Persistent DS

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prerequisite

tau pointer

hayooo.. yang ngga tau berarti
ketauan ngga dengerin lecture Bu Inge
lecture pertama pisan

tau linked list

tau segment tree

In computing, a **persistent data structure** is a data structure that always preserves the previous version of itself when it is modified.

buat ngilustrasiin, pake soal ini aja

diberikan linked list, ada dua tipe operasi :

1. update x value pertama

2. print linked list setelah update ke-k

bruteforce :
bikin linked list baru tiap update,
simpen ke array untuk tiap “versi”

kalo x kecil, bisa manfaatin linked list sebelomnya

allocate cuma x space baru, terus elemen ke-x point ke element ke-(x+1) dari linked list sebelomnya

pasti ga ngerti

contoh
awal = {1, 2, 3, 4, 5, 6}
update 1 : x = 3, {12, 13, 14}
update 2 : x = 2, {20, 21}
update 3 : x = 5, {1, 2, 3, 4, 5}
update 4 : x = 1, {100}

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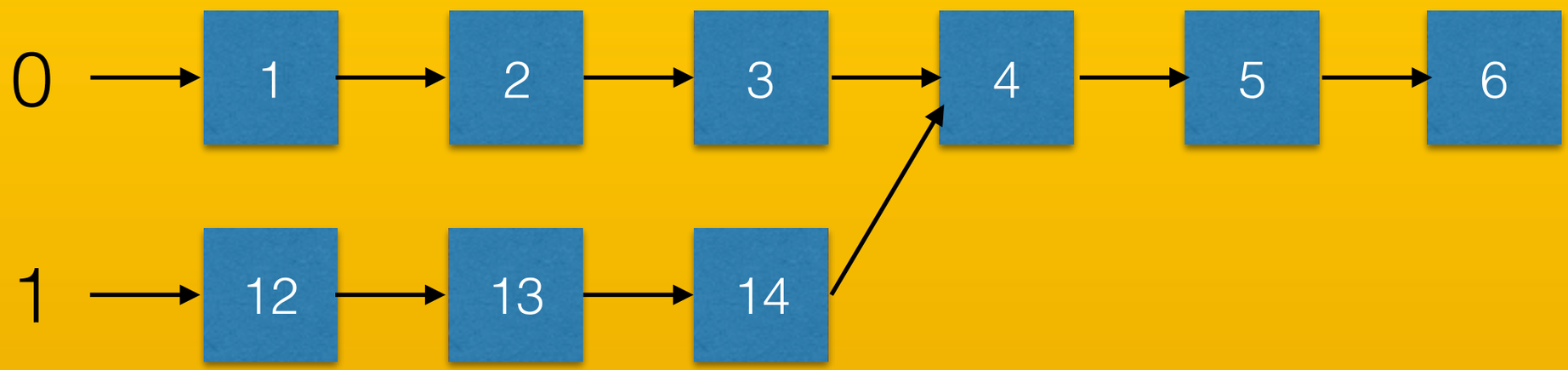

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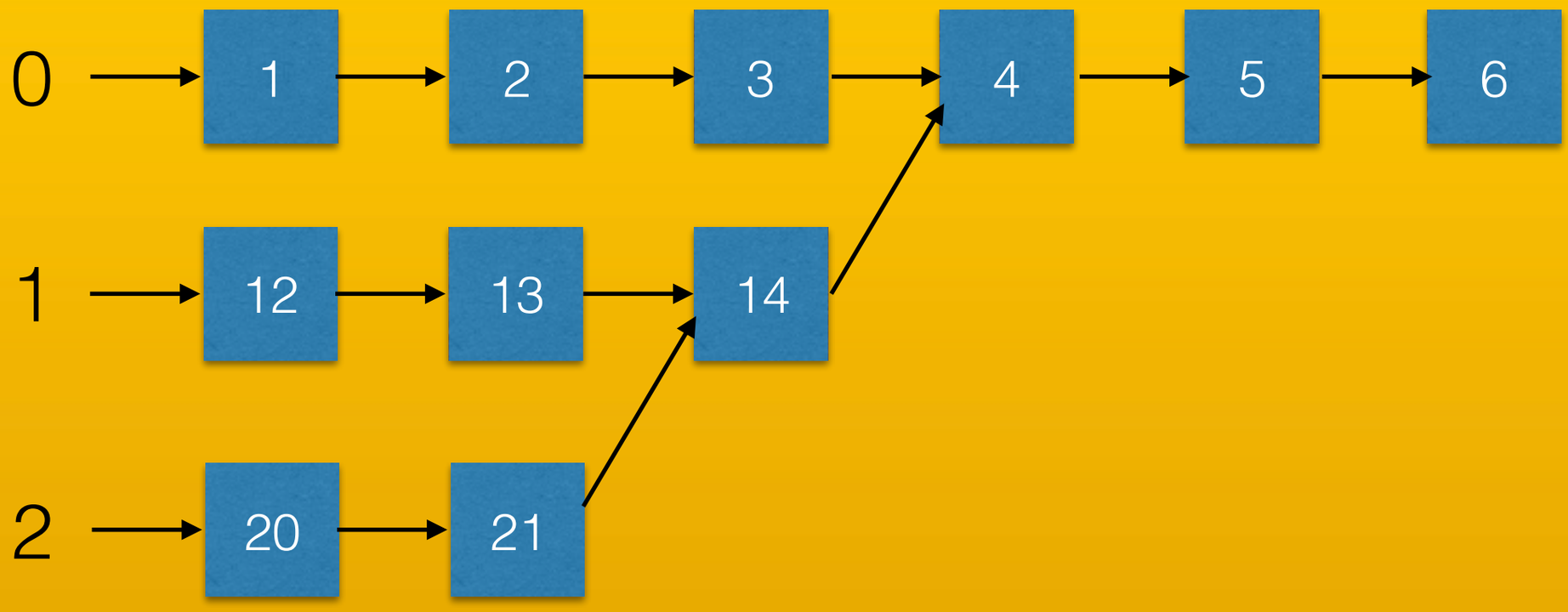

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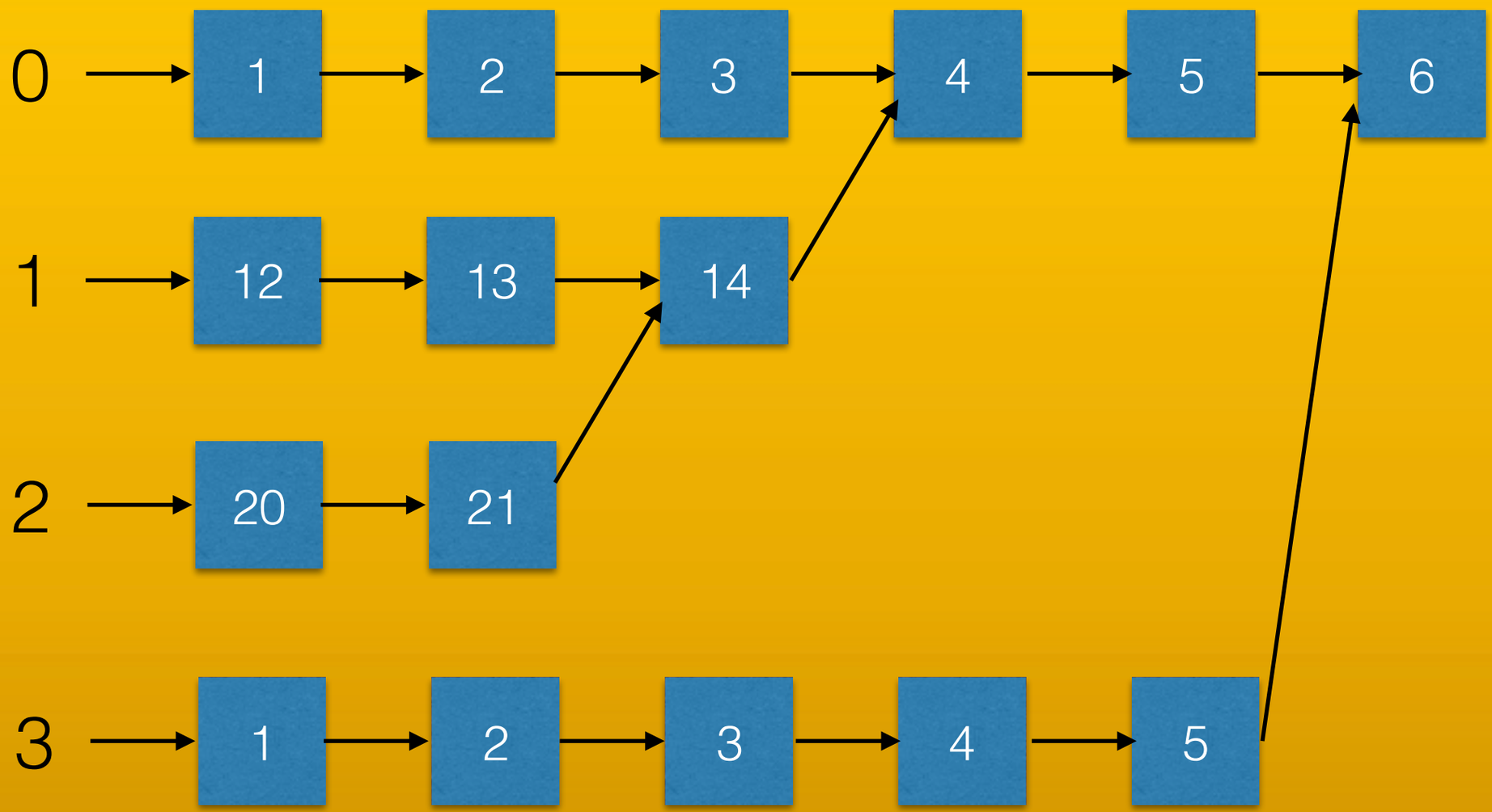

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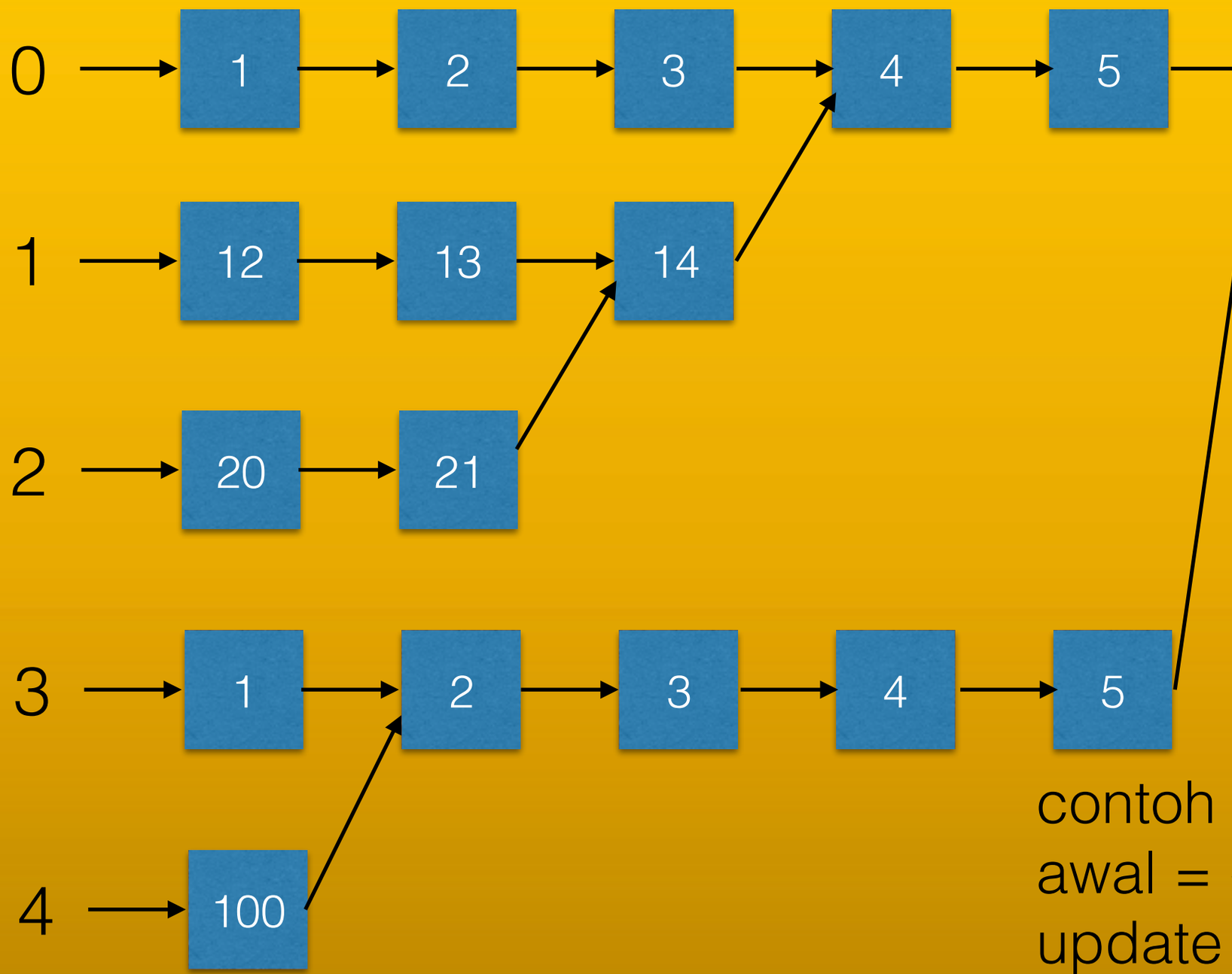

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bikin classnya dulu

```
class Node {
  int val;
  Node* next;

  Node(int val, Node* next): val(val), next(next) {}
}
```

```
Node* last[N];
Node* head[Q];

void init(vector<int> awal) {
  last[N - 1] = new Node(awal[N - 1], NULL);
  for (int i = N - 2; i >= 0; --i) {
    last[i] = new Node(awal[i], last[i + 1]);
  }
}

Node* insert(int index, vector<int> x) {
  if (index >= x.size()) {
    return last[index];
  }
  Node* now = new Node(x[index], insert(index + 1, x));
  return now;
}
```

kalo ada update vector x (katakan update ke-k) tinggal

```
head[k] = insert(0, x);
```

```
Node* last[N];
Node* head[Q];

void init(vector<int> awal) {
  last[N - 1] = new Node(awal[N - 1], NULL);
  for (int i = N - 2; i >= 0; --i) {
    last[i] = new Node(awal[i], last[i + 1]);
  }
}

Node* insert(int index, vector<int> x) {
  if (index >= x.size()) {
    return last[index];
  }
  Node* now = new Node(x[index], insert(index + 1, x));
  return last[index] = now; // jangan lupa diupdate last nya
}
```

kalo ada update vector x (katakan update ke-k) tinggal

```
head[k] = insert(0, x);
```

```
Node* last[N];
Node* head[Q];

void print(Node* now) {
   if (now == NULL) {
      return;
   }
   printf(“%d “, now->val);
   print(now->next);
}
```

kalo ada query print linked list ke-k tinggal

```
print(head[k]);
```

nah sekarang
persistent segment tree

kasih motivasi dulu

dikasih array A isinya N bilangan, dikasih Q query.

querynya itu dikasih tiga bilangan x,y,z. lu harus jawab ada berapa i yang memenuhi x≤i≤y, A[i] ≤ z

0 ≤ A[i] ≤ N, biar ga butuh kompres2an

range trees?

O(lg^2 N) per query?

gamau, maunya

O(lg N) per query

nih, misalkan kita punya infinite time dan memory buat precomputation sebelum query

kita bisa bikin N^2 segment tree untuk tiap interval (i,j)

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tiap segment tree nyimpen ada berapa occurences untuk tiap bilangan HANYA pada range itu

```
int query(int ix, int L, int R, int z) {
  if (R == z) return tree[ix];
  int M = (L + R) >> 1;
  if (z <= M) return query(ix*2+1, L, M, z);
  else return tree[ix*2+1] + query(ix*2+2, M+1, R, z);
}
```

nah tapi preprocessingnya jadi O(N^3) kan

mahal parah

optimisasi 1 :

instead of N^2 segment tree, bisa kita optimize jadi N segment tree doang

node ke-k dari segment tree (i,j)

=

node ke-k dari segment tree (1,j)

-

node ke-k dari segment tree (1,i-1)

still, kita masih punya N segment tree, which means O(N^2) preprocessing

optimisasi 2 :

dari segment tree (1,i) ke segment tree (1,i+1), cuma log(N) node yang berubah

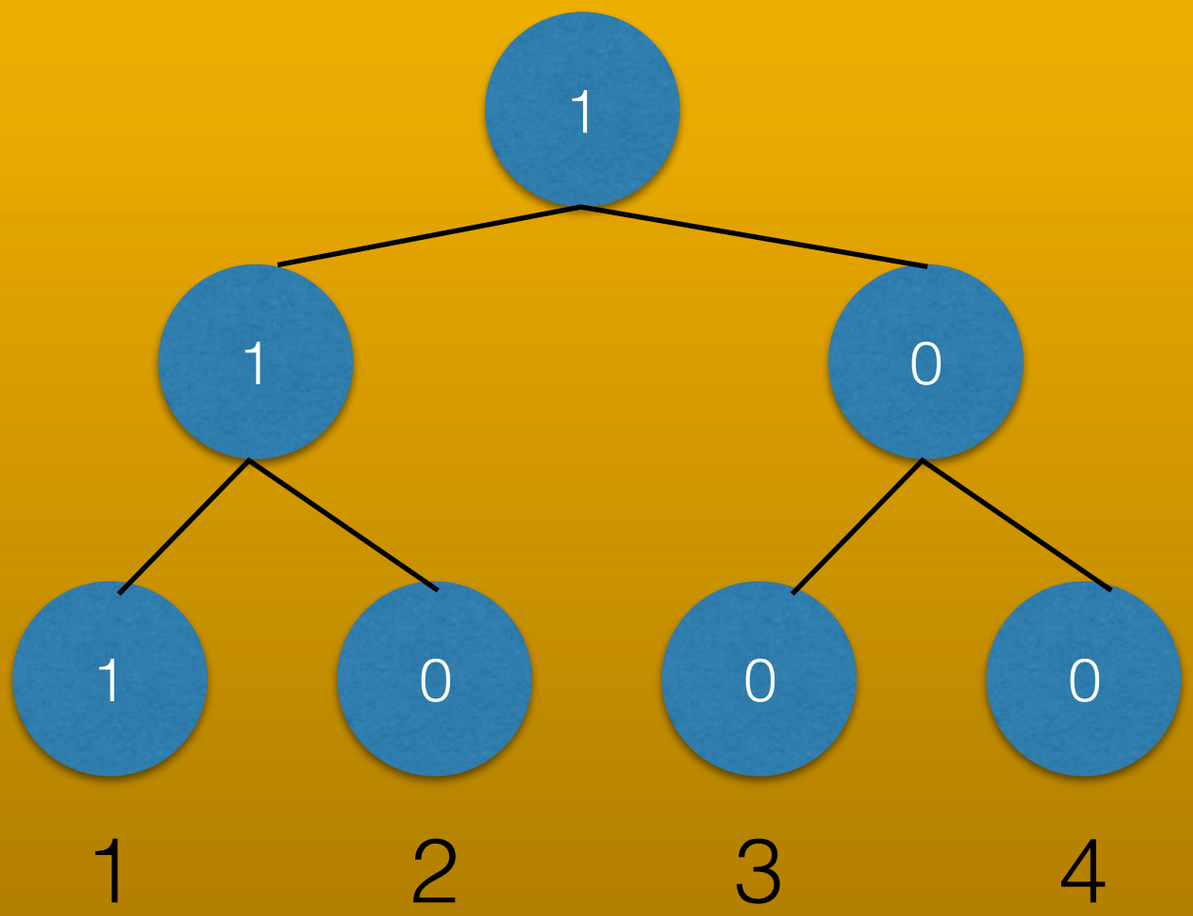
misal

$$A = \{1,2,3,1\}$$

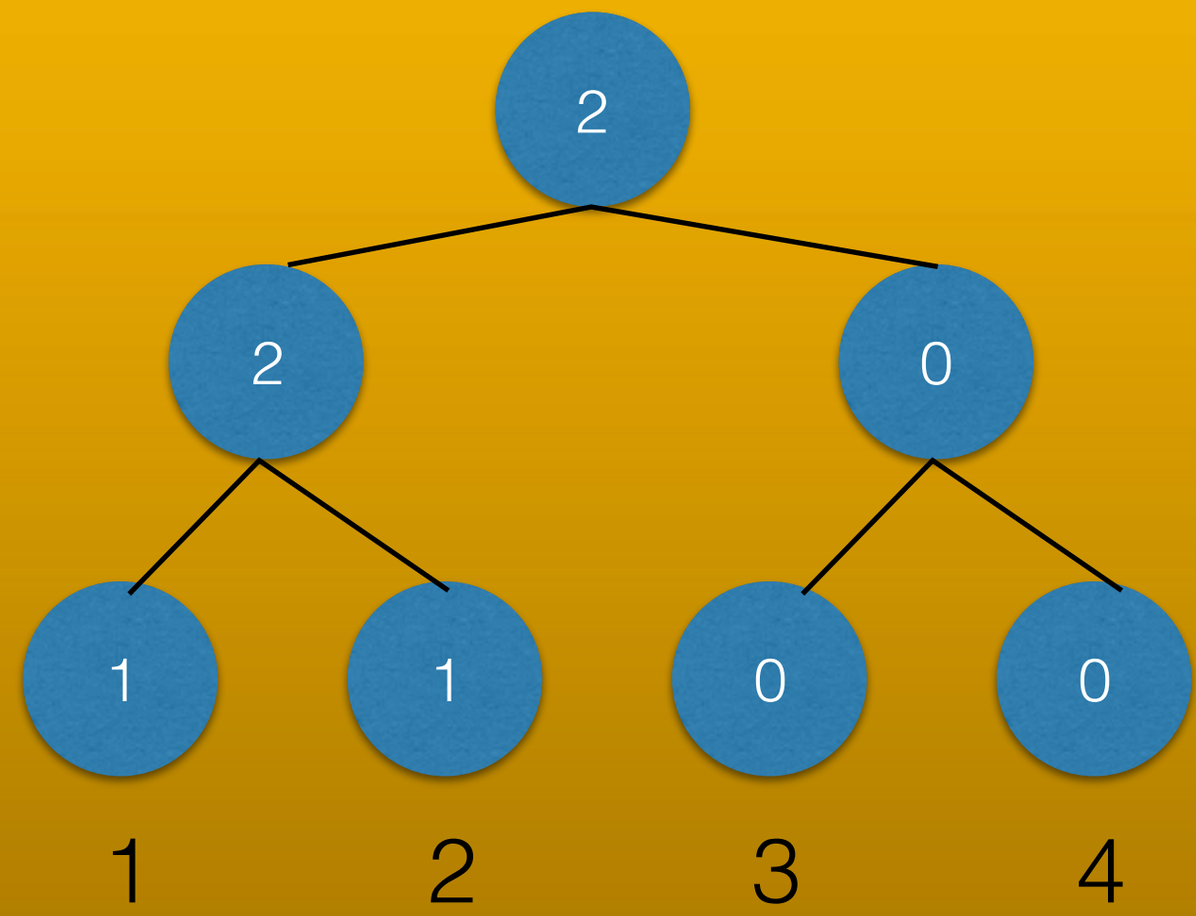
misal

A = {1,2,3,1}

segtree (1,1)



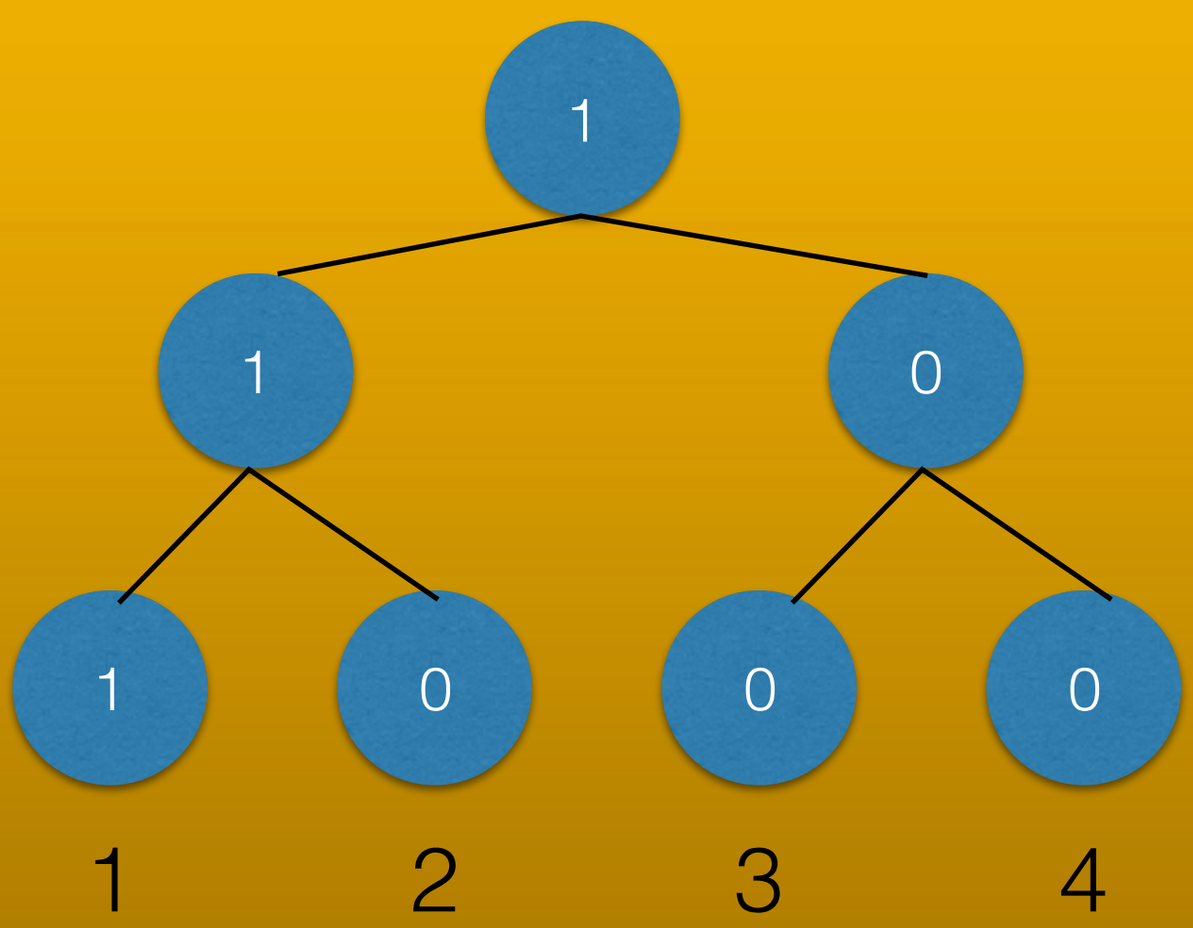
segtree (1,2)



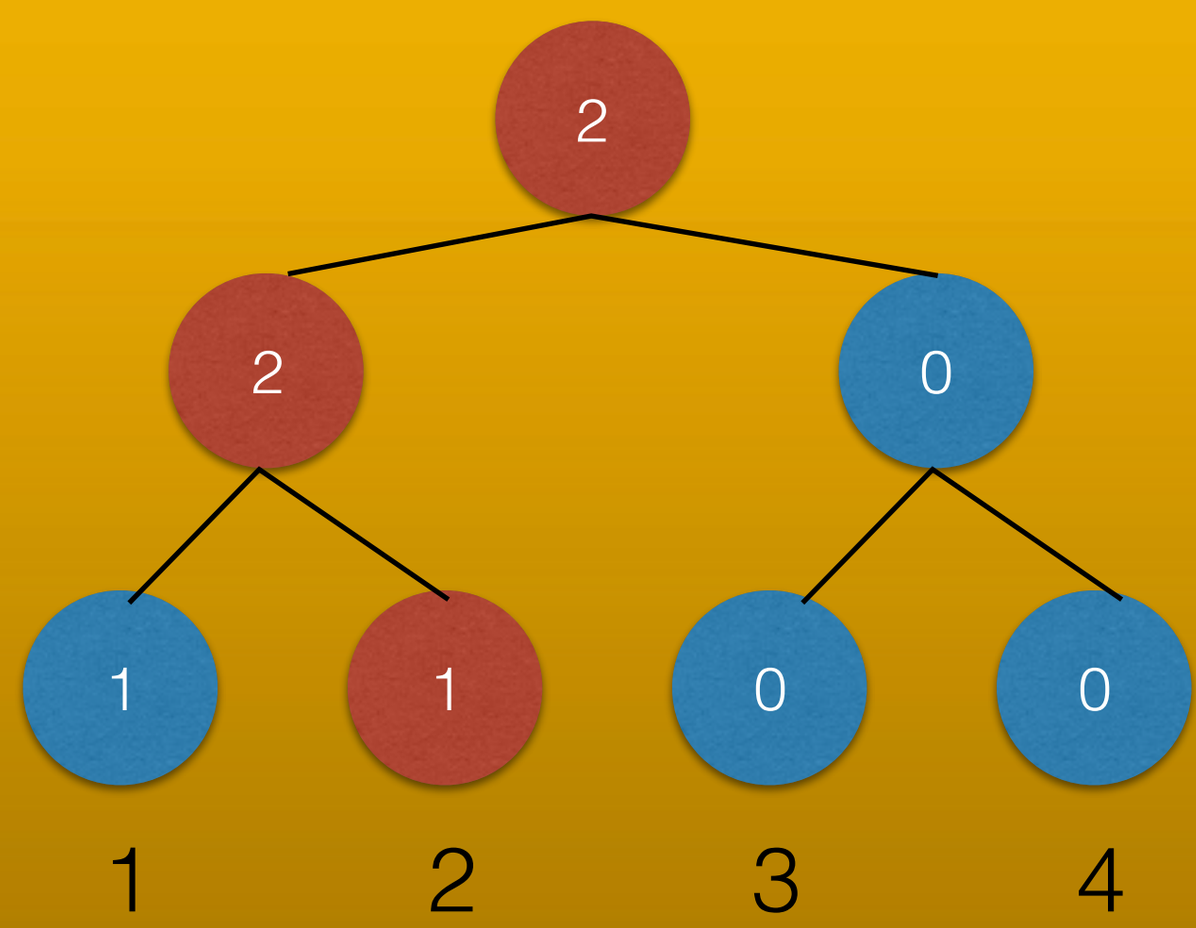
misal

A = {1,2,3,1}

segtree (1,1)



segtree (1,2)



pas compute segtree (1,2), kalo range sebuah node ga mencakup 2, pake node dari segtree (1,1)

bikin classnya dulu

```
class Node {
  int val;
  Node* left;
  Node* right;

  Node(int val, Node* left, Node* right):
    val(val), left(left), right(right) {}
}
```

```
Node* last[4 * N];
Node* root[N];

Node* insert(int ix, int L, int R, int z) {
  if (z < L || R < z) return last[ix];
  if (L == R) {
    Node* now = new Node(last[ix]->val + 1, NULL, NULL);
  } else {
    int M = (L + R) >> 1;
    Node* now = new Node(last[ix]->val + 1,
      insert(ix*2+1, L, M, z), insert(ix*2+2, M+1, R, z));
  }
  return last[ix] = now;
}
```

panggilnya `root[k] = insert(0,0,N-1,A[k]);`

querynya

```
Node* last[4 * N];
Node* root[N];

int query(Node* u, Node* v, int ix, int L, int R, int z) {
  if (L == R) return v->val - u->val;
  int M = (L + R) >> 1;
  if (z > M)
    return v->left->val - u->left->val +
           query(u->right, v->right, ix*2+2, M+1, R, z);
  return query(u->left, v->left, ix*2+1, L, M, z);
}


int x, y, z; // find how many integers < z in A[x..y]
int ans = query(root[x-1], root[y], 0, 0, N-1, z);
```

coba latihan

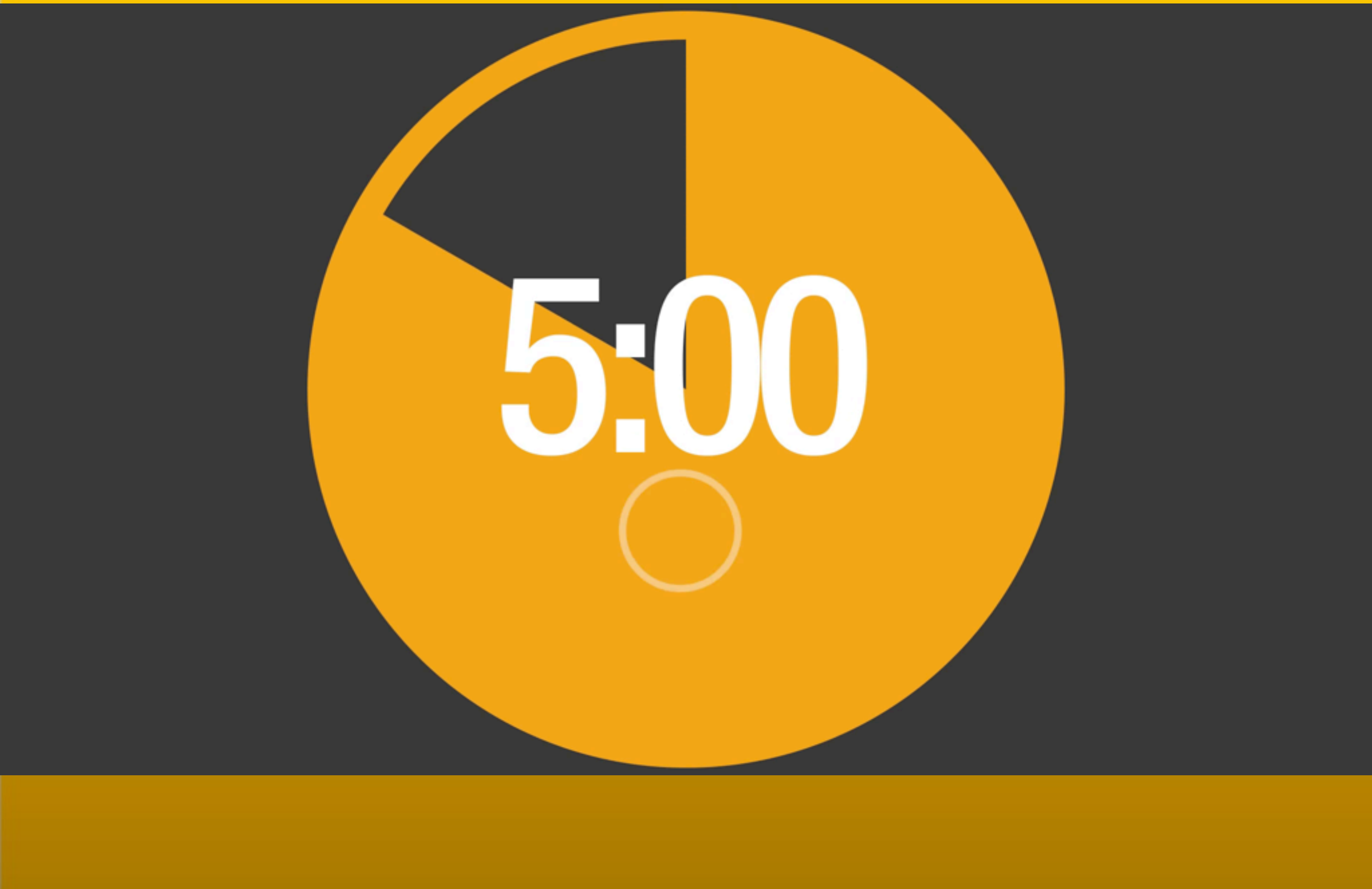
SPOJ MKTHNUM

gw cukup yakin ada yang udah pernah baca

dikasih array N dan Q query.
querynya dikasih tiga bilangan x,y,k.
Tentuin bilangan ke-k dari {A[x], A[x+1], A[x+2], …, A[y]} kalo disort

ada solusi pake range trees + binser, O(N lg^3 N)

tapi sekarang coba cari solusi O(N lg N) nya


5:00

EOF

Q&A?