Phil/LPS 31 Introduction to Inductive Logic
Lecture 3

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Topics

- Sentential Logic: Inference
- Deductive Logic
- Validity
- Soundness

Sentential Logic: Inference

Remember that the formal symbols of sentential logic are:

1. p, q, r, s and t as symbols for sentences. If we need more than 5 symbols (rarely!), then add the following countably many symbols p_1, p_2, p_3, ...
2. $\vee$ for "or", $\neg$ for "not" since the other symbols for "and" and "if..., then..." can be defined from these. (More of this later)
3. (for left bracket and) for right bracket.

The transformation rules (also known as "syntactic rules") are:

1. Any sentence p is a formula.
2. If p is a formula, then $\neg p$ is formula.
3. If p is a formula and q is a formula, then $(p \vee q)$ is a formula.

The closure condition states that a formula is anything that results from 1 or from a finite number of applications of rule 2 or rule 3. Nothing else is a formula of sentential logic.

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- This set of sentences consisting of (1) premises and (2) a conclusion which are connected by (3) inference is called an argument.
- We wish to use sentential logic to represent the structure of good arguments in natural language.

From Sentential Logic to Deductive Logic

- The process of drawing a conclusion or inferring a claim is called deduction if the premises entail the conclusion. To understand what the word "entail" means, we need the truth table for the truth function $(p \to q)$, which is read as "If p, then q". See Homework 1 for why $(p \to q)$ is truth-functionally equivalent to $(\neg p \vee q)$.

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- Then we say that a formula F entails another formula G in sentential logic just in case $F \to G$ is a tautology.

- From the truth table for $(p \to q)$, we see that $(F \to G)$ is a tautology just in case either the formula F is false or the formula G is true because $(p \to q)$ is false just in case p is true and q is false.

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- Deductive logic is great for mathematics (where almost every sentence is either a theorem or refutable) but it has its limited use in science. (More of this later!)
- Sentential Logic can be turned into a deductive logic by adding rules of inference that preserve truth.
- The rules of inference that preserve truth are those rules of inference for which the relation between the premises and conclusion in an argument is entailment.

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  4. If the conclusion is false, at least one of the premises is false.

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- An argument that is (1) valid and (2) has true premises is a sound argument.
- I hope you see that deductive logic cannot help you to determine whether an argument in English is sound. Bummer! Whether the premises are true relies on domain knowledge, i.e., knowledge of a specific, specialized discipline or field.