Fuzzy Controllers

Controller Type Typical Operation

Mamdani (linguistic) controller *Direct* closed-loop controller with either fuzzy or singleton consequents

Takagi-Sugeno (TS) or **Takagi-** *Supervisory* controller - as a self

Sugeno-Kang controller (TSK) tuning device

The controller can be used with the process in two modes:

- *Feedback* mode when the controller will act as a control device
- *Feedforward* mode where the controller can be used as a prediction device

All inputs to, and outputs from, the controller are in the form of linguistic variables. In many ways, a fuzzy controller maps the input variables into a set of output linguistic variables.

Key Differences

A zero-order TSK model can be viewed as a special case of a Mamdani system in which each rule is specified by a fuzzy singleton or some pre-fuzzified consequent. In the TSK model, each rule has a crisp output and the overall output is obtained by a weighted average. This avoids the time consuming process of defuzzification that is required in a Mamdani model. The weighted average operator is replaced by a weighed sum to reduce the computation further.

Mamdani Controllers

Rules contain membership functions for both antecedents and consequent.

If $e(k)$ is $positive(e)$ and $\Delta e(k)$ is $positive(\Delta e)$ then $\Delta u(k)$ is $positive(\Delta u)$

The Good

This method is regarded widely for capturing expert knowledge and facilitates an intuitively plausible description of the knowledge.

The Bad

This method involves the computation of a two-dimensional shape by summing, or more accurately integrating across a continuously varying function. This can be expensive.

Example

Mamdani Conntroller:

1. If *temperature* is `cold` then *speed* is `minimal`
2. If *temperature* is `cool` then *speed* is `slow`
3. If *temperature* is `pleasant` then *speed* is `medium`
4. If *temperature* is `warm` then *speed* is `fast`
5. If *temperature* is `hot` then *speed* is `blast`

Full example done in notes 03.

Takagi-Sugeno Controllers

Rules contain membership functions for antecedents and linear functions in the consequent.

If $e(k)$ is $positive(e)$ and $\Delta e(k)$ is $positive(\Delta e)$ then $\Delta u(k) = \alpha e(k) + \beta \Delta e(k) + \delta$ where α, β and δ are obtained from empirical observations by relating the behaviour of the errors and change in errors over a fixed range of changes in control.

A zero order Takagi-Sugeno Model will be given as $R: (x_{1} \text{ is } \mu_{A}(x_{1}), \dots, x_{k} \text{ is } \mu_{A}(x_{k})) \rightarrow y = k)$

TSK Assumptions

1. Complex technological processes may be described in terms of interacting, yet simpler sub processes. This is the mathematical equivalent of fitting a piece-wise linear equation to a complex curve.
2. The output variable(s) of a complex physical system (e.g. in the sense that it can take a number of input variables to produce 1 or more outputs) can be related to the system's input variable in a linear manner provided the output space can be subdivided into a number of distinct regions.

Example

Zero-order TSK Controller:

1. If *temperature* is `cold` then *speed* is $k_{1}=0$
2. If *temperature* is `cool` then *speed* is $k_{2}=30$
3. If *temperature* is `pleasant` then *speed* is $k_{3}=50$
4. If *temperature* is `warm` then *speed* is $k_{4}=70$
5. If *temperature* is `hot` then *speed* is $k_{5}=100$

Temperature is 16 degrees.

Fuzzification: Temperature is `cool` and `pleasant`

$$\mu_{\text{cool}}(T) = \max{(\min{(\frac{T-0}{12.5-0}, \frac{17.5-T}{17.5-12.5})}, 0)}$$
$$\mu_{\text{cool}}(16) = \max{(\min{(\frac{16-0}{12.5-0}, \frac{17.5-16}{17.5-12.5})}, 0)} = 0.3$$

$$\mu_{\text{pleasant}}(T) = \max{(\min{(\frac{T-15}{17.5-15}, \frac{20-T}{20-17.5})}, 0)}$$
$$\mu_{\text{pleasant}}(16) = \max{(\min{(\frac{16-15}{17.5-15}, \frac{20-16}{20-17.5})}, 0)} = 0.4$$

	μ_{cold}	μ_{cool}	$\mu_{pleasant}$	μ_{warm}	μ_{hot}
Temp=16	0	0.3	0.4	0	0
Fire Rule (#)	no (#1)	yes (#2)	yes (#3)	no (#4)	no (#5)

Inference: Rule #2 and #3 are firing and are essentially the fuzzy patches made out of the cross products of $$\text{cool} \times \text{slow}$$ $$\text{pleasant} \times \text{medium}$$

Composition:

- $\mu_{cool} = 0.3 \rightarrow k_{2}$
- $\mu_{pleasant} = 0.4 \rightarrow k_{3}$

Defuzzification:

COG: $$\frac{\mu_{cool} \times k_{2} + \mu_{pleasant} \times k_{3}}{\mu_{cool}+\mu_{pleasant}} = \frac{0.3 \times 30 + 0.4 \times 50}{0.3+0.4} = 41.43\text{ RPM}$$

Mean of Maxima: $$\frac{0.4 \times 50}{0.4} = 50 \text{ RPM}$$

Comparison

Controller	TSK	Mamdani
COG	41.43	36.91
Mean of Maxima	50	50

Controller	TSK	Mamdani
COG	12%	0%
Mean of Maxima	35%	35%

A zero-order Sugeno fuzzy model can be viewed as a special case of the Mamdani fuzzy inference system in which each rule is specified by the fuzzy singleton or a pre-defuzzified consequent.

In Sugeno's model, each rule has a crisp output. The overall input is obtained by the weighted average. This avoids the time consuming process of defuzzification required in a Mandani model. The weighted average operator is replaced by a weighted sum to reduce computation further.