Phil/LPS 31 Introduction to Inductive Logic
Lecture 12

David Mwakima
dmwakima@uci.edu
Department of Logic and Philosophy of Science
University of California, Irvine

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Topics

- Joint Probability
- Marginal Probability
- Probabilistic Independence
- Conditional Probability
- Bayes' Theorem

Joint Probability and Marginal Probability

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▶ We call $P(A \cap B)$, the joint probability of A and B.

▶ In the context of joint probability, and a concept we shall introduce later of conditional probability, $P(A)$ and $P(B)$ are called the marginal probability of A and B, respectively. The reason for the adjective "marginal" will become clear in a minute.

Joint Probability and Marginal Probability: First Illustration

	Spades	Hearts	Diamonds	Clubs	
Ace	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{13}$
2	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{13}$
3	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{13}$
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Jack	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{13}$
Queen	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{13}$
King	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{52}$	$\frac{1}{13}$
	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1

Joint Probability and Marginal Probability: First Illustration

- What is $P(\text{King})$, $P(\text{Spades})$, $P(\text{Ace})$, $P(\text{Diamonds})$, $P(7)$, $P(\text{Hearts})$? All these are examples of marginal probabilities

- What is $P(\text{King of Spades})$, $P(\text{Ace of Diamonds})$, $P(7 \text{ of Hearts})$? All these are are examples of joint probabilities

- What is $P(\text{King}) \times P(\text{Spades})$, $P(\text{Ace}) \times P(\text{Diamonds})$, $P(7) \times P(\text{Hearts})$? Are the joint probabilities equal to the product of the respective marginal probabilities?

- We say that the suit and rank of a card are probabilistically independent.

Joint Probability and Marginal Probability: Second Illustration

Eye Color	Hair Color				
	Black	Brunette	Red	Blond	
Brown	0.11	0.20	0.04	0.01	0.37
Blue	0.03	0.14	0.03	0.16	0.36
Hazel	0.03	0.09	0.02	0.02	0.16
Green	0.01	0.05	0.02	0.03	0.11
	0.18	0.48	0.12	0.21	1

Joint Probability and Marginal Probability: Second Illustration

▶ What is P(Brown eye color), P(Black Hair), P(Blue eye color), P(Red Hair color), P(Green eye color), P(Blond Hair color)? All these are examples of marginal probabilities.

▶ What is P(Brown eyes and Black Hair), P(Blue eyes and Red Hair), P(Green eyes and Blond Hair)? All these are are examples of joint probabilities.

▶ What is P(Brown Eye color) $\times$ P(Black Hair), P(Blue eye color) $\times$ P(Red Hair color), P(Green Eyes) $\times$ P(Blond Hair)? Are the joint probabilities equal to the product of the respective marginal probabilities?

▶ We say that eye-color and hair color are probabilistically dependent.

Probabilitistic Independence

▶ We say two events E and F are probabilistically independent, or simply just independent, if:
$$P(E \cap F) = P(E) \times P(F)$$

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▶ For that we need the concept of conditional probability.

▶ But first let us talk about the theorem of total probability.

Joint Probability and Marginal Probability: Some Facts

We begin with some facts. Consider the two illustrations once more.

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▶ The columns and rows of the table of joint probabilities are two different partitions of the set we are considering in a given context. For example, in the first illustration, the deck of cards is partitioned by suit column-wise and by rank row-wise. The set of people included in the eye and hair color data is partitioned by hair color column-wise and by eye-color row-wise.

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▶ The row totals are the marginal probabilities of each set in the partition by row (rank or eye-color). This is also an example of the theorem of total probability.

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- ▶ The grand total of the marginal probabilities (by row or by column) is 1 (as it should be since the union of the sets in a partition is the whole set).

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- Suppose $\{E_1, E_2, E_3, \ldots, E_n\}$ is partition of E. The theorem of total probability says that:

$$P(E) = P(E \cap E_1) + P(E \cap E_2) + P(E \cap E_3) + \cdots + P(E \cap E_n)$$
$$= \sum_{i}^{n} P(E \cap E_i)$$

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▶ Suppose you have a bag containing 13 yellow and 17 white marbles. You decide to sample one marble at a time, with replacement. What is P(Yellow marble), P(White marble | Yellow marble), P(Yellow marble | White marble)?

Conditional Probability

▶ Now suppose you decide to sample one marble at a time, without replacement? This kind of sampling introduces dependency between events. We need to condition on the new information we are given (or that we obtain) once we see the color of the marble we sampled earlier in the sequence. What is $P(\text{White marble} \mid \text{Yellow marble})$, $P(\text{Yellow marble} \mid \text{White marble})$?

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▶ Exercise. Consider the deck of cards illustration from earlier in the slides. Suppose you're drawing cards from a standard deck of 52 cards with replacement. What is:
$P(\text{Ace of Hearts} \,|\, \text{King of Spades})$,
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Event E is independent of F if and only $P(E) = P(E \,|\, F)$.

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  Event E is independent of F if and only $P(E) = P(E \mid F)$.

▶ So the independence between two events captures, in a sense, what we mean when we say that the occurrence of one event E doesn't give any information we can conditionalize upon in order to learn about the chances of another event F occurring.

Joint Probability from Conditional Probability

▶ Suppose we sample twice with replacement from the bag with 13 yellow marbles and 17 white marbles. What is:
$P(2$ white marbles in a row$)$,
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Rule 10 $P(E \cap F) = P(F) \times P(E|F)$

▶ From Rule 10 we define the conditional probability of E given F as:

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

Bayes' Theorem

► A very powerful (immediate) consequence of the definition of conditional probability is Bayes' Theorem, named after Rev. Thomas Bayes. In Homework 6 you will derive Bayes' Theorem. Here I just state this remarkable theorem, which we shall get a lot of mileage out of.

$$P(E \mid F) = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|E^c)P(E^c)}$$

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▶ A slightly more general form of Bayes Theorem says that if $\{E_1, E_2, E_3, E_4, \ldots, E_n\}$ is a partition of S, then for any $E_i \subset S$:

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▶ Memorize this!