Computational Mathematics Assignment 3

Student name: Davy Nolan
Student number: 17330208

Q 4.26 (Done in matlab)

```
matA = [-2 1 0; 1 -2 1; 0 1 -1.5];
matB = [4 -1 0 1 0; -1 4 -1 0 1; 0 -1 4 -1 0; 1 0 -1 4 -1; 0 1 0 -1 4];

fprintf("\nInfinityNorm(matA) = %d",InfinityNorm(matA));
fprintf("\nInfinityNorm(matB) = %d",InfinityNorm(matB));


function N = InfinityNorm(A)
    [m,n] = size(A);
    rows = [];
    for i=1 : n
        row = 0;
        for j=1 : m
            row = row + abs(A(i,j));
        end
        rows = [rows, row];
    end
    N = max(rows(:));
end
```

Execution:

```
InfinityNorm(matA) = 4
InfinityNorm(matB) = 7>>
```

Q 6.13

Wind Speed (MPH)	14	22	30	38	46
Electric Power (W)	320	490	540	500	480

Lagrange Polynomials

First-order polynomial:

$$f(x) = \left(\frac{(x - x_2)}{(x_1 - x_2)}\right) \times y_1 + \left(\frac{(x - x_1)}{(x_2 - x_1)}\right) \times y_2$$

Second-order polynomial:

$$f(x) = \left(\frac{(x - x_2) \times (x - x_3)}{(x_1 - x_2) \times (x_1 - x_3)}\right) \times y_1 + \cdots + \left(\frac{(x - x_1) \times (x - x_2)}{(x_3 - x_1) \times (x_3 - x_2)}\right) \times y_3$$

General formula:

$$f(x) = \sum_{i=1}^{n} y_i l_i(x)$$

Where:

$$\ell_j(x) := \prod_{\substack{0 \le m \le k \\ m \ne j}} \frac{x - x_m}{x_j - x_m} = \frac{(x - x_0)}{(x_j - x_0)} \cdots \frac{(x - x_{j-1})}{(x_j - x_{j-1})} \frac{(x - x_{j+1})}{(x_j - x_{j+1})} \cdots \frac{(x - x_k)}{(x_j - x_k)},$$

Determine fourth-order polynomial in the Lagrange form that passes through the points..
use this polynomial to calculate the power at a wind speed of 26mph..

Fourth-order polynomial:
f(x) = ((x-x2)*(x-x3)*(x-x4)*(x-x5))/((x1-x2)*(x1-x3)*(x1-x4)*(x1-x5)) * y1
+ ((x-x1)*(x-x3)*(x-x4)*(x-x5))/((x2-x1)*(x2-x3)*(x2-x4)*(x2-x5)) * y2
+ ((x-x1)*(x-x2)*(x-x4)*(x-x5))/((x3-x1)*(x3-x2)*(x3-x4)*(x3-x5)) * y3
+ ((x-x1)*(x-x2)*(x-x3)*(x-x5))/((x4-x1)*(x4-x2)*(x4-x3)*(x4-x5)) * y4
+ ((x-x1)*(x-x2)*(x-x3)*(x-x4))/((x5-x1)*(x5-x2)*(x5-x3)*(x5-x4)) * y5

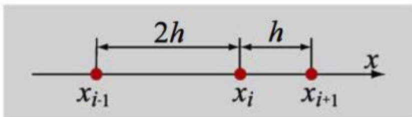
f(26) = ((26-22)*(26-30)*(26-38)*(26-46))/((14-22)*(14-30)*(14-38)*(14-46)) * 320
+ ((26-14)*(26-30)*(26-38)*(26-46))/((22-14)*(22-30)*(22-38)*(22-46)) * 490
+ ((26-14)*(26-22)*(26-38)*(26-46))/((30-14)*(30-22)*(30-38)*(30-46)) * 540
+ ((26-14)*(26-22)*(26-30)*(26-46))/((38-14)*(38-22)*(38-30)*(38-46)) * 500
+ ((26-14)*(26-22)*(26-30)*(26-38))/((46-14)*(46-22)*(46-30)*(46-38)) * 480

f(26) = (-12.5) + (229.6875) + (379.6875) + (-78.125) + (11.25)

f(26) = 530W

Q 8.7

8.7 Derive a finite difference approximation formula for $f''(x_i)$ using three points x_{i-1}, x_i, and x_{i+1}, where the spacing is such that $x_i - x_{i-1} = 2h$ and $x_{i+1} - x_i = h$.



Taylor Series:

$$f(x) = f(a) + \left(\frac{f'(a)}{1!}\right)(x-a) + \left(\frac{f''(a)}{2!}\right)(x-a)^2 + \left(\frac{f'''(a)}{3!}\right)(x-a)^3$$

Taylor Series for expansion point x_{i+1}:

$$f(x_{i+1}) = f(x_i) + f'(x_i)((x_{i+1}) - x_i) + \left(\frac{f''(x_i)}{2!}\right)\left((x_{i+1}) - x_i\right)^2$$

We can sub in h for $(x_i + 1) - x_i$:

$$f(x_i + 1) = f(x_i) + f'(x_i)(h) + \left(\frac{f''(x_i)}{2!}\right)(h)^2$$

Taylor Series for expansion point x_{i-1}:

$$f(x_{i-1}) = f(x_i) + f'(x_i)(x_i - (x_{i-1})) + \left(\frac{f''(x_i)}{2!}\right)(x_i - (x_{i-1}))^2$$

We can sub in 2h for $(x_i + 1) - x_i$:

$$f(x_i + 1) = f(x_i) + f'(x_i)(2h) + \left(\frac{f''(x_i)}{2!}\right)(2h)^2$$

Adding both equations:

$$f(x_{i+1}) + f(x_{i-1}) = f(x_i) + f'(x_i)(h) + \left(\frac{f''(x_i)}{2!}\right)(h)^2 +$$
$$f(x_i) + f'(x_i)(2h) + \left(\frac{f''(x_i)}{2!}\right)(2h)^2$$

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) - f'(x_i)(h) + (5)\left(\frac{f''(x_i)}{2!}\right)(h)^2$$

Solve for $f''(x_i)$:

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) - f'(x_i)(h) + (5)\left(\frac{f''(x_i)}{2!}\right)(h)^2$$

$$(5)f''(x_i)(h)^2 = 2(\, f(x_{i+1}) + f(x_{i-1}) + 2f(x_i) + f'(x_i)(h)\,)$$

$$f''(x_i)(h)^2 = \frac{2(\, f(x_{i+1}) + f(x_{i-1}) + 2f(x_i) + f'(x_i)(h)\,)}{5}$$

Include truncation error:

$$f''(x_i) = \frac{2(\, f(x_{i+1}) + f(x_{i-1}) + 2f(x_i) + f'(x_i)(h)\,)}{5} + O(h^2)$$

Q 8.9

Year	1980	1990	2000	2002	2003	2006	2008
#Males	413395	511227	618182	638182	646493	665647	677807
#Females	54284	104194	195537	215005	225042	256257	276417

(a)

$$f'(x_{i+2}) = \left(\frac{(x_{i+2}) - (x_{i+1})}{(x_i - x_{i+1})(x_i - x_{i+2})}\right) \times y_i +$$

$$\left(\frac{((x_{i+2}) - x_i)}{((x_{i+1}) - x_i)((x_{i+1}) - (x_{i+2}))}\right) \times \; y_{i+1} +$$

$$\left(\frac{((2x_{i+2}) - x_i - (x_{i+1}))}{(((x_{i+2}) - x_i)(((x_{i+2}) - x_i)((x_{i+2}) - (x_{i+1})))}\right) \times \; y_{i+2}$$

```
roc_male_2006 = ((2006-2003) / ((2002-2003)*(2002-2006)))*(638182) ...
                +((2006-2002) / ((2003-2002)*(2003-2006)))*(646493) ...
                +(((2*2006)-2002-2003) / ((2006-2002)*(2006-2003)))*(665647);

fprintf("%d",roc_male_2006);
```

```
>> Q4_a
4.939917e+03>>
```

$$\approx 4939.917$$

```
roc_female_2006 = ((2006-2003) / ((2002-2003)*(2002-2006)))*(215005) + ...
                  ((2006-2002) / ((2003-2002)*(2003-2006)))*(225042) + ...
                  (((2*2006)-2002-2003) / ((2006-2002)*(2006-2003)))*(256257);
fprintf("\n%d",roc_female_2006);
```

```
>> Q4_a

10681>>
```

$$\approx 10681$$

(b)

$$f'(x_{i+1}) = \left(\frac{(x_{i+1}) - (x_{i+2})}{(x_i - x_{i+1})(x_i - x_{i+2})}\right) \times y_i +$$

$$\left(\frac{\left((2x_{i+1}) - x_i - (x_{i+2})\right)}{\left((x_{i+1}) - x_i\right)\left((x_{i+1}) - (x_{i+2})\right)}\right) \times \ y_{i+1} +$$

$$\left(\frac{\left((x_{i+1}) - x_i\right)}{\left(((x_{i+2}) - x_i\right)\left(((x_{i+2}) - x_{i+1})\right)}\right) \times \ y_{i+2}$$

Male:

4939.916666 = ((2006-2008) / ((2003-2006)*(2003-2008)))*(646,493) + (((2*2006)-2003-2008) / ((2006-2003)*(2006-2008)))*(665,647) + ((2006-2003) / ((2008-2003)*(2008-2006)))*(X)

4939.916666 = (-86199.0666667) + (-110941.166667) + (0.3X)

X = ((4949.916666) + (86199.0666667) + (110941.166667)) / 0.3

X = 673600.499979

Error: |1 - (677,807 / 673600.499979)| = 0.006244
= 0.6244%

Female:

10,681 = ((2006-2008) / ((2003-2006)*(2003-2008)))*(225,042) + (((2*2006)-2003-2008) / ((2006-2003)*(2006-2008)))*(256,257) + ((2006-2003) / ((2008-2003)*(2008-2006)))*(X)

10,681 = (-30005.6) + (-42709.5) + (0.3X)

X = ((10,681) + (30005.6) + (42709.5)) / 0.3

X = 277,987

Error: |1 - (276,419 / 277,987)| = 0.00564
= 0.564%