### Phil/LPS 31 Introduction to Inductive Logic Lecture 15

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### **Topics**

- ▶ Part 1: Decision Problems under Ignorance
  - Ordinal Utilities
  - Dominance Principles
  - Maximin
- Part 2: Decision Problems under Information
  - Cardinal Utilities
  - ► Expected Utility and Risk
  - Principles of Rational Choice under Information

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- Decision problems under information are also known as decision problems under certainty or risk. The relevant sense of "certainty" here is that one is certain about the probability distribution of states. So one can compute the risk associated with taking a decision.

### Part 2: Decision Problems under Ignorance

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- ▶ We write  $A_i \succ A_j$  to mean Act *i* is preferred more than Act *j*.
- ▶ We write  $A_i \sim A_j$  to mean Act i is preferred equally to Act j.

▶ Let  $A_1$ ,  $A_2$ ,  $A_3$  denote the acts of bringing white wine, red wine or rosé to the dinner party, respectively. And let  $S_1$ ,  $S_2$  denote the state in which the host serves fish or chicken, respectively.

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- Suppose that given  $S_1$ , we can order our preferences as  $A_1 \succ A_3 \succ A_2$ . This means that if our host serves fish, we would prefer bringing white wine more than we would prefer bringing either rosé or red wine; and we would prefer to bring rosé more than we would prefer to bring red wine.

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- ▶ Here we see that 4 > 3 > 2. So this utility function respects the preference ordering of the acts. 4, 3 and 2 are ordinal utilities.

▶ Suppose now that the host serves chicken, S₂. You think that if the host serves chicken you'd much rather bring white wine than either red wine or rosé. Assume also that if you can't find white wine at Trader Joe's you'd much rather bring rosé than red wine.

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  - 3 Verify that your utility function respects your preference ordering.

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  - (4) Provide no information about the strength of preferences.

### Making Decisions with Ordinal Utilities

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White	4	5
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- ▶ **Strict Dominance**:  $A_i \succ A_j$  if and only if (1)  $u(A_i|S_n) \ge u(A_j|S_n)$  for every state  $S_n$  (at least as good) and (2) there exists a state  $S_m$  such that  $u(A_i|S_m) > u(A_j|S_m)$  (at least one better).

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  - Does the strong dominance principle imply the weak dominance principle?

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  - How would you decide in this case?

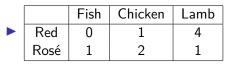
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- According to this principle, one should MAXimise the MINimal value obtainable with each act. If the worst possible outcome of one alternative is better than that of another, then the former should be chosen.



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Red	0	1	4
Rosé	1	2	1

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  - ▶ What is the worst possible outcome for  $A_2$ ? How about  $A_3$ ?
  - ▶ Does  $A_3 \succeq A_2$ ?
  - ▶ Why would an agent choose  $A_3$ ?

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  - ▶ Does  $A_3 \succeq A_2$ ?
  - Why would an agent choose A<sub>3</sub>?
- ► There are other principles of rational choice in the context of decisions under ignorance. But we shall not cover them in this introductory course. The book by Martin Peterson An Introduction to Decision Theory is highly recommended for this.

Part 1: Decision Problems under Information

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- ▶ This means that the agent can also calculate the expected value of functions of these states, namely, consequences of an act.
- However, not just any concept of utility will do. We have seen that because ordinal utilities cannot be added or multiplied, we cannot use them to calculate expected values. Further ordinal utilities do not quantify the strength of preference, they simply respect the ordering of our preferences.

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  - (1) Like ordinal utilities, cardinal utilities are unique up to strictly increasing transformations.
  - (2) Cardinal utilities are invariant under positive scaling and positive translation. Barrett and Huttegger call this positive affine transformation So cardinal utilities are like hours and seconds or temperature intervals.

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  - (2) Cardinal utilities are invariant under positive scaling and positive translation. Barrett and Huttegger call this positive affine transformation So cardinal utilities are like hours and seconds or temperature intervals.
  - (3) From (2) we can calculate expected utilities using cardinal utilities.

- Cardinal utilities are the right concept of utility to quantify the strength of preference.
- ► They also enjoy the right kind of properties that make calculations of expected values possible.
- Some of these properties are:
  - (1) Like ordinal utilities, cardinal utilities are unique up to strictly increasing transformations.
  - (2) Cardinal utilities are invariant under positive scaling and positive translation. Barrett and Huttegger call this positive affine transformation So cardinal utilities are like hours and seconds or temperature intervals.
  - (3) From (2) we can calculate expected utilities using cardinal utilities.
  - (4) Provide information about the strength of preferences.

## Decision Problems Under Risk

# Expected Utility and Risk

$$U(A_1) = u(A|S_1)P(S_1) + u(A|S_2)P(S_1) + \dots + u(A|S_n)P(S_n)$$
  
=  $\sum_{i=1}^{n} u(A|S_i)P(S_i)$ 

# Maximize Expected Utility