Mathematics Assignment 1

CS1003 Homework 1 Davy Nolan

Q1

$A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

[*The determinant of a 2x2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $ad - bc = \det(A)$]

(i) $\det A = (3)(2) - (1)(4) = 2$

$\det B = (1)(3) - (4)(2) = -5$

(ii) [Transpose = interchange rows and columns]

$$AB = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} (3 \ \ 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (3 \ \ 1)\begin{pmatrix} 4 \\ 3 \end{pmatrix} \\ (4 \ \ 2)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (4 \ \ 2)\begin{pmatrix} 4 \\ 3 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 3(1) + 1(2) & 3(4) + 1(3) \\ 4(1) + 2(2) & 4(4) + 2(3) \end{pmatrix}$$

$$AB = \begin{pmatrix} 5 & 15 \\ 8 & 22 \end{pmatrix} \Rightarrow \boxed{(AB)^T = \begin{pmatrix} 5 & 8 \\ 15 & 22 \end{pmatrix}}$$

$B^T = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ $A^T = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$

$$B^T A^T = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \times \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1\ 2)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (1\ 2)\begin{pmatrix} 4 \\ 2 \end{pmatrix} \\ (4\ 3)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (4\ 3)\begin{pmatrix} 4 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1(3)+2(1) & 1(4)+2(2) \\ 4(3)+3(1) & 4(4)+3(2) \end{pmatrix}$$

$$\boxed{B^T A^T = \begin{pmatrix} 5 & 8 \\ 15 & 22 \end{pmatrix}}$$ ✓ $(AB)^T = B^T A^T$

Q2

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 5 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

(i) $$AB = \begin{pmatrix} (1\ 2\ 3)\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} & (1\ 2\ 3)\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} & (1\ 2\ 3)\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \\ \cancel{1(2)+2(2)+3(1)} & \cancel{1(5)+2(1)+3(2)+} & \\ (2\ 1\ 3)\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} & (2\ 1\ 3)\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} & (2\ 1\ 3)\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \\ (1\ 1\ 2)\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} & (1\ 1\ 2)\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} & (1\ 1\ 2)\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1(2)+2(2)+3(1) & 1(5)+2(1)+3(2) & 1(3)+2(1)+3(2) \\ 2(2)+1(2)+3(1) & 2(5)+1(1)+3(2) & 2(3)+1(1)+3(2) \\ 1(2)+1(2)+2(1) & 1(5)+1(1)+2(2) & 1(3)+1(1)+2(2) \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 13 & 11 \\ 9 & 17 & 13 \\ 6 & 10 & 8 \end{pmatrix} = AB$$

$$BA = \begin{pmatrix} (2\ 5\ 3)\begin{pmatrix}1\\2\\1\end{pmatrix} & (2\ 5\ 3)\begin{pmatrix}2\\1\\1\end{pmatrix} & (2\ 5\ 3)\begin{pmatrix}3\\3\\2\end{pmatrix} \\ (2\ 1\ 1)\begin{pmatrix}1\\2\\1\end{pmatrix} & (2\ 1\ 1)\begin{pmatrix}2\\1\\1\end{pmatrix} & (2\ 1\ 1)\begin{pmatrix}3\\3\\2\end{pmatrix} \\ (1\ 2\ 2)\begin{pmatrix}1\\2\\1\end{pmatrix} & (1\ 2\ 2)\begin{pmatrix}2\\1\\1\end{pmatrix} & (1\ 2\ 2)\begin{pmatrix}3\\3\\2\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2(1)+5(2)+3(1) & 2(2)+5(1)+3(1) & 2(3)+5(3)+3(2) \\ 2(1)+1(2)+1(1) & 2(2)+1(1)+1(1) & 2(3)+1(3)+1(2) \\ 1(1)+2(2)+2(1) & 1(2)+2(1)+2(1) & 1(3)+2(3)+2(2) \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 12 & 27 \\ 5 & 6 & 11 \\ 7 & 6 & 13 \end{pmatrix} = BA$$

(ii) $AB \neq BA$, matrix multiplication is not commutative unless a square matrix is being multiplied by the identity matrix 'I'. Neither A nor B are the identity matrix, therefore, AB does not equal BA.

Q3

$$x + y + z = 2$$
$$2x + 3y + z = 3$$
$$-x + y + 2z = 6$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \\ -1 & 1 & 2 & 6 \end{pmatrix}$$

~~① Add 2 times the 1st~~

Row 1 = R1 Row 2 = R2 Row 3 = R3

~~② Add~~

① Add $-2R1$ to R2:

$$\begin{array}{lrrrr} R2: & 2 & 3 & 1 & 3 \\ -2R1: & -2 & -2 & -2 & -4 \\ \hline \text{new}R2: & 0 & 1 & -1 & -1 \end{array} \qquad \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ -1 & 1 & 2 & 6 \end{pmatrix} = A$$

② Add R1 to R3:

$$\begin{array}{lrrrr} R3: & -1 & 1 & 2 & 6 \\ R1: & 1 & 1 & 1 & 2 \\ \hline \text{new}R3: & 0 & 2 & 3 & 8 \end{array} \qquad \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & 3 & 8 \end{pmatrix} = A$$

③ Add $-2R2$ to R3:

$$\begin{array}{lrrrr} R3: & 0 & 2 & 3 & 8 \\ -2R2: & 0 & -2 & 2 & 2 \\ \hline \text{new}R3: & 0 & 0 & 5 & 10 \end{array} \qquad \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 5 & 10 \end{pmatrix} = A$$

4. Multiply R3 by $\frac{1}{5}$:

$\frac{1}{5}R3: \ 0 \ 0 \ 1 \ 2$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix} = A$$

5. Add R3 to R2:

$$\begin{array}{lrrrr} R2: & 0 & 1 & -1 & -1 \\ R3: & 0 & 0 & 1 & 2 \\ \hline & 0 & 1 & 0 & 1 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} = A$$

6. Add $-1R3$ to R1:

$$\begin{array}{lrrrr} R1: & 1 & 1 & 1 & 2 \\ -1R3: & 0 & 0 & -1 & -2 \\ \hline \text{new } R1: & 1 & 1 & 0 & 0 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} = A$$

7. Add $-1R2$ to R1:

$$\begin{array}{lrrrr} R1: & 1 & 1 & 0 & 0 \\ -1R2: & 0 & -1 & 0 & -1 \\ \hline \text{new } R1 & 1 & 0 & 0 & -1 \end{array}$$

Reduced row-echelon form

$$\boxed{\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} = A}$$

8. Rewrite equations:

$$\boxed{\begin{aligned} x &= -1 \\ y &= 1 \\ z &= 2 \end{aligned}}$$