Exercise 6.3.1

We will prove by contradiction that in the teleportation protocol,

$$[qq] + 2[c \to c] \geq C[q \to q] \tag{1}$$

where we cannot have $C > 1$. Suppose that there exists some protocol that has resource inequality as

$$[qq] + 2[c \to c] \geq C[q \to q] \tag{2}$$

If we combine infinite entanglement and the super dense code together, we have

$$[q \to q] + \infty[qq] \geq 2[c \to c] + \infty[qq] \tag{3}$$

If we excute protocol with eq. (3) first and then excute protocol with eq. (2), we would achieve the following resource inequality,

$$[q \to q] + \infty[qq] \geq 2[c \to c] + \infty[qq] \geq C[q \to q] + \infty[qq] \tag{4}$$

Note that an infinite amount of entanglement is still available after excuting protocol with eq. (2) because it only consumes a finite amount of entanglement. From eq. (4), we could have an overall protocol that achieves the following inequality,

$$[q \to q] + \infty[qq] \geq C[q \to q] + \infty[qq] \tag{5}$$

We can continue the process similar with the process from eq. (3) to (5) for k times so that we could implement the following inequality,

$$[q \to q] + \infty[qq] \geq C^k[q \to q] + \infty[qq] \tag{6}$$

The result of eq. (6) is that one noiseless quantum channel and an infinite amount of entanglement can generate an infinite amount of noiseless quantum channel and an infinite amount of entanglement. However, the protocol stated above is impossible since entanglement cannot extend the capacity of noiseless qubit channel, so it is hard to doing unlimited quantum communication with only one channel and many ebits. Therefore, we cannot have $C > 1$ in eq. (1).