

Phil/LPS 31 Introduction to Inductive Logic

Lecture 1

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Topics

- ▶ Logic in General
- ▶ Sentential Logic
- ▶ Truth-functional connectives
- ▶ Truth tables
- ▶ Deductive Logic

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- ▶ This distinction between “form” and “content” is crucial to understand a logic in general because logicians distinguish between the formal aspect of a language (**syntax**) and its interpretation (**semantics**).
- ▶ Finally, the word “system” means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also **belong to the representation**.

Toy Example of a Logic

- ▶ Here's a toy example of a logic.

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- ▶ It turns out that saying what “propositions” are is a **hard** philosophical problem. So we’ll stick to sentences!

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 4. Having 1001 strands of hair does not make you bald. How about 1000, 999, . . . ,?

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- ▶ What's the difference between the sentences in **Case 1** and **Case 2**?

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- ▶ Sentential logic captures the sentence structure of a fragment of natural language using truth-functional connectives.

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- ▶ The **closure condition** simply states that nothing else is a formula of sentential logic.