Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

Integrated Computer Science Programme **Trinity Term 2016**
B.A. (Mod.) Computer Science and Business
Year 1 Annual Examinations

Computational Mathematics

Monday 9th May 2016 **RDS** **14.00 – 16.00**

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Instructions to Candidates:

(i) A total of FOUR questions should be attempted.

(ii) All questions carry equal marks.

Materials Permitted for this Examination:

(i) Use of non-programmable calculators and log-tables is permitted.

Question 1.

The power output of a solar cell varies with the voltage it puts out. The voltage V_{mp} at which the output power is maximum is given by the equation:

$$e^{(qV_{mp}/k_BT)}\left(1+\frac{qV_{mp}}{k_BT}\right)=e^{(qV_{OC}/k_BT)}$$

where V_{OC} is the open circuit voltage, T is the temperature in Kelvin, q = 1.6022×10^{-19} C is the charge on an electron, and k_B = 1.3806×10^{-23} J/K is Boltzmann's constant.

For V_{OC} = 0.5 V and room temperature (T = 297 K), determine the voltage V_{mp} at which the power output of the cell is a maximum by writing a MATLAB program in a script file that uses the fixed-point iteration method to find the root.

For a starting point, use V_{mp}= 0.5 V. To terminate the iterations, use the Estimated Relative Error(ε) $\leq$0.001 .

[25 Marks]

Question 2.

Write a user-defined MATLAB function that decomposes an $n \times n$ matrix into a lower triangular matrix $[L]$ and an upper triangular matrix $[U]$ (such that $[A] = [L][U]$) using the Gaussian Elimination Method (without pivoting). For the function name and arguments, use [L,U] = LUdecompGauss(A), where the input argument A is the matrix to be decomposed and the output arguments L and U are the corresponding upper and lower triangular matrices.

[25 Marks]

Question 3.

The power generated by a windmill varies with the wind speed. In an experiment, the following five measurements were obtained:

Wind Speed (Kmph)	14	22	30	38	46
Electric Power (W)	320	490	540	500	480

Determine the fourth-order polynomial in the Lagrange form that passes through the points. Use the polynomial to calculate the power at a wind speed of 26 Kmph.

[25 Marks]

Question 4.

A particular finite difference formula for the first derivative of a function is:

$$f'(x_i) = \frac{-f(x_{i+3}) + 9f(x_{i+1}) - 8f(x_i)}{6h}$$

where the points $x_i, x_{i+1}, x_{i+2}\ and\ x_{i+3}$ are all equally spaced with step size h. What is the order of the truncation or discretization error?

[25 Marks]

Question 5.

The central span of the Golden Gate bridge is 1260m long and the towers' height from the roadway is 150m. The shape of the main suspension cables can be approximately modeled (as a catenary) by the equation:

$$f(x) = C\left(\frac{e^{x/C} + e^{-x/C}}{2} - 1\right) \quad for \ -630 \leq x \leq 630m$$

where C=1347.

By using the equation $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$, determine the length (L) of the main suspension cables using three-point Gaussian Quadrature.

The given Gauss points and coefficients for the interval [-1, 1] are:

$$x_1 = -0.77459667, \qquad x_2 = 0, \qquad x_3 = 0.77459667$$

$$C_1 = 0.5555556, \qquad C_2 = 0.8888889, \qquad C_3 = 0.5555556$$

[25 Marks]

Formula Sheet

1. Lagrange Polynomials:

$$f(x) = \sum_{i=1}^{n} y_i L_i(x) = \sum_{i=1}^{n} y_i \prod_{j=1, j \neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

2. Taylor Series:

$$f(x_{i+1}) = f(x_i) + h \left.\frac{df}{dx}\right|_{x=x_i} + \frac{h^2}{2} \left.\frac{d^2 f}{dx^2}\right|_{x=x_i} + \frac{h^3}{6} \left.\frac{d^3 f}{dx^3}\right|_{x=\xi_i}$$

where h is the stepsize and $\xi_i \in [x_i, x_{i+1}]$.

3. Gaussian Quardature:

$$\int_{-1}^{1} f(x)dx \approx C_1 f(x_1) + C_2 f(x_2) + C_3 f(x_3) + \ \dots + C_n f(x_n)$$

For the change of variable:

$$\int_{a}^{b} f(x)dx \rightarrow \int_{-1}^{1} g(t)dt$$

then:

$$x = \frac{1}{2}[t(b-a) + a + b] \text{ and } dx = \frac{1}{2}(b-a)dt$$