School of Physics and Astronomy

MSci Project Proposal

Bayesian Hierarchical Modelling of Helium Signatures in Red Giant Stars

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Abstract

The amount of Helium in our galaxy is poorly understood; however, modern asteroseismology techniques allow us to dissect the internal compositions of stars and shed light on the enrichment of Helium. Small signatures of Helium cause 'glitches' in the frequencies of oscillation of red giants. This project aims to produce a new estimate of an enrichment law for Helium as a function of metallicity, whereupon variations will be explored to test new forms of this law. This will be achieved by utilising Bayesian statistics to construct a hierarchical model followed by a comparison of results to chemical evolution models of the galaxy.

1 Introduction

A few minutes after the Big Bang, recently formed neutrons and protons combined to create nuclei of Deuterium and Helium. It would not be for another 300,000 years before the universe became sufficiently cool to allow electrons to become bound to these nuclei and form neutral atoms. In this early universe, around 23% of all baryonic matter was Helium.[1]
Since then, the amount of Helium has increased due to fusion within stars, but measuring this abundance is a task that has eluded astronomers for decades. However, modern techniques of asteroseismology allow astrophysicists to probe the internal compositions of stars, providing insight into the enrichment of Helium.
Asteroseismic data can be used to detect Helium within the connective envelope of red giant stars. Small signatures of Helium produce subtle 'glitches' in the frequencies of oscillation modes of the star. Over the duration of this project, these 'glitches' will be identified for an ensemble of a few thousand stars; the data will be used to produce a Bayesian hierarchical model, from which a new estimate for a Helium Enrichment law will be derived.

2 Aims

This project ultimately aims to produce a law for Helium enrichment as a function of metallicity. From there, other forms of enrichment laws will be considered and compared to models for validation.
For this project, the statistical programming language Stan will be employed to produce a Bayesian hierarchical model. The resulting relation for Helium enrichment will be tested through comparison with chemical evolution models, which can be created using another programming tool: PyChem.

3 Background and Motivation

To achieve the aims set out previously, one must gain an understanding of the physical theory (in order to appreciate the data) and the statistical analysis (in order to process the data).

3.1 Asteroseismology

The interior of a star may be one of the least accessible places in the galaxy and, as such, attempting to peer beyond the surface poses a challenge to astronomers. We have a detailed knowledge of our own planet's interior due to the measurement of seismic activity at the surface (geo-seismology). The movement of tectonic plates and resulting oscillations allow measurements of the speed of sound beneath the surface. From these measurements, the temperature and density can be found and subsequently, other features can be deduced.

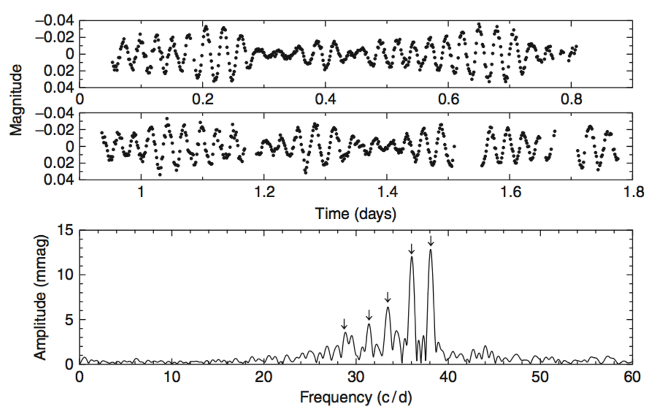

Figure 1: The variations in luminosity (upper two panels) of a pulsating star and the corresponding Fourier spectrum, identifying the component frequencies.[2]

Stars, much like our own planet, exhibit seismic effects, the study of which is known as asteroseismology. These stellar oscillations are caused by stochastic turbulence within the convective zones of the star's interior. When this excitation reaches the surface, the resulting rarefaction or compression causes variation in light, radial velocity, and line profile. These variations can be detected many light-years away, spectroscopically or photometrically.[2]

When the entire star vibrates, it is known as a normal mode oscillation and provides the most information of the interior. The frequencies of stellar models are matched to observed frequencies, allowing the deduction of physical parameters and interior structure. The superposition of normal mode frequencies are viewed as sound waves passing through the various layers of the star.

Shown in the upper two panels in figure 1 is the variation in light as a star pulsates. The frequencies can then be identified by performing a discrete Fourier transform (shown in equation 1), resulting in the spectrum in the lower panel of figure 1. Often the analysis is carried forward by further processing; fitting sinusoids to the data, optimising frequencies, amplitudes and phases, etc.

$$F_N(f) = \sum_{k=1}^{N} x(t_k) e^{i2\pi f t_k} \tag{1}$$

To determine the physical parameters of the star, the next stage is to identify the modes and angular degrees of these modes in the frequency spectra. The mode numbers contain information regarding the path length through the interior and the frequencies contain information of the 'speed of sound' along that path. Assuming the simplest normal mode (radial), the modes of oscillation are described by three indices l, m, n, where l is the angular degree, m is the harmonic order and n is the radial order.[3] The different paths of these modes can be seen in figure 2.

However, the oscillation modes of a star are not spherically symmetric, and the l mode frequency is offset primarily due to the star's rotation. This is called frequency splitting, and can be used to determine rotation and other properties caused by lack of spherical symmetry.

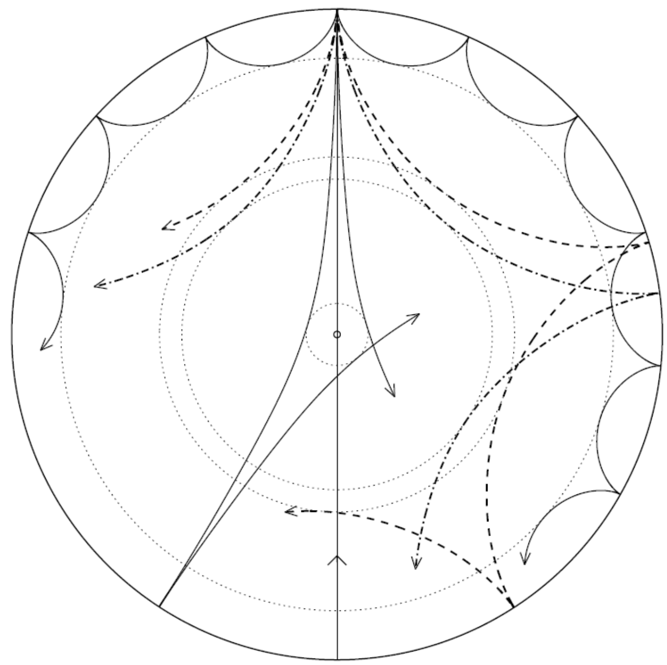

Figure 2: Cross-sectional diagram of a star showing the mode penetration. Low angular degree modes penetrate deeper.[3]

This project will be analysing data from Red Giants as their oscillations are similar to main sequence stars and characterisation of these modes leads to reliable estimations of mass and radius. Red Giants contain two resonant cavities: the core and the envelope. Gravity (g) modes only propagate in the core and acoustic (p) modes propagate through the convective envelope.[4] Predictions have been made that discontinuities in sound-speed exist and disturb the observed oscillations. In Red Giants this discontinuity is the region in which Helium undergoes its second ionisation, known as the HeII envelope[5].[6] This exists towards the surface of the star, meaning it is only acoustic modes that need to be considered.

The discontinuity in sound-speed is what produces the all-important 'glitch', and is due to the rapid variation in chemical composition as Helium becomes fully ionised. The characterisation of this glitch allows tighter constraints on the chemical composition of the star and potential for deducing the abundance of Helium.

3.2 Bayesian statistics

Due to the relatively small signature of the Helium glitch within the observation of the star's oscillation, the uncertainty for any single star will be large. Therefore, to produce reliable results, one must leverage the power of the ensemble and use data from thousands of Red Giants. In order to interpret data from such a large sample size, a statistical model must be created.

This will take the form of a Bayesian hierarchical model: a statistically rigorous way of making scientific inferences about a population based on many individual observations.[7] When working with stellar observations, one can regard many of their parameters as having some kind of relationship. This lends itself to a hierarchical model, in which one can construct probability distributions that capture the dependencies between relevant variables while keeping models simple.[8]

The first step in constructing a hierarchical model is to estimate a parameter and, using similar previous experiments, define a prior distribution. A likelihood (or sampling distribution) is what ties the model to the data, determining how likely it is, given the prior, for the model to produce the data one has. This results in what is called a posterior, which acts as a new prior, updated with information contained within the data.

This is shown in the form of Bayes theorem in equation 2:

$$p(\theta|y) \propto p(y|\theta)p(\theta) \tag{2}$$

where $p(\theta)$ is the prior density for parameter θ, $p(y|\theta)$ is the likelihood (model) and $p(\theta|y)$ is the posterior density.

One can expand this model by adding another level to the hierarchy: the parameters defining the distribution of parameter θ can also defined by a distribution of their shared dependencies. This is what makes the model hierarchical.

4 Method

4.1 Constructing the model

Due to the large amount of data and subsequent processing required, the majority of time will be spent programming. For statistical analysis, the language Stan will be used and interfaced with using Python. 'PyStan' as this package is known, allows the development and implementation of statistical models, utilising Bayesian inference.

The hierarchical model produced using PyStan will be applied to a few thousand stars and from this, a new estimate for a law of Helium enrichment will be determined. While the Helium signature is detected only as a small deviation within the stellar oscillations, by using data from many thousands of Red Giants, the uncertainty will be mitigated.

4.2 Verifying results

Once this law is established, other forms will be explored to fully scrutinise the process of Helium enrichment. The results will then be verified by comparing to chemical evolution models of the galaxy. These models describe chemical abundances and how the most common chemical elements evolve in space and time.[9] The models will be developed using another Python-interface programming tool called PyChem.

Milestone	Due date
Completion of Bayesian Hierarchical model	14/12/2018
Produce estimate of He Enrichment law	25/01/2019
Explore other forms and compare to chemical evolution models	29/03/2019

Table 1: Table showing each milestone in the project and the deadline to achieve it.

5 Plan

To effectively produce the desired results within the given time frame, some milestones will need to be identified. These are shown in table 1 and if met by these due dates, the aims set out in section 2 will be achieved.
As a theory experiment, it is unlikely that the method will need to be adapted over the course of the project. Because of this, a contingency plan is not needed. Due to the use of established data, any result will be worthy of discussion. Even a null result showing no relation between He enrichment and another parameter (or parameters) is a discovery in itself.

6 Predictions

If successful, this project will result in a new understanding of Helium enrichment and the abundance of the element in our galaxy. From galactic chemical evolution models, the most likely outcome is that Helium enrichment will very with metallicity. This is a measure of the abundance of elements heavier than Hydrogen and Helium within a star.
Alternatively, some models claim that abundance ratios of elements in the Milky Way are determined by galactic parameters: the Initial Mass Function (IMF) and the Star Formation Rate (SFR).[10] These determine the evolution of a local cluster of stars; the former being a metric to describe the initial distribution of mass and the latter being the total mass of stars formed per year. Therefore, these could also be dependencies of Helium enrichment.

7 Final thoughts

This project has the potential to shed new light on an area of astronomy that, due to advances in asteroseismic techniques, has only recently become accessible. Utilising the methods specified in this report will be key to accomplishing this; producing a relation to describe Helium enrichment which may then better our understanding of the inner workings of the stars.

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