Phil/LPS 31 Introduction to Inductive Logic Lecture 1

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April 3rd 2023

Topics

- ▶ Logic in General
- ▶ Sentences
- ▶ Truth-functional connectives
- ▶ Sentential logic

Logic in General

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▶ Finally, the word "system" means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also belong to the representation. The symbols that belong to the representation are called, you guessed it, formulas!

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- ► Closure condition: A formula is anything that results from 1 or a finite number of applications of rule 2 or rule 3. Nothing else is a formula.

- ► Verify that all these are examples of formulas of the logic: *S*, *ab*, *aaSbb*, *aabb*.

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- It turns out that saying what "propositions" are is a hard philosophical problem. So we'll stick to sentences!

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  4. Having 1001 strands of hair does not make you bald. How about 1000, 999, . . . ?

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▶ What's the difference between the sentences in Case 1 and Case 2?

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