Phil/LPS 31 Introduction to Inductive Logic
Lecture 5

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Topics

- Inference in Quantified Relational Logic
- Models and Counterexamples

Inference in Quantified Relational Logic

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  $\therefore$2. Someone is wise. ($\exists x(Wx)$) (From 1 and By Existential Generalization)

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  $\therefore$ 2. If Ruth Barcan Marcus is a logician, then she is wise. $(La \rightarrow Wa)$ (From 1 and By Universal Instantiation)

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4. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to infer a sentence G using other truth preserving rules of inference. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

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- So combining the first and second arguments we can conclude that something is not wise if and only if not everything is wise, i.e, $((\exists x(\neg(Wx))) \leftrightarrow (\neg\forall x(Wx)))$

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  3. Infer $(\forall x(\mathcal{F}))$ using the Useful equivalence.

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  6. So $\neg\neg(\forall x(Tx \rightarrow Sx))$, which is logically equivalent to $(\forall x(Tx \rightarrow Sx))$.

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- This will require a tool which we will call a model or intended interpretation of some sentences in quantified relational logic.
- I emphasize the word sentence because truth and falsity are properties of sentences, not formulas. A sentence in quantified relational logic is a formula with no free variables.

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- Thus, the model for sentential logic can be visualized as a table, which specifies the truth value conditions of the sentences in sentential logic.

Models and Counterexamples: Sentential Logic

Formula	Truth value conditions
(F)	1 if F is true; 0 otherwise.
$(\neg F)$	1 if F is false, 0 otherwise.
$(F \wedge G)$	1 if both F and G are true, 0 otherwise.
$(F \vee G)$	0 if both F and G are false, 1 otherwise.
$(F \rightarrow G)$	1 if either F is false or G is true; 0 otherwise.
$(F \leftrightarrow G)$	1 if truth value of F $=$ truth value of G; 0 otherwise.

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- But the idea is the same as the model for sentential logic. We will specify the truth value conditions for sentences in quantified relational logic. Importantly, the truth conditions for truth functional connectives in sentential logic carry over.
- The crucial difference now is that we add more structure to the model in order to account for (1) terms, (2) predicates and relations, (3) the order in which the terms stand in various relations and (4) quantifiers.

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- To account for $\forall x \mathcal{F}$ we consider every x-variant $\mathcal{F}'$ of $\mathcal{F}$ and to account for $\exists x \mathcal{F}$ we consider some x-variant $\mathcal{F}'$ of $\mathcal{F}$, soon to be made precise.

Models and Counterexamples: Quantified Relational Logic

Finally we can specify the form of an interpretation or model of quantified relational logic.

Symbols	Interpretation
Constant	Term in the universe of discourse
Predicate, P	$[P] = \{a \,\vert\, Pa\}$ where a is a term
Relation, R	$[R] = \{< a, b > \,\vert\, Rab\}$ a and b terms

Sentence	Truth value conditions in the model
Pa	1 if $a \in [P]$, 0 otherwise.
Rab	1 if $< a, b > \in [R]$, 0 otherwise.
$\exists x \mathcal{F}$	1 if for some x-variant $\mathcal{F}'$ of $\mathcal{F}$, the truth value of $\mathcal{F}'$ is 1.
$\forall x \mathcal{F}$	1 if for every x-variant $\mathcal{F}'$ of $\mathcal{F}$, the truth value of $\mathcal{F}'$ is 1.

Models and Counterexamples: Quantified Relational Logic

- Here is an example of how we use models to check whether sentences in quantified relational logic are true in model. It is important to emphasize that a sentence in quantified relational logic is true relative to a model.