Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering and Systems Sciences

Department of Computer Science

Integrated Computer Science **Trinity Term 2018**
BA (Mod) Computer Science and Business
Year 3 Annual Examinations

CS3081 Computational Mathematics

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Wednesday 9th May 2018 RDS Main Hall 09.30-11.30

Instructions to Candidates:

(i) There are THREE questions in this exam paper.

(ii) A total of TWO questions should be attempted.

(iii) All questions carry equal marks.

(iv) Exam Paper is not to be removed from the venue

Materials Permitted for this Examination:

(i) Use of non-programmable calculators and log-tables is permitted.
You must note the make and model of your calculator on your answer book.

(ii) There is a Formula Sheet appended to this paper.

Question 1.

Determining the square root of a number p, $\sqrt{p}$, is the same as finding a solution to the equation $f(x) = x^2 - p = 0$.
Write a MATLAB user-defined function that determines the square root of a positive number by solving the equation using Newton's method. Name the function $X_S = SquareRoot(p)$. The output argument X_S is the answer and the input argument p is the number whose square root is to be determined. Ensure that you comment your program extensively.

The program should include the following features:

- It should check if the number is positive. If not, the program should stop and display an error message.

- The starting value of x for the iterations should be p.

- The iterations should stop when the estimated relative error is smaller than 0.00001.

- The number of iterations should be limited to 20. If a solution is not obtained in 20 iterations, the program should stop and display an error message.

[50 Marks]

Question 2.

The power generated by a windmill varies with the wind speed. In an experiment, the following five measurements were obtained:

Wind Speed (Kmph)	14	22	30	38	46
Electric Power (W)	320	490	540	500	480

a) Derive a general expression for the (n-1)th order Lagrange polynomial passing through n points.

[35 Marks]

b) Use the result from part a) to calculate the power at a wind speed of 26 Kmph.

[15 Marks]

Question 3.

The following data show the number of female and male physicians in the U.S. for various years (American Medical Association):

Year	1980	1990	2000	2002	2003	2006	2008
Number of males	413,395	511,227	618,182	638,182	646,493	665,647	677,807
Number of females	54,284	104,194	195,537	215,005	225,042	256,257	276,417

a) Derive a three-point backward difference formula for the derivative with unequally spaced points and use it to calculate the rate of change of both male and female physicians in 2006.

[20 Marks]

b) Derive a three-point central difference formula for the derivative with unequally spaced points and use it with the result of part a) to calculate (predict) the number of male and female physicians in 2008.

[20 Marks]

c) Compare your answers with the given data and calculate the percentage error in both cases.

[10 Marks]

Formula Sheet

1. Lagrange Polynomials:

$$f(x) = \sum_{i=1}^{n} y_i L_i(x) = \sum_{i=1}^{n} y_i \prod_{j=1, j\neq i}^{n} \frac{(x-x_j)}{(x_i-x_j)}$$

2. Three-point backward difference formula for the derivative with unequally spaced points:

$$f'(x_{i+2}) = \frac{x_{i+2}-x_{i+1}}{(x_i-x_{i+1})(x_i-x_{i+2})} y_i + \frac{x_{i+2}-x_i}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})} y_{i+1} + \frac{2x_{i+2}-x_i-x_{i+1}}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})} y_{i+2} \quad (8.37)$$

Three-point central difference formula for the derivative with unequally spaced points:

$$f'(x_{i+1}) = \frac{x_{i+1}-x_{i+2}}{(x_i-x_{i+1})(x_i-x_{i+2})} y_i + \frac{2x_{i+1}-x_i-x_{i+2}}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})} y_{i+1} + \frac{x_{i+1}-x_i}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})} y_{i+2} \quad (8.36)$$