Phil/LPS 31 Introduction to Inductive Logic
Lecture 5

David Mwakima
dmwakima@uci.edu
Department of Logic and Philosophy of Science
University of California, Irvine

April 12th 2023

Topics

- Inference in Quantified Relational Logic
- Models and Counterexamples

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x(Gx))$

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x(Gx))$
  2. $(\forall x(\neg Gx))$

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x(Gx))$
  2. $(\forall x(\neg Gx))$
  3. $(\forall x((\neg Qx) \rightarrow (\exists y(Sxy))))$

Inference in Quantified Relational Logic

- In order to do inference in quantified relational logic we need the notions of (1) the scope of a quantifier and (2) an instance of a formula in quantified relational logic.
- The scope of quantifier are all the free variables, which it binds, in a formula.
- An instance of a formula in quantified relational logic is a formula that is obtained by freeing a bound variable within the scope of a quantifier and uniformly replacing every occurrence of this free variable by a constant.
- For each of these formulas: what is the scope of each quantifier? Provide an instance of each formula.
  1. $(\exists x(Gx))$
  2. $(\forall x(\neg Gx))$
  3. $(\forall x((\neg Qx) \rightarrow (\exists y(Sxy))))$
  4. $(\forall x \forall y((Lxy) \rightarrow (\exists z((Lxz) \wedge (Lzy)))))$

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
    1. All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  1. All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.
  2. From an instance of a formula $\mathcal{F}$ infer $(\exists x \mathcal{F})$ where the formula $\mathcal{F}$ is the result of uniformly replacing with variable x any constant in $\mathcal{F}$. This rule is known as Existential Generalization.

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  1. All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.
  2. From an instance of a formula $\mathcal{F}$ infer $(\exists x \mathcal{F})$ where the formula $\mathcal{F}$ is the result of uniformly replacing with variable x any constant in $\mathcal{F}$. This rule is known as Existential Generalization.
- Here is an instance of an inference that uses Existential Generalization:

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  1. All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.
  2. From an instance of a formula $\mathcal{F}$ infer $(\exists x \mathcal{F})$ where the formula $\mathcal{F}$ is the result of uniformly replacing with variable x any constant in $\mathcal{F}$. This rule is known as Existential Generalization.
- Here is an instance of an inference that uses Existential Generalization:
  1. Hypatia is wise. (Wh)

Inference in Quantified Relational Logic

- Finally we can state the inference rules of quantified relational logic which turn it into a deductive logic.
  1. All the truth-preserving rules of inference covered in sentential logic are truth-preserving rules of inference in quantified relational logic.
  2. From an instance of a formula $\mathcal{F}$ infer $(\exists x \mathcal{F})$ where the formula $\mathcal{F}$ is the result of uniformly replacing with variable x any constant in $\mathcal{F}$. This rule is known as Existential Generalization.
- Here is an instance of an inference that uses Existential Generalization:
  1. Hypatia is wise. (Wh)

  $\therefore$2. Someone is wise. $(\exists x(Wx))$ (From 1 and By Existential Generalization)

Inference in Quantified Relational Logic

- Inference rules continued from previous slide:

Inference in Quantified Relational Logic

- Inference rules continued from previous slide:
  3. From $\forall x \mathcal{F}$ you may infer any of its instances. This rule is known as Universal Instantiation

Inference in Quantified Relational Logic

- Inference rules continued from previous slide:
  3. From $\forall x \mathcal{F}$ you may infer any of its instances. This rule is known as Universal Instantiation
- Here is an instance of an inference that uses Universal Instantiation.

Inference in Quantified Relational Logic

- Inference rules continued from previous slide:
  3. From $\forall x \mathcal{F}$ you may infer any of its instances. This rule is known as Universal Instantiation
- Here is an instance of an inference that uses Universal Instantiation.
  1. All logicians are wise. $(\forall x(Lx \to Wx))$

Inference in Quantified Relational Logic

- Inference rules continued from previous slide:
  3. From $\forall x \mathcal{F}$ you may infer any of its instances. This rule is known as Universal Instantiation
- Here is an instance of an inference that uses Universal Instantiation.
  1. All logicians are wise. $(\forall x(Lx \rightarrow Wx))$

  $\therefore$2. If Ruth Barcan Marcus is a logician, then she is wise. $(La \rightarrow Wa)$ (From 1 and By Universal Instantiation)

Inference in Quantified Relational Logic

4. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to infer a sentence G using other truth preserving rules of inference. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

- Here is an instance of Existential Supposition:

Inference in Quantified Relational Logic

4. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to infer a sentence G using other truth preserving rules of inference. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

- Here is an instance of Existential Supposition:
  1. Someone is wise. $(\exists x(Wx))$

Inference in Quantified Relational Logic

4. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to infer a sentence G using other truth preserving rules of inference. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

- Here is an instance of Existential Supposition:
    1. Someone is wise. ($\exists x(Wx)$)
    2. Suppose Cavendish is wise. (Wc) (From 1. and Existential Supposition)

Inference in Quantified Relational Logic

4. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to infer a sentence G using other truth preserving rules of inference. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

- Here is an instance of Existential Supposition:
  1. Someone is wise. ($\exists x(Wx)$)
  2. Suppose Cavendish is wise. (Wc) (From 1. and Existential Supposition)
  3. Wise people use logic. (Additional Premise)

Inference in Quantified Relational Logic

4. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to infer a sentence G using other truth preserving rules of inference. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

- Here is an instance of Existential Supposition:
  1. Someone is wise. ($\exists x(Wx)$)
  2. Suppose Cavendish is wise. (Wc) (From 1. and Existential Supposition)
  3. Wise people use logic. (Additional Premise)

  $\therefore$ 4. Cavendish uses logic. (G)

Inference in Quantified Relational Logic

4. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to infer a sentence G using other truth preserving rules of inference. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

- Here is an instance of Existential Supposition:
  1. Someone is wise. $(\exists x(Wx))$
  2. Suppose Cavendish is wise. (Wc) (From 1. and Existential Supposition)
  3. Wise people use logic. (Additional Premise)

  $\therefore$ 4. Cavendish uses logic. (G)
- These are the only truth-preserving rules of inference that we will avail for ourselves in quantified relational logic but other rules of inference can be derived from these.

Models and Counterexamples

- We know why the truth-preserving rules of sentential logic are truth-preserving.

Models and Counterexamples

- We know why the truth-preserving rules of sentential logic are truth-preserving.
- Since $\exists$ and $\forall$ are not truth-functional, in order to say why the rule Existential Generalization, the rule Universal Instantiation and the rule Existential Supposition are truth-preserving rules of inference, we need to discuss the semantics, interpretation or meaning of sentences in quantified relational logic.

Models and Counterexamples

- We know why the truth-preserving rules of sentential logic are truth-preserving.
- Since $\exists$ and $\forall$ are not truth-functional, in order to say why the rule Existential Generalization, the rule Universal Instantiation and the rule Existential Supposition are truth-preserving rules of inference, we need to discuss the semantics, interpretation or meaning of sentences in quantified relational logic.
- This will require a tool which we will call a model or intended interpretation of some sentences in quantified relational logic.

Models and Counterexamples