Phil/LPS 31 Introduction to Inductive Logic Lecture 15

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Topics

- ▶ Part 1: Decision Problems under Ignorance
 - Ordinal Utilities
 - Dominance Principles
 - Maximin
- Part 2: Decision Problems under Information
 - Cardinal Utilities
 - ► Expected Utility and Risk
 - Principles of Rational Choice under Information

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- Decision problems under information are also known as decision problems under certainty or risk. The relevant sense of "certainty" here is that one is certain about the probability distribution of states. So one can compute the risk associated with taking a decision.

Part 2: Decision Problems under Ignorance

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- ▶ We write $A_i \succ A_j$ to mean Act *i* is preferred more than Act *j*.
- ▶ We write $A_i \sim A_j$ to mean Act i is preferred equally to Act j.

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- ▶ Here we see that 4 > 3 > 2. So this utility function respects the preference ordering of the acts. 4, 3 and 2 are ordinal utilities.

▶ Suppose now that the host serves chicken, S₂. You think that if the host serves chicken you'd much rather bring white wine than either red wine or rosé. Assume also that if you can't find white wine at Trader Joe's you'd much rather bring rosé than red wine.

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 - 1 Write down the preference ordering on the Acts.
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 - (4) Provide no information about the strength of preferences.

Making Decisions with Ordinal Utilities

From the previous exercises we obtain the following desirability table for acts based on our ordinal utility function.

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White	4	5
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- ▶ **Strict Dominance**: $A_i \succ A_j$ if and only if (1) $u(A_i|S_n) \ge u(A_j|S_n)$ for every state S_n (at least as good) and (2) there exists a state S_m such that $u(A_i|S_m) > u(A_j|S_m)$ (at least one better).

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 - How would you decide in this case?

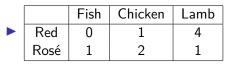
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- ► The maximin principle focuses on the worst possible outcome of each alternative act. Essentially, we're asking what's the worst that can happen?
- According to this principle, one should MAXimise the MINimal value obtainable with each act. If the worst possible outcome of one alternative is better than that of another, then the former should be chosen.



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 - ▶ Why would an agent choose A_3 ?

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 - ▶ What is the worst possible outcome for A_2 ? How about A_3 ?
 - ▶ Does $A_3 \succeq A_2$?
 - Why would an agent choose A₃?
- ► There are other principles of rational choice in the context of decisions under ignorance. But we shall not cover them in this introductory course. The book by Martin Peterson An Introduction to Decision Theory is highly recommended for this.

Part 1: Decision Problems under Information

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- ▶ This means that the agent can also calculate the expected value of functions of these states, namely, consequences of an act.
- However, not just any concept of utility will do. We have seen that because ordinal utilities cannot be added or multiplied, we cannot use them to calculate expected values. Further ordinal utilities do not quantify the strength of preference, they simply respect the ordering of our preferences.

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 - (3) From (2) we can calculate expected utilities using cardinal utilities.
 - (4) Provide information about the strength of preferences.

Decision Problems Under Risk

Expected Utility and Risk

$$U(A_1) = u(A|S_1)P(S_1) + u(A|S_2)P(S_1) + \dots + u(A|S_n)P(S_n)$$

= $\sum_{i=1}^{n} u(A|S_i)P(S_i)$

Maximize Expected Utility