Historical Introduction to Bohm (1952)

David Mwakima

March 13th 2023

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- Wave-Particle Duality for **photons**.

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- Successfully explained the Bohr-Sommerfeld model of the atom. See Bacciagalupi (2017, 49 & 51)

The Classical Wave Equation

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- What's the corresponding plane wave equation for matter waves if de Broglie's hypothesis is right?

Schröndinger Wave Equation for Matter (1926)

- $\Psi(x,t)$ is the non-relativistic plane wave equation of a single particle that satisfies the Schröndinger Wave Equation:

$$\boxed{\widehat{H}|\Psi(x,t)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi(x,t)\rangle}$$

where

$$\widehat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)$$

is the Hamiltonian, $\hbar$ is the reduced Planck constant $\frac{h}{2\pi}$ and $V(x,t)$ is the potential function. Compare Bacciagaluppi and Valentini (2017, 59 – 60)

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- Solving the *time-independent* Schröndinger Wave Equation has the consequence that if $|\Psi(x, t_0)\rangle_S = |\psi(x)\rangle_S$ is the state of system S at time t_0, then S has a determinate value of energy E since $|\psi(x)\rangle_S$ is an eigenstate of the Hamiltonian acting on $|\Psi(x, t_0)\rangle_S$. (Special case of Eigenvalue-Eigenstate link)

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- If a system is in a superposition of energy states

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \Psi_n(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{\frac{iE_n t}{\hbar}}$$

  , then **when a measurement is made**, the probability that a system is in energy state E_n is $|c_n|^2$. This is a special case of rule 4.II in the standard collapse formulation of Quantum Mechanics.

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- On this picture, the physical interpretation of the wave function in terms of uncertainty in **position** and **energy/momentum** (when a measurement is made) applies only to the single particle (electron) case.
- What is the meaning of Ψ for the N particle case if Ψ is in configuration space? (This point motivated de Broglie's no-collapse formulation of Quantum Mechanics)

Einstein-Podolsky-Rosen (EPR) 1935

Assume realism and locality hold.

- Realism: Particles have determinate positions (and electrons have definite spins) prior to any measurement. (Compare Rule 3 of the Standard Formulation)
- Locality (Einstein 1927 Solvay Congress):
  - Relativity of simultaneity. Two observers in inertial frames that are space-like separated can disagree on which events are simultaneous. Since there is no privileged inertial reference frame, there is no fact of the matter about whether two events are simultaneous.
  - c is the limiting velocity of the propagation of signals.

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  - c is the limiting velocity of the propagation of signals.
- Incomplete: If Born's interpretation is right, then the indeterminacy of quantum mechanics is "a matter of ignorance". In that case we need to find the "extra information" (the hidden variable) that, together with the wave-function, would enable us to predict with certainty the outcome of any experiment.

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  [T]he statistical quantum theory would . . . take an approximately analogous position to the statistical mechanics within the framework of classical mechanics. (Einstein 1949: 672)

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- In 1932, von Neumann published "a proof" that no deterministic theory could be compatible with quantum mechanics.
- Is this a **no-go theorem** for "hidden variable" theories of Quantum Mechanics?

Reinterpretating the Wave Function?

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- And this, by implication (given the EPR argument), means rejecting Born's **interpretation** of the wave function and providing a different interpretation.

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- In both, this involves formulating a deterministic theory of Quantum Mechanics – no-collapse dynamics (Rule 4.II).
- And this, by implication (given the EPR argument), means rejecting Born's **interpretation** of the wave function and providing a different interpretation.
- Let's see how this works.

de Broglie Pilot Wave Theory (1927)

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$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{ikx} dk$$

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$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{ikx} dk$$

- Here the function $A(k)$ can be chosen to modulate the amplitude of the waves in the superposition depending on their wave number k.

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- One can show that if we take real part of $\psi(x)$ for $A(k)$ defined in $[k_0 - \Delta k, k_0 + \Delta k]$ to be of the form:

$$\left(\frac{1}{2\pi\alpha}\right)^{\frac{1}{4}} e^{-\frac{x^2}{4\alpha}} \cos(k_0 x)$$

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  [I]t is acceptable to adopt the following point of view: one will assume the existence, as distinct realities, of the material point and of the continuous wave represented by the function Ψ, and one will take it as a postulate that the motion of the point is determined as a function of the phase...One then conceives the continuous wave as guiding the motion of the particle. It is a pilot wave. Quoted from Bacciagaluppi and Valentini (2017, 71)

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- Criticizing Schröndinger's work, de Broglie writes, "it seems a little paradoxical to construct a configuration space with coordinates of points that do not exist." Quoted in Bacciagaluppi and Valentini (2017, 76)

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  In the last number of the Journal de Physique, a paper by de Broglie has appeared... de Broglie attempts here to reconcile the full determinism of physical processes with the dualism between waves and corpuscles...even if this paper by de Broglie is off the mark (and I hope that actually), still it is very rich in ideas and very sharp, and on a much higher level than the childish papers by Schröndinger, who even today still thinks he may abolish material points.

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- Why "singularities"?

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Heisenberg Uncertainty Principle (1927)

- The probability

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- $\boxed{\Delta x \Delta p = \frac{\hbar}{2}}$ is the Heisenberg Uncertainty Principle 1927.

General Uncertainty Principle

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- One can take a different interpretation of the Heisenberg Uncertainty Principle based on the principle of complementarity due to Bohr (1927) in terms of non-commutativity of the Hermitian operators representing the two observables.

de Broglie Pilot Wave Theory (1927)

- Take it as a postulate and regard Ψ as a **physically-real** pilot wave guiding the motion of the **physically-real** particle according to the equation

$$\vec{v} \propto \nabla S \tag{2}$$

where S is the phase of Ψ. See Bacciagaluppi and Valentini (2017, 72, 76ff)

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  *In brief, in our hypotheses, each wave Ψ determines a 'class of motions', and each one of these motions is governed by equation (2) when one knows the initial position of the corpuscle. If one ignores this initial position, then the formula (1) gives the probability for the presence of the corpuscle in the element of volume $d\tau$ at the instant t. The wave Ψ then appears as both a **pilot wave** (Führungsfeld of Mr. Born) and a **probability wave**.*

de Broglie Pilot Wave Theory and Bohmian Mechanics

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- For de Broglie (2) is the fundamental equation of motion. One can write the dynamics of a particle in classical relativistic terms provided one includes a variable proper mass.
- In the non-relativistic limit an additional quantum potential is required. This was Bohm's proposal in the series of papers of 1952.

THANK YOU