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1. Sort by $W_i + S_i$ decreasing and if same by W_i by decreasing

We can use counting sort for this and cost $O(N + W_{max} + S_{max})$

2. Remove unimportant item

If the occurrence of one item with the weight W_i is above $\frac{W_{max} + S_{max}}{W_i}$

We don't need to consider that item again

Using harmonic series, this will make the remaining item at most $S \lg S$

$O(N)$

3. Use $DP[i][j]$ which define by:

The largest total weight with item from i to n with the available remaining weight possible is j .

$$O((S_{max} + W_{max}) \cdot \lg(S_{max} + W_{max}) \cdot S_{max}) = O((S_{max}^2 + S_{max} \cdot W_{max}) \cdot \lg(S_{max} + W_{max}))$$

4. The base case happen when $i = \text{number of item}$

If $i = \text{number of item}$, the largest weight is 0

5. $DP[i][j] = \max(DP[i+1][j], W_i + DP[i+1][\min(j - W_i, S_i)])$

We need to make sure that for the latter case, $W_i \leq j$ is always happen

6. So the answer should be in $DP[0][\text{inf}]$

In this case, $\text{inf} = W_{max} + S_{max}$

7. Backtrack the items

Keep track for every DP state, whether they come from the path that pick item $_i$ or not

$O(S_{max} + W_{max})$

Final Complexity:

$$\begin{aligned} &= O(N + W_{max} + S_{max} + N + (S_{max}^2 + S_{max} \cdot W_{max}) \cdot \lg(S_{max} + W_{max}) + S_{max} + W_{max}) \\ &= O(N + W_{max} + S_{max} + (S_{max}^2 + S_{max} \cdot W_{max}) \cdot \lg(S_{max} + W_{max})) \end{aligned}$$

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