Exercise 3.6.1

Consider the composite quantum state $|\Phi^+\rangle_{AB}$,

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B) \tag{1}$$

The $|+\rangle$ and $|-\rangle$ states are given by

$$\begin{aligned} |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \\ \iff |0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle), |1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \end{aligned} \tag{2}$$

Substitute eq. (2) into eq. (1), we have

$$\begin{aligned} |\Phi^+\rangle_{AB} =& \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|+\rangle_A + |-\rangle_A) \otimes \frac{1}{\sqrt{2}}(|+\rangle_B + |-\rangle_B)\right. \\ &\left. + \frac{1}{\sqrt{2}}(|+\rangle_A - |-\rangle_A) \otimes \frac{1}{\sqrt{2}}(|+\rangle_B - |-\rangle_B)\right] \\ =& \frac{1}{2\sqrt{2}}(|+\rangle_A|+\rangle_B + |+\rangle_A|-\rangle_B + |-\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B \\ &+ |+\rangle_A|+\rangle_B - |+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B) \\ =& \frac{1}{2\sqrt{2}}(2|+\rangle_A|+\rangle_B + 2|-\rangle_A|-\rangle_B) \\ =& \frac{1}{\sqrt{2}}(|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B) \end{aligned} \tag{3}$$