

Exercise 6.2.1

If we have a resource inequality $[c \rightarrow c] \geq [cc]$, it means that

- The noiseless classical bit channel can simulate the shared randomness

Note that the resource inequality would not indicate that the shared randomness cannot simulate the noiseless channel. To illustrate the first statements above, we could consider the following procedure. Suppose that we have a protocol that consists of following steps,

- Step 1: Alice prepares a random classical bit with probability of getting 0 and 1 are both $1/2$
- Step 2: Alice performs experiments and send her result to Bob through the classical noiseless channel. Then Bob also has a random classical bit with probability of getting 0 and 1 are both $1/2$; Alice and Bob would get same result, so the probability of getting 00 and 11 are $1/2$, which shows they share randomness.

To illustrate the second statements above, we could consider a counter-example. If Alice prepares a random classical bit with probability of getting 0 and 1 are $1/3$ and $2/3$, respectively, then she performs experiments and sends her result to Bob through the classical noiseless channel. Then Bob also has a random classical bit with probability of getting 0 and 1 are $1/3$ and $2/3$, and the probability of getting 00 and 11 are $1/3$ and $2/3$, respectively. However, a shared randomness can only produce the outcome of 00 and 11 with probability as $1/2$, so the shared randomness cannot simulate the noiseless classical channel.

Therefore, with analysis above, we conclude that we have the resource inequality $[c \rightarrow c] \geq [cc]$.