Phil/LPS 31 Introduction to Inductive Logic
Lecture 1

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April 3rd 2023

Topics

- Logic in General
- Sentences
- Truth-functional connectives
- Sentential logic

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- This distinction between "form" and "content" is crucial to understand a logic in general because logicians distinguish between the formal aspect of a language (syntax) and its content, meaning or interpretation (semantics).
- Finally, the word "system" means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also belong to the representation. The symbols that belong to the representation are called, you guessed it, formulas!

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- Closure condition: A formula is anything that results from 1 or a finite number of applications of rule 2 or rule 3. Nothing else is a formula.
- Verify that all these are examples of formulas of the logic: S, ab, $aaSbb$, $aabb$.

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- It turns out that saying what "propositions" are is a hard philosophical problem. So we'll stick to sentences!

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  4. Having 1001 strands of hair does not make you bald. How about 1000, 999, . . . ?

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- What's the difference between the sentences in Case 1 and Case 2?

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- Sentential logic is the logic for representing the sentence structure of a fragment of natural language using truth-functional connectives.

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  3. If F is a formula and G is a formula, then $(F \vee G)$ is a formula.

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- The closure condition states that a formula of sentential logic is anything that results from 1 or from a finite number applications of rule 2 or rule 3. Nothing else is a formula of sentential logic.