Phil/LPS 31 Introduction to Inductive Logic
Lecture 2

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Topics

- Sentential Logic: Formulas
- Sentential Logic: Main connectives
- Sentential Logic: Paraphrasing English
- Truth functions and Truth tables
- Uses of Truth Tables

Sentential Logic: Formulas

Remember that the formal symbols of sentential logic are:

1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)
3. (for left bracket and) for right bracket.

The transformation rules (also known as “syntactic rules”) are:

1. Any sentence p is a formula.
2. If p is a formula, then $\neg p$ is formula.
3. If p is a formula and q is a formula, then $(p \vee q)$ is a formula.

The closure condition states that a formula is anything that results from 1 or from a finite number of applications of rule 2 or rule 3. Nothing else is a formula of sentential logic.

Sentential Logic: Formulas

Are the following formulas in sentential logic?

1. q
2. t
3. $\neg q$
4. $\neg\neg r$
5. $(\neg\neg\neg p \vee q)$
6. $(p \vee (r \vee \neg q))$
7. $((q \vee (t \vee s)) \vee (r \vee \neg p))$

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- So using brackets is not just a stipulation of our logic, it turns out that having brackets helps us parse the formula correctly in order to identify the main connective. In (1) we can identify the main connective but in (2) we cannot. This is why brackets matter.

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- Since the focus of this class is not sentential logic, we will not spend a lot of time doing these paraphrases in class. But! See Homework 1.

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- To understand what truth functions are, remember that we agreed that sentences in sentential logic will be unambiguously true or false (not both).
- So truth functions will take as input the truth value (true or false) of the paraphrased sentences in sentential logic and return as output a unique a truth value (true or false).

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- But! We shall see that using 0 and 1 will help us see the difference between deductive logic and inductive logic more clearly.

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- From a truth table, one can see the unique output of a truth function for all of the possible inputs.
- Here is the truth table for the simplest truth function. This is the truth function takes the truth value of p and returns its truth value.

p	
1	1
0	0

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- Here is the truth table for the truth function that takes the truth value of p and returns the truth value of $\neg p$.

p	$\neg p$
1	0
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- Here is the truth table for the truth function that takes the truth value of p and q and returns the truth value of $(p \vee q)$.

p	q	$(p \vee q)$
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- The simplicity of the idea of truth tables, understates just how very very powerful truth tables are!

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- The focus of this class will not be on truth tables. But! See Homework 1.

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- Two formulas are truth-functionally equivalent if their truth functions are identical.

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- See Homework 1 for more exercises.