SISO

1. If *temperature* is `cold` then *speed* is `minimal`
2. If *temperature* is `cool` then *speed* is `slow`
3. If *temperature* is `pleasant` then *speed* is `medium`
4. If *temperature* is `warm` then *speed* is `fast`
5. If *temperature* is `hot` then *speed* is `blast`

Consider that the temperature is at 16 and we want our knowledge base to compute the speed.

Fuzzification

Crisp value $\rightarrow$ Linguistic variables.
16C $\rightarrow$ Cool/Pleasant

Temp	Cold	Cool	Pleasant	Warm	Hot
0	Y*	N	N	N	N
5	Y	Y	N	N	N
10	N	Y	N	N	N
12.5	N	Y*	N	N	N
15	N	Y	N	N	N
17.5	N	N	Y*	N	N
20	N	N	N	Y	N
22.5	N	N	N	Y*	N
25	N	N	N	Y	N
27.5	N	N	N	N	Y
30	N	N	N	N	Y*

The fuzzification of the crisp temperature gives the following membership for the Temperature fuzzy set:

$$trimf(x; a, b, c) = \max{(\min{(\frac{x-a}{b-a}, \frac{c-x}{c-b})}, 0)}$$

$$\mu_{\text{cool}}(T) = \max{(\min{(\frac{T-0}{12.5-0}, \frac{17.5-T}{17.5-12.5})}, 0)}$$
$$\mu_{\text{cool}}(16) = \max{(\min{(\frac{16-0}{12.5-0}, \frac{17.5-16}{17.5-12.5})}, 0)} = 0.3$$

$$\mu_{\text{pleasant}}(T) = \max{(\min{(\frac{T-15}{17.5-15}, \frac{20-T}{20-17.5})}, 0)}$$
$$\mu_{\text{pleasant}}(16) = \max{(\min{(\frac{16-15}{17.5-15}, \frac{20-16}{20-17.5})}, 0)} = 0.4$$

	μ_{cold}	μ_{cool}	$\mu_{pleasant}$	μ_{warm}	μ_{hot}
Temp=16	0	0.3	0.4	0	0
Fire Rule (#)	no (#1)	yes (#2)	yes (#3)	no (#4)	no (#5)

Inference

Rules containing the linguistic variables

Rules 2 \& 3

Rule #2 and #3 are firing and are essentially the fuzzy patches made out of the cross products of $$\text{cool} \times \text{slow}$$ $$\text{pleasant} \times \text{medium}$$

RPM	Slow	Medium
10	N	N
20	Y	N
30	Y*	N
40	Y	N
50	N	Y*
60	N	N

The fuzzification of the crisp volatile gives the following membership for the Voltage fuzzy set:

$$trimf(x; a, b, c) = \max{(\min{(\frac{x-a}{b-a}, \frac{c-x}{c-b})}, 0)}$$

$$\mu_{\text{slow}}(V) = \max{(\min{(\frac{x-10}{30-10}, \frac{50-x}{50-30})}, 0)}$$

$$\mu_{\text{medium}}(V) = \max{(\min{(\frac{x-40}{50-40}, \frac{60-x}{60-50})}, 0)}$$

Using the min inference method, the output `slow` is clipped off at 0.3 and `medium` at 0.4.

Speed	Slow	Medium
10	0	0
12.5	0.125	0
15	0.25	0
17.5	0.3	0
20	0.3	0
22.5	0.3	0
25	0.3	0
27.5	0.3	0
30	0.3	0
32.5	0.3	0
35	0.3	0
37.5	0.3	0
40	0.3	0
42.5	0.3	0.25
45	0.25	0.4
47.5	0.125	0.4
50	0	0.4
52.5	0	0.4
55	0	0.4
57.5	0	0.25
60	0	0

Composition

Create new membership function of the alpha levelled functions for `cool` and `pleasant`

The cool and pleasant sets have an output of \$0.3\$ and \$0.4\$ respectively. Using the max composition method, the fuzzy sets for slow and medium have to be given an \$\alpha\$-level cut for these output values respectively.

Speed	Slow	Medium	Output of 2 Rules
10	0	0	0
12.5	0.125	0	0.125
15	0.25	0	0.25
17.5	0.3	0	0.3
20	0.3	0	0.3
22.5	0.3	0	0.3
25	0.3	0	0.3
27.5	0.3	0	0.3
30	0.3	0	0.3
32.5	0.3	0	0.3
35	0.3	0	0.3
37.5	0.3	0	0.3
40	0.3	0	0.3
42.5	0.3	0.25	0.3
45	0.25	0.4	0.4
47.5	0.125	0.4	0.4
50	0	0.4	0.4
52.5	0	0.4	0.4
55	0	0.4	0.4
57.5	0	0.25	0.25
60	0	0	0

Defuzzification

Examine the fuzzy sets of `slow` and `medium` and contain a speed value in rpm.

Now we have to find a way to obtain one single number from the curve. One number corresponding to the speed of the air conditioner's motor.

Centroid

Weighted Speed = \$\text{output of 2 rules} \times \text{speed}\$

Speed	Slow	Medium	Output of 2 Rules	Weighted Speed
10	0	0	0	0
12	0.125	0	0.125	0

Speed	Slow	Medium	Output of 2 Rules	Weighted Speed
12.5	0.25	0	0.25	1.5625
15	0.3	0	0.3	3.75
17.5	0.3	0	0.3	5.25
20	0.3	0	0.3	6
22.5	0.3	0	0.3	6.75
25	0.3	0	0.3	7.5
27.5	0.3	0	0.3	8.25
30	0.3	0	0.3	9
32.5	0.3	0	0.3	9.75
35	0.3	0	0.3	10.5
37.5	0.3	0	0.3	11.25
40	0.3	0	0.3	12
42.5	0.3	0.25	0.3	12.75
45	0.25	0.4	0.4	18
47.5	0.125	0.4	0.4	19
50	0	0.4	0.4	20
52.5	0	0.4	0.4	21
55	0	0.4	0.4	22
57.5	0	0.25	0.25	14.375
60	0	0	0	0

Sum of output is 5.925
Sum of weighted speed is 218.687

The computation leads to a single value for the speed - an average computed with respect to the centre of gravity of the output fuzzy set.

Computed speed is $\frac{218.687}{5.925}=36.91$RPM

Mean of maxima

Speed	Slow	Medium	Output of 2 Rules	Weighted Speed
45	0.25	0.4	0.4	18
47.5	0.125	0.4	0.4	19
50	0	0.4	0.4	20
52.5	0	0.4	0.4	21
55	0	0.4	0.4	22

Sum of output is 2
Sum of weighted speed is 100
Computed speed it $\frac{100}{2}=50$RPM

MISO

We have a pole of length l, mass m as its head and mass M at its base, both connected by a weightless shaft. As application of a force F is required to control the pole at the base, which can be moved on a horizontal axis.

It is possible to approach the control of the cart pole without the use of differential equations by rules like:

`If` θ is $\text{approximately_zero}(a_{z})$ `&` $d \theta / dt$ is $\text{approximately_zero}(a_{z})$
`Then` F is $\text{approximately_zero}(a_{z})$

Fuzzification

Consider the fuzzy partition of the linguistic variable *angle* (θ) expressed through the linguistic terms *negative* (n_{θ}), *approximately zero* (az_{θ}) and *positive* (p_{θ}).

The same can be said about the angular velocity (Θ) and for the applied force (F)

- *negative* (n_{Θ}), *approximately zero* (az_{Θ}) and *positive* (p_{Θ})
- *negative* (n_{F}), *approximately zero* (az_{F}) and *positive* (p_{F}).

There are nine possible rules for the partitioned fuzzy sets of angle, angular velocity and force.

	n_{Θ}	**az_{Θ}**	**p_{Θ}**
n_{θ}	n_{f}	n_{f}	az_{f}
az_{θ}	n_{f}	az_{f}	p_{f}
p_{θ}	az_{f}	p_{f}	p_{f}

The rule base must be refined further to fine tune the controle. More terms are added, comprising:

- negative big
- negative medium
- negative small
- approximately zero
- positive small
- positive medium
- positive big

1. If θ is a_{z} and Θ is a_{z} then F is a_{z}
2. If θ is p_{s} and Θ is p_{s} then F is p_{s}
3. If θ is p_{s} and Θ is a_{z} then F is p_{s}
4. If θ is a_{z} and Θ is n_{s} then F is p_{s}
5. If θ is n_{s} and Θ is n_{b} then F is p_{s}

6. If θ is p_{b} and Θ is p_{s} then F is p_{m}
7. If θ is p_{m} and Θ is a_{z} then F is p_{m}
8. If θ is a_{z} and Θ is n_{m} then F is p_{m}
9. If θ is p_{b} and Θ is a_{z} then F is p_{b}
10. If θ is a_{z} and Θ is n_{b} then F is p_{b}
11. If θ is p_{s} and Θ is p_{b} then F is n_{s}
12. If θ is a_{z} and Θ is p_{s} then F is n_{s}
13. If θ is n_{s} and Θ is n_{s} then F is n_{s}
14. If θ is n_{s} and Θ is a_{z} then F is n_{s}
15. If θ is a_{z} and Θ is p_{m} then F is n_{m}
16. If θ is n_{m} and Θ is a_{z} then F is n_{m}
17. If θ is n_{b} and Θ is n_{s} then F is n_{m}
18. If θ is a_{z} and Θ is p_{b} then F is n_{b}
19. If θ is n_{b} and Θ is a_{z} then F is n_{b}

	n_{b}	n_{m}	n_{s}	a_{z}	p_{s}	p_{m}	p_{b}
θ_{max}	-67.5	-45	-22.5	0	22.5	45.0	67.5
θ_{min}^{1}	-45	-22.5	0	22.5	45	67.5	90
θ_{min}^{2}	-90	-67.5	-45	-22.5	0	22.5	45
Θ_{max}	-33.75	-22.5	-11.25	0	11.25	22.5	33.75
Θ_{min}^{1}	-22.5	-11.25	0	11.25	22.5	33.75	45
Θ_{min}^{2}	-45	-33.75	-22.5	-11.25	0	11.25	22.5
F_{max}	-7.5	-5	-2.5	0	2.5	5	7.5
F_{min}^{1}	-5	-2.5	0	2.5	5	7.5	10
F_{min}^{2}	-10	-7.5	-5	-2.5	0	2.5	5

Consider $\theta=36$ (p_{s} and p_{s}) and $\Theta=-2.25$ (a_{z}).

$$\mu_{ps}(\theta) = \max{(\min{(\frac{\theta-a}{b-a}, \frac{c-\theta}{c-b})}, 0)}$$ $$\mu_{ps}(36) = \max{(\min{(\frac{36-0}{22.5-0}, \frac{45-36}{45-22.5})}, 0)}$$ $$\mu_{ps}(36) = \max{(\min{(1.6, 0.4)}, 0)} = 0.4$$

$$\mu_{pm}(\theta) = \max{(\min{(\frac{\theta-a}{b-a}, \frac{c-\theta}{c-b})}, 0)}$$ $$\mu_{pm}(36) = \max{(\min{(\frac{36-22.5}{45-22.5}, \frac{67.5-36}{67.5-45})}, 0)}$$ $$\mu_{pm}(36) = \max{(\min{(0.6, 1.4)}, 0)} = 0.6$$

$$\mu_{az}(\Theta) = \max{(\min{(\frac{\Theta-a}{b-a}, \frac{c-\Theta}{c-b})}, 0)}$$ $$\mu_{az}(-2.25) = \max{(\min{(\frac{-2.25-(-11.25)}{0-(-11.25)}, \frac{11.25-(-2.25)}{11.25-0})}, 0)}$$ $$\mu_{az}(-2.25) = \max{(\min{(0.8, 1.2)}, 0)} = 0.8$$

1. If θ is p_{s} and Θ is a_{z} then F is p_{s}
2. If θ is p_{m} and Θ is a_{z} then F is p_{m}

Inference

$$\mu_{ps}(F) = \max{(\min{(\frac{F-a}{b-a}, \frac{c-F}{c-b})}, 0)}$$ $$\mu_{ps}(F) = \max{(\min{(\frac{F-0}{2.5-0}, \frac{5-F}{5-2.5})}, 0)}$$

$$\mu_{pm}(F) = \max{(\min{(\frac{F-a}{b-a}, \frac{c-F}{c-b})}, 0)}$$ $$\mu_{pm}(F) = \max{(\min{(\frac{F-2.5}{5-2.5}, \frac{7.5-F}{7.5-5})}, 0)}$$

For rule number 3, we take the α level cut at $\min{(0.4, 0.8)}=0.4$
For rule number 7, we take the α level cut at $\min{(0.6, 0.8)}=0.6$

F	μ_{ps}	μ_{pm}
0	0	0
0.5	0.2	0
1	0.4	0
1.5	0.4	0
2	0.4	0
2.5	0.4	0
3	0.4	0.2
3.5	0.4	0.3
4	0.4	0.6
4.5	0.4	0.6
5	0.2	0.6
5.5	0	0.6
6	0	0.6
6.5	0	0.4
7	0	0.2
7.5	0	0

Composition

F	μ_{ps}	μ_{pm}	Output
0	0	0	0
0.5	0.2	0	0.2
1	0.4	0	0.4
1.5	0.4	0	0.4
2	0.4	0	0.4
2.5	0.4	0	0.4
3	0.4	0.2	0.4
3.5	0.4	0.3	0.4
4	0.4	0.6	0.6
4.5	0.4	0.6	0.6
5	0.2	0.6	0.6
5.5	0	0.6	0.6
6	0	0.6	0.6
6.5	0	0.4	0.4
7	0	0.2	0.2
7.5	0	0	0

Defuzzification

Maximum Criterion Model

F	Output	Weighted Output
4	0.6	2.4
4.5	0.6	2.7
5	0.6	3
5.5	0.6	3.3
6	0.6	3.6

Any value of the force between 4 and 6 will do to steady the cartpole.

Mean of Maximum

F	Output	Weighted Output
4	0.6	2.4
4.5	0.6	2.7
5	0.6	3
5.5	0.6	3.3
6	0.6	3.6

Sum of output is 3
Sum of weighted output is 15
$$F=\frac{15}{3} = 5$$

COG

F	Output	Weighted Output
0	0	0
0.5	0.2	0.1
1	0.4	0.4
1.5	0.4	0.6
2	0.4	0.8
2.5	0.4	1
3	0.4	1.2
3.5	0.4	1.4
4	0.6	2.4
4.5	0.6	2.7
5	0.6	3
5.5	0.6	3.3
6	0.6	3.6
6.5	0.4	3.9
7	0.2	4.2
7.5	0	0

Sum of output is 6.2
Sum of weighted output is 24.5
$$F=\frac{24.5}{6.2} = 3.95$$