Phil/LPS 31 Introduction to Inductive Logic
Lecture 13

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Topics

- Odds
- Expected Value (Discrete Case Only)

Recap: Probability Rules

Rule 1 $P(S) = 1$

Rule 2 $0 \leq P(E) \leq 1$

Rule 3 If $E \cap F = \emptyset$, then $P(E \cup F) = P(E) + P(F)$ (Special Disjunction Rule)

Rule 4 $P(E \cup E^c) = 1$

Rule 5 $P(E^c) = 1 - P(E)$

Rule 6 $P(\emptyset) = 0$

Rule 7 $P(E) = P(E \cap F) + P(E \cap F^c)$

Rule 8 $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ (General Disjunction Rule)

Rule 9 $P(E \cap F) = P(E) \times P(F)$ if E and F are independent.

Rule 10 $P(E \cap F) = P(E) \times P(F|E)$ if E and F are dependent.

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- If $\{E_1, E_2, \ldots, E_n\}$ is a partition of E, then from the theorem of total probability:

$$\begin{aligned} P(E \mid F) &= \frac{P(F \mid E)P(E)}{P(F \mid E_1)P(E_1) + \cdots + P(F \mid E_n)P(E_n)} \\ &= \frac{P(F \mid E)P(E)}{\sum_{i=1}^{n} P(F \mid E_i)P(E_i)} \end{aligned}$$

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- Odds are a useful way of talking about the chances of an event and many people often go back and forth between probability and odds as if they mean the same thing. They don't!
- In this lecture we will see why and also introduce ways of going (1) from probability to odds and (2) from odds to probability.

From Probabilities to Odds

- If the probability of an event E is p, then we know by Rule 5, that the probability of E^c is $1 - p$. We define the odds in favor, or simply just odds, of E as:

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  (1) The probability that Manchester City win after being ahead by 2 - 0 at half time is 0.8.
  (2) The probability that a family has two children who are both girls is $\frac{1}{2}$.

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  (4) If $\mathcal{O}(E) = 1$, then $P(E) = P(E^c)$

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