An Introduction to MESA

February 5, 2019

This document presents a brief introduction into the methods and equations used by MESA (Modules for Experiments in Stellar Astrophysics). In order to produce and evolve the stellar models in the program, a number of assumptions are made, which are made to allow simulations to be performed more easily by a computer.

1. Stars are spherical (no oblate hyper-rotating stars).
2. The interiors of stars act like conventional fluids, and as such can be described using standard equations of fluid dynamics.
3. The star is isolated and evolves in the absence of any external influencing factors – stars in binary systems are often sufficiently separate that they may be considered as being isolated.
4. All forces except gravity are ignored (i.e. presence of magnetic fields ignored). This is simplified earlier to consider the case of a non-rotating system.
5. The star is in thermal equilibrium throughout (this doesn't hold for the solar corona).

With these conditions, there are a number of equations that MESA solves during each step of the evolution.

Structure Equations

1. Conservation of Mass
$$\frac{dm}{dr} = 4\pi r^2 \rho \tag{1}$$
2. Hydrostatic Equilibrium (Conservation of Momentum)
$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho \tag{2}$$
3. Conservation of Energy
$$\frac{dL}{dm} = \epsilon = \epsilon_{nuc} - \epsilon_{\nu} - T\frac{\partial \epsilon}{\partial t} \tag{3}$$
where ϵ is an energy production rate and the final term represents energy loss due to gravitational collapse.
4. Temperature Gradient
$$\frac{dT}{dr} = -\nabla \frac{Gm\rho}{r^2}\frac{T}{P} \tag{4}$$
where $\nabla \equiv \frac{d \ln T}{d \ln P}$.

One must also additionally consider the criterion for convection to occur. Simplistically, if one perturbs a mass of material from its equilibrium point, it may either oscillate about this point (convectively stable), or will continue to depart from the original value (convectively unstable). The convective parcel is modelled using a mixing length theory, where the mixing length l_m is given

$$l_m = \alpha_{mlt} H_P = \alpha_{mlt} \left| \frac{P}{dp/dr} \right| \tag{5}$$

and α_{mlt} is a mixing parameter and H_P is the pressure scale height.

Composition Equations The overall evolution of a star is ultimately driven by the change in the chemical (isotopic) abundances of the various elements X_i.

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho} \left(\sum_j r_{ji} - \sum_k r_{ik} \right) - \frac{\partial}{\partial m} \left(D_{conv} \frac{\partial X_i}{\partial m} \right) \tag{6}$$

The two summation terms represent the rates of production (for a particular product) and destruction respectively, whilst D_{conv} is the diffusive coefficient.

Boundary & Initial Conditions For the differential equations to be solved, some conditions need to be applied that constrain the model to physically possible scenarios. For example, at the core of the star $m = L = r = 0$, whilst at the surface $M = M_\star$ and

$$L = 4\pi R_\star^2 \sigma T_\star^4. \tag{7}$$

Likewise, the pressure due to gravity P_g is given

$$P_g = \frac{2}{3} \left(\frac{g_s}{\kappa} - \frac{F}{c} \right) \tag{8}$$

where g_s, κ and F are the surface gravity, opacity and flux respectively. In addition, we also apply the constraint that the is no leakage of elements at the extrema of the system, such that $\frac{dX_i}{dr} = 0$.

MESA allows models to be started from previously saved 'photos', however we can also model a star from primordial conditions: a uniform composition is present with $T\frac{\partial s}{\partial t} = 0$ and we apply pre-main sequence conditions.