Phil/LPS 31 Introduction to Inductive Logic
Lecture 13

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Topics

- Odds
- Expected Value (Discrete Case Only)

Recap: Probability Rules

Rule 1 $P(S) = 1$

Rule 2 $0 \leq P(E) \leq 1$

Rule 3 If $E \cap F = \emptyset$, then $P(E \cup F) = P(E) + P(F)$ (Special Disjunction Rule)

Rule 4 $P(E \cup E^c) = 1$

Rule 5 $P(E^c) = 1 - P(E)$

Rule 6 $P(\emptyset) = 0$

Rule 7 $P(E) = P(E \cap F) + P(E \cap F^c)$

Rule 8 $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ (General Disjunction Rule)

Rule 9 $P(E \cap F) = P(E) \times P(F)$ if E and F are independent.

Rule 10 $P(E \cap F) = P(E) \times P(F|E)$ if E and F are dependent.

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- A slightly more general form of Bayes Theorem says that if $\{E_1, E_2, E_3, E_4, \ldots, E_n\}$ is a partition of an event E, then for any $E_i \subset E$:

$$\boxed{P(E_i \mid F) = \frac{P(F|E_i)P(E_i)}{\sum_{i=1}^{n} P(F|E_i)P(E_i)}}$$

\end{center}

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- Odds are a useful way of talking about the chances of an event and many people often go back and forth between probability and odds as if they mean the same thing. They don't!
- In this lecture we will see why and also introduce ways of going (1) from probability to odds and (2) from odds to probability.

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  (2) The odds of having two girls, if the probability that a family has two children who are both girls is $\frac{1}{4}$.

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- Exercise. What are the ways that probabilities are different from odds given these properties of odds?

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- We will see later that the Odds Ratio provides a good measure of association (or dependence) between two groups in the same way that we used joint and marginal probabilities to assess independence between groups.

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- Solving this equation for p we get:

$$p = \frac{\mathcal{O}(E)}{1+\mathcal{O}(E)}$$

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- See Homework 7 for more exercises.

Expected Value (Discrete Case)

- The number of telephone calls received at a remote call center per minute show the following frequencies:

Number of Calls	Count
1	10
2	8
3	4
4	4
5	2
6	2
7	1
8	1
TOTAL	32

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- Using the probability distribution, we can then ask useful questions such as: what is the expected value or expectation of the random variable.

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- Here is the data for your convenience.

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- The expected value of A is given by:

$$
\begin{aligned}
E(A) &= a_1 p_1 + a_2 p_2 + a_3 p_3 + \cdots + a_n p_n \\
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- Commit this formula to memory!