Exercise 2.1.1

For a given information source $\{x \in \mathcal{X}, p(x)\}$, the entropy $H(X)$ is given by

$$H(X) = -\sum_{x \in \mathcal{X}} p_X(x) \log p_X(x) \tag{1}$$

For a certain information source with each variable x_i is uniformly random, the probability distribution is

$$p(x_i) = \frac{1}{|\mathcal{X}|} \text{ for all } x_i \in \mathcal{X} \tag{2}$$

where $|\mathcal{X}|$ is the size of the variable's alphabet. Substitute eq. (2) into eq. (1), we get the entropy for the uniform random variable as

$$\begin{aligned} H(X) &= -\sum_{i} \frac{1}{|\mathcal{X}|} \log \left(\frac{1}{|\mathcal{X}|} \right) \\ &= -|\mathcal{X}| \cdot \frac{1}{|\mathcal{X}|} \log \left(\frac{1}{|\mathcal{X}|} \right) \\ &= -\log \left(\frac{1}{|\mathcal{X}|} \right) = \log |\mathcal{X}| \end{aligned} \tag{3}$$