Phil/LPS 31 Introduction to Inductive Logic
Lecture 9

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Topics

- Operations on Sets
- Relationships between Sets

Recap from last time

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- We are going to proceed "naïvely", which means that we will not proceed axiomatically but with the awareness that what sets are is a delicate matter.

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  - (left bracket and) right bracket.
  - Set-forming operators: $\cup$ (union), $\cap$ (intersection), $\mathcal{P}$ (powerset)

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- Sometimes it is impossible to list all of the members or elements of a set. They might be too many! In this case, if there is a predicate that describes what all the members of the set are, we can write:

$$\{x \in \mathcal{U} \mid Px\}$$

  which is read as "The set of those x's in $\mathcal{U}$ that are P's" or "The set x's in $\mathcal{U}$ such that Px". Here $\mathcal{U}$ is called the universe of discourse, i.e., the terms that the variables range over in a given context.

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- The point here is that one can write down what a given set is either by: (1) listing its members; or (2) specifying the condition of membership to that set.

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- So although the complement always exists, we limit its size by only looking at the complement of A in $\mathcal{U}$. What the universe of discourse is will vary with context. For example, in probability theory which we will talk about later, $\mathcal{U}$ has a special name: the sample space.

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  - $A = \{1, 2\}, \mathcal{U} = \{1, 2, 3, 4, 5, 6\}$
  - $A = \{2, 4, 6\}, \mathcal{U} = \{1, 2, 3, 4, 5, 6\}$
  - $A = \{3, 5, 6\}, \mathcal{U} = \{1, 2, 3, 4, 5, 6\}$
  - $A = \{n \in \mathbb{N} \mid n \text{ is even}\}, \mathcal{U} = \mathbb{N}$

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- Think of the conditional $\rightarrow$ here. $(p \rightarrow q)$ is false just in case p is true but q is false. Similarly, A is not a subset of B if there is a counterexample, i.e., $a \in A$ but $a \notin B$.

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  - $A = \emptyset$, $B = \{1, 2, 3\}$

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  - $C = \{1, 2, 3, 4\}$