Exercise 3.4.1

In quantum mechanics, the measurement result is the eigenvalues of the observable. For observable X, its eigenvectors are $|+\rangle$ and $|-\rangle$ with eigenvalue as $+1$ and -1, respectively. Thus, if we measure observable X, we would get two possible measurement outputs $+1$ and -1 corresponding two post-measurement states $|+\rangle$ and $|-\rangle$, respectively.

With above information, we can write observable X in the diagonal representation,

$$X = |+\rangle\langle+| - |-\rangle\langle-| \tag{1}$$

According to the definition of von Neumann measurement, since $\langle+|-\rangle = 0$, the $\{|+\rangle, |-\rangle\}$ is an orthonormal basis, so the measurement operators of X are $\Pi_+ = |+\rangle\langle+|$ and $\Pi_- = |-\rangle\langle-|$.