Exercise 3.3.4

The Pauli matrix Y in the computational basis is

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{1}$$

The eigenvalue λ of Y is the solution of the following equation,

$$|Y - \lambda I| = \begin{vmatrix} -\lambda & -i \\ i & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0 \iff \lambda = \pm 1 \tag{2}$$

The corresponding eigenstates are given by

$$Y|v_1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \iff |v_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{3a}$$

$$Y|v_2\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = -\begin{pmatrix} c \\ d \end{pmatrix} \iff |v_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{3b}$$