Phil/LPS 31 Introduction to Inductive Logic
Lecture 5

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April 12th 2023

Topics

- Inference in Quantified Relational Logic
- Models and Counterexamples

Inference in Quantified Relational Logic

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  $\therefore$2. Someone is wise. $(\exists x(Wx))$ (From 1 and By Existential Generalization)

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- Inference rules continued from previous slide:

3. From $\exists x \mathcal{F}$, you may suppose that $\mathcal{F}(a)$ and use $\mathcal{F}(a)$ together with other premises to show a sentence G. Here $\mathcal{F}(a)$ is the result of uniformly replacing every free occurrence of x in $\mathcal{F}$ with an arbitrary constant a, which has not occurred before. This rule is known as Existential Supposition.

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- These are the only truth-preserving rules of inference that we will avail for ourselves in quantified relational logic but other rules of inference can be derived from these.

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- This will require a tool which we will call a model or intended interpretation of some sentences in quantified relational logic.

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