Phil/LPS 31 Introduction to Inductive Logic
Lecture 15

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May 22nd 2023

Topics

- Part 1: Decision Problems under Ignorance
  - Ordinal Utilities
  - Dominance Principles
  - Maximin
- Part 2: Decision Problems under Information
  - Cardinal Utilities
  - Expected Utility and Risk
  - Principles of Rational Choice under Information

Recap: Decision Problems: Acts, States, Consequences, Utility/Loss Functions

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- In any case, what is the rational choice for you?

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- Decision problems under information are also known as decision problems under certainty or risk. The relevant sense of "certainty" here is that one is certain about the probability distribution of states. So one can compute the risk associated with taking a decision.

Part 2: Decision Problems under Ignorance

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- We write $A_i \sim A_j$ to mean Act i is preferred equally to Act j.

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- Suppose that given S_1, we can order our preferences as $A_1 \succ A_3 \succ A_2$. This means that if our host serves fish, we would prefer bringing white wine more than we would prefer bringing either rosé or red wine; and we would prefer to bring rosé more than we would prefer to bring red wine.

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- Consider the following utility function: $u(A_1 \mid S_1) = 4$, $u(A_2 \mid S_1) = 2$ and $u(A_3 \mid S_1) = 3$

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- Here we see that $4 > 3 > 2$. So this utility function respects the preference ordering of the acts. 4, 3 and 2 are ordinal utilities.

Ordinal Utilities

- Suppose now that the host serves chicken, S_2. You think that if the host serves chicken you'd much rather bring white wine than either red wine or rosé. Assume also that if you can't find white wine at Trader Joe's you'd much rather bring rosé than red wine.

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- Exercise.
  1. Write down the preference ordering on the Acts.
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  3. Verify that your utility function respects your preference ordering.

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  (4) Provide no information about the strength of preferences.

Making Decisions with Ordinal Utilities

- From the previous exercises we obtain the following desirability table for acts based on our ordinal utility function.

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White	4	5
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  - Decide on the basis of the strong dominance principle.
  - Decide on the basis of the maximin principle.

Strict and Weak Dominance Principles

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- A widely accepted dominance principle in decision theory prescribes that dominated acts must not be chosen.

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- **Weak Dominance**: $A_i \succeq A_j$ if and only if $u(A_i|S) \geq u(A_j|S)$ for every state S (at least as good)
- **Strict Dominance**: $A_i \succ A_j$ if and only if (1) $u(A_i|S_n) \geq u(A_j|S_n)$ for every state S_n (at least as good) and (2) there exists a state S_m such that $u(A_i|S_m) > u(A_j|S_m)$ (at least one better).

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White	3	4	1
Red	2	1	4
Rosé	3	4	4

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  - Does the strong dominance principle imply the weak dominance principle?

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  - How would you decide in this case?

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- The maximin principle focuses on the worst possible outcome of each alternative act. Essentially, we're asking what's the worst that can happen?
- According to this principle, one should MAXimise the MINimal value obtainable with each act. If the worst possible outcome of one alternative is better than that of another, then the former should be chosen.

Maximin Principle

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	Fish	Chicken	Lamb
Red	0	1	4
Rosé	1	2	1

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  - Does $A_3 \succeq A_2$?
  - Why would an agent choose A_3?

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- Exercise. Refer to the desirability table for the dinner party example above to answer the following questions.
  - What is the worst possible outcome for A_2? How about A_3?
  - Does $A_3 \succeq A_2$?
  - Why would an agent choose A_3?
- There are other principles of rational choice in the context of decisions under ignorance. But we shall not cover them in this introductory course. The book by Martin Peterson *An Introduction to Decision Theory* is highly recommended for this.

Part 1: Decision Problems under Information

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- This means that the agent can also calculate the expected value of functions of these states, namely, consequences of an act.
- However, not just any concept of utility will do. We have seen that because ordinal utilities cannot be added or multiplied, we cannot use them to calculate expected values. Further ordinal utilities do not quantify the strength of preference, they simply respect the ordering of our preferences.

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Decision Problems Under Risk

Expected Utility and Risk

$$
\begin{aligned}
U(A_1) &= u(A|S_1)P(S_1) + u(A|S_2)P(S_1) + \ldots u(A|S_n)P(S_n) \\
&= \sum_{i}^{n} u(A \mid S_i)P(S_i)
\end{aligned}
$$

Maximize Expected Utility