

TRINITY COLLEGE DUBLIN  
School of Computer Science and Statistics

**Week 4 Questions**

ST3009: Statistical Methods for Computer Science

For each problem, explain/justify how you obtained your answer in order to obtain full credit. In fact, most of the credit for each problem will be given for the derivation/model used as opposed to the final answer.

**Question 1.** A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you lose \$1.00. Calculate:

- (a) The expected value of the amount you win    **-0.0667**
- (b) The variance of the amount you win.        **1.0889**

**Question 2.** Suppose you carry out a poll following an election. You do this by selecting  $n$  people uniformly at random and asking whether they voted or not, letting  $X_i = 1$  if person  $i$  voted and  $X_i = 0$  otherwise. Suppose the probability that a person voted is 0.6.

- (a) Calculate  $E[X_i]$  and  $Var(X_i)$ .

Let  $Y = \sum_{i=1}^n X_i$ .

- (c) What is  $E[Y]$  ? Is it the same as  $E[X]$  or different, and why ?
- (d) What is  $E[\frac{1}{n}Y]$  ?
- (e) What is the variance of  $\frac{1}{n}Y$  (express in terms of  $Var(X)$ ) ?

Hints: use linearity of the expectation and the fact that people are sampled independently.

**Question 3.** Suppose that 2 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let  $X_i$  equal 1 if the  $i$ 'th ball selected is white, and let it equal 0 otherwise.

- (a) Give the joint probability mass function of  $X_1$  and  $X_2$     **0.3590**
- (b) Are  $X_1$  and  $X_2$  independent ? (Use the formal definition of independence to determine this)
- (c) Calculate  $E[X_2]$     **0.3846**
- (d) Calculate  $E[X_2|X_1 = 1]$     **0.3333**