Phil/LPS 31 Introduction to Inductive Logic Lecture 1

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Topics

- ► Logic in General
- Sentences
- ► Truth-functional connectives
- ► Sentential logic

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- ► Finally, the word "system" means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also belong to the representation. The symbols that belong to the representation are called, you guessed it, formulas!

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- Closure condition: Nothing else is a formula.
- Verify that all these are examples of formulas of the logic: S, ab, aaSbb, aabb.

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- ▶ It turns out that saying what "propositions" are is a hard philosophical problem. So we'll stick to sentences!

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 - 4. Having 1001 strands of hair does not make you bald. How about 1000, 999, ...,?

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- Sentential logic is the logic for representing the sentence structure of a fragment of natural language using truth-functional connectives.

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- ► The closure condition simply states that nothing else is a formula of sentential logic.