Phil/LPS 31 Introduction to Inductive Logic
Lecture 15

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Topics

- Part 1: Decision Problems under Ignorance
  - Ordinal Utilities
  - Dominance Principles
  - Maximin
- Part 2: Decision Problems under Information
  - Cardinal Utilities
  - Expected Utility and Risk
  - Principles of Rational Choice under Information

Recap: Decision Problems: Acts, States, Consequences, Utility/Loss Functions

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- In any case, what is the rational choice for you?

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- Decision problems under information are also known as decision problems under certainty or risk. The relevant sense of "certainty" here is that one is certain about the probability distribution of states. So one can compute the risk associated with taking a decision.

Part 2: Decision Problems under Ignorance

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- We write $A_i \sim A_j$ to mean Act i is preferred equally to Act j.

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- Suppose that given S_1, we can order our preferences as $A_1 \succ A_3 \succ A_2$. This means that if our host serves fish, we would prefer bringing white wine more than we would prefer bringing either rosé or red wine; and we would prefer to bring rosé more than we would prefer to bring red wine.

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- An ordinal utility function is an assignment of utilities to acts that respects the preference ordering or ranking of the acts.
- Consider the following utility function: $u(A_1 \mid S_1) = 4$, $u(A_2 \mid S_1) = 2$ and $u(A_3 \mid S_1) = 3$

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- Here we see that $4 > 3 > 2$. So this utility function respects the preference ordering of the acts. 4, 3 and 2 are ordinal utilities.

Ordinal Utilities

- Suppose now that the host serves chicken, S_2. You think that if the host serves chicken you'd much rather bring white wine than either red wine or rosé. Assume also that if you can't find white wine at Trader Joe's you'd much rather bring rosé than red wine.

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- Exercise.
  1. Write down the preference ordering on the Acts.
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  3. Verify that your utility function respects your preference ordering.

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  (4) Provide no information about the strength of preferences.

Making Decisions with Ordinal Utilities

- From the previous exercises we obtain the following desirability table for acts based on our ordinal utility function.

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White	4	5
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  - Decide on the basis of the strong dominance principle.
  - Decide on the basis of the maximin principle.

Strict and Weak Dominance Principles

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- A widely accepted dominance principle in decision theory prescribes that dominated acts must not be chosen.

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- **Weak Dominance**: $A_i \succeq A_j$ if and only if $u(A_i|S) \geq u(A_j|S)$ for every state S (at least as good)
- **Strict Dominance**: $A_i \succ A_j$ if and only if (1) $u(A_i|S_n) \geq u(A_j|S_n)$ for every state S_n (at least as good) and (2) there exists a state S_m such that $u(A_i|S_m) > u(A_j|S_m)$ (at least one better).

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Red	2	1	4
Rosé	3	4	4

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  - Does the strong dominance principle imply the weak dominance principle?

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  - How would you decide in this case?

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- The maximin principle focuses on the worst possible outcome of each alternative act. Essentially, we're asking what's the worst that can happen?
- According to this principle, one should MAXimise the MINimal value obtainable with each act. If the worst possible outcome of one alternative A_i is better than the worst possible outcome of another alternative A_j, then choose A_i.

Maximin Principle

	Fish	Chicken	Lamb
Red	0	1	4
Rosé	1	2	1

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  - Does $A_3 \succeq A_2$?
  - Why would an agent choose A_3?

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  - What is the worst possible outcome for A_2? How about A_3?
  - Does $A_3 \succeq A_2$?
  - Why would an agent choose A_3?
- There are other principles of rational choice in the context of decisions under ignorance. What we have covered so far is more than enough for an introductory course to inductive logic. The book by Martin Peterson *An Introduction to Decision Theory* is highly recommended reading!

Part 1: Decision Problems under Information

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- This means that the agent can also calculate the expected value of functions of these states, namely, consequences of an act.
- However, not just any concept of utility will do. We have seen that because ordinal utilities cannot be added or multiplied with each other, we cannot use them to calculate expected values. Further ordinal utilities do not quantify the strength of preference, they simply respect the ordering of our preferences.

Cardinal Utilities

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- They also enjoy the right kind of properties that make calculations of expected values possible.

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  (1) Like ordinal utilities, cardinal utilities are unique up to strictly increasing transformations.
  (2) Cardinal utilities are invariant under positive scaling and positive translation. Barrett and Huttegger call this positive affine transformation. Basically cardinal utilities are like hours and seconds.
  (3) From (2) we can calculate expected utilities using cardinal utilities.
  (4) Provide information about the strength of preferences.

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- Suppose that the states in a decision problem are listed as $\{S_1, S_2, \ldots, S_n\}$ and that there is a probability distribution on the states that assigns them probabilities $\{P(S_1), P(S_2), \ldots, P(S_n)\}$. Then the expected utility of an act A denoted as $U(A)$ is given by:

$$\begin{aligned} U(A) &= u(A|S_1)P(S_1) + u(A|S_2)P(S_2) + \cdots + u(A|S_n)P(S_n) \\ &= \sum_i^n u(A \mid S_i)P(S_i) \end{aligned}$$

Expected Utility

- Consider the dinner party example again. This time suppose that the entries are cardinal utilities.

	Fish	Chicken
White	4	5
Red	2	1
Rosé	3	3

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- Exercise. Suppose you know that there are even odds that the host will serve fish or chicken. Calculate the expected value of A_1, A_2 and A_3

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- More specifically, the weighted average of the loss values we attach to unfavorable consequences of an act where the weights on the consequences are determined by the probability distribution on states is the risk of that act.
- Suppose that the states in a decision problem are listed as $\{S_1, S_2, \ldots, S_n\}$ and that there is a probability distribution on the states that assigns them probabilities $\{P(S_1), P(S_2), \ldots, P(S_n)\}$. Let the loss of an act A_i given a state S_i be denoted by $L(A_i \mid S_i)$. Then the risk of an act A denoted by $R(A)$ is given by:

$$\begin{aligned} R(A) &= L(A|S_1)P(S_1) + L(A|S_2)P(S_2) + \cdots + L(A|S_n)P(S_n) \\ &= \sum_{i}^{n} L(A \mid S_i)P(S_i) \end{aligned}$$

Risk

- Consider President Biden's *hypothetical* loss function.

	Deal	No Deal
Invoke^c	0	-10
Invoke	-1	-1

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- Exercise. Suppose you President Biden knows that the odds are 3:5 that a deal will be reached with the U.S. House of Representative Majority Leader Kevin McCarthy. What is the risk of the possible acts that Biden can take to avoid having the Federal Government default on its debt?

Maximizing Expected Utility

- The cardinal principle (you see what I did there?) of decision problems under certainty is the principle of maximizing expected utility if you're using a cardinal utility function.

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- The cardinal principle (you see what I did there?) of decision problems under certainty is the principle of maximizing expected utility if you're using a cardinal utility function.
- This means that in any decision problem, choose the act that maximizes expected utility with respect to your probability distribution on states.

Maximizing Expected Utility

- For the following exercise, refer to the following desirability table.

	Fish	Chicken	Lamb
White	3	4	1
Red	2	1	4
Rosé	3	4	4

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	Fish	Chicken	Lamb
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- Suppose you know that because of rising tariffs on fish imports from Canada and recent shortages of lamb, your host is likely to serve chicken with probability 0.8, fish with probability 0.15 and lamb with probability 0.05. What wine should you choose to bring to the dinner party according to the principle of maximizing expected utility?

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- The principle of minimizing risk is to risk as the principle of maximizing expected utility is to expected utility.
- Let us call a person risk averse if given a choice between two actions with risk, they will tend to choose the act that is less risky. That is, they choose to minimize the risk.
- There is an on-going debate about what the implications of risk for rationality. See Lara Buchak's *Risk and Rationality*!

Minimizing Risk

Consider President Biden's modified *hypothetical* loss function modified in such a way that President Biden does nothing.

	Deal	No Deal
Invoke	-1	-1
Do Nothing	0	-10

Exercise. Suppose that President Biden knows that the odds are 3:5 that a deal will be reached with the U.S. House of Representative Majority Leader Kevin McCarthy. What act would you advise President Biden to take in order to avoid having the Federal Government default on its debt?