case F is true and the formula G is false. This is the powerful idea behind deductive logic.

p	q	$(p \rightarrow q)$
1	1	1
1	0	0
0	1	1
0	0	1

From Sentential Logic to Deductive Logic

- A deductive logic is a logic where the relation of inference between the premises and the conclusion is entailment, i.e., the premises entail the conclusion.

From Sentential Logic to Deductive Logic

- A deductive logic is a logic where the relation of inference between the premises and the conclusion is entailment, i.e., the premises entail the conclusion.
- Another way to think of a deductive logic is that a deductive logic is a logic of truth-preserving reasoning. With a deductive logic it is impossible to begin with true premises then reason to, or infer, false conclusions!

From Sentential Logic to Deductive Logic

- A deductive logic is a logic where the relation of inference between the premises and the conclusion is entailment, i.e., the premises entail the conclusion.
- Another way to think of a deductive logic is that a deductive logic is a logic of truth-preserving reasoning. With a deductive logic it is impossible to begin with true premises then reason to, or infer, false conclusions!
- Deductive logic is great for mathematics (where almost every sentence is either a theorem or refutable) but deductive logic has limited use in experimental science, even real life! (More of this later.)

From Sentential Logic to Deductive Logic

- A deductive logic is a logic where the relation of inference between the premises and the conclusion is entailment, i.e., the premises entail the conclusion.
- Another way to think of a deductive logic is that a deductive logic is a logic of truth-preserving reasoning. With a deductive logic it is impossible to begin with true premises then reason to, or infer, false conclusions!
- Deductive logic is great for mathematics (where almost every sentence is either a theorem or refutable) but deductive logic has limited use in experimental science, even real life! (More of this later.)
- Sentential Logic can be turned into a deductive logic by adding rules of inference that preserve truth.

From Sentential Logic to Deductive Logic

- The rules of inference that preserve truth are those rules of inference for which the relation between the premises and conclusion in an argument is entailment.

From Sentential Logic to Deductive Logic

- The rules of inference that preserve truth are those rules of inference for which the relation between the premises and conclusion in an argument is entailment.
- Some rules of inference, which preserve truth, are:

From Sentential Logic to Deductive Logic

- The rules of inference that preserve truth are those rules of inference for which the relation between the premises and conclusion in an argument is entailment.
- Some rules of inference, which preserve truth, are:
  1. From a formula F, infer $(F \vee G)$ for any formula G.

From Sentential Logic to Deductive Logic

- The rules of inference that preserve truth are those rules of inference for which the relation between the premises and conclusion in an argument is entailment.
- Some rules of inference, which preserve truth, are:
  1. From a formula F, infer $(F \vee G)$ for any formula G.
  2. From a formula $\neg F$ and $(F \vee G)$, infer G.

From Sentential Logic to Deductive Logic

- The rules of inference that preserve truth are those rules of inference for which the relation between the premises and conclusion in an argument is entailment.
- Some rules of inference, which preserve truth, are:
  1. From a formula F, infer $(F \vee G)$ for any formula G.
  2. From a formula $\neg F$ and $(F \vee G)$, infer G.
- Verify that $(F \rightarrow (F \vee G))$ and $((\neg F \wedge (F \vee G)) \rightarrow G)$ are tautologies.

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.
- Consider the first rule of inference, which says from a formula F, infer $(F \vee G)$ for any formula G. Here is an instance of this argument form:

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.
- Consider the first rule of inference, which says from a formula F, infer $(F \vee G)$ for any formula G. Here is an instance of this argument form:
  1. I will have bacon for breakfast.

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.
- Consider the first rule of inference, which says from a formula F, infer $(F \vee G)$ for any formula G. Here is an instance of this argument form:
  1. I will have bacon for breakfast.

  $\therefore$2. Either I will have bacon for breakfast or I will have toast for breakfast.

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.
- Consider the first rule of inference, which says from a formula F, infer $(F \vee G)$ for any formula G. Here is an instance of this argument form:
  1. I will have bacon for breakfast.

  $\therefore$ 2. Either I will have bacon for breakfast or I will have toast for breakfast.
- Now consider the second rule of inference, which says from a formula $\neg F$ and $(F \vee G)$, infer G. Here is an instance of this argument form:

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.
- Consider the first rule of inference, which says from a formula F, infer $(F \vee G)$ for any formula G. Here is an instance of this argument form:
  1. I will have bacon for breakfast.

  $\therefore$ 2. Either I will have bacon for breakfast or I will have toast for breakfast.
- Now consider the second rule of inference, which says from a formula $\neg F$ and $(F \vee G)$, infer G. Here is an instance of this argument form:
  1. Either I will have bacon for breakfast or I will have toast for breakfast.

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.
- Consider the first rule of inference, which says from a formula F, infer $(F \vee G)$ for any formula G. Here is an instance of this argument form:
  1. I will have bacon for breakfast.

  $\therefore$ 2. Either I will have bacon for breakfast or I will have toast for breakfast.
- Now consider the second rule of inference, which says from a formula $\neg F$ and $(F \vee G)$, infer G. Here is an instance of this argument form:
  1. Either I will have bacon for breakfast or I will have toast for breakfast.
  2. I will not have bacon for breakfast.

From Sentential Logic to Deductive Logic

- Intuitively, these rules of inference make sense when we think about them using sentences in English.
- Consider the first rule of inference, which says from a formula F, infer $(F \vee G)$ for any formula G. Here is an instance of this argument form:
  1. I will have bacon for breakfast.

  $\therefore$ 2. Either I will have bacon for breakfast or I will have toast for breakfast.
- Now consider the second rule of inference, which says from a formula $\neg F$ and $(F \vee G)$, infer G. Here is an instance of this argument form:
  1. Either I will have bacon for breakfast or I will have toast for breakfast.
  2. I will not have bacon for breakfast.

  $\therefore$ 3. I will have toast for breakfast.

Validity and Soundness

- The goal of using deductive logic is to give "good" arguments.

Validity and Soundness

- The goal of using deductive logic is to give "good" arguments.
- We will use two criteria to evaluate arguments for their "goodness". One criterion is called validity, the other criterion is called soundness.

Validity and Soundness

- The goal of using deductive logic is to give "good" arguments.
- We will use two criteria to evaluate arguments for their "goodness". One criterion is called validity, the other criterion is called soundness.
- An argument is valid if the conclusion is inferred from the premises using a truth-preserving rule of inference. The following are all equivalent ways of saying that an argument is valid:

Validity and Soundness

- The goal of using deductive logic is to give "good" arguments.
- We will use two criteria to evaluate arguments for their "goodness". One criterion is called validity, the other criterion is called soundness.
- An argument is valid if the conclusion is inferred from the premises using a truth-preserving rule of inference. The following are all equivalent ways of saying that an argument is valid:
  1. Any truth value assignment that makes the premises true must make the conclusion true.

Validity and Soundness

- The goal of using deductive logic is to give "good" arguments.
- We will use two criteria to evaluate arguments for their "goodness". One criterion is called validity, the other criterion is called soundness.
- An argument is valid if the conclusion is inferred from the premises using a truth-preserving rule of inference. The following are all equivalent ways of saying that an argument is valid:
  1. Any truth value assignment that makes the premises true must make the conclusion true.
  2. No truth value assignment makes the premises true and the conclusion false.

Validity and Soundness

- The goal of using deductive logic is to give "good" arguments.
- We will use two criteria to evaluate arguments for their "goodness". One criterion is called validity, the other criterion is called soundness.
- An argument is valid if the conclusion is inferred from the premises using a truth-preserving rule of inference. The following are all equivalent ways of saying that an argument is valid:
  1. Any truth value assignment that makes the premises true must make the conclusion true.
  2. No truth value assignment makes the premises true and the conclusion false.
  3. It is impossible to assign truth values to the premises in such a way that all the premises are true and the conclusion is false.

Validity and Soundness

- The goal of using deductive logic is to give "good" arguments.
- We will use two criteria to evaluate arguments for their "goodness". One criterion is called validity, the other criterion is called soundness.
- An argument is valid if the conclusion is inferred from the premises using a truth-preserving rule of inference. The following are all equivalent ways of saying that an argument is valid:
  1. Any truth value assignment that makes the premises true must make the conclusion true.
  2. No truth value assignment makes the premises true and the conclusion false.
  3. It is impossible to assign truth values to the premises in such a way that all the premises are true and the conclusion is false.
  4. If the conclusion is false, at least one of the premises is false.

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:
  1. Either I will have bacon for breakfast or I will have toast.

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:
  1. Either I will have bacon for breakfast or I will have toast.
  2. I will not have bacon.

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:
  1. Either I will have bacon for breakfast or I will have toast.
  2. I will not have bacon.

  $\therefore$3. I will have toast for breakfast.

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:
  1. Either I will have bacon for breakfast or I will have toast.
  2. I will not have bacon.

  $\therefore$3. I will have toast for breakfast.
- Another instance of this argument form is:

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:
  1. Either I will have bacon for breakfast or I will have toast.
  2. I will not have bacon.

  $\therefore$3. I will have toast for breakfast.
- Another instance of this argument form is:
  1. Either $2 + 2 = 5$ or the moon is made of green cheese.

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:
  1. Either I will have bacon for breakfast or I will have toast.
  2. I will not have bacon.

  $\therefore$3. I will have toast for breakfast.
- Another instance of this argument form is:
  1. Either $2 + 2 = 5$ or the moon is made of green cheese.
  2. $2 + 2 \neq 5$

Validity and Soundness

- Let me stress that the validity of an argument is a formal notion. This means that whether an argument is valid depends on (1) the formulas in question and (2) whether the formulas in the premises entail the the formula in the conclusion.
- Because validity is a formal notion, we say that an argument in English is valid just in case it is an instance of a argument form in deductive logic in which the rule of inference is truth-preserving.
- Consider the argument form which says that from a formula $\neg F$ and $(F \vee G)$, infer G. An instance of this argument form is:
  1. Either I will have bacon for breakfast or I will have toast.
  2. I will not have bacon.

  $\therefore$3. I will have toast for breakfast.
- Another instance of this argument form is:
  1. Either $2 + 2 = 5$ or the moon is made of green cheese.
  2. $2 + 2 \neq 5$

  $\therefore$3. The moon is made of green cheese.

Validity and Soundness

- Wait. What?

Validity and Soundness

- Wait. What?
- That is the right reaction here.

Validity and Soundness

- Wait. What?
- That is the right reaction here.
- Although both instances of the argument form are valid, one of the premises in the second instance is false. This is the premise that says "Either $2 + 2 = 5$ or the moon is made of green cheese."

Validity and Soundness

- Wait. What?
- That is the right reaction here.
- Although both instances of the argument form are valid, one of the premises in the second instance is false. This is the premise that says "Either $2 + 2 = 5$ or the moon is made of green cheese."
- An argument that is (1) valid and (2) all the premises are true is a sound argument.

Validity and Soundness

- Wait. What?
- That is the right reaction here.
- Although both instances of the argument form are valid, one of the premises in the second instance is false. This is the premise that says "Either $2 + 2 = 5$ or the moon is made of green cheese."
- An argument that is (1) valid and (2) all the premises are true is a sound argument.
- Convince yourself that deductive logic cannot help you to determine whether an argument in English is sound. Bummer! Whether the premises are true relies on domain knowledge, i.e., knowledge of a specific, specialized discipline or field; it does not depend on logic.