CS3081-1

TRINITY COLLEGE DUBLIN
THE UNIVERSITY OF DUBLIN

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

Integrated Computer Science Programme
B.A. (Mod.) Business and Computing

Year 3 Annual Examinations **Trinity Term 2015**

Computational Mathematics

Fergal Shevlin

Friday May 15th 2015 **Sports Centre** **14:00–16:00h**

Instructions to Candidates:

Answer **both** questions in **Part A** and **two** questions out of three in **Part B.** All questions carry equal marks.

Suggestion: Take 20 minutes to read all five questions. This leaves 100 minutes for answering questions worth 100 marks in total. So if a part of a question is worth five marks, spend five minutes answering it.

Materials permitted for this examination:
Mathematical formulae and tables booklet—available from the invigilators.
Graph paper—available from the invigilators.
Non-programmable calculator—indicate make and model.

Part A

Question 1. (i) In the IEEE 754 standards for *half*, *single* and *double* precision binary floating point number systems, how many binary digits are specified for sign p, significand s and exponent e?

[6 marks]

(ii) Calculate (in decimal) the approximate minimum non-zero positive values that can be expressed in the above *half* and *single* systems with an exponent bias of $2^{e-1}-1$, when the numbers are:
(a) normalized; (b) not normalized.

[8 marks]

(iii) Write a program fragment to plot the differences between division by 10_{10} and multiplication by 0.1_{10}, implemented as double precision floating point numbers, for a subset of the range of integers between 1×10^0 and 1×10^{30}. Explain your choice of starting value and sampling interval such that the program is relatively efficient. Assume an appropriate plotting method is available.

[6 marks]

(iv) Derive an expression for the relative error of subtraction.

[5 marks]

Question 2. (i) Use the simple iterative method to find the root of the polynomial $f(x) = x^3 - 30x^2 + 2552$ in the vicinity of $x = 11.8620$ to four decimal places of precision.

[10 marks]

(ii) What can $\psi'(\omega)$, the slope of the iteration function at the actual root, tell us about the likelihood of convergence when the initial approximation is close to ω? Show how the condition is derived. Can we be certain about it?

[10 marks]

(iii) How can a root-finding problem be solved as an optimization problem? Write a program fragment which uses an optimization method to find the root of $f(x)$ specified above, assuming the appropriate optimization method is available.

[5 marks]

Part B

Question 3. (i) For a system of equations $F_i(x_j) = 0$ where $i = 1 \ldots m$ and $j = 1 \ldots n$, which are *non-linear* in the unknowns x_j, explain an approach through which a solution can be approximated iteratively.

[13 marks]

(ii) How might it be possible to determine that a reasonable solution for the above will be found in advance of iterative approximation?

[5 marks]

(iii) Write a program fragment which finds an approximate solution for a non-linear system of equations. Assume an appropriate method is available.

[7 marks]

Question 4. (i) Specify and name quadrature formulae of polynomial degrees 1, 2 and 3 that can be used in numerical integration.

[9 marks]

(ii) Use the composite trapeziodal rule to numerically integrate $\int_0^1 e^x dx$ with intervals $h_0 = 1, h_1 = \frac{h_0}{2}, h_2 = \frac{h_1}{2}$. Note the true solution is $e - 1$.

[8 marks]

(iii) Combine the above estimates using Richardson's deferred approach to the limit with h^2 extrapolation.

[8 marks]

Question 5. (i) Describe in detail some parameters which should be passed to a method used to solve a system of ordinary differential equations.

[15 marks]

(ii) Describe in detail some values which should be returned from a method used to solve a system of ordinary differential equations.

[10 marks]