CS1231 Assignment 1

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Tutorial Group 30

Problem 1

1.a.

$\forall x(\forall y(Loves(x, \text{santa}) \wedge Reindeer(y) \rightarrow Loves(x, y)))$

1.b.

$\exists! x(Loves(x, Mary) \wedge Loves(\text{John}, \text{Mary}))$

1.c.

$\forall x \forall y(Reindeer(y) \wedge \neg \exists z(Reindeer(z) \wedge Loves(x, y) \wedge Loves(x, z)))$

Problem 2

2.a

1. $\forall x(Musician(x) \rightarrow \neg Singer(x))$
2. $\exists x(Singer(x) \rightarrow Dancer(x))$

2.b

1. $\forall x(Actor(x) \rightarrow Musician)$
2. $\forall x(Musician(x) \rightarrow \neg Singer(x))$
3. $\forall x(Actor(x) \rightarrow \neg Singer(x))$ (*by Law of Transitivity*)

   $\therefore \text{Conclusion 3 is } true$

Problem 3

1. $a \vee b$ *(W1)*
2. $\neg a$ *(W2)*
3. If a chest contain cobra, then it won't contain treasure *(C1)*
4. $W1 \oplus \neg W2$ *(C2)*
5. If W1
   1. $\text{W2 is } true$ *(by C2)*

    2. $\neg a$ (by W2)
    3. b *(by W1 and W2)*
6. If $\neg \text{W1}$ $(\neg a \wedge \neg b)$ *(by De Morgan's Law)*
    1. $\neg \text{W2}$ *(C2)* (a) *by conjunction intro)*
    2. $a \wedge \neg a \wedge \neg b$
    3. $false \wedge \neg b$ *(by negation law)*
    4. $false$ *(by Identity law)*
    5. This case won't happen.
7. It is guaranteed to find treasure in chest B (5.3).

Problem 4

4.a

1. Let n be represent of the form $d_k d_{k-1} \dots d_2 d_1 d_0$
2. $n = d_k \cdot 10^k + d_{k-1} \cdot 10^{k-1} + \dots + d_2 \cdot 10^2 + d_1 \cdot 10^1 + d_0 \cdot 10^0$
3. *Proof (by Direct Proof).*
4. $SumDigits(n) \equiv n \ (mod\ 9)$
5. $$d_k + d_{k-1} + \dots + d_2 + d_1 + d_0 \equiv d_k \cdot 10^k + d_{k-1} \cdot 10^{k-1} + \dots + d_2 \cdot 10^2 + d_1 \cdot 10^1 + d_0 \cdot 10^0 \ (mod\ 9)$$
6. $d_k + d_{k-1} + \dots + d_2 + d_1 + d_0 \equiv d_k \cdot 1^k + d_{k-1} \cdot 1^{k-1} + \dots + d_2 \cdot 1^2 + d_1 \cdot 1^1 + d_0 \cdot 1^0 \ (mod\ 9)$ *(based on Theorem Corollary 8.4.4 (Epp))*
7. $d_k + d_{k-1} + \dots + d_2 + d_1 + d_0 \equiv d_k + d_{k-1} + \dots + d_2 + d_1 + d_0 \ (mod\ 9)$
8. $\therefore SumDigits(n) \equiv n \ (mod\ 9)$

4.b

1. $SumDigits(3m) \leq SumDigits(m) \cdot SumDigits(3)$ *(by P3)*
2. $SumDigits(3m) \leq 100 \cdot 3$
3. $SumDigits(3m) \leq 300$
4. $SumDigits(44m) \leq SumDigits(41m) + SumDigits(3m)$ *(by P2)*
5. $SumDigits(44m) \leq SumDigits(41) \cdot SumDigits(m) + SumDigits(3m)$ *(by P3)*
6. $SumDigits(44m) - SumDigits(41) \cdot SumDigits(m) \leq SumDigits(3m)$ *(by basic algebra)*
7. $800 - 5 \cdot 100 \leq SumDigits(3m)$
8. $300 \leq SumDigits(3m)$
9. $300 \leq SumDigits(3m) \leq 300$
10. $\therefore SumDigits(3m) = 300$ *(by basic algebra)*

Problem 5

1. *Proof (by Induction):*
    1. *Base Case:* $P(1)$
    2. There will be only one configuration which is red-blue (blue-red is the same because it form a circle)
    3. Choose the red ball so it will be *successful* trip.

4. *Induction Hypothesis $P(k) \text{ for } \forall k \in \mathbb{Z}(k > 1)$ *:
5. Let the next ball of a ball is the ball that next to it in clockwise direction.
6. There must be exist a red ball that have a blue ball as its next ball.
    1. *Proof (by Contradiction)*:
    2. For all red ball, it doesn't have a blue ball as its next ball. *(the negation)*
    3. The only case 6.2 will happen if all the ball is red color.
    4. 6.3 won't happen because there are n number of red ball and n number of blue ball.
    5. The contradiction is false. Therefore the initial statement is true.
7. If we delete those 2 balls, we get $P(k-1)$.
8. So the ball that we choose in $P(k-1)$ is also become the ball we choose in $P(k)$
9. Let x is the red ball we remove in 7.
10. Let r is the amount of red ball until the ball before x.
11. Let s is the amount of blue ball until the ball before x.
12. Because $P(k-1)$ is successful, therefore, $r \geq s$.
13. At the ball $x, r+1 \geq s$
14. And at the ball after $x, r+1 \geq s+1$
15. For the balls after that, the amount of red ball always greater than the amount of blue balls since the amount of both balls at $P(k)$ is 1 greater than the one in $P(k-1)$.
16. $\therefore P(k-1) \rightarrow P(k)$.