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Statistical Methods for CS

Weekly Questions $\boxed{\text{Week 3}}$

Q1 (a) Since each roll is an independent random event, there is a $\frac{1}{6}$ chance for each roll.

$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$$
$$1, 1, 2, 2, 3, 3 \Rightarrow \frac{1}{6^6} = 0.000021433$$

(b) Total number of outcomes from 6 rolls

$$\Rightarrow 6^6 = 46{,}656$$

Since 4 of the 6 rolls must be three, the remaining 2 rolls can be any of the 5 remaining numbers (1, 2, 4, 5, 6)

$$\Rightarrow 5 \times 5 = 25 \text{ combinations}$$

$$\Rightarrow \frac{6!}{(4!)(2!)} = 15 \Rightarrow 15 \times 25 = 375 \text{ possible outcomes}$$

There are 375 possible outcomes where there are exactly 4 threes.

$$\Rightarrow \frac{375}{46{,}656} \approx \boxed{0.008}$$

(c) Total no. of outcomes $= 6^6 = 46,656$

Since 1 roll must be one, the remaining 5 rolls can be any of the 5 remaining outcomes (2,3,4,5,6).

$\Rightarrow 5 \times 5 \times 5 \times 5 \times 5 = 3,125$ combinations

$\Rightarrow \frac{6!}{(5!)(1!)} = 6 \quad \Rightarrow 6 \times 3125 = 18,750$

possible outcomes with a single one.

$\Rightarrow \frac{18,750}{46,656} = \boxed{0.4019}$

(d) Total no. of outcomes $= 6^6 = 46,656$

Goal is to get no. of outcomes where there are no ones and minus it from total.

$\Rightarrow 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15,625$

$46656 - 15625 = 31,031$ possible outcomes with one or more 1's

$\frac{31,031}{46,656} = \boxed{0.6651}$

Q2

Total number of outcomes $= 6 \times 20 = 120$

Since 20 of the total outcomes are A,
$\Rightarrow P(A) = \frac{20}{120} = 0.1667$

Since 1 of the total outcomes are B,
$\Rightarrow P(B) = \frac{1}{120} = 0.0083.$

Since $P(A) \times P(B) = 0.1667 \times 0.0083 = 0.0014$
and $P(A \cap B) = \frac{1}{120} = 0.0083$

$$\Rightarrow P(A) \times P(B) \neq P(A \cap B)$$

Therefore, $\boxed{\text{A and B are NOT Independent.}}$

Q3 (a) The probability will be $\frac{1}{n-k+1}$ on the k^{th} try.

To calculate the probability that it was exactly this try, we multiply all the probabilities that the previous tries were unsuccessful

$\Rightarrow \frac{n-1}{n} \times \frac{n-2}{n-1}$ (if the first 2 tries were unsuccessful and so on....)

(b) $\frac{5}{6} \times \frac{4}{5} \times \frac{1}{4} = \boxed{0.1667}$

(c) Since incorrect passwords will not be ~~deleted~~ deleted, the denominator does not need to be decremented.

$\Rightarrow \frac{1}{n} \times \frac{n-1}{n}$ (k-1 times)

(d) $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \boxed{0.1157}$ (Since each try is an independent random event)

Q4

(a) Goal is to find probability that the robot does not get flagged and minus from 1.

$\Rightarrow 0.3 \times 0.3 \times 0.3 = 0.027$

$1 - 0.027 = \boxed{0.973}$

(b) Same method as previous part.

$\Rightarrow 0.95 \times 0.95 \times 0.95 = 0.8574$

$1 - 0.8574 = \boxed{0.1426}$

(c) Use Baye's Rule

$P(R) = 0.1$, prob. of a robot

$P(F) = P(F|R)P(R) + P(F|R')P(R')$, prob of being flagged.

$\Rightarrow 0.973 \times 0.1 + 0.142625 \times 0.9$
$= 0.2256625$

$$P(R|F) = \frac{P(F|R)P(R)}{P(F)} = \frac{0.973 \times 0.1}{0.2256625}$$

$$\approx \boxed{0.4313}$$