Phil/LPS 31 Introduction to Inductive Logic
Lecture 5

David Mwakima
dmwakima@uci.edu
University of California, Irvine

April 12th 2023

Topics

- Inference in Quantified Relational Logic
- Models and Counterexamples

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x Gx)$

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x Gx)$
  2. $(\forall x \neg Gx)$

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x Gx)$
  2. $(\forall x \neg Gx)$
  3. $\forall x(\neg Qx \rightarrow (\exists y Sxy))$

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x Gx)$
  2. $(\forall x \neg Gx)$
  3. $\forall x(\neg Qx \rightarrow (\exists y Sxy))$
  4. $\forall x \forall y(Lxy \rightarrow \exists z(Lxz \wedge Lzy))$

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  - All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  - All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.
  - From an instance of a formula $\mathcal{F}$ infer $(\exists x \mathcal{F})$ where the formula $\mathcal{F}$ is the result of uniformly replacing with variable x any constant in $\mathcal{F}$. This rule is known as Existential Introduction.

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  - All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.
  - From an instance of a formula $\mathcal{F}$ infer $(\exists x \mathcal{F})$ where the formula $\mathcal{F}$ is the result of uniformly replacing with variable x any constant in $\mathcal{F}$. This rule is known as Existential Introduction.
  - From $\exists x \mathcal{F}$, infer $\mathcal{F}(a)$, where $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with a constant a. This rule is known as Existential Exploitation.

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  - All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.
  - From an instance of a formula $\mathcal{F}$ infer $(\exists x \mathcal{F})$ where the formula $\mathcal{F}$ is the result of uniformly replacing with variable x any constant in $\mathcal{F}$. This rule is known as Existential Introduction.
  - From $\exists x \mathcal{F}$, infer $\mathcal{F}(a)$, where $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with a constant a. This rule is known as Existential Exploitation.
- These are the only truth-preserving rules of inference that we will avail for ourselves in quantified relational logic.

Models and Counterexamples

- We know why the truth-preserving rules of sentential logic are truth-preserving.

Models and Counterexamples

- We know why the truth-preserving rules of sentential logic are truth-preserving.
- Since the operator $\exists$ is not truth-functional, in order to say why the rule Existential Introduction and the rule Existential Exploitation are a truth-preserving rules of inference, we need to discuss the semantics, interpretation or meaning of sentences in quantified relational logic.

Models and Counterexamples

- We know why the truth-preserving rules of sentential logic are truth-preserving.
- Since the operator $\exists$ is not truth-functional, in order to say why the rule Existential Introduction and the rule Existential Exploitation are a truth-preserving rules of inference, we need to discuss the semantics, interpretation or meaning of sentences in quantified relational logic.
- This will require a tool which we will call a model or intended interpretation of some sentences in quantified relational logic.

Models and Counterexamples