

Phil/LPS 31 Introduction to Inductive Logic

Lecture 9

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Topics

- ▶ Operations on Sets
- ▶ Relationships between Sets

Recap from last time

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- ▶ We shall not cover the axioms of set theory. There are more advanced classes in the Mathematics Department and the Logic and Philosophy of Science (LPS) Department that cover that.
- ▶ We are going to proceed "naïvely", which means that we will not proceed axiomatically but with the awareness that what sets are is a delicate matter.

Operations with Sets: Writing down sets

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 - ▶ (left bracket and) right bracket.
 - ▶ Set-forming operators: \cup (**union**), \cap (**intersection**), \mathcal{P} (**powerset**)

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- ▶ Sometimes it is impossible to list all of the members or elements of a set. They might be too many! In this case, if there is a predicate that describes what all the members of the set are, we can write:

$$\{x \in \mathcal{U} \mid Px\}$$

which is read as “The set of those x ’s in \mathcal{U} that are P ’s” or “The set x ’s in \mathcal{U} such that Px ”. Here \mathcal{U} is called the universe of discourse, i.e., the terms that the variables range over in a given context.

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- ▶ The point here is that one can write down what a given set is either by: (1) **listing its members**; or (2) **specifying the condition of membership** to that set.

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- ▶ But this definition isn't complete because we didn't specify what the universe of discourse \mathcal{U} is. Unless we specify what \mathcal{U} is, A^c isn't quite well defined. A^c can be “too big” to be a set. Remember the Russell Paradox?

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- ▶ So although the complement always exists, we limit its size by only looking at the complement of A in \mathcal{U} . What the universe of discourse is will vary with context. For example, in probability theory which we will talk about later, \mathcal{U} has a special name: **the sample space**.

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- ▶ So, suppose we're dealing with the set of possible outcomes of the roll of one die. Here $\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$
- ▶ Let $A = \{1, 2, 3, 4\}$. Then A^c in \mathcal{U} is everything that is in \mathcal{U} but **not** in A . $A^c = \{5, 6\}$. To verify that A^c is the complement of A in \mathcal{U} , we need to check that the members of A^c are in \mathcal{U} but not in A .
- ▶ Exercise. Write down A^c in \mathcal{U} in the following examples:
 - ▶ $A = \{1, 2\}$, $\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$
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 - ▶ $A = \{3, 5, 6\}$, $\mathcal{U} = \{1, 2, 3, 4, 5, 6\}$
 - ▶ $A = \{n \in \mathbb{N} \mid n \text{ is even}\}$, $\mathcal{U} = \mathbb{N}$

Relationships between sets: Subset

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- ▶ Think of the conditional \rightarrow here. $(p \rightarrow q)$ is false just in case p is true but q is false. Similarly, A is **not** a subset of B if there is a **counterexample**, i.e., $a \in A$ but $a \notin B$.

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