Phil/LPS 31 Introduction to Inductive Logic
Lecture 8

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Topics

- Recap of the problem of induction
- Motivating Problem: Monty Hall Problem
- Naïve Set Theory

Recap of the problem of induction

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  In vain do you pretend to have learned the nature of bodies from your past experience. Their secret nature, and consequently all their effects and influence, may change without any change in their sensible qualities. This happens sometimes, and with regard to some objects: why may it not happen always, and with regard to all objects? What logic, what process or argument secures you against this supposition? My practice, you say, refutes my doubts. But you mistake the purport of my question. As an agent, I am quite satisfied in the point; but as a philosopher...I want to learn the foundation of this inference.

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- In the case of deductive logic, the justification of the rules of inference there was that the good rules of inference are precisely those rules of inference that are truth-preserving. But we have seen that these rules are truth-preserving because they are non-ampliative.
- Do we have a similar criterion for selecting the inductive rules of inference that are good? In other words, is inductive logic possible?

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- There is no inherent self-contradiction in any of these approaches.
- So the real question is deciding, in a non-circular way, which of these rules of inductive inference are reliable.

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- So the good rules of inductive inference will be justified by their consequences.
- There is a parallel here with the justification of deductive logic. We justified a formal system of deductive logic by showing that it had the favorable consequence, namely, that the good rules of inference were precisely those that were truth preserving.

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- For example, a utility can simply be "getting it right" or as the next example shows, "Winning a car on a game show."; while a loss can simply be "getting it wrong" or as the next example shows, "Winning a goat on a game show."

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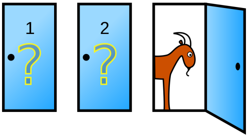
- For example, a utility can simply be "getting it right" or as the next example shows, "Winning a car on a game show."; while a loss can simply be "getting it wrong" or as the next example shows, "Winning a goat on a game show."
- So will want an inductive logic that either maximizes expected utility or minimizes expected loss (i.e., risk). Seen this way, inductive logic is the logic of reliable on-going scientific inquiry or rational deliberation. See Rudolf Carnap (1971) "Inductive Logic and Rational Decisions"

Motivating Problem: Monty Hall Problem

- We have three doors that can be labelled 1,2,3. Only of these doors has a car behind it. Monty Hall asks you to pick a door. Suppose you pick door 1. Monty, who knows what's behind each door, opens door 2 and reveals a goat. He then poses the question "would you like to stick with your choice or switch to door three?". With three doors, you have a 1/3 chance of winning the car straight away. But if Monty has opened door 2 (thereby revealing a goat), does that change your chance of winning the car when you switch?



Poll: Should you Switch?

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- Let's see your responses!

Motivating Problem: Ask Marilyn

In her September 9th, 1990 column for *Parade Magazine*, Marilyn vos Savant answered a question posed by reader Charles F. Whitaker,

Ask Marilyn™

BY MARILYN VOS SAVANT

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?
—Craig F. Whitaker, Columbia, Md.

Yes; you should switch. The first door has a one-third chance of winning, but the second door has a two-thirds chance. Here's a good way to visualize what happened. Suppose there are a *million* doors, and you pick door No. 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?

Motivating Problem: Ask Marilyn Reactions

Here were some of the reactions to her answer:

I'll come straight to the point. In the following question and answer, you blew it!

"Suppose you're on a game show and given a choice of three doors. Behind one is a car; behind the others are goats. You pick Door No. 1, and the host, who knows what's behind them, opens No. 3, which has a goat. He then asks if you want to pick No. 2. Should you switch?"

You answered, "Yes. The first door has a 1/3 chance of winning, but the second has a 2/3 chance."

Let me explain: If one door is shown to be a loser, that information changes the probability to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and, in the future, being more careful.

—Robert Sachs, Ph.D.,
George Mason University, Fairfax, Va.

You blew it, and you blew it big! I'll explain: After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your answer or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!

—Scott Smith, Ph.D., University of Florida

Your answer to the question is in error. But if it is any consolation, many of my academic colleagues also have been stumped by this problem.

—Barry Pasternack, Ph.D.,
California Faculty Association

Motivating Problem: Ask Marilyn Reactions Contd.

Reactions continued from previous slide:

You are in error—and you have ignored good counsel—but Albert Einstein earned a dearer place in the hearts of the people after he admitted his errors.

—Frank Rose, Ph.D., University of Michigan

I have been a faithful reader of your column and have not, until now, had any reason to doubt you. However, in this matter, in which I do have expertise, your answer is clearly at odds with the truth.

—James Rauff, Ph.D, Millikin University

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?

—Charles Reid, Ph.D., University of Florida

Your logic is in error, and I am sure you will receive many letters on this topic from high school and college students. Perhaps you should keep a few addresses for help with future columns.

—W. Robert Smith, Ph.D. Georgia State University

You are utterly incorrect about the game-show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively toward the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?

—E. Ray Bobo, Ph.D., Georgetown University

I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake.

—Kent Ford, Dickinson State University

Maybe women look at math problems differently than men.

—Don Edwards, Sunriver, Ore.

You are the goat!

—Glenn Calkins Western State College

You're wrong, but look at the positive side. If all those Ph.D.s were wrong, the country would be in very serious trouble.

—Everett Harman, Ph.D., U.S. Army Research Institute

Gasp! If this controversy continues, even the *postman* won't be able to fit into the mailroom. I'm receiving thousands of letters, nearly all insisting that I'm wrong, including one from the deputy director of the Center for Defense Information and another from a research mathematical statistician from the National Institutes of Health! Of the letters from the general public, 92% are against my answer; and of the letters from universities, 65% are against my answer. Overall, nine out of 10 readers completely disagree with my reply.

But math answers aren't determined by votes. For those readers new to all this, here's the original question and answer in full, to which the first readers responded:

"Suppose you're on a game show, and you're given a choice of three doors. Behind one door is a car; behind the others, goats. You pick a door—say, No. 1—and the host, who knows what's behind the doors, opens another door—say, No. 3—which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to switch your choice?"

I answered, "Yes, you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good way to visualize what happened. Suppose there are a *million* doors, and you pick door No. 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door No. 777,777. You'd switch to that door pretty fast, wouldn't you?"

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- Recall the juxtaposition of deductive inference and inductive inference:

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Entailment	Conditional Probability
Apriori justification	Aposteriori justification
Monotonic	Non-monotonic

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- For the rest of the quarter we shall spend our time doing probability theory, analyzing the different interpretations of probability and seeing how conditional probability can be applied in the context of inductive logic as we have defined it.

Introduction to Probability Theory: Naïve Set Theory

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- So we shall begin our study of probability theory with the study of special sets which obey certain rules of combination.
- Remember how we started studying deductive logic by saying what the rules of formation for formulas were. Here we're going to begin by saying what ways of putting sets together lead to the sorts of sets which probability can handle.

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- Convince yourself of the force of this paradox!

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  - Set-forming operators: $\cup$, $\cap$, $\mathcal{P}$

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- The intersection of two sets A and B is denoted by $A \cap B$ is the set of those elements or elements in both A and B. Think of the conjunction $\wedge$.

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- We say two sets A and B are equal, written $A = B$, if $A \subset B$ and $B \subset A$. Think of the biconditional $(p \leftrightarrow q)$.
- To every set A, there is a set A^c called the complement of A of those elements that do not belong to A. Think of the negation $(\neg p)$

Introduction to Probability Theory: Naïve Set Theory

- Here's a useful table to help you remember these set operations. In each case, the truth conditions of the formula in sentential logic determine the membership relation of the corresponding set operation.

Sentential Logic	Set Theory
$\neg p$	A^c
$(p \vee q)$	$(A \cup B)$
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- Memorize this!