$V = v(x, t)$ is the Potential
function

$\Longleftrightarrow \quad F = -\dfrac{\partial V}{\partial x}$

$H = \dfrac{p^2}{2m} + V(x,t).$ (Hamiltonian)

$\quad = KE + PE.$

$\underline{\partial m v}$

$\hbar = \dfrac{h}{2\pi}$ is the Reduced
Planck Constant.

$E = hf.$ $v = \lambda f$ $h =$

$\quad = \dfrac{hv}{\lambda} = (mv)v \quad f = \dfrac{v}{\lambda} \quad$ But $\lambda = \dfrac{h}{mv}$ so

$\quad \ = $

Summary

$$\Psi(x,t) = \psi(x)\phi(t) = \psi(x)e^{-iEt/\hbar}$$

$\psi(x)$ - Eigenfunction

$\phi(t) = e^{-iEt/\hbar}$ - Time dependence

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Total Energy

Time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Energy eigenvalue equation: $\quad H\psi(x) = E\psi(x)$

Hamiltonian operator: $\quad H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$

The Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

1) Specify the potential energy function $V(x,t)$

2) Solve the Schrödinger Equation to find $\Psi(x,t)$

But what does $\Psi(x,t)$ represent?

Interpreting the coefficients

$$\langle E\rangle = \int \Psi^*(x,t)H\Psi(x,t)dx$$

$$\langle E^2\rangle = \int \Psi^*(x,t)H^2\Psi(x,t)dx$$

$$H\psi(x) = E\psi(x) \qquad H^2\psi(x) = E^2\psi(x) \qquad \int \psi_m(x)^*\psi_n(x)dx = \delta_{mn}$$

Summary so far

Solutions to the time-independent Schrödinger equation exist only for certain values of the energy: $E_1, E_2, E_3, \ldots, E_n$ *(energy is quantized)*

Corresponding to each energy eigenvalue is an eigenfunction:
$\psi_1(x), \psi_2(x), \psi_3(x), \ldots, \psi_n(x)$ *given a potential $V(x, t)$*

For each eigenvalue there is also a corresponding wave function:
$\Psi_1(x, t), \Psi_2(x, t), \Psi_3(x, t), \ldots, \Psi_n(x, t)$

$$\Psi_n(x, t) = \psi_n(x) e^{-\frac{iE_n t}{\hbar}}$$

If the system is described by the wave function $\Psi_n(x, t)$, it is said to be in the **quantum state n.**

Each quantum state wave function $\Psi_n(x, t)$ is a particular solution to the general Schrödinger equation for the potential $V(x)$

$$\Psi_n(x, t) = \psi_n(x) e^{-\frac{iE_n t}{\hbar}}$$

Superposition of states

$$\Psi(x,t) = \sqrt{\frac{2}{a}} \sum_{n=1}^{2} c_n \sin\left(\frac{n\pi x}{a}\right) e^{-\frac{iE_n t}{\hbar}} = \frac{1}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) e^{-\frac{i\hbar\pi^2 t}{2ma^2}} + \frac{1}{\sqrt{a}} \sin\left(\frac{2\pi x}{a}\right) e^{-\frac{i2\hbar\pi^2 t}{ma^2}}$$

$$E = hf \longrightarrow E = \hbar\omega \longrightarrow \omega = E/\hbar \qquad \boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}}$$

$$e^{-\frac{iE_1 t}{\hbar}} = e^{-i\omega_1 t} \longrightarrow \omega_1 = \frac{\hbar\pi^2}{2ma^2} \longrightarrow \omega_2 = 4\omega_1$$
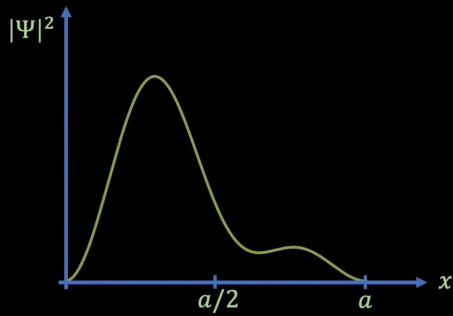
Energy uncertainty

$$\boxed{\Psi(x,t) = \frac{1}{\sqrt{2}} \psi_1(x) e^{-i\omega_1 t} + \frac{1}{\sqrt{2}} \psi_2(x) e^{-4i\omega_1 t}}$$

$$\sigma_E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2} = \frac{3\hbar^2\pi^2}{4ma^2} \neq 0$$

$$\langle E \rangle = P(E_1)E_1 + P(E_2)E_2 = \frac{1}{2}\left(\frac{\hbar^2\pi^2}{2ma^2}\right) + \frac{1}{2}\left(\frac{2\hbar^2\pi^2}{ma^2}\right) = \frac{5\hbar^2\pi^2}{4ma^2}$$

$$\langle E^2 \rangle = P(E_1)E_1^2 + P(E_2)E_2^2 = \frac{1}{2}\left(\frac{\hbar^2\pi^2}{2ma^2}\right)^2 + \frac{1}{2}\left(\frac{2\hbar^2\pi^2}{ma^2}\right)^2 = \frac{17\hbar^4\pi^4}{8m^2a^4}$$

Probability Density

$$|\Psi(x,t)|^2 = \frac{1}{a}\left[\sin^2\left(\frac{\pi x}{a}\right) + 2\cos(3\omega_1 t)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi x}{a}\right) + \sin^2\left(\frac{2\pi x}{a}\right)\right]$$

Wave Number

$$y = A \sin(B(x + c)) + D$$

$A :=$ amplitude.

$B :=$ wavelength.

$V = f\lambda.$

Period $:= \dfrac{2\pi}{B.}$ Frequency $= \dfrac{1}{period}.$

The smaller the
  B the higher the period.

$C :=$ Phase-shift. If $C = \dfrac{\pi}{2} \Rightarrow$ $\dfrac{\sin(x + \frac{\pi}{2})}{\underset{\cos(x)}{=}}$

$D :=$ vertical-shift.

$\varphi \quad \psi$

Matter wave

$$\psi(x) = A \cos(k_o x)$$

$$k \equiv \frac{2\pi}{\lambda}$$

$$= \frac{p}{\hbar}$$

$$= \frac{\sqrt{2mE}}{\hbar}$$

Here E is
the kinetic
energy.

$$\hbar = \frac{h}{2\pi}$$

$$\lambda = \frac{h}{p} \text{ (De Broglie).}$$

wave
number.
(intuitively
  how
many waves
in a cyle
  of 2π
(rad)
Spatial
  frequency

$$p = mv$$
$$= \sqrt{2m \tfrac{1}{2}mv^2}$$

$$\vec{V}_{wave} = \frac{\omega}{k} = f\lambda$$

$$= \frac{E}{h}\left(\frac{h}{p}\right)$$

$$= \frac{E}{p}$$

$$= \frac{1}{2}mv_p^2 \cdot \frac{1}{mv_p}$$

$$= \frac{1}{2}\vec{V}_{particle}$$

(Classical Wave Equation doesn't work. Since ω and k are constants)
in the phase equation
$$y(x,t) = A\cos(kx + \omega t)$$

De Broglie Wave Packet approach.

Suppose both ω and k are changing.

$$\vec{V} \propto \nabla S$$

$$S = kx + \omega t .$$

$$\Delta S = \Delta kx + \Delta \omega t$$

$$V_{group} = \frac{\Delta \omega}{\Delta k} = \frac{d\omega}{dk} \text{ in the limit}$$
$$\Delta \omega \to 0$$
$$\Delta k \to 0$$

Phase function of a Wave.

Fortunately the solutions themselves are easy enough to describe: they are of the form*

$$f(x,t) = A \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

so that λ is the wavelength, T is the period of the wave, and $f(x,t)$ is equal to the amplitude of the wave at the point x at time t, while A is a constant. The argument of this function,

$$\varphi(x,t) = \frac{2\pi}{\lambda}x - \frac{2\pi}{T}t$$

is the *phase*. With this definition we can write

$$f(x,t) = f(\varphi(x,t))$$

so as to consider f as a function of its phase alone. So two waves f_1 and f_2 have a *phase difference* $\Delta\varphi$ if

$$f_1(x,t) = f_1(\varphi(x,t) + \Delta\varphi)$$
$$f_2(x,t) = f_2(\varphi(x,t))$$

So when you add sinusoidal waves like these what you get is dependent on $\Delta\varphi$. E.g if you consider the sum $f(\varphi(x,t) + \Delta\varphi) + f(\varphi(x,t))$ with f as above for different values of $\Delta\varphi$ you get

for $\Delta\varphi = 0$, $f(\varphi(x,t) + 0) + f(\varphi(x,t)) = 2f(\varphi(x,t))$

for $\Delta\varphi = \pi$, $f(\varphi(x,t) + \pi) + f(\varphi(x,t)) = -f(\varphi(x,t)) + f(\varphi(x,t)) = 0$ since $\sin(\phi + \pi) = -\sin(\phi)$ for all ϕ.

$$v_{group} = \frac{d\omega}{dk} \qquad \text{We need to determine } \omega \text{ for a freely moving particle}$$

$$E = hf$$
$$\omega = 2\pi f$$

$$\omega = \frac{2\pi E}{h} = \frac{E}{\hbar} = \frac{p^2}{2m\hbar} = \frac{\hbar k^2}{2m}$$

$$E = \frac{1}{2}mv^2$$
$$p = mv$$

$$E = \frac{p^2}{2m}$$

$$p = \frac{h}{\lambda}$$
$$k = \frac{2\pi}{\lambda}$$

$$p = \frac{hk}{2\pi} = \hbar k$$

$$v_{group} = \frac{d\omega}{dk} \qquad \omega = \frac{\hbar k^2}{2m}$$

$$v_{group} = \frac{d}{dk}\left(\frac{\hbar k^2}{2m}\right) = \frac{\hbar k}{m} = \frac{p}{m} = \frac{mv_p}{m} = v_p$$

$$p = \hbar k = mv_p \qquad \boxed{v_{group} = v_p}$$

$$\boxed{v_{wave} = \frac{v_p}{2}} \qquad \boxed{v_{group} = v_p}$$
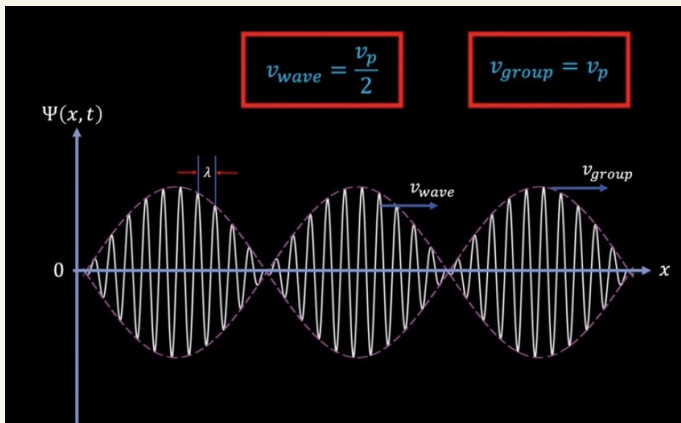
A different interpretation of Heisenberg's Uncertainty principle

commutativity means of the triangle

$$f = g \circ \pi$$

(g following π)
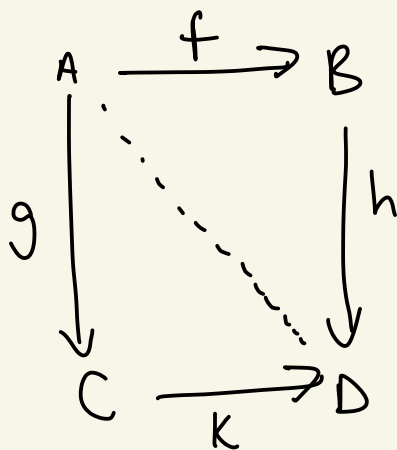
Commutativity means

$$h \circ f = k \circ g$$

✓ It is not <u>uncertainty</u> but due to a principle of <u>complementarity</u>

Bohr (1927) Como, Italy.
and Bohr (1927) in Solvay
Conference.

In physics, **complementarity** is a conceptual aspect of quantum mechanics that Niels Bohr regarded as an essential feature of the theory.[1][2] <u>The complementarity principle holds that objects have certain pairs of complementary properties which cannot all be observed or measured simultaneously.</u> An example of such a pair is position and momentum. Bohr considered one of the foundational truths of quantum mechanics to be the fact that setting up an experiment to measure one quantity of a pair, for instance the position of an electron, excludes the possibility of measuring the other, yet understanding both experiments is necessary to characterize the object under study. In Bohr's view, the behavior of atomic and subatomic objects cannot be separated from the measuring instruments that create the context in which the measured objects behave. Consequently, there is no "single picture" that unifies the results obtained in these different experimental contexts, and only the "totality of the phenomena" together can provide a completely informative description.[3]

√ "Führungsfeld" := guiding field

$\Psi(x)$

$\vec{v}_{group}$
$\vec{v}_{wave}$
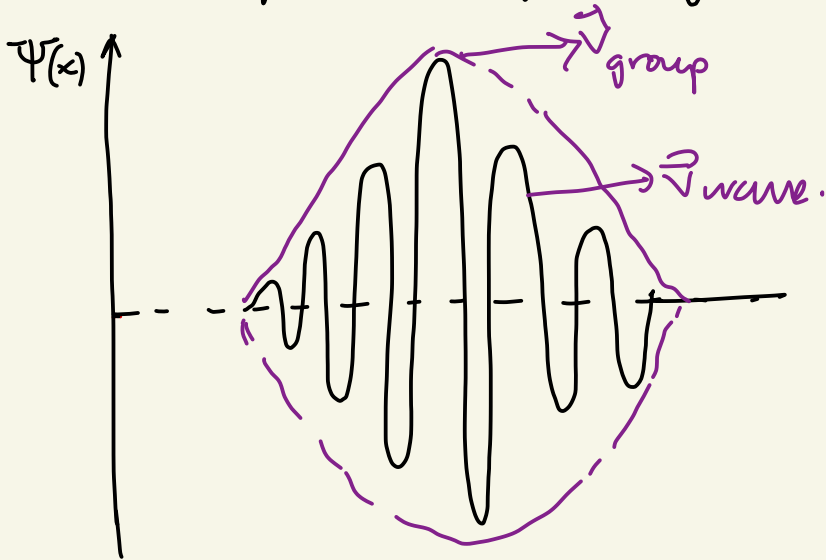
BOHMIAN MECHANICS

This demonstrates that it is wrong to claim that the predictions of quantum theory are incompatible with the existence of hidden variables, with an underlying deterministic model in which quantum randomness arises from averaging over ignorance. Bohmian mechanics provides us with just such a model: For any quantum experiment we merely take as the relevant Bohmian system the combined system, including the system upon which the experiment is performed as well as all the measuring instruments and other devices used to perform the experiment (together with all other systems with which these have significant interaction over the course of the experiment). We then obtain the "hidden variables" model by regarding the initial configuration of this big system as random in the usual quantum mechanical way, with distribution given by $|\psi|^2$. The guiding equation for the big system then transforms the initial configuration into the final configuration at the conclusion of the experiment. It then follows that this final configuration of the big system, including in particular the orientation of instrument pointers, will also be distributed in the quantum mechanical way. Thus our deterministic Bohmian model yields the usual quantum predictions for the results of the experiment.