Phil/LPS 31 Introduction to Inductive Logic
Lecture 14

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Topics

- ▶ Recap: Is inductive logic possible?
- ▶ Introduction to Decision Theory
- ▶ Utilities and Losses

Recap: Is inductive logic possible?

Here's Hume (1748) *An Enquiry Concerning Human Understanding*:

> *In vain do you pretend to have learned the nature of bodies from your past experience. Their secret nature, and consequently all their effects and influence, may change without any change in their sensible qualities. This happens sometimes, and with regard to some objects: why may it not happen always, and with regard to all objects? What logic, what process or argument secures you against this supposition? My practice, you say, refutes my doubts. But you mistake the purport of my question. As an agent, I am quite satisfied in the point; but as a philosopher...I want to learn the foundation of this inference.*

Recap: Is inductive logic possible?

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▶ Do we have a similar criterion for selecting the inductive rules of inference that are good? In other words, is inductive logic possible?

Is probability the very guide of life?

But to us, probability is the very guide of life.

Joseph Butler (1736) *The Analogy of Religion, Natural and Revealed, to the Constitution and Course of Nature*

Is probability the true logic for this world?

They say that Understanding ought to work by the rules of right reason. These rules are, or ought to be, contained in Logic; but the actual science of Logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the the True Logic for this world is the Calculus of Probabilities, which takes account of the magnitude of the probability (which is, or which ought to be in a reasonable man's mind). This branch of Math., which is generally thought to favor gambling, dicing, and wagering, and therefore highly immoral, is the only "Mathematics for Practical [People]', as we ought to be.

James Clerk Maxwell's Letter to Lewis Campbell, c. July 1850

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▶ Such a characterization will give us a precise of way of saying what we mean when we say that good rules of inductive inference are those rules that lead to "favorable consequences most of the time."

▶ The way we do this is by merging: (1) probability theory (which gives us the "most of the time part" using expected value) and (2) decision theory (which gives principles we can use to "evaluate consequences", namely the concepts of utility and loss).

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The probability of any event is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the chance of the thing expected upon it's happening.

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▶ This is quite remarkable! It means that you can: (1) use the expected value of the losses/gains on your decisions to calibrate your probabilities; and (2) you can use probabilities to decide upon which actions are "rational" to take based on their expected losses (risks) or expected gains.

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▶ Let us see how we can do this more formally.

Decision Problems: Illustration 1

▶ Suppose you are invited to a dinner party and your host asks you to bring some wine. You have to choose between white wine or red wine. It is also known that red wine goes well with red meat (like beef, lamb and veal) while white wine goes well with white meat (like chicken or fish). So you want the wine you bring to match the type of dish that will be served.

Decision Problems: Illustration 1

▶ You cannot bring both to the dinner party because you're on a
budget. So you are faced with a decision problem which we
may represent in the following decision table (the left table)
and corresponding desirability table (the right table).

	Fish	Lamb
White	The right wine	The wrong wine
Red	An odd wine	The right wine

	Fish	Lamb
White	5	2
Red	1	5

Decision Problems: Illustration 2

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▶ Treasure Department Secretary Janet Yellen says that if President Biden invoked the 14th Amendment, it would create a constitutional crisis.

▶ The President is faced with two choices to make sure that the American Federal Government lives up to its obligations and avoids default: invoke the 14th amendment or reach a deal with U.S. House of Representative leaders.

▶ President Biden has a decision problem.

	Deal	No Deal
Invoke^c	Default^c	Default
Invoke	Default^c	Crisis

	Deal	No Deal
Invoke^c	0	-10
Invoke	-1	-1

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- Acts correspond to choices that an agent may make when faced with a decision problem. It is assumed that the space of possible acts is a partition of the space of choices that an agent may take, in the sense that the choices are mutually exclusive. We use A, with or without numerical subscripts to denote acts.

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▶ A Utility Function is an assignment of cardinal utilities to consequences. The distinction between cardinal utilities and ordinal utilities is in your required reading for this week.

▶ Here is the utility function for the dinner party example:

	Fish	Lamb
White	5	2
Red	1	5

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▶ Here is President Biden's *hypothetical* loss function.

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Decision Problems: Rational Choice

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▶ Since consequences are joint functions of acts and states, we can calculate the expected value of taking an act A_i, given state S_i.

▶ This will lead to the concept of expected utility or expected loss (or risk)