# J00M.1—Shape of an Arch

Problem

The shape of an arch is determined by the condition that each brick is held in place by the normal force of its neighbors, with no need for mortar or "glue." To model this consider a thin course of bricks shaped so the normal force exerted on each brick by the neighbor on either side supports the brick against the uniform gravitational acceleration g. Then imagine the limit where the arch is a thin line with height $y = y(x)$ as a function of horizontal position x. The constant mass per unit length along the line of the arch is μ.
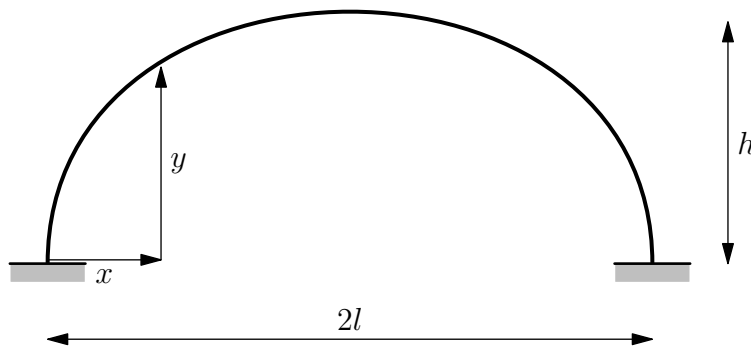

Find $y(x)$ for an arch with horizontal width $2l$ and height h. Give the equations that determine the constants in your solution, but you need not solve for the constants.

J00M.2—Linked Rods Falling

Problem

The centers of two thin rods with mass m and length l are connected by a thread, and the two ends are connected by a very short flexible thread. The threads have negligible mass. The other ends of the rods are free to slide without friction on a horizontal table. The plane of the rods is normal to the table, so the gravitational acceleration g is in the plane of the rods, as shown in the drawing. The rods are at rest, and make angle θ with the table. When the thread connecting the centers is cut the rods fall vertically until they hit the table.
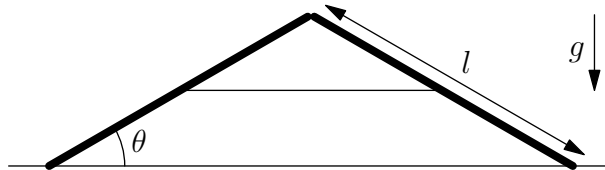

a)   Find the speed at which the connected ends of the rods are falling immediately before they hit the table.

b)   Find the tension in the thread that connects the ends of the rods immediately before the rods hit the table.

J00M.3—Orbiting Mass on a String

Problem

A mass m_1 slides without friction on a horizontal table. The mass is tied to a string with negligible mass that passes without friction through a small hole. A mass m_2 is tied to the other end of the string. The uniform gravitational acceleration g is normal to the table.
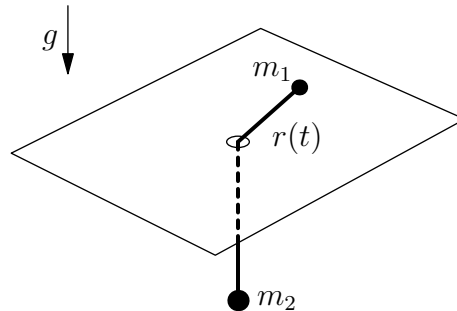
The orbit of m_1 is only slightly perturbed from circular. The masses m_1 and m_2 are chosen so the orbit is closed, with one maximum and one minimum of the distance $r(t)$ of m_1 from the hole, when computed to first order in the departure from a circular orbit. Find m_2 in terms of the other parameters.

J00E.1—A Two-Wire Transmission Line

Problem

A transmission line consists of a pair of conducting wires each of radius a whose centers are distance b apart. The space surrounding the wires has unit dielectric contant and permeability. Deduce the capacitance C per unit length.

[From this you could deduce the inductance L per unit length using $LC = 1/c^2$, the impedance $Z = \sqrt{L/C} = 1/cC$, and the sensitivity of the impedance to an error δb in the wire spacing, *etc.*]

J00E.2—Pitching Pennies into a Magnet

Problem

If one pitches a penny into a large magnet, eddy currents are induced in the penny, and their interaction with the magnetic field results in a repulsive force, according to Lenz' law. Estimate the minimum velocity needed for a penny to enter a long, solenoid magnet with central field $B = 1$ T and diameter $D = 0.1$ m.

You may suppose that the "penny" is actually a thin ring (torus) of radius a, cross-section area πb^2 where $b \ll a$, mass density ρ and conductivity σ. The "penny" moves so that its axis always coincides with that of the magnet, as shown in the figure below. Ignore gravity. The speed of the "penny" is low enough that the magnetic field caused by the eddy currents may be neglected compared to that of the solenoid. Equivalently, you may assume that the magnetic diffusion time is small.
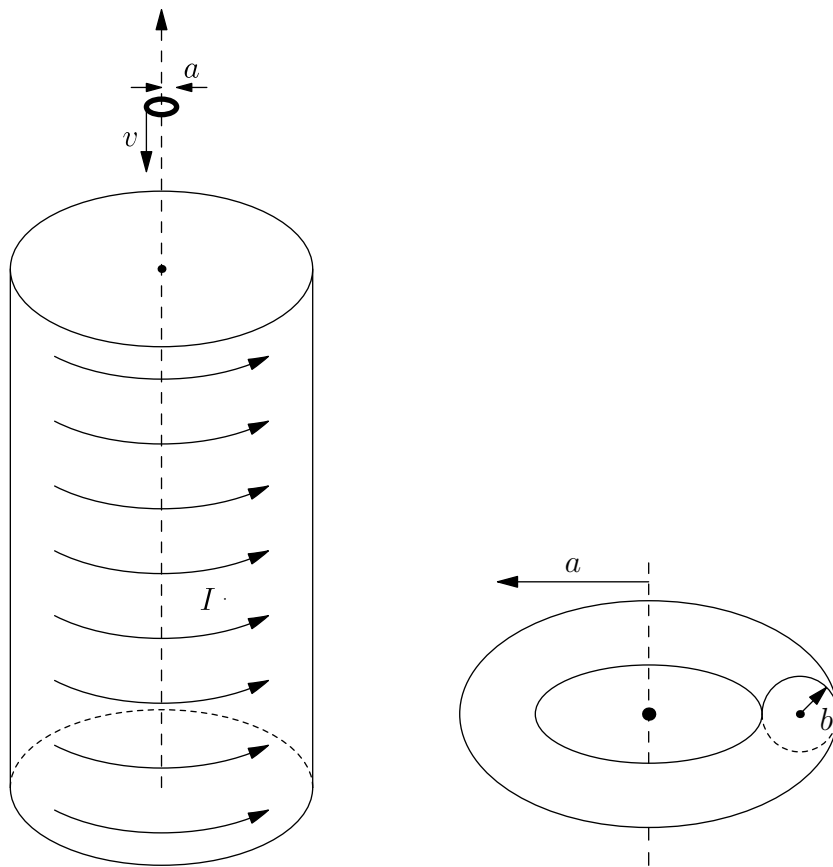
J00E.3—A Phased Antenna Array

Problem

Two "short" dipole antennas form a small "phased array" as shown in the figure. The second dipole is placed a distance $\Delta = \lambda/2$ away from the first along the y axis. The two dipoles are parallel to one another and are driven $180°$ out of phase of one another.
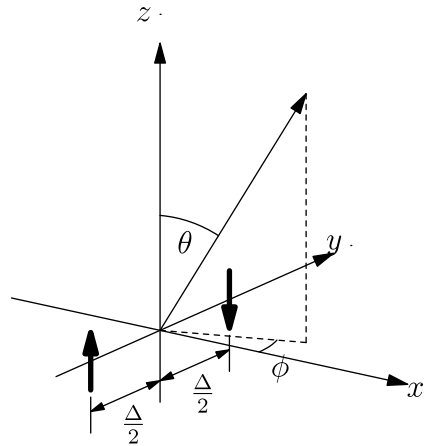
Each antenna is a center-fed dipole radiator formed from two wires, each of length $d/2 \ll \lambda$ and driven by a current source as shown in the figure below. The wires are aligned parallel to the z axis $(\theta = (0, \pi))$. The current source produces a time-dependent current given by $I(t) = I_0 e^{-i\omega t}$. You may assume that the charge that enters the wires is uniformly distributed along their lengths.
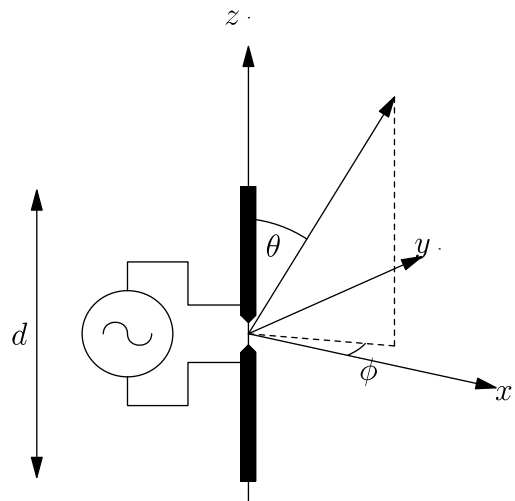
Calculate the time-averaged angular distribution of the radiated power for this arrangement in the radiation zone as a function of θ and ϕ, *i.e.*, calculate $\langle dP(\theta, \phi)/d\Omega \rangle$.

J00Q.1—Scattering From a Central Potential

Problem

A particle of mass m and energy $\hbar^2 k^2 / 2m$ scatters in a central potential $V(r)$ which is everywhere positive and vanishes rapidly as $r \to \infty$. Let $d\sigma / d\Omega$ be the differential cross section as computed in the Born approximation. For precisely backwards scattering you are given

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{back}} = A \, \frac{\exp(-4\lambda k)}{k^2}$$

where A, λ are given parameters.

a)   Calculate $d\sigma / d\Omega$ in the same approximation for arbitrary scattering angle.

b)   Calculate $V(r)$.

J00Q.2—Two Interacting Particles

Problem

Two interacting particles have Hamiltonian $H = H_0 + H'$, where

$$H_0 = -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) + V(\vec{r}_1) + V(\vec{r}_2),$$

$$V(\vec{r}) = \frac{1}{2}k|\vec{r}|^2,$$

$$H' = \epsilon(x_1 x_2 + y_1 y_2 - 2z_1 z_2).$$

Find the ground state energy to lowest non-vanishing order in ϵ.

J00Q.3—Electron on a Ring

Problem

An electron moves on a circular ring of radius R in the x-y plane. A constant uniform magnetic field B is applied in the z direction. Calculate the energy levels. Take the gyromagnetic ratio to be $g_s = 2$.

J00T.1—Ideal Gas In a Magnetic Field

Problem

A container C consists of two equal cubes joined together with a thin pipe through which gas can flow. We insert a monatomic, paramagnetic gas in C and it is at equilibrium at temperature T. The spin of each atom is $1/2$ and its magnetic moment is $g\mu_B$. The mass of each atom is m.

Next we place one of the cubes between the poles of a magnet which produces a field, H, in this cube, but not in the other. Assuming that there is no change in temperature and that the gas can be considered to be 'ideal', what is the ratio of the pressures in the two cubes?
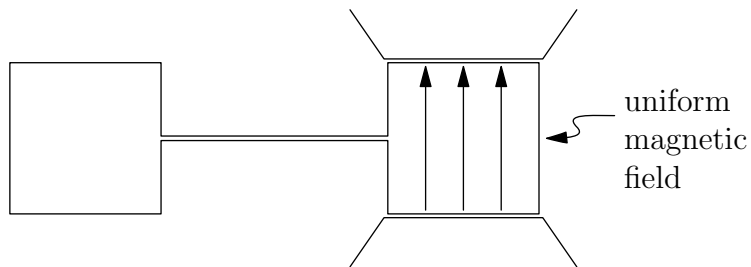
J00T.2—Masses on a Piston of Gas

Problem

An absent-minded physicist has a container of gas, but has forgotten whether it is Argon (Ar) or Nitrogen (N_2).

An experiment is performed using a piston of area A and some small masses which are slowly placed on top of the piston one by one, as shown in the figure. The weight of the piston is negligible. The walls are insulating and the gas is 'ideal'.

(adiabatic piston & walls)

gas h

Area A
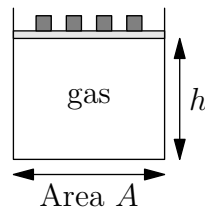
The system is initially at rest, without masses on the piston. As the masses are added the height h is measured; the following relation is observed to describe the total masses as a function of piston height:

$$M_0 + M(h) \propto h^{-n},$$

where M_0 is a constant.

If the gas were N_2 what would the exponent n be? What is M_0?

J00T.3—Bose-Einstein Condensation by Evaporation

Problem

The object of this problem is to find a method for producing Bose-Einstein condensation by means of evaporation. Consider a thermally insulating container of volume V containing N noninteracting particles obeying Bose-Einstein statistics.

a)   Find the equilibrium particle distribution in momentum space in terms of the temperature and chemical potential when the particle number density is less than the critical value $\rho_c(T)$ for Bose-Einstein condensation.

b)   Find the equilibrium particle distribution in momentum space when the particle number density is larger than $\rho_c(T)$.

c)   The critical density $\rho_c(T)$ for Bose-Einstein condensation scales with temperature as T^γ. Use this distribution (plus some elementary scaling) to find γ.

d)   Suppose, at first, that the density ρ is at the critical density $\rho_c(T)$. We allow some of the particles to escape. The few particles that escape have mean energy Ae, where e is the mean energy of the particles that remain and A is some experimentally determined constant. You are allowed to assume that the change in ρ is small. The system, which is thermally insulated, is then allowed to return to equilibrium. The question to be answered is this: For what values of A does the system remain in the condensed phase?