1.

$$
\begin{aligned}
\Phi(T) &= |2 \cdot T.num - T.size| \\
\text{let } x_i &= \text{the size of table deleted in step i} \\
c_i &= 1 + x_i \\
\hat{c}_i &= c_i + \Phi(D_i) - \Phi(D_{i-1}) \\
&= 1 + x_i + |2 \cdot T_i.num - T_i.size| - |2 \cdot (T_i.num - 1) - (T_i.size - x_i)| \\
&= 1 + x_i + 2 \cdot (T_i.num - (T_i.num - 1)) + (-T_i.size - (-T_i.size - x_i)) \\
&= 1 + x_i + 2 + -x_i \\
&= 3
\end{aligned}
$$

2.

Using accounting method:

$$\text{Enqueue}(x) = \$3$$
$$\text{Dequeue}(x) = \$0$$

Proof:

The money in the bank:

$$= \text{the number of element in stack1} +$$
$$\text{number of element that haven't been popped (stack1 + stack2)}$$
$$= 2 \cdot \text{element in stack1} + \text{element in stack2}$$

$\text{Enqueue}(x)$:

We need to pay \$3 for each operation

\$1 for insert

\$1 for later moving the number from stack 1 to stack 2

\$1 for later pop from stack 2

$\hat{c} = \$3 = \Theta(1)$

$\text{Dequeue}(x)$:

We need to pay \$0 for each operation

\$1 from the bank for each element moved from stack1 to stack2

\$1 from the bank for popping from stack 2

$\hat{c} = \$0 = \Theta(0)$

3.

$$\Phi'(D_i) = \Phi(D_i) - \Phi(D_0)$$

Because of $\Phi(D_i) \geq \Phi(D_0)$

$$\Phi'(D_i) \geq 0$$

In Φ:

$$\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1})$$

In Φ':

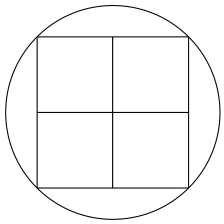
$$\hat{c}_i = c_i + \Phi'(D_i) - \Phi'(D_{i-1})$$

$$\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_0) - (\Phi(D_{i-1} - \Phi(D - 0))$$

$$\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1})$$

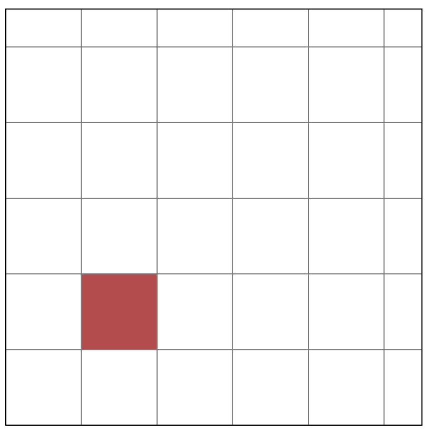
4.

Create 4 squares inside of a circle with r radius



Each square have a side length of $\frac{r\sqrt{2}}{2}$

Create squares with that size inside of square size $l \cdot l$



So we have $(\frac{l}{\frac{r\sqrt{2}}{2}})^2 = (\frac{2 \cdot l}{r\sqrt{2}})^2 \approx \lceil \frac{2 \cdot l^2}{r^2} \rceil$ smaller squares

If we choose any point inside the red square as the center point of the circle, the whole red square is covered.

This will be a coupon collecting problem with $\lceil \frac{2 \cdot l^2}{r^2} \rceil$ squares

$= O(\lceil \frac{2 \cdot l^2}{r^2} \rceil)$

$= O(\lceil \frac{l^2}{r^2} \rceil)$