

# Phil/LPS 31 Introduction to Inductive Logic

## Lecture 1

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# Topics

- ▶ Logic in General
- ▶ Sentences
- ▶ Truth-functional connectives
- ▶ Sentential logic

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- ▶ Finally, the word “system” means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also **belong to the representation**. The symbols that belong to the representation are called, you guessed it, **formulas**!

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- ▶ Verify that all these are examples of formulas of the logic:  $S$ ,  $ab$ ,  $aaSbb$ ,  $aabb$ .

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## Sentential Logic: Sentences

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- ▶ It turns out that saying what “propositions” are is a **hard** philosophical problem. So we'll stick to sentences!

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  4. Having 1001 strands of hair does not make you bald. How about 1000, 999, ...?

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- ▶ What's the difference between the sentences in **Case 1** and **Case 2**?



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- ▶ Sentential logic is the logic for representing the sentence structure of a fragment of natural language **using** truth-functional connectives.

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