Phil/LPS 31 Introduction to Inductive Logic Lecture 13

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Topics

- Odds
- Expected Value (Discrete Case Only)

Recap: Probability Rules

Rule 1 $P(S) = 1$

Rule 2 $0 \leq P(E) \leq 1$

Rule 3 If $E \cap F = \emptyset$, then $P(E \cup F) = P(E) + P(F)$ (Special Disjunction Rule)

Rule 4 $P(E \cup E^c) = 1$

Rule 5 $P(E^c) = 1 - P(E)$

Rule 6 $P(\emptyset) = 0$

Rule 7 $P(E) = P(E \cap F) + P(E \cap F^c)$

Rule 8 $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ (General Disjunction Rule)

Rule 9 $P(E \cap F) = P(E) \times P(F)$ if E and F are independent.

Rule 10 $P(E \cap F) = P(E) \times P(F|E)$ if E and F are dependent.

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▶ If $\{E_1, E_2, \ldots, E_n\}$ is a partition of E, then from the theorem of total probability:

$$\begin{aligned}
P(E \mid F) &= \frac{P(F \mid E)P(E)}{P(F \mid E_1)P(E_1) + \cdots + P(F \mid E_n)P(E_n)} \\
&= \frac{P(F \mid E)P(E)}{\sum_{i=1}^{n} P(F \mid E_i)P(E_i)}
\end{aligned}$$

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▶ In this lecture we will see why and also introduce ways of going (1) from probability to odds and (2) from odds to probability.

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  (2) The odds of having two girls, if the probability that a family has two children who are both girls is $\frac{1}{4}$.

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▶ Exercise. What are the ways that probabilities are different from odds given these properties of odds?

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▶ We will see later that the Odds Ratio provides a good measure of association (or dependence) between two groups in the same way that we used joint and marginal probabilities to assess independence between groups.

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▶ Solving this equation for p we get:
$$p = \frac{\mathcal{O}(E)}{1 + \mathcal{O}(E)}$$

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▶ See Homework 7 for more exercises.

Expected Value (Discrete Case)

► The number of telephone calls received at a remote call center per minute show the following frequencies:

Number of Calls	Count
1	10
2	8
3	4
4	4
5	2
6	2
7	1
8	1
TOTAL	32

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- Using the probability distribution, we can then ask useful questions such as: what is the expected value or expectation of the random variable.

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- Here is the data for your convenience.

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▶ Commit this formula to memory!