Phil/LPS 31 Introduction to Inductive Logic
Lecture 1

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Topics

- Logic in General
- Sentences
- Truth-functional connectives
- Sentential logic

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- Finally, the word "system" means that given (1) these symbols and (2) rules of transforming these symbols; we can get (3) other symbols that also belong to the representation. The symbols that belong to the representation are called, you guessed it, formulas!

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- Verify that all these are examples of formulas of the logic: S, ab, $aaSbb$, $aabb$.

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- It turns out that saying what "propositions" are is a hard philosophical problem. So we'll stick to sentences!

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  4. Having 1001 strands of hair does not make you bald. How about 1000, 999, . . . ?

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- What's the difference between the sentences in Case 1 and Case 2?

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