Phil/LPS 31 Introduction to Inductive Logic
Lecture 1

David Mwakima
dmwakima@uci.edu
Department of Logic and Philosophy of Science
University of California, Irvine

April 3rd 2023

Topics

- Logic in General
- Sentences
- Truth-functional connectives
- Sentential logic

Logic in General

- A logic is a formal system for representing something whose structure, as opposed to content, we wish to describe.

Logic in General

- A logic is a formal system for representing something whose structure, as opposed to content, we wish to describe.
- The word “formal” here means that in logic we are concerned with form, i.e., uninterpreted symbols or formulas, not material content. Inevitably, this will involve both abstraction and idealization.

Logic in General

- A logic is a formal system for representing something whose structure, as opposed to content, we wish to describe.
- The word "formal" here means that in logic we are concerned with form, i.e., uninterpreted symbols or formulas, not material content. Inevitably, this will involve both abstraction and idealization.
- This distinction between "form" and "content" is crucial to understand a logic in general because logicians distinguish between the formal aspect of a language (syntax) and its content, meaning or interpretation (semantics).

Logic in General

- A logic is a formal system for representing something whose structure, as opposed to content, we wish to describe.
- The word "formal" here means that in logic we are concerned with form, i.e., uninterpreted symbols or formulas, not material content. Inevitably, this will involve both abstraction and idealization.
- This distinction between "form" and "content" is crucial to understand a logic in general because logicians distinguish between the formal aspect of a language (syntax) and its content, meaning or interpretation (semantics).
- Finally, the word "system" means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also belong to the representation. The symbols that belong to the representation are called, you guessed it, formulas!

Toy Example of a Logic[1]

- The symbols are a, b and S.

[1] This example is from Gerstein, "Introduction to Mathematical Structures and Proofs" p. 3

Toy Example of a Logic[1]

- The symbols are a, b and S.
- The rules of transformation are:

[1] This example is from Gerstein, "Introduction to Mathematical Structures and Proofs" p. 3

Toy Example of a Logic[1]

- The symbols are a, b and S.
- The rules of transformation are:
  1. S is a formula.

[1] This example is from Gerstein, "Introduction to Mathematical Structures and Proofs" p. 3

Toy Example of a Logic[1]

- The symbols are a, b and S.
- The rules of transformation are:
  1. S is a formula.
  2. The result of replacing S with aSb is a formula.

[1] This example is from Gerstein, "Introduction to Mathematical Structures and Proofs" p. 3

Toy Example of a Logic[1]

- The symbols are a, b and S.
- The rules of transformation are:
  1. S is a formula.
  2. The result of replacing S with aSb is a formula.
  3. The result of deleting S and closing up any resulting space is a formula.

[1] This example is from Gerstein, "Introduction to Mathematical Structures and Proofs" p. 3

Toy Example of a Logic[1]

- The symbols are a, b and S.
- The rules of transformation are:
  1. S is a formula.
  2. The result of replacing S with aSb is a formula.
  3. The result of deleting S and closing up any resulting space is a formula.
- Closure condition: A formula is anything that results from 1 or a finite number of applications of rule 2 or rule 3. Nothing else is a formula.

[1] This example is from Gerstein, "Introduction to Mathematical Structures and Proofs" p. 3

Toy Example of a Logic[1]

- The symbols are a, b and S.
- The rules of transformation are:
  1. S is a formula.
  2. The result of replacing S with aSb is a formula.
  3. The result of deleting S and closing up any resulting space is a formula.
- Closure condition: A formula is anything that results from 1 or a finite number of applications of rule 2 or rule 3. Nothing else is a formula.
- Verify that all these are examples of formulas of the logic: S, ab, $aaSbb$, $aabb$.

[1]This example is from Gerstein, "Introduction to Mathematical Structures and Proofs" p. 3

Sentential Logic: Sentences

- Sentential logic is a logic for representing the sentence structure of a fragment or piece of natural language. The natural language in our case will be English.

Sentential Logic: Sentences

- Sentential logic is a logic for representing the sentence structure of a fragment or piece of natural language. The natural language in our case will be English.
- Philosophers like to distinguish between sentences and propositions.

Sentential Logic: Sentences

- Sentential logic is a logic for representing the sentence structure of a fragment or piece of natural language. The natural language in our case will be English.
- Philosophers like to distinguish between sentences and propositions.
- A sentence is a linguistic expression (a declarative statement in some language), which expresses a proposition.

Sentential Logic: Sentences

- Sentential logic is a logic for representing the sentence structure of a fragment or piece of natural language. The natural language in our case will be English.
- Philosophers like to distinguish between sentences and propositions.
- A sentence is a linguistic expression (a declarative statement in some language), which expresses a proposition.
- Different sentences can express the same proposition, e.g., if they are in different languages.

Sentential Logic: Sentences

- Sentential logic is a logic for representing the sentence structure of a fragment or piece of natural language. The natural language in our case will be English.
- Philosophers like to distinguish between sentences and propositions.
- A sentence is a linguistic expression (a declarative statement in some language), which expresses a proposition.
- Different sentences can express the same proposition, e.g., if they are in different languages.
  - Theluji ni nyeupe. (Swahili)

Sentential Logic: Sentences

- Sentential logic is a logic for representing the sentence structure of a fragment or piece of natural language. The natural language in our case will be English.
- Philosophers like to distinguish between sentences and propositions.
- A sentence is a linguistic expression (a declarative statement in some language), which expresses a proposition.
- Different sentences can express the same proposition, e.g., if they are in different languages.
  - Theluji ni nyeupe. (Swahili)
  - Snow is white. (English)

Sentential Logic: Sentences

- Sentential logic is a logic for representing the sentence structure of a fragment or piece of natural language. The natural language in our case will be English.
- Philosophers like to distinguish between sentences and propositions.
- A sentence is a linguistic expression (a declarative statement in some language), which expresses a proposition.
- Different sentences can express the same proposition, e.g., if they are in different languages.
  - Theluji ni nyeupe. (Swahili)
  - Snow is white. (English)
- It turns out that saying what "propositions" are is a hard philosophical problem. So we'll stick to sentences!

Sentential Logic: Sentences

- I assume we all know what a sentence is.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.
  2. Kamala Harris was not a California Senator.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.
  2. Kamala Harris was not a California Senator.
  3. Michelle Yeoh won an Oscar Award.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.
  2. Kamala Harris was not a California Senator.
  3. Michelle Yeoh won an Oscar Award.
- The following examples are not sentences (in our sense) because their truth value is either context dependent or vague:

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.
  2. Kamala Harris was not a California Senator.
  3. Michelle Yeoh won an Oscar Award.
- The following examples are not sentences (in our sense) because their truth value is either context dependent or vague:
  1. He did it yesterday.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.
  2. Kamala Harris was not a California Senator.
  3. Michelle Yeoh won an Oscar Award.
- The following examples are not sentences (in our sense) because their truth value is either context dependent or vague:
  1. He did it yesterday.
  2. This is black.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.
  2. Kamala Harris was not a California Senator.
  3. Michelle Yeoh won an Oscar Award.
- The following examples are not sentences (in our sense) because their truth value is either context dependent or vague:
  1. He did it yesterday.
  2. This is black.
  3. There will be a sea battle tomorrow.

Sentential Logic: Sentences

- I assume we all know what a sentence is.
- But! For this class (or, more specifically classical logic) a sentence must be unambiguously true or false. This is just a convenient restriction.
- Examples of sentences:
  1. The number two is an even prime number.
  2. Kamala Harris was not a California Senator.
  3. Michelle Yeoh won an Oscar Award.
- The following examples are not sentences (in our sense) because their truth value is either context dependent or vague:
  1. He did it yesterday.
  2. This is black.
  3. There will be a sea battle tomorrow.
  4. Having 1001 strands of hair does not make you bald. How about 1000, 999, . . . ?

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.
  4. If today is Tuesday, then tomorrow is Wednesday.

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.
  4. If today is Tuesday, then tomorrow is Wednesday.

- Each of these sentences, is really a compound sentence of two simpler sentences joined together by a "connective". But so are these sentences, call these Case 2:

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.
  4. If today is Tuesday, then tomorrow is Wednesday.

- Each of these sentences, is really a compound sentence of two simpler sentences joined together by a "connective". But so are these sentences, call these Case 2:
  1. David is teaching until his students are having fun. (True)

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.
  4. If today is Tuesday, then tomorrow is Wednesday.

- Each of these sentences, is really a compound sentence of two simpler sentences joined together by a "connective". But so are these sentences, call these Case 2:
  1. David is teaching until his students are having fun. (True)
  2. Ukraine is fighting Russia until Russia occupies Crimea. (False)

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.
  4. If today is Tuesday, then tomorrow is Wednesday.

- Each of these sentences, is really a compound sentence of two simpler sentences joined together by a "connective". But so are these sentences, call these Case 2:
  1. David is teaching until his students are having fun. (True)
  2. Ukraine is fighting Russia until Russia occupies Crimea. (False)
  3. The students are having fun because the students like to study logic. (True)

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.
  4. If today is Tuesday, then tomorrow is Wednesday.

- Each of these sentences, is really a compound sentence of two simpler sentences joined together by a "connective". But so are these sentences, call these Case 2:
  1. David is teaching until his students are having fun. (True)
  2. Ukraine is fighting Russia until Russia occupies Crimea. (False)
  3. The students are having fun because the students like to study logic. (True)
  4. The students like to study logic because the discussions are early in the morning. (False)

Sentential Logic: Truth-functional connectives

- Consider the following sentences, call these Case 1:
  1. The number two is an even number and the number two is prime.
  2. Kamala Harris was not California Senator.
  3. Either Michelle Yeoh won an Oscar Award for Best Actress or Ana de Armas won an Oscar Award for Best Actress.
  4. If today is Tuesday, then tomorrow is Wednesday.

- Each of these sentences, is really a compound sentence of two simpler sentences joined together by a "connective". But so are these sentences, call these Case 2:
  1. David is teaching until his students are having fun. (True)
  2. Ukraine is fighting Russia until Russia occupies Crimea. (False)
  3. The students are having fun because the students like to study logic. (True)
  4. The students like to study logic because the discussions are early in the morning. (False)

- What's the difference between the sentences in Case 1 and Case 2?

Sentential Logic: Truth-functional connectives

- In Case 1, the truth-value of the compound sentence is uniquely and fully determined by the truth-values of the simpler sentences it is composed of.

Sentential Logic: Truth-functional connectives

- In Case 1, the truth-value of the compound sentence is uniquely and fully determined by the truth-values of the simpler sentences it is composed of.
- The connectives (and, not, or, if..., then...) in Case 1 are said to be truth-functional. See Homework 1 for more examples.

Sentential Logic: Truth-functional connectives

- In Case 1, the truth-value of the compound sentence is uniquely and fully determined by the truth-values of the simpler sentences it is composed of.
- The connectives (and, not, or, if..., then...) in Case 1 are said to be truth-functional. See Homework 1 for more examples.
- In contrast, the connectives (until, because) in Case 2 are not truth-functional. In order to determine the truth-value of the compound sentences which involve them, we need extra information beyond the truth-value of the simpler sentences. See Homework 1 for more examples.

Sentential Logic: Truth-functional connectives

- In Case 1, the truth-value of the compound sentence is uniquely and fully determined by the truth-values of the simpler sentences it is composed of.
- The connectives (and, not, or, if..., then...) in Case 1 are said to be truth-functional. See Homework 1 for more examples.
- In contrast, the connectives (until, because) in Case 2 are not truth-functional. In order to determine the truth-value of the compound sentences which involve them, we need extra information beyond the truth-value of the simpler sentences. See Homework 1 for more examples.
- Sentential logic is the logic for representing the sentence structure of a fragment of natural language using truth-functional connectives.

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.
- The formal symbols of sentential logic are:

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.
- The formal symbols of sentential logic are:
  1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, $\ldots$.

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.
- The formal symbols of sentential logic are:
  1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
  2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.
- The formal symbols of sentential logic are:
  1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
  2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)
  3. (for left bracket and) for right bracket.

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.
- The formal symbols of sentential logic are:
  1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
  2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)
  3. (for left bracket and) for right bracket.
- In order to state the transformation rules (also known as “syntactic rules”) of sentential logic we shall introduce the symbols F, G and H to talk about formulas in sentential logic at a meta-level:

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.
- The formal symbols of sentential logic are:
  1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
  2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)
  3. (for left bracket and) for right bracket.
- In order to state the transformation rules (also known as "syntactic rules") of sentential logic we shall introduce the symbols F, G and H to talk about formulas in sentential logic at a meta-level:
  1. Any sentence p is a formula.

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.
- The formal symbols of sentential logic are:
  1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
  2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)
  3. (for left bracket and) for right bracket.
- In order to state the transformation rules (also known as "syntactic rules") of sentential logic we shall introduce the symbols F, G and H to talk about formulas in sentential logic at a meta-level:
  1. Any sentence p is a formula.
  2. If F is a formula, then $\neg F$ is formula.

Sentential Logic: The Logic Itself

- We are now in a position to describe sentential logic itself. I need: (1) formal symbols, (2) rules of transformation for getting formulas and (3) a closure condition.

- The formal symbols of sentential logic are:
  1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
  2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)
  3. (for left bracket and) for right bracket.

- In order to state the transformation rules (also known as “syntactic rules”) of sentential logic we shall introduce the symbols F, G and H to talk about formulas in sentential logic at a meta-level:
  1. Any sentence p is a formula.
  2. If F is a formula, then $\neg F$ is formula.
  3. If F is a formula and G is a formula, then $(F \vee G)$ is a formula.

Sentential Logic: The Logic Itself

- The closure condition states that a formula of sentential logic is anything that results from 1 or from a finite number applications of rule 2 or rule 3. Nothing else is a formula of sentential logic.