Phil/LPS 31 Introduction to Inductive Logic
Lecture 5

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Topics

- Inference in Quantified Relational Logic
- Models and Counterexamples

Inference in Quantified Relational Logic

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  4. $\forall x \forall y(Lxy \rightarrow \exists z(Lxz \wedge Lzy))$

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- These are the only truth-preserving rules of inference that we will avail for ourselves in quantified relational logic.

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- Since the operator $\exists$ is not truth-functional, in order to say why Existential Introduction is a truth-preserving rule of inference, we need to discuss the semantics, interpretation or meaning of sentences in quantified relational logic.
- This will require a tool which we will call a model or intended interpretation of some sentences in quantified relational logic.

Models and Counterexamples