

Phil/LPS 31 Introduction to Inductive Logic

Lecture 1

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Topics

- ▶ Logic in General
- ▶ Sentences
- ▶ Truth-functional connectives
- ▶ Sentential logic

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- ▶ Finally, the word “system” means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also **belong to the representation**. The symbols that belong to the representation are called, you guessed it, **formulas**!

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- ▶ Verify that all these are examples of formulas of the logic: S , ab , $aaSbb$, $aabb$.

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 - ▶ Schnee ist weiß (German)
 - ▶ Snow is white (English)
- ▶ It turns out that saying what “propositions” are is a **hard** philosophical problem. So we’ll stick to sentences!

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 4. Having 1001 strands of hair does not make you bald. How about 1000, 999, ...?

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- ▶ What's the difference between the sentences in **Case 1** and **Case 2**?

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