Phil/LPS 31 Introduction to Inductive Logic
Lecture 5

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Topics

- Inference in Quantified Relational Logic
- Models and Counterexamples

Inference in Quantified Relational Logic

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  4. $(\forall x \forall y((Lxy) \to \exists z((Lxz) \wedge (Lzy))))$

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- These are the only truth-preserving rules of inference that we will avail for ourselves in quantified relational logic.

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- Since the operator $\exists$ is not truth-functional, in order to say why the rule Existential Introduction and the rule Existential Exploitation are a truth-preserving rules of inference, we need to discuss the semantics, interpretation or meaning of sentences in quantified relational logic.
- This will require a tool which we will call a model or intended interpretation of some sentences in quantified relational logic.

Models and Counterexamples