Exercise 3.4.4

Consider operator $Z_0 = Z - \langle Z \rangle I$ and $X_0 = X - \langle X \rangle I$, we could find that

$$\begin{aligned}
[Z_0, X_0] &= Z_0 X_0 - X_0 Z_0 \\
&= (Z - \langle Z \rangle)(X - \langle X \rangle) - (X - \langle X \rangle)(Z - \langle Z \rangle) \\
&= ZX - \langle X \rangle Z - \langle Z \rangle X + \langle Z \rangle \langle X \rangle I - XZ + \langle Z \rangle X + \langle X \rangle Z - \langle X \rangle \langle Z \rangle I \\
&= ZX - XZ \\
&= [Z, X]
\end{aligned} \tag{1}$$

Also,

$$\begin{aligned}
[Z, X] = ZX - XZ &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\
&= 2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = 2i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = 2iY
\end{aligned} \tag{2}$$