

# Phil/LPS 31 Introduction to Inductive Logic

## Lecture 3

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# Topics

- ▶ Sentential Logic: Arguments and Inference
- ▶ Deductive Logic
- ▶ Validity
- ▶ Soundness

# Sentential Logic: Arguments and Inference

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- ▶ This set of sentences consisting of (1) premises and (2) a conclusion which are connected by (3) inference is called an **argument**.
- ▶ We wish to use sentential logic to represent the structure of **good** arguments made in natural language.

# From Sentential Logic to Deductive Logic

- The process of drawing or inferring a conclusion is called **deduction** if the premises **entail** the conclusion.

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- ▶ The process of drawing or inferring a conclusion is called **deduction** if the premises **entail** the conclusion.
- ▶ To understand what the word “entail” means, we need the truth table for the truth function  $(p \rightarrow q)$ , which is read as “If  $p$ , then  $q$ ”. See **Homework 2** for why  $(p \rightarrow q)$  is truth-functionally equivalent to  $(\neg p \vee q)$ .

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- ▶ From the truth table for  $(p \rightarrow q)$ , we see that  $(F \rightarrow G)$  is false just in case  $F$  is true **and** the formula  $G$  is false. This is the **powerful idea** behind deductive logic.

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- ▶ Deductive logic is great for mathematics (where almost every sentence is either a theorem or refutable) but deductive logic has limited use in experimental science, even real life! (More of this later.)
- ▶ Sentential Logic can be turned into a deductive logic by adding **rules of inference** that preserve truth.

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- ▶ Verify that  $(F \rightarrow (F \vee G))$  and  $((\neg F \rightarrow (F \vee G)) \rightarrow G)$  are tautologies.

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  4. If the conclusion is false, at least one of the premises is false.

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- ▶ An argument that is (1) valid and (2) all the premises are true is a **sound argument**.
- ▶ Convince yourself that deductive logic cannot help you to determine whether an argument in English is sound. Bummer! Whether the premises are true relies on domain knowledge, i.e., knowledge of a specific, specialized discipline or field; it does not depend on logic.