University of Birmingham
School of Physics and Astronomy

Y4 Project

Bayesian Hierarchical Modelling of Helium Signatures in Red Giant Stars

Project Proposal

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Abstract

Asteroseismology provides a means of determining the helium abundance in the galaxy, which is poorly understood, when applied to to He II ionisation region in stars. We will aim to conduct a statistical analysis of a large population of Red Giant stars. This will include the use of hierarchical Bayesian models constructed using stan to produce a prediction for the helium enrichment law for stars. A dependence on metallicity is expected to some degree, though any result will be significant – large variation from the models may suggest an incomplete knowledge of stellar physics.

Keywords: Asteroseismology, Bayesian Statistics, Hierarchical Modelling, HBM, Stars: Helium

1 Introduction & Aims

Despite large advances in our understanding of the universe and stellar evolution over the past century, knowledge about the abundance of helium in our galaxy, the second-most plentiful element in the universe, remains elusive.

This project will make use of powerful and novel statistical techniques, though the use of probabilistic programming languages, in an effort to develop a helium enrichment law that can be applied to the larger population. The use of a large sample size allows a significant improvement in the accuracy on any previous estimates by exploiting the power of the statistical ensemble.

2 Background

Currently there are no adequate laws to describe the helium composition of stars, which introduces a great deal of uncertainty when considering stellar models and their applications, from considering the evolution of the galaxy to that of our own planet.

Helium abundance in stars is notoriously hard to measure. It was first observed in the spectrum of the sun over 100 years ago, [1] however these emissions are produced in the corona and chromosphere, the outermost layers of the sun. It is not possible to use these regions as a measure of the helium abundance in the sun; partly because the composition is likely to be different to the interior regions, but also as thermodynamic equilibrium is not necessarily maintained. [2]

Similarly, another potential technique for measuring He abundance involves looking directly at absorption spectra, however this is not feasible since stars like the sun are not sufficiently hot to allow the He atoms to be excited.

Historically, due to the resulting dearth of knowledge pertaining to the helium abundance, the mass fraction Y has previously been adjusted as a free parameter in stellar models, or simply related directly to metal mass fraction Z, where $X + Y + Z = 1$ and X is the hydrogen mass fraction. However this is imperfect due to the lack of detail in the models themselves: near-surface layers cannot be modelled properly and this leads to a large potential source of error.

The advent of asteroseismology however offers the ability to probe for this information by looking at the subtle effect that ionised helium has on the oscillations within stars, a technique that has been explored by previous studies. [3,4,5,6]

2.1 Asteroseismology

Traditional optical based observations of stars can only provide us with a small amount of information, such as temperature and metallicity (contribution of elements heavier that He). The development of asteroseismology provides a sophisticated tool that is sensitive to the internal structures of stars, thereby allowing us to extract more detail about stellar processes.

Information in asteroseismology comes from studying the subtle oscillations caused by standing waves trapped within the star. There are two main types of oscillation, pressure and gravity modes, with the possibility of additional modes that are mixed in nature. This project will only deal with pressure modes – a more comprehensive description of the other types can be found in the review by Chaplin and Miglio. [7]

Throughout the project we will be using asteroseismic data provided by Red Giant Branch (RGB) stars. These are evolved low to intermediate-mass stars that have previously exhausted the nuclear hydrogen fuel in their cores and moved off the main sequence. Demands placed by conservation of hydrostatic equilibrium require that if the star's inner regions contract in order to maintain hydrogen fusion, the outer layers will expand. These RGB stars can be referred to as 'Solar-like Oscillators' because they have a similar structure to the sun, with a deep radiative region and an outer convective layer.

2.1.1 Pressure Modes

Pressure (or p-modes) originate from pressure gradients within the star that provide a restoring force to a perturbation. They are the stellar analogues to the p-waves that are observed within the earth following earthquakes. There are two parameters we use to describe different modes: n, the overtone number (describes number of radial oscillation nodes); and the angular degree l, which corresponds to the wavelength of the oscillation on the star's surface. The first few terms can be interpreted as such:

- $l = 0$ Radial mode (expansion only in radial component)
- $l = 1$ Dipole Mode (one hemisphere expands whilst opposite contracts)
- $l = 2$ Quadrupole Mode
- *etc.*

Current observations are sensitive to oscillations of high radial order (large n), where the frequencies ν_{nl}

can be approximated by[7]

$$\nu_{nl} \simeq \Delta\nu \left(n + \frac{l}{2} + \epsilon\right) - \Delta\nu^2 \left\{\frac{A[l(l+1)] - B}{\nu_{nl}}\right\} \quad (1)$$

where ϵ is a boundary condition determined by properties of the oscillation cavity, along with the coefficients A and B. At this point, looking at the first few terms, one may see how the analogy of sound waves in an organ pipe becomes relevant. The frequency spacing $\Delta\nu$ is given by

$$\Delta\nu = \left(2\int_0^R \frac{dr}{c}\right)^{-1} \quad (2)$$

and represents the inverse of the acoustic diameter. This is the time taken for sound (of speed c) to travel across the stellar diameter (i.e. $2R$).

2.1.2 Visualising Oscillations

The large number of observed oscillations are often more easily interpreted on a power spectrum. Figure 1 shows a frequency power spectrum to illustrate the different oscillation modes, showing a clear envelope with a frequency of maximum power, ν_{max}; this value is linked to the conditions of the near-surface regions. Since successive oscillations of the same angular degree l are spread over a large range of frequencies, an Échelle Diagram (Figure 2) provides a useful means of viewing the frequencies (by 'wrapping around') modulo the large separation $\Delta\nu$. The x-axis in Figure 1 becomes the y-axis of Figure 2.

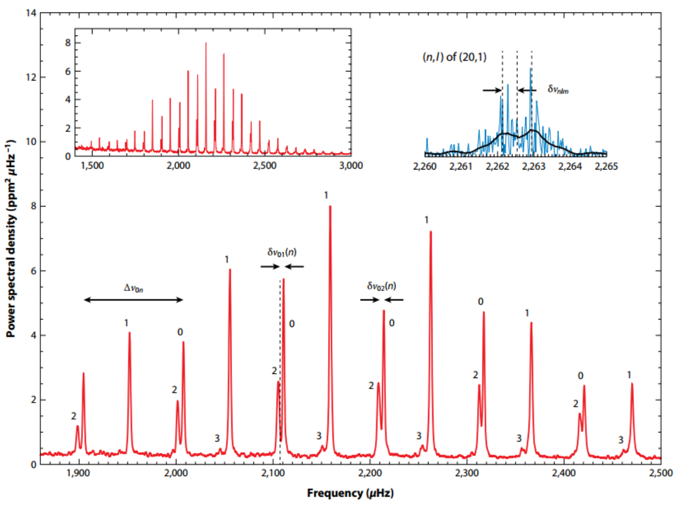

Figure 1: *A power spectrum for the a solar-like star, showing the regular spacing between successive oscillation modes.*[7]

The advantage of the Échelle Diagram is that the different angular modes roughly form a line. Deviation to form a slight curve results from the structure of the star, however there is a much weaker oscillatory signal that decays with increasing frequency, most easily visible in the $l = 0, 1$ modes of the figure. It is this signal that will be looking to exploit during our analysis, resulting from the presence of the Helium II ionisation region.

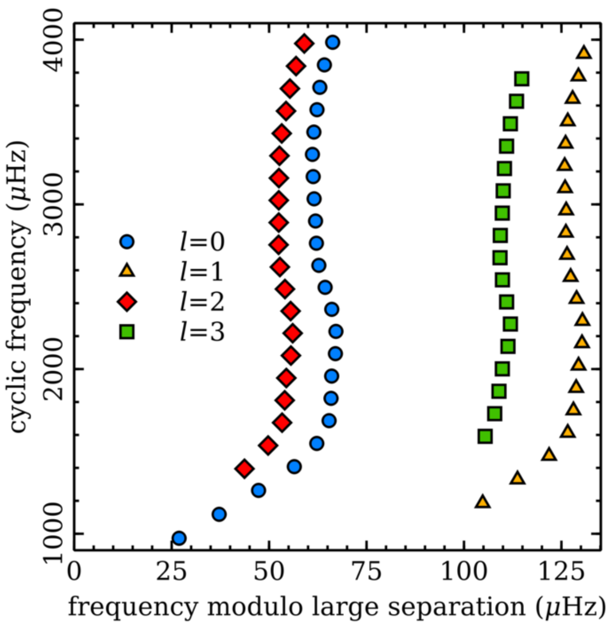

Figure 2: *An example Échelle Diagram for the Sun, using data from the Birmingham Solar Oscillations Network (BiSON).*[8]

2.2 Helium in the Universe

2.2.1 Production

The amount of helium found throughout the universe has been increasing ever since the Big Bang; it is important to note the main processes that lead to the release of the element into the environment, briefly summarised below:

1. **Big Bang Nucleogenesis** - Fixed amounts of H, He and Li were produced shortly after the big bang as the universe cooled. This provides a lower bound on the helium abundance that can be prescribed in our models: estimates vary with uncertainty but the figure is thought to be $Y_P \sim 24\%$.[9]

2. **AGB Stars** - The asymptotic giant branch is the final phase of evolution for stars like the sun. Helium in a star can be produced by the 'pp-chain',

CNO or triple-α processes, dependent on stellar mass and stage of evolution.[2] Upon evolving onto the AGB, the star becomes unstable as all remaining nuclear fuel is depleted, and the outer layers which contain a significant portion of He are gradually sloughed off, forming a planetary nebula and releasing He into the local environment.

3. **Type Ia Supernovae** - These supernovae occur when a white dwarf in a binary system accretes sufficient material from a companion star that its mass breaks the Chandrasekhar Limit.[10] A runaway thermonuclear explosion results that scatters helium into the interstellar medium.

One may expect Type II Supernovae (exhibited by high-mass stars) to be a significant method of scattering He due to their highly explosive nature. In fact, most of the He is destroyed by the fusion of successively heavier elements. Furthermore, during the supernova event, the majority of the He residing in the outermost layers of the star is destroyed during the creation of more nuclides.

2.2.2 Helium II Ionisation

The overarching signal in p-mode oscillations is driven by the larger scale structures of the star, however more localised effects can be caused by sharp changes in the interior stellar stratification, where the adiabatic exponent

$$\gamma_1 = \left(\frac{d\ln P}{d\ln\rho}\right)_s \tag{3}$$

rapidly changes, where P is the pressure and ρ the density at constant entropy s. This primarily occurs at the boundary of the convective region, but also in Helium II ionisation regions (containing the species He^+), which is the target of our investigation. The change in γ_1 introduces what is referred to as an 'acoustic glitch' by causing a small shift in the eigenfrequencies of the lower order (in l, n) oscillation.[11,12] This manifests in the recorded signal in the form of a decaying sinusoid, visible in Figure 3.

The effect due to He II is smaller than that due to the convective boundary zone. As a result, the use of RGB stars is important since they have a very deep convective envelope, so the two effect should not contaminate each other and be distinguishable.[13]

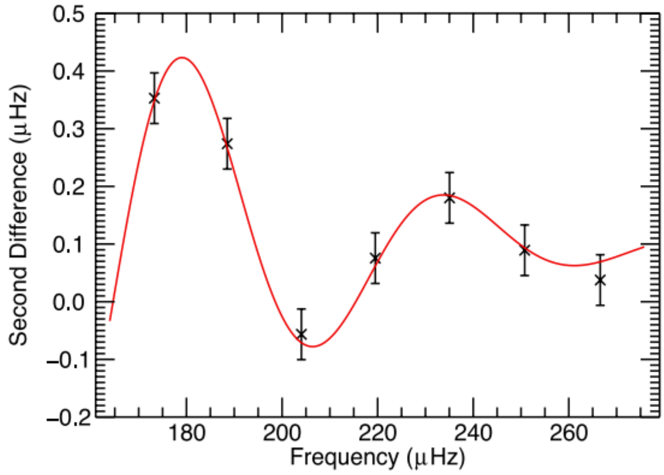

Figure 3: *Example of $l = 0$ frequency differences caused by the acoustic glitch due to the presence of He II.*[14]

2.3 Hierarchical Bayesian Modelling (HBM)

The basis for our statistical analysis comes from Bayes' Theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} \tag{4}$$

where A and B are two separate events and $P(A|B)$ is the probability of event A occuring given that B is also true. We wish to estimate a particular parameter θ about a population based on data x from individual observations thus

$$\underset{posterior}{p(\theta|x)} \propto \underset{likelihood}{p(x|\theta)} \quad \underset{prior}{p(\theta)} \tag{5}$$

- Likelihood - likeliness of observed data given a specific value of θ.
- Posterior - output probability of different θ given the data and model.
- Prior - known information about the system and parameter values before data is taken.

Currently however, this only considers a single 'level' of parameters and is not a hierarchical model. We can make one by introducing new variables. As well as stating that some variable x may follow a normal distribution $x|\theta \sim N(\theta, 1)$, θ (known as a latent variable) itself may also follow $\theta|\mu \sim N(\mu, 1)$ whilst μ is governed by a standard normal distribution $N(0, 1)$. In this situation, μ is known as a hyperparameter whilst $\mu \sim N(0, 1)$ is the hyperprior. This leads to a simple hierarchical model which can be presented as[15]

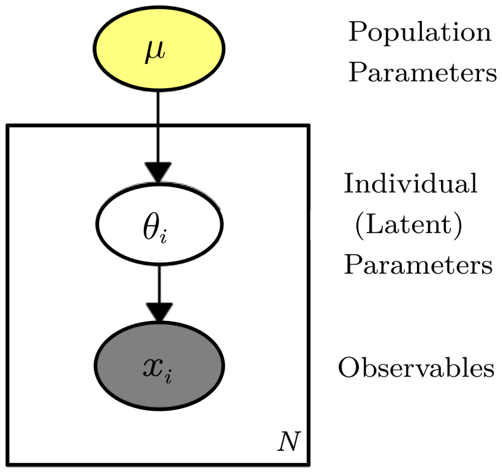

Figure 4: *Graph representation of Equation 6. (Own work)*

$$p(\mu, \theta|x) \propto p(x|\theta, \mu)\, p(\theta|\mu)\, p(\mu) \tag{6}$$

which in theory can be extended as necessary to include many more parameters. This can also be illustrated in graph form (Figure 4).

It soon becomes quickly apparent that solving to find the relevant probability density functions in a multi-dimensional parameter space becomes very intensive. To perform these calculations, it is possible to employ a sampling algorithm.

MCMC

One such sampling algorithm is the Markov Chain Monte-Carlo (MCMC) algorithm. The steps involved for a simple method can briefly be summarised thus:

- Randomly select a starting point θ_0
- Propose to move to a trial state θ'
- Calculate posterior probability for trial state
- Sample a random number $\lambda \sim U[0,1]$
- Accept/reject move to new state based on $\alpha = \frac{f(\theta')}{f(\theta_0)}$ by comparing with value of λ.
- If $\lambda \leq \alpha$, move to new state θ' and repeat process until sample size reached

More description about MCMC can be found online.[16]

3 Methods & Plan

The initial task will be to construct a program that can call `Stan`[17], which is a probabilistic programming language with a similar syntax to `C++`. Stan is able to use MCMC to perform Bayesian inference, and hence construct our HBM. Our programs will be written in python and we will be able to interface with Stan using `PyStan`[18]. These models will then be applied to real data and compared to see what (if any) variables appear to be linked to the helium enrichment. By making use of a large sample size N, it is possible to to make significant gains in reducing the uncertainty, since it scales with $\frac{1}{\sqrt{N}}$. Later techniques may include using the `pychem` module, which is able to simulate galactic chemical evolution as another tool for comparison.

3.1 Expected Results

Based upon the large dataset that we will be using during this project for our analysis, it should be possible to extract a result since this data has been used in previous established work (for example Vrard et al. 2015).[13] Since so little is known about helium enrichment laws, any result we are able to obtain will be significant so there is no need for a formal contingency plan. Possibilities therefore include a null result, suggesting there is no relation, however we most likely expect at least some dependence on the metallicity of a star. However there is likely to be local variation from the model throughout the population due to fluctuating star formation histories in different regions of the galaxy. More significant scatter from an obtained model may suggest our models and understanding of galactic and stellar evolution are incomplete.

3.2 Time Plan

Since the entirety of this project is computer oriented, it is not practical to provide concrete deadlines for when particular goals can be achieved. Nonetheless, the following table briefly summarises the key deadlines throughout the year as well as a couple of near-term targets.

Date	Deadline / Target
Nov. onwards	Start understanding and developing use of HBM
Sem. 1, Mid-Nov	*Project Work 1*
End of Sem. 1	Aim to have initial model finished to allow full data analysis
Sem. 2, Wk. 1-2	*Seminar*
Sem. 2, Mid-Feb	*Project Work 2* - split to focus on statistical and astro aspects?
Sem. 2, Week 11	*Finish Report*
Sem. 3, Week 1	*Project Viva*

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