Phil/LPS 31 Introduction to Inductive Logic
Lecture 4

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Topics

- Quantified Relational Logic: Motivation
- Quantified Relational Logic: Variables and Quantifiers
- Quantified Relational Logic: Predicates and Relations
- Quantified Relational Logic: The Logic Itself

Quantified Relational Logic: Motivation

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  2. Ruth Barcan Marcus is a logician.

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  ∴3 Ruth Barcan Marcus is wise.

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- Now consider the following argument:
  1 All logicians are wise.
  2 Ruth Barcan Marcus is a logician.
  $\therefore$3 Ruth Barcan Marcus is wise.
- This argument is valid, (and sound!) but sentential logic cannot explain why.

Quantified Relational Logic: Motivation

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  1a. Hypatia is wise.

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- So we need a logic with more expressive power to represent not only the truth-functional structure of sentences in a natural language like English, but also their internal or grammatical structure. This logic is called quantified relational logic or first order logic.

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- So we need a logic with more expressive power to represent not only the truth-functional structure of sentences in a natural language like English, but also their internal or grammatical structure. This logic is called quantified relational logic or first order logic.
- This logic will allow us to say why the argument we started with earlier is deductively valid.

Quantified Relational Logic: Variables and Quantifiers

Consider the following sentences:

(1) Hypatia is wise.

(2) Cavendish is wise.

(3) Hypatia is female.

(4) Cavendish is female.

- Fundamental to quantified relational logic is the notion of a variable and a quantifier, which we use to generalize.

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- Look at (1) and (2), if we want to generalize from these two sentences, we can drop the proper names ‘Hypatia’ and ‘Cavendish’ and simply say that:

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- Look at (1) and (2), if we want to generalize from these two sentences, we can drop the proper names 'Hypatia' and 'Cavendish' and simply say that:
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- If the universe of discourse involves only Hypatia and Cavendish we generalize and say that:

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- Look at (1) and (2), if we want to generalize from these two sentences, we can drop the proper names 'Hypatia' and 'Cavendish' and simply say that:
  (6) Someone is wise.
- If the universe of discourse involves only Hypatia and Cavendish we generalize and say that:
  (7) Everyone is wise.

Quantified Relational Logic: Variables and Quantifiers

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- In quantified relational logic, we paraphrase (6) and (7) as:

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- In quantified relational logic, we paraphrase (6) and (7) as:
  (8) $(\exists x\ (x \text{ is wise}))$ Read as "There exists an x such that x is wise."

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  (8) $(\exists x\ (x \text{ is wise}))$ Read as "There exists an x such that x is wise."
  (9) $(\forall x\ (x \text{ is wise}))$ Read as "For all x, x is wise." or "Every x is wise."

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- In your Homework 3 you will practice with these sorts of paraphrases. But this is not the main focus of the class. I just want to cover this because we will need it to understand things we will talk about later in week 2 or early week 3.

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  (10) $4^2 - 2^2 = (4-2)(4+2)$

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- Since (10) is true for some numbers, namely 4 and 2, we can generalize and say:

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- In fact, (10) is true not just for some two numbers but any two numbers. So we can generalize even further and say:
(12) $(\forall x \forall y(x^2 - y^2 = (x-y)(x+y)))$
- We will not have to represent anything crazy like (11) and (12) for this class, but you will need to have an idea similar to what happens in high school algebra to understand what is going on.

Quantified Relational Logic: Variables and Quantifiers

Let us go back to some of our original sentences:

(1) Hypatia is wise.

(2) Cavendish is wise.

We said we can represent (1) and (2) as either as:

(6) Someone is wise, or

(7) Everyone is wise.

Or symbolically as:

(8) $(\exists x\ (x \text{ is wise}))$

(9) $(\forall x\ (x \text{ is wise}))$

- The symbols $\exists$ and $\forall$ are called quantifiers. x is a called a variable that is bound by that quantifier.

Quantified Relational Logic: Variables and Quantifiers

Using quantifiers and variables generalize the other two sentences we started with assuming that the universe of discourse includes only Hypatia and Cavendish.

(3) Hypatia is female.

(4) Cavendish is female.

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Quantified Relational Logic: Predicates and Relations

Now consider all four of our original sentences:

(1) Hypatia is wise.

(2) Cavendish is wise.

(3) Hypatia is female.

(4) Cavendish is female.

- Notice that there is variation in proper names ("Hypatia", "Cavendish") and what comes after the proper name ("is wise", "is female"), which are both one place relations.

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Now consider all four of our original sentences:

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- Notice that there is variation in proper names ("Hypatia", "Cavendish") and what comes after the proper name ("is wise", "is female"), which are both one place relations.
- But! what is is common between all four sentences is their logical form. They are all of the logical form:

  $<$subject$>$ + $<$one place relation$>$

Quantified Relational Logic: Predicates and Relations.

(1) Hypatia is wise.

(2) Cavendish is wise.

(3) Hypatia is female.

(4) Cavendish is female.

- We have already seen how to generalize when the subject of a sentence is a proper name. We introduced symbols for quantifiers ($\exists$ and $\forall$) and variables (x, y).

Quantified Relational Logic: Predicates and Relations.

(1) Hypatia is wise.

(2) Cavendish is wise.

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- We have already seen how to generalize when the subject of a sentence is a proper name. We introduced symbols for quantifiers ($\exists$ and $\forall$) and variables (x, y).
- How do we generalize sentences which have the same logical form:

$$<\text{subject}> + <\text{one place relation}> \ ?$$

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- We introduce the relational symbol W for the relation "is wise" and paraphrase (1) and (2) as:

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  (14) (Wx) Read as "x is W"

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  (14) (Wx) Read as "x is W"

  (15) (Wy) Read as "y is W"

Quantified Relational Logic: Predicates and Relations.

(1) Hypatia is wise.

(2) Cavendish is wise.

(3) Hypatia is female.

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- Introduce the relational symbol F for the relation “is female” and paraphrase (3) and (4):

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(1) Hypatia is wise.

(2) Cavendish is wise.

(3) Hypatia is female.

(4) Cavendish is female.

- So the structure of (1), (2) can be represented most generally using quantified relational logic as either:

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- So the structure of (1), (2) can be represented most generally using quantified relational logic as either:

  (16) $(\exists x(Wx))$ Read as "There exists an x and x is W", or

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  (16) $(\exists x(Wx))$ Read as "There exists an x and x is W", or
  (17) $(\forall x(Wx))$ Read as "Every x is W".

Quantified Relational Logic: Predicates and Relations

Using appropriate symbols for quantifiers, variables and relations, paraphrase or formalize the following sentences in quantified relational logic:

(3) Hypatia is female.

(4) Cavendish is female.

Quantified Relational Logic: Predicates and Relations

Now consider the following sentences:

(1) Hypatia is wise.

(18) Hypatia possesses wisdom.

(19) Ottoline met Russell.

- While (1) and (18) appear to mean the same thing, they have different logical form. (1) is of the form:

  $$<\text{subject}> + <\text{one place relation}>$$

  while (18) is of the form:

  $$<\text{subject}> + <\text{two place relation}> + <\text{object}>$$

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- Similarly (19) is also of the form:

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- The numbers one and two that tell us how many subjects or objects ("one place" or "two place") a relation needs in a full grammatical sentence of English are called the arity of the relation.

Quantified Relational Logic: Predicates and Relations

There is a special name for relations whose arity is one. They are called predicates. “is wise”, “is female” are all predicates.

To paraphrase sentences like:

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we introduce symbols for 2-place or binary relations and use different variables for all the distinct subjects or objects that stand in the relation.

- So (18) can be paraphrased as:

  (Pxy) Read as "x possesses y"

  Here we chose the variable x to stand for Hypatia and the variable y to stand for wisdom and the relational symbol P to stand for the two place relation $< x$ possesses $y >$.

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- Once we made this choice, the correct paraphrase was (Pxy) not (Pyx)! So with relations order matters!

Quantified Relational Logic: Predicates and Relations

(18) Hypatia possesses wisdom.

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- Once we have paraphrased (18) as (Pxy) we can use quantifiers and variables to generalize (18) as:

$$(\exists x \exists y (Pxy))$$

Quantified Relational Logic: Predicates and Relations

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Using the symbol M for the relation $< x$ met $y >$ paraphrase (19) with full generality.

Quantified Relational Logic: The Logic Itself

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  - Upper case letters $P, Q, R, S, T, \ldots, A, B, C, D, \ldots, M, \ldots$ of the English alphabet as symbols for relations (one or two place only).

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  - Truth-functional connectives: $\vee$, $\wedge$, $\neg$, $\rightarrow$, $\leftrightarrow$
  - Quantifiers: $\forall$, $\exists$
  - Brackets: (for left bracket and) for right bracket.

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    1. For any predicate symbol P and for any two place relation symbol R, given any constants a or b or any variables x or y, (Pa), (Px), (Rab), (Rxy) are formulas. In (Px) and (Rxy) formulas, x and y are said to "free" variables because there are no quantifiers to which they are bound.

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  2. If (Px) is a formula, then $(\exists x(Px))$ is a formula and $(\forall x(Px))$ is a formula. Rule 2 is known as binding any free variable in (Px) by a quantifier.

Quantified Relational Logic: The Logic Itself

- The transformation rules are:
  1. For any predicate symbol P and for any two place relation symbol R, given any constants a or b or any variables x or y, (Pa), (Px), (Rab), (Rxy) are formulas. In (Px) and (Rxy) formulas, x and y are said to "free" variables because there are no quantifiers to which they are bound.
  2. If (Px) is a formula, then $(\exists x(Px))$ is a formula and $(\forall x(Px))$ is a formula. Rule 2 is known as binding any free variable in (Px) by a quantifier.
  3. If Rxy is a formula, then $(\exists x(Rxy))$ is a formula and $(\forall x(Rxy))$ is a formula. Here y is a free variable but x is bound.

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In order to talk about formulas at a meta-level we use the symbols $\mathcal{F}, \mathcal{G}$ and $\mathcal{H}$ and say:

4. If $\mathcal{F}$ is a formula by rule 3, then the result of binding any free variable in $\mathcal{F}$ and closing everything up with one left and one right parenthesis is a formula.
5. A formula $\mathcal{F}$ formed by either rule 1, 2, 3 or 4 is called an atomic formula.
6. If $\mathcal{F}$ and $\mathcal{G}$ are atomic formulas then $(\neg\mathcal{F})$, $(\mathcal{F} \vee \mathcal{G})$, $(\mathcal{F} \wedge \mathcal{G})$, $(\mathcal{F} \rightarrow \mathcal{G})$ and $(\mathcal{F} \leftrightarrow \mathcal{G})$ are complex formulas.
7. If $\mathcal{H}$ is a complex formula, then the result of binding any free variable in $\mathcal{H}$ and closing everything up with one left and one right parenthesis is a formula.

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- The Closure Condition says that a formula of quantified relational logic is either an atomic formula or a complex formula built from atomic formulas by finite applications of rule 5, 6 and 7. Nothing else is a formula.

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- The Closure Condition says that a formula of quantified relational logic is either an atomic formula or a complex formula built from atomic formulas by finite applications of rule 5, 6 and 7. Nothing else is a formula.
- We have now characterized the syntax of quantified relational logic. Next time we will introduce some inference rules for doing deduction in quantified relational logic, and talk about the interpretation or semantics of quantified relational logic very briefly.