Davy Nolan CS1003 Homework II

Q1

$$x + y + z = 2$$
$$2x + y + 0z = 3$$
$$x - y - 3z = 0$$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 3 \\ 1 & -1 & -3 & 0 \end{pmatrix} \begin{matrix} R1 = \text{Row } 1 \\ R2 = \text{Row } 2 \\ R3 = \text{Row } 3 \end{matrix}$$

① Add $-2R1$ to $R2$

$$\begin{array}{rrrrr} R2: & 2 & 1 & 0 & 3 \\ -2R1: & -2 & -2 & -2 & -4 \\ \hline & 0 & -1 & -2 & -1 \end{array} \qquad \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 1 & -1 & -3 & 0 \end{pmatrix}$$

② Add $R2$ to $R1$:

$$\begin{array}{rrrrr} R1: & 1 & 1 & 1 & 2 \\ R2: & 0 & -1 & -2 & -1 \\ \hline & 1 & 0 & -1 & 1 \end{array} \qquad \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & -1 & -2 & -1 \\ 1 & -1 & -3 & 0 \end{pmatrix}$$

③ Multiply $R2$ by -1

$$0 \quad 1 \quad 2 \quad 1 \qquad \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & -3 & 0 \end{pmatrix}$$

④ Add $-R1$ to $R3$:

$$\begin{array}{rrrrr} R3: & 1 & -1 & -3 & 0 \\ -R1: & -1 & 0 & 1 & -1 \\ \hline & 0 & -1 & -2 & -1 \end{array} \qquad \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & -1 \end{pmatrix}$$

⑤ Add R2 to R3:

R3: 0 −1 −2 −1
R2: 0 1 2 1
0 0 0 0

$$\begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x - z = 1$ $\qquad z = x - 1$

$y + 2z = 1$ $\qquad z = \frac{1-y}{2}$

Q2

$$\begin{pmatrix} 2 & 4 & 1 \\ 3 & 3 & 2 \\ 4 & 1 & 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc:ccc} 2 & 4 & 1 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 4 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \begin{matrix} R1 \\ R2 \\ R3 \end{matrix}$$

① Multiply R1 by $\frac{1}{2}$:

$1 \quad 2 \quad \frac{1}{2} : \frac{1}{2} \; 0 \; 0$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 4 & 1 & 4 & 0 & 0 & 1 \end{array}\right)$$

② Add $-3R1$ to $R2$:

$R2: \; 3 \; 3 \; 2 : 0 \; 1 \; 0$

$-3R1: \; -3 \; -6 \; -\frac{3}{2} : -\frac{3}{2} \; 0 \; 0$

$0 \; -3 \; \frac{1}{2} : -\frac{3}{2} \; 1 \; 0$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -3 & \frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 4 & 1 & 4 & 0 & 0 & 1 \end{array}\right)$$

③ Add $-4R1$ to $R3$:

$R3: \; 4 \; 1 \; 4 : 0 \; 0 \; 1$

$-4R1: \; -4 \; -8 \; -2 : -2 \; 0 \; 0$

$0 \; -7 \; 2 : -2 \; 0 \; 1$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -3 & \frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & -7 & 2 & -2 & 0 & 1 \end{array}\right)$$

④ Multiply R2 by $-\frac{1}{3}$:

$0 \; 1 \; -\frac{1}{6} : \frac{1}{2} \; -\frac{1}{3} \; 0$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & -7 & 2 & -2 & 0 & 1 \end{array}\right)$$

⑤ Add 7R2 to R3:

$$R3: 0\ -7\ 2 : -2\ 0\ 1$$
$$7R2: 0\ 7\ -\tfrac{7}{6} : \tfrac{7}{2}\ -\tfrac{7}{3}\ 0$$
$$0\ 0\ \tfrac{5}{6} : \tfrac{3}{2}\ -\tfrac{7}{3}\ 1$$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{5}{6} & \frac{3}{2} & -\frac{7}{3} & 1 \end{array}\right)$$

⑥ Add -2R2 to R1:

$$R1: 1\ 2\ \tfrac{1}{2} : \tfrac{1}{2}\ 0\ 0$$
$$-2R2: 0\ -2\ \tfrac{1}{3} : -1\ \tfrac{2}{3}\ 0$$
$$1\ 0\ \tfrac{5}{6} : -\tfrac{1}{2}\ \tfrac{2}{3}\ 0$$

$$\left(\begin{array}{ccc:ccc} 1 & 0 & \frac{5}{6} & -\frac{1}{2} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{5}{6} & \frac{3}{2} & -\frac{7}{3} & 1 \end{array}\right)$$

⑦ Multiply R3 by $\frac{6}{5}$:

$$0\ 0\ 1 : \tfrac{9}{5}\ -\tfrac{14}{5}\ \tfrac{6}{5}$$

$$\left(\begin{array}{ccc:ccc} 1 & 0 & \frac{5}{6} & -\frac{1}{2} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{array}\right)$$

⑧ Add $\frac{1}{6}$R3 to R2:

$$R2: 0\ 1\ -\tfrac{1}{6} : \tfrac{1}{2}\ -\tfrac{1}{3}\ 0$$
$$\tfrac{1}{6}R3: 0\ 0\ \tfrac{1}{6} : \tfrac{3}{10}\ -\tfrac{7}{15}\ \tfrac{1}{5}$$
$$0\ 1\ 0 : \tfrac{4}{5}\ -\tfrac{4}{5}\ \tfrac{1}{5}$$

$$\left(\begin{array}{ccc:ccc} 1 & 0 & \frac{5}{6} & -\frac{1}{2} & \frac{2}{3} & 0 \\ 0 & 1 & 0 & \frac{4}{5} & -\frac{4}{5} & \frac{1}{5} \\ 0 & 0 & 1 & \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{array}\right)$$

⑨ Add $-\frac{5}{6}$R3 to R1:

$$R1: 1\ 0\ \tfrac{5}{6} : -\tfrac{1}{2}\ \tfrac{2}{3}\ 0$$
$$-\tfrac{5}{6}R3: 0\ 0\ -\tfrac{5}{6} : -\tfrac{3}{2}\ \tfrac{7}{3}\ -1$$
$$1\ 0\ 0 : -2\ 3\ -1$$

$$\left(\begin{array}{ccc:ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & \frac{4}{5} & -\frac{4}{5} & \frac{1}{5} \\ 0 & 0 & 1 & \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{array}\right)$$

⑩ Inverse $= \begin{pmatrix} -2 & 3 & -1 \\ \frac{4}{5} & -\frac{4}{5} & \frac{1}{5} \\ \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{pmatrix}$

Q3

$$3x - y = 5$$
$$2x + 3y = 1$$

$$\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} = A$$

$$\det(A) = \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix} = (3)(3) - (-1)(2)$$

$$= 11$$

$$x = \frac{\begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix}}{11} = \frac{16}{11}$$

$$y = \frac{\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix}}{11} = \frac{-7}{11}$$