Department of Physics and Astronomy
University of Southern California

Graduate Screening Examination
Part II
Saturday, April 13, 2013

Do not separate this page from the problem pages.
Fill out and turn in at the end of the exam.

Student ____________________
Fill in your L-#

The exam is **closed book**. Use only the paper provided and *make sure that each page is signed with your L-number*. Do not write answers to different problems on the same page. Mark each page with the problem number. Staple *separately* your answers to *each* problem.

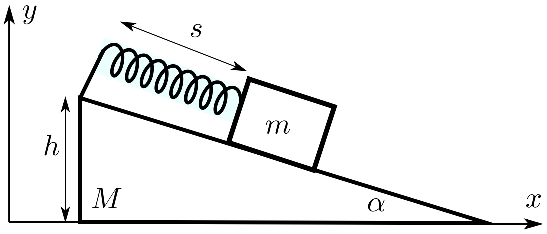
Solve 3 problems of your choice. Do not turn in more than 3 problems.

The total time allowed **2 hrs.**

Please, indicate problems you are turning in:

☐ **II-1** ☐ **II-2** ☐ **II-3** ☐ **II-4**

II-1. (Classical Mechanics)



A block of mass m is attached to a wedge of mass M by a spring with spring constant k. The inclined frictionless surface of the wedge makes an angle α to the horizontal. The wedge is free to slide on a horizontal frictionless surface, as shown above.

(i) Given the relaxed length of the spring alone is d, find the value s_0 when both the block and the wedge are at rest.

(ii) Find the Lagrangian of the system as a function of the x coordinate of the wedge and the length of the spring s. Write the equations of motion.

(iii) What is the natural frequency of small oscillations?

II-2. (E & M)

A hollow sphere of inner radius R is initially uncharged, but a pure point dipole $\vec{p}$ sits at its center. Subsequently, the sphere is grounded.

(i) Calculate the final potential $\Phi(r, \theta)$ for $r < R$.

(ii) Calculate the final surface charge density on the sphere.

(iii) How much charge flows from ground to the sphere?

(iv) What would be the surface charge density if the sphere were not grounded?

II-3. (Quantum Mechanics)

Find the energy levels and the wave function of two harmonic oscillators of masses m_1 and m_2, having identical frequencies ω, and coupled by the interaction $\frac{1}{2}k(x_1 - x_2)^2$. Your answer should include:

(i) the full Hamiltonian,

(ii) a desription in words of the path you will follow to solve the problem, the final goal, and the steps to answer parts (iii) and (iv),

(iii) the computation of the spectrum,

(iv) the full set of eigenstates.

II-4. (Mathematical Methods)

The gamma function is defined as

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt\,, \qquad \mathrm{Re}\, z > 0\,.$$

(i) Prove that for positive integer n,

$$\Gamma(n+1) = n!\,.$$

(ii) Show that

$$\Gamma(z+1) = z^{z+1} \int_0^\infty e^{z(\ln s - s)} ds\,.$$

(iii) For large z, the dominant contribution to the integration in (ii) comes from the maximum of function $g(s) = \ln s - s$. Use this observation to prove the Stirling's formula

$$n! \approx \sqrt{2\pi}\, n^{n+1/2} e^{-n}\,, \qquad n \gg 1\,.$$

Hint: Perform the Taylor series expansion of $g(s)$ around its maximum and use the leading terms to evaluate the integration.