Phil/LPS 31 Introduction to Inductive Logic
Lecture 15

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Topics

- Ordinal Utilities
- Strict Dominance Principle
- Cardinal Utilities
- Expected Utility and Risk
- Principles of Rational Choice

Decision Problems: Rational Choice

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- The relevant sense of "information" here is information about the probability distribution of states.
- The relevant sense of "uncertainty" here is that we don't know the probabilities with which the states will occur with certainty.

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- We write $A_i \sim A_j$ to mean Act i is preferred equally to Act j.

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- Suppose that given S_1, we can order our preferences as $A_1 \succ A_3 \succ A_2$. This means that if our host serves fish, we would prefer bringing white wine more than we would prefer bringing either rosé or red wine; and we would prefer to bring rosé more than we would prefer to bring red wine.

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- Here we see that $4 > 3 > 2$. So this utility function respects the preference ordering of the acts. 4, 3 and 2 are ordinal utilities.

Ordinal Utilities

- Suppose now that the host serves chicken, S_2. You think that if the host serves chicken you'd much rather bring white wine than either red wine or rosé. Assume also that if you can't find white wine at Trader Joe's you'd much rather bring rosé than red wine.

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  3. Verify that your utility function respects your preference ordering.

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  (4) Provide no information about the strength of preferences.

Making Decisions with Ordinal Utilities

- From the previous exercises we obtain the following desirability table for acts based on our ordinal utility function.

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White	4	5
Red	2	1
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  - Decide on the basis of the maximin principle.

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- We say that A_1 "dominates" both A_2 and A_3.
- Similarly A_3 "dominates" A_2.
- A widely accepted dominance principle in decision theory prescribes that dominated acts must not be chosen.

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- Weak Dominance: $A_i \succeq A_j$ if and only if $u(A_i|S) \geq u(A_j|S)$ for every state S (at least as good)

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- Weak Dominance: $A_i \succeq A_j$ if and only if $u(A_i|S) \geq u(A_j|S)$ for every state S (at least as good)
- Strict Dominance: $A_i \succ A_j$ if and only if (1) $u(A_i|S_n) \geq u(A_j|S_n)$ for every state S_n (at least as good) and (2) there exists a state S_m such that $u(A_i|S_m) > u(A_j|S_m)$ (at least one better).

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	Fish	Chicken	Lamb
White	3	4	1
Red	2	1	4
Rosé	3	4	4

Strict and Weak Dominance Principle

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White	3	4	1
Red	2	1	4
Rosé	3	4	4

- Exercise. Refer to the desirability table for the dinner party example to answer the following questions.

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White	3	4	1
Red	2	1	4
Rosé	3	4	4

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Red	2	1	4
Rosé	3	4	4

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  - $A_1 \succeq A_3$?
  - $A_3 \succ A_1$?
  - $A_3 \succeq A_2$?
  - Does the strong dominance principle imply the weak dominance principle?

Maximin Principle

Cardinal Utilities

Minimax: MINimize the MAXimum regret

Decision Problems Under Risk

Expected Utility and Risk

$$
\begin{aligned}
U(A_1) &= u(A|S_1)P(S_1) + u(A|S_2)P(S_1) + \ldots u(A|S_n)P(S_n) \\
&= \sum_{i}^{n} u(A \mid S_i)P(S_i)
\end{aligned}
$$

Maximize Expected Utility