

Phil/LPS 31 Introduction to Inductive Logic

Lecture 3

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Topics

- ▶ Sentential Logic: Arguments and Inference
- ▶ Deductive Logic
- ▶ Validity
- ▶ Soundness

Sentential Logic: Arguments and Inference

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- ▶ The reasons we give, hoping to convince or persuade someone to accept a claim, are called **premises**. We hope that if someone believes the premises then they will **infer** or draw **the conclusion**, which we claim.
- ▶ This set of sentences consisting of (1) premises and (2) a conclusion which are connected by (3) inference is called an **argument**.
- ▶ We wish to use sentential logic to represent the structure of **good** arguments made in natural language.

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- ▶ To understand what the word “entail” means, we need the truth table for the truth function $(p \rightarrow q)$, which is read as “If p , then q ”. See **Homework 2** for why $(p \rightarrow q)$ is truth-functionally equivalent to $(\neg p \vee q)$.

p	q	$(p \rightarrow q)$
1	1	1
1	0	0
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- ▶ We say that a formula F **entails** another formula G in sentential logic just in case $F \rightarrow G$ is a **tautology**.
- ▶ From the truth table for $(p \rightarrow q)$, we see that $(F \rightarrow G)$ is a tautology just in case either the formula F is false or the formula G is true because $(p \rightarrow q)$ is false just in case p is true and q is false.

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- ▶ The rules of inference that preserve truth are those rules of inference for which the relation between the premises and conclusion in an argument is entailment.

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 4. If the conclusion is false, at least one of the premises is false.

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 2. $2 + 2 \neq 5$
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- ▶ Although the second instance of the argument form is valid, the premises are **false**.
- ▶ An argument that is (1) valid and (2) has true premises is a **sound argument**.
- ▶ I hope you see that deductive logic cannot help you to determine whether an argument in English is sound. Bummer! Whether the premises are true relies on domain knowledge, i.e., knowledge of a specific, specialized discipline or field.