Phil/LPS 31 Introduction to Inductive Logic
Lecture 15

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May 22nd 2023

Topics

- Part 1: Decision Problems under Ignorance
  - Ordinal Utilities
  - Dominance Principles
  - Maximin
- Part 2: Decision Problems under Information
  - Cardinal Utilities
  - Expected Utility and Risk
  - Principles of Rational Choice under Information

Recap: Decision Problems: Acts, States, Consequences, Utility/Loss Functions

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- Decision problems under information are also known as decision problems under certainty or risk. The relevant sense of "certainty" here is that one is certain about the probability distribution of states. So one can compute the risk associated with taking a decision.

Part 2: Decision Problems under Ignorance

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- We write $A_i \sim A_j$ to mean Act i is preferred equally to Act j.

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- Suppose that given S_1, we can order our preferences as $A_1 \succ A_3 \succ A_2$. This means that if our host serves fish, we would prefer bringing white wine more than we would prefer bringing either rosé or red wine; and we would prefer to bring rosé more than we would prefer to bring red wine.

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- Here we see that $4 > 3 > 2$. So this utility function respects the preference ordering of the acts. 4, 3 and 2 are ordinal utilities.

Ordinal Utilities

- Suppose now that the host serves chicken, S_2. You think that if the host serves chicken you'd much rather bring white wine than either red wine or rosé. Assume also that if you can't find white wine at Trader Joe's you'd much rather bring rosé than red wine.

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  3. Verify that your utility function respects your preference ordering.

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  4. Provide no information about the strength of preferences.

Making Decisions with Ordinal Utilities

- From the previous exercises we obtain the following desirability table for acts based on our ordinal utility function.

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White	4	5
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  - Decide on the basis of the strong dominance principle.
  - Decide on the basis of the maximin principle.

Strict and Weak Dominance Principles

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- We say that A_1 "dominates" both A_2 and A_3.
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- A widely accepted dominance principle in decision theory prescribes that dominated acts must not be chosen.

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- **Weak Dominance**: $A_i \succeq A_j$ if and only if $u(A_i|S) \geq u(A_j|S)$ for every state S (at least as good)
- **Strict Dominance**: $A_i \succ A_j$ if and only if (1) $u(A_i|S_n) \geq u(A_j|S_n)$ for every state S_n (at least as good) and (2) there exists a state S_m such that $u(A_i|S_m) > u(A_j|S_m)$ (at least one better).

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Red	2	1	4
Rosé	3	4	4

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  - Does the strong dominance principle imply the weak dominance principle?

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  - How would you decide in this case?

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- The maximin principle focuses on the worst possible outcome of each alternative act. Essentially, we're asking what's the worst that can happen?
- According to this principle, one should MAXimise the MINimal value obtainable with each act. If the worst possible outcome of one alternative is better than that of another, then the former should be chosen.

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  - Does $A_3 \succeq A_2$?
  - Why would an agent choose A_3?

- There are other principles of rational choice in the context of decisions under ignorance. But we shall not cover them in this introductory course. The book by Martin Peterson *An Introduction to Decision Theory* is highly recommended for this.

Part 1: Decision Problems under Information

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- This means that the agent can also calculate the expected value of functions of these states, namely, consequences of an act.
- However, not just any concept of utility will do. We have seen that because ordinal utilities cannot be added or multiplied, we cannot use them to calculate expected values. Further ordinal utilities do not quantify the strength of preference, they simply respect the ordering of our preferences.

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Cardinal Utilities

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- They also enjoy the right kind of properties that make calculations of expected values possible.
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  (3) From (2) we can calculate expected utilities using cardinal utilities.
  (4) Provide information about the strength of preferences.

Expected Utility

- The expected utility of an act is the weighted average of the consequences of that act where the weights on the consequences are determined by the probability distribution on states.

$$\begin{aligned} U(A) &= u(A|S_1)P(S_1) + u(A|S_2)P(S_2) + \cdots + u(A|S_n)P(S_n) \\ &= \sum_{i}^{n} u(A \mid S_i)P(S_i) \end{aligned}$$

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- In other words, take the utility of that action given each state of the world multiplied by the probability that the state obtains and sum everything up

Expected Utility

- Consider the dinner party example again. This time the entries are cardinal utilities.

	Fish	Chicken
White	4	5
Red	2	1
Rosé	3	3

Expected Utility

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- Exercise. Suppose you know that there are even odds that the host will serve fish or chicken calculate the expected value of A_1, A_2 and A_3

Risk

- The risk is of an act is just like the expected value of the act, except that we're using a loss function to assign loss values to possible consequences of an act.

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Risk

- Consider President Biden's *hypothetical* loss function.

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Invoke^c	0	-10
Invoke	-1	-1

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- Exercise. Suppose you that President Biden knows that the odds are 3:5 that a deal will be reached with the U.S. House of Representative Majority Leader Kevin McCarthy. What is the risk of the possible acts that Biden can take to avoid having the Federal Government default on its debt?

Maximizing Expected Utility

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Maximizing Expected Utility

- The cardinal principle (you see what I did there?) of decision problems under risk is the principle of maximizing expected utility if you're using a cardinal utility function.
- This means that in any decision problem, choose the act that maximizes expected utility with respect to your probability distribution on states.

Maximizing Expected Utility

- For the following exercise, refer to the following desirability table.

	Fish	Chicken	Lamb
White	3	4	1
Red	2	1	4
Rosé	3	4	4

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- Suppose you know that because of rising tariffs on fish imports from Canada and recent shortages of lamb, you host is likely to serve chicken with probability 0.8, fish with probability 0.15 and lamb with probability 0.05. What wine should you choose to bring to the dinner party according to the principle of maximizing expected utility?

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- Let us call a person risk averse if given a choice between two actions with risk, they will tend to choose the act that is less risky. That is, they choose to minimize the risk.
- There is an on-going debate about what the implications of risk for rationality. See Lara Buchak's *Risk and Rationality*.

Minimizing Risk

Consider President Biden's modified *hypothetical* loss function with the additional action that President Biden does nothing.

	Deal	No Deal
Invoke^c	0	-10
Invoke	-1	-1
Do Nothing	0	-10