Phil/LPS 31 Introduction to Inductive Logic
Lecture 14

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Topics

- Recap: Is inductive logic possible?
- Introduction to Decision Theory
- Utilities and Losses
- Expected Utility and Risk
- Principles of Rational Choice

Recap: Is inductive logic possible?

Here's Hume (1748) *An Enquiry Concerning Human Understanding*:

> *In vain do you pretend to have learned the nature of bodies from your past experience. Their secret nature, and consequently all their effects and influence, may change without any change in their sensible qualities. This happens sometimes, and with regard to some objects: why may it not happen always, and with regard to all objects? What logic, what process or argument secures you against this supposition? My practice, you say, refutes my doubts. But you mistake the purport of my question. As an agent, I am quite satisfied in the point; but as a philosopher...I want to learn the foundation of this inference.*

Recap: Is inductive logic possible?

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- In the case of deductive logic, the justification of the rules of inference there was that the good rules of inference are precisely those rules of inference that are truth-preserving. But we have seen that these rules are truth-preserving because they are non-ampliative.
- Do we have a similar criterion for selecting the inductive rules of inference that are good? In other words, is inductive logic possible?

Is probability the very guide of life?

But to us, probability is the very guide of life.

Joseph Butler (1736) *The Analogy of Religion, Natural and Revealed, to the Constitution and Course of Nature*

Is probability the true logic for this world?

> *They say that Understanding ought to work by the rules of right reason. These rules are, or ought to be, contained in Logic; but the actual science of Logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the the True Logic for this world is the Calculus of Probabilities, which takes account of the magnitude of the probability (which is, or which ought to be in a reasonable man's mind). This branch of Math., which is generally thought to favor gambling, dicing, and wagering, and therefore highly immoral, is the only "Mathematics for Practical [People]', as we ought to be.*

James Clerk Maxwell's Letter to Lewis Campbell, c. July 1850

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- Such a characterization will give us a precise of way of saying what we mean when we say that good rules of inductive inference are those rules that lead to "favorable consequences most of the time."
- The way we do this is by merging: (1) probability theory (which gives us the "most of the time part" using expected value) and (2) decision theory (which gives principles we can use to "evaluate consequences", namely the concepts of utility and loss).

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  The probability of any event is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the chance of the thing expected upon it's happening.

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- This is quite remarkable! It means that you can: (1) use the expected value of the losses/gains on your decisions to calibrate your probabilities; and (2) you can use probabilities to decide upon which actions are "rational" to take based on their expected losses (risks) or expected gains.

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- Let us see how we can do this more formally.

Decision Problems: Illustration 1

- Suppose you are invited to a dinner party and your host asks you to bring some wine. You have to choose between white wine or red wine. It is also known that red wine goes well with red meat (like beef, lamb and veal) while white wine goes well with white meat (like chicken or fish). So you want the wine you bring to match the type of dish that will be served.

Decision Problems: Illustration 1

- You cannot bring both to the dinner party because you're on a budget. So you are faced with a decision problem which we may represent in the following decision table (the left table) and corresponding desirability table (the right table).

	Fish	Lamb
White	The right wine	The wrong wine
Red	An odd wine	The right wine

	Fish	Lamb
White	5	2
Red	1	5

Decision Problems: Illustration 2

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- Treasure Department Secretary Janet Yellen says that if President Biden invoked the 14th Amendment, it would create a constitutional crisis.
- The President is faced with two choices to make sure that the American Federal Government lives up to its obligations and avoids default: invoke the 14th amendment or reach a deal with U.S. House of Representative leaders.

Decision Problems: Illustration 2

- President Biden has a decision problem.

	Deal	No Deal
Invoke^c	Default^c	Default
Invoke	Default^c	Crisis

	Deal	No Deal
Invoke^c	0	-10
Invoke	-1	-1

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- Acts correspond to choices that an agent may make when faced with a decision problem. It is assumed that the space of possible acts is a partition of the space of choices that an agent may take, in the sense that the choices are mutually exclusive. We use A, with or without numerical subscripts to denote acts.

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- Consequences are the joint outcomes of choosing to act one way given one of the states obtains.
- A Utility Function is an assignment of cardinal utilities to consequences. The distinction between cardinal utilities and ordinal utilities is in your required reading for this week.
- Here is the utility function for the dinner party example:

	Fish	Lamb
White	5	2
Red	1	5

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- So we can in principle solve most decision problems using either utilities or losses. It just involves a shift in perspective.
- But! the principles that make a choice rational will not be same. We'll come back to this later.
- Here is President Biden's *hypothetical* loss function.

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- This will lead to the concept of expected utility or expected loss (or risk)

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- Suppose you know that there are even odds that the host will serve red meat or white meat, what is the expected utility of each act that you may take?
- Now suppose that because of the rising cost of beef, the host will serve white meat with probability 0.8, what is the expected utility of each act that you may take?

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- What do think is the best decision here?

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- Suppose President Biden after returning from his trip from Japan knows that the odds that U.S. House of Representatives will agree to a deal are $3:5$, what is the risk of each act that President Biden may take?
- How would you advise President Biden?