Phil/LPS 31 Introduction to Inductive Logic
Lecture 16

David Mwakima
dmwakima@uci.edu
Department of Logic and Philosophy of Science
University of California, Irvine

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- Interpretations of Probability
  - Frequentist Interpretation of Probability

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- Similarly, when we introduced quantified relational logic we added formal symbols for variables, constants and quantifiers to capture more of the structure of sentences than sentential logic could capture. Here, the interpretation of the symbols was given using a model.

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- The Kolmogorov Axioms (or rules) for probability functions are formal in the same way that the truth-preserving rules of deductive logic are formal.
- By "in the same way", I mean that both are formulated in a general way that allows the possibility of many, and different, applications or interpretations.

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- The situation is similar in the case of probability functions characterized by the Kolmogorov Axioms. There are lots of applications or interpretations because there are lots of things that satisfy, faithfully represent or model the Kolmogorov Axioms.

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- What these examples have in common is that they all appear to interpret or apply probability functions in different contexts and to address different problems.

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- Weather forecasters (and the Instructor) seem to be talking about their beliefs. The beliefs here can be objective (based on precise measurements of wind patterns) or subjective (the Instructor doesn't know for sure).

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- For each of these interpretations, I want you to know: (1) the main ideas behind it; (2) the main advantages and drawbacks; (3) how evidence is evaluated.

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- First elaborated with precision and in detail by the English logician John Venn in the 19th century and later by von Mises.
- To say that a random experiment has a stable objective probability $P(E)$ associated with one of its possible outcomes E is to say that it has a characteristic tendency to do so.
- The magnitude of this tendency is measured by the limiting or long-run relative frequency with which the outcome in question is actually produced.

$$P(E) = \lim_{n \to \infty} \frac{\text{Number of times E occurs in } n \text{ trials}}{n}$$

Frequentist Interpretation of Probability

- Consider an ordinary coin that is being flipped in the standard way. As it is flipped repeatedly a sequence of outcomes is generated:

$$
\begin{gathered}
< H, T, H, T, T, T, H, H, T, T, H, T, T, \\
T, T, H, T, H, T, T, T, H, H, H, H, \cdots >
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- We can associate with this sequence of results a sequence of relative frequencies as follows:

$$<1/1, 1/2, 2/3, 2/4, 2/5, 2/6, 3/7, 4/8, 4/9, 4/10, 5/11, 5/12,$$
$$5/13, 5/14, 5/15, 6/16, 7/18, 7/19, 7/20, 7/21, 8/22, 9/23,$$
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- The frequentist interpretation involves an idealization which says that in the limit, the relative frequencies in this sequence approach 1/2.

Frequentist Interpretation of Probability: Advantages

- It is objective. The half life of C^{14} atoms is 5730 years. This means that the relative frequency of spontaneous disintegration of C^{14} within 5730 years is $1/2$. We can use this as an objective measure of the probability that the next sample of C^{14} will decay and for carbon dating.

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- It is a faithful interpretation of the Kolmogorov Axioms for probability functions. This means that it can be shown that the identification of probability with the limit of relative frequencies is consistent with the Kolmogorov Axioms. $P(E)$ is whatever this limit is (if it exists).

Frequentist Interpretation of Probability: Disadvantages

- "The long run is a misleading guide to current affairs. In the long run we are all dead," wrote John Maynard Keynes in his 1923 work, *A Tract on Monetary Reform*. Although this is not what Keynes meant, a similar point has often been made against the frequentist interpretation. The point is that the use of the limit definition invites the objection that we can never in principle, not just in practice, observe an infinite sequence.

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- The observed relative frequency in any finite sample is irrelevant to the limiting frequency. This should remind us of Hume's problem of induction.

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- If, however, probability is defined as a limiting frequency where the event is repeated potentially infinite many times, then it does not seem to make any sense to talk about probabilities of single occurrences (an election, tomorrow). This problem of the single case raises a problem about the applicability of the frequency interpretation of probability.