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Statistical Methods for CS

Weekly Questions | Week 5

Q1 (a) Chance of winning:

$\Rightarrow$ 2 balls of same colour

$\Rightarrow \frac{5}{10} * \frac{4}{9} = 0.2222$

2 colours $\Rightarrow 0.2222 \times 2 = 0.4444$

The expected value of a discrete random variable X taking values in $\{x_1, x_2, \dots, x_n\}$ is defined as:

$$E[X] = \sum_{i=1}^{n} x_i P(X = x_i)$$

$$(0.4444 * 1.1) + ((1 - 0.4444) * (-1)) \approx \boxed{-0.0667}$$

(b)

$$Var(X) = E[X^2] - E[X]^2$$

$\Downarrow$

Variance of X formula

Using the mean from the last question, we can calculate the variance of the amount we win.

$E[X^2] = (0.4444 * (1.1)^2) + ((1 - 0.4444) * (-1)^2$
$= 1.093324$

$E[X]^2 = (-0.06676)^2 = 0.00444889$

$E[X^2] - E[X]^2 = 1.093324 - 0.00444889$
≈ 1.0889

Q2 (a) $E[X_i] = 0.6 * 1 + 0.4 * 0 = 0.6 \Rightarrow \boxed{0.6n}$

$Var(X_i) = E[X_i^2] - E[X_i]^2$

$\Rightarrow ((0.6 * (1)^2) + (0.4 * (0)^2)) - (0.6n)^2$
$= \boxed{0.6n - (0.6n)^2}$

(c) $Y = \sum_{i=1}^{n} X_i$

$E[Y]$ is the expected value of the sum of people ~~who~~ who voted.
$\Rightarrow E[X_1 + X_2 + \dots + X_n]$

$E[X]$ is the expected value of a person voting.

(d) $E[\frac{1}{n} Y]$ is the expected value of a fraction of the people who have voted.
(e.g) if 10 people ~~which~~ are in this survey and 4 people voted, Y is 4 and n is 10
$\Rightarrow \frac{1}{10} 4 = 0.4$

(e)

Q3 (a) We have to work out the chances ~~of~~ of all possible combinations of X_i and X_2.
If X_1 and X_2 are both 0, both balls are red $\Rightarrow \frac{8}{13} * \frac{7}{12} = \frac{14}{39}$
Repeating this....

$\frac{8}{13} \times \frac{5}{12} = \frac{10}{39}$ when $X_1 = 0, X_2 = 1$

$\frac{5}{13} * \frac{8}{12} = \frac{10}{39}$ when $X_1 = 1, X_2 = 0$

$\frac{5}{13} * \frac{4}{12} = \frac{5}{39}$ when $X_1 = 1, X_2 = 1$

	$X_1 = 0$	$X_1 = 1$	$P(X_2 = x)$
$X_2 = 0$	$\frac{14}{39}$	$\frac{10}{39}$	$\frac{24}{39}$
$X_2 = 1$	$\frac{10}{39}$	$\frac{5}{39}$	$\frac{15}{39}$
$P(X1 = x)$	$\frac{8}{13}$	$\frac{5}{13}$	1

(b) Independence

$$\Rightarrow P(X \cap Y) = P(X)P(Y)$$

2 events are independent of each other if the probability of 1 event occurring has no impact on the probability of the other event occurring.

Using the table from the last question,
$P(X_1 = 0 \cap X_2 = 0) = \frac{14}{39}$

$\Rightarrow$ This should equal $P(X_1 = 0)P(X_2 = 0)$ but it does not as after every event, the ball is not replaced.

$\therefore$ Since ~~these~~ events X_1 and X_2 affect the probability of each other, they are not independent.

(c) $E[X] = \sum_{i=1}^{n} x_i P(X = x_i)$

$E[X_2] = (1 * \frac{5}{39}) + (0 * \frac{10}{39}) + (1 * \frac{10}{39}) + (0 * \frac{14}{39})$

≈ 0.3846

(d) $E[X_2] = (1 * \frac{5}{39}) + (0 * \frac{10}{39})$

$= \frac{5}{39}$

$E[X_2] = \frac{5}{39} \div \frac{5}{13} \approx 0.3333$

(Since we are given $X_1 = 1$, we are only concerned with possibilities where $X_1 = 1$)