Exercise 5.2

Following Eq.(5.5)-Eq.(5.10) in the textbook, the Fourier transformation of state $|j\rangle$ can be written as

$$|j\rangle \rightarrow \frac{1}{2^{n/2}}\left(|0\rangle + e^{2\pi i 0.j_n}|1\rangle\right)\left(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle\right)\cdots\left(|0\rangle + e^{2\pi i 0.j_1\ldots j_n}|1\rangle\right) \tag{1}$$

According to the textbook, $j_1 \ldots j_n$ is binary representation of integer j, and

$$0.j_l j_{l+1} \ldots j_m = j_l/2 + j_{l+1}/4 + \ldots + j_m/2^{m-l+1} \tag{2}$$

If we have a $n-$qubit state $|j\rangle = |0\rangle = |00\ldots 0\rangle$, then $j_k = 0$ for any $1 \leq k \leq n$ in the binary representation. Also, it always have $0.j_k \ldots j_n = 0$ for any k from eq.(2). Thus, the Fourier transformation of $n-$qubit state $|00\ldots 0\rangle$ is given by

$$|00\ldots 0\rangle \rightarrow \frac{1}{2^{n/2}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)\cdots(|0\rangle + |1\rangle) = |+\rangle^{\otimes n} \tag{3}$$

where $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.