Assignment 2

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TG30

Question 1

1.a

$$
\begin{aligned}
&= (R \circ R)(aba) \\
&= (R)(R(aba)) \\
&= R(R(aba)) \\
&= R(bba) \\
&= (bbb)
\end{aligned}
$$

1.b

No
$R(ab) = R(bb) = bb$

1.c

Yes
By definition C is onto iff $\forall y \in \mathbb{N}\ \exists x\ C(x) = y$
x is a string of length y consist of only 'a' character
QED

Question 2

2.a

If there is only 1 dice
$\text{Expected value} = (\$2 * (2 + 3 + 5) - \$3 * (1 + 4 + 6))/6$
$\text{Expected value} = -\$\frac{13}{6}$

If there is 100 die
$\text{Expected value} = 100 * (-\$\frac{13}{6})$
$\text{Expected value} = -\$216.\overline{6}$
$\text{Expected value} = -\$217$

2.b

$$
\begin{aligned}
\mathbb{E} &= 20 + 10 * (\frac{1}{3} + (-\frac{1}{4} * \frac{2}{3})) \\
\mathbb{E} &= 20 + 10 * (\frac{1}{3} + (-\frac{2}{12})) \\
\mathbb{E} &= 20 + 10 * (\frac{1}{6}) \\
\mathbb{E} &= 20 + (\frac{5}{3}) \\
\mathbb{E} &= 21.\overline{6}
\end{aligned}
$$

Question 3

$$
\begin{aligned}
\mathbb{P}(A) &= 1 - \mathbb{P}(\neg A) \\
\mathbb{P}(A) &= 1 - (\frac{5}{6})^6 \\
\mathbb{P}(A) &= 0.6651
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(B) &= 1 - \mathbb{P}(\neg B) \\
\mathbb{P}(B) &= 1 - ((\frac{5}{6})^{12} + \frac{1}{6} \cdot (\frac{5}{6})^{11} \cdot \binom{12}{1}) \\
\mathbb{P}(B) &= 0.6187
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(C) &= 1 - \mathbb{P}(\neg C) \\
\mathbb{P}(C) &= 1 - ((\frac{5}{6})^{18} + \frac{1}{6} \cdot (\frac{5}{6})^{17} \cdot \binom{18}{1} + \frac{1}{36} \cdot (\frac{5}{6})^{16} \cdot \binom{18}{2}) \\
\mathbb{P}(C) &= 0.5973 \\
\therefore \mathbb{P}(A) &\text{ has the highest probability of winning}
\end{aligned}
$$

Question 4

Let A, B, C be the vertices of the triangle.
Let D, E, F be the *midPoint* of $\overline{AB}, \overline{BC}, \overline{AC}$ respectively.
There will be 4 new smaller equilateral triangle with side of 5 cm which is $\triangle ADF, \triangle BDE, \triangle CEF$, and $\triangle DEF$.
By PHP, with these smaller triangle as the holes and the dots as the pigeon, it's guaranteed that at least one triangle have at least 2 dots.

Question 5

5.a

$$
\begin{aligned}
&= \mathbb{P}(\neg \text{rain} \cap \text{heavyTrafic} \cap \neg \text{late}) \\
&= \mathbb{P}(\neg \text{rain}) \cdot \mathbb{P}(\text{heavyTraffic} \mid \neg \text{rain}) \cdot \mathbb{P}(\neg \text{late} \mid (\text{heavyTraffic} \cap \neg \text{rain})) \\
&= \frac{3}{4} \cdot \frac{1}{5} \cdot \frac{3}{4} \\
&= \frac{9}{80}
\end{aligned}
$$

5.b

$$
\begin{aligned}
&= \mathbb{P}(late) \\
&= \mathbb{P}(late \mid heavyTraffic) + \mathbb{P}(late \mid \neg heavyTraffic) \\
&= \mathbb{P}(late \mid heavyTraffic \mid rain) + \mathbb{P}(late \mid heavyTraffic \mid \neg rain) + \mathbb{P}(late \mid \neg heavyTraffic \mid rain) + \mathbb{P}(late \mid \neg heavyTraffic \mid \neg rain) \\
&= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{5} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{10} \cdot \frac{4}{5} \cdot \frac{3}{4} \\
&= \frac{1}{24} + \frac{3}{80} + \frac{1}{24} + \frac{3}{50} \\
&= \frac{217}{1200}
\end{aligned}
$$

5.c

$$
\begin{aligned}
&= \mathbb{P}(rain|late) \\
&= \frac{\mathbb{P}(rain \cap late)}{\mathbb{P}(late)} \\
&= \frac{\mathbb{P}(rain \cap late \cap heavyTraffic) + \mathbb{P}(rain \cap late \cap \neg heavyTraffic)}{\mathbb{P}(late)} \\
&= \frac{\frac{1}{4} \cdot (\frac{1}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{4})}{\frac{217}{1200}} \\
&= \frac{100}{217}
\end{aligned}
$$