

# Adaptive Phasor Estimation Technique During Off-Nominal Frequency

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**Abstract**— the use of digital multifunction protection relays and their proper integration on power system and smart grid is an important element. However, the performance of digital relays depends on many factors and needs to be carefully evaluated. These factors may be caused correct operations with less performance or completely incorrect operations. Frequency deviation is one of those factors which results from unbalanced demand load and generation levels. Load characteristics and generation control response to demand load changes will effect on amount and duration of frequency deviations. Frequency decreases when demand load is higher than generation, whereas, it increases when load is less than generation. Protection relays are provided with frequency tracking to reduce the effects of frequency deviations on accuracy of relays decisions. In this paper traditional frequency tracking technique used on protection relays is modified to give a better performance during off-nominal frequency variations. The performance of the proposed algorithm is compared with traditional tracking using Least Square Error (LSQ), and Discrete Fourier transform (DFT) algorithm without any tracking, which are investigated in MATLAB environment.

**Keywords**—Numerical Relays; Frequency Deviation; Phasor Estimation; Digital Signal Processing.

## I. INTRODUCTION

Phasor estimation is essential and vital in numerical protection relays [1-2]. Reliable detection and isolation for power system faults require rapidly and accurately phasor estimation on numerical protection relays [3, 4].

Frequency deviation results from unbalanced demand load and generation levels. Load characteristics and generation control response to demand load changes will effect on amount and duration of frequency deviations. Frequency decreases when demand load is higher than generation, whereas, it increases when load is less than generation [5].

As frequency deviation is a phenomenon that happens quite often, it is necessary to have a correct estimation of the possible errors due to it. Large frequency deviations especially occur during emergency states of the system. In such cases it is especially important that the protection operates correctly. Some works had been done in estimating the impact of

frequency deviation on the performance of protection relays. These works showed that frequency deviation can lead to mal-trip for these types of relays [6-9].

Several phasor estimation techniques have been proposed, the most common ones are Discrete Fourier Transform (DFT) and Least Square Algorithm (LSQ). DFT filter has gained importance for measuring fundamental and harmonic content of a waveform. DFT filter can be implemented in recursive and non-recursive forms. Recursive DFT is commonly used algorithm. Errors are introduced during phase and magnitude estimation using DFT filter due to sampled signal dynamics and frequency deviations [10].

The estimation errors of the phasor will be affected by the window length. Practically, speed and accuracy cannot be both simultaneously satisfied in phasor estimation. A fixed shorter window will result in an unacceptable estimation error in some complicated fault transients. On the other hand, a fixed longer window is not necessary for some simple fault transients, which may result in additional delay in the tripping process [11, 12].

Many researches present several techniques for improving the estimation of phasor. Gauss-Newton's Method [13], Kalman Filter [14-17], Prony Algorithm and Matrix Pencil Method [18-21].

This paper presents evaluation of Least Square Error (LSQ) phasor estimation technique with two various frequency tracking algorithms under off-nominal frequency situation, compared with DFT Discrete Fourier transform algorithm without any frequency tracking. Basic concept of numerical relays is covered in section II. Phasor estimation techniques mathematics is reviewed in section III. In Section IV methods of frequency tracking is revised. Simulation results comparing the proposed algorithm with Least Square Error (LSQ) and Discrete Fourier transform (DFT) algorithms are carried out in Section V. Finally Section VI concludes up with the key points analyzed in this paper.

## II. NUMERICAL RELAY BASIC ARCHITECTURE

To provide proper performance protection, there are some principles must be taken on consideration like reliability, selectivity, Speed of operation and simplicity.

The block diagram of typical numerical relay is shown in Fig. 1 below.

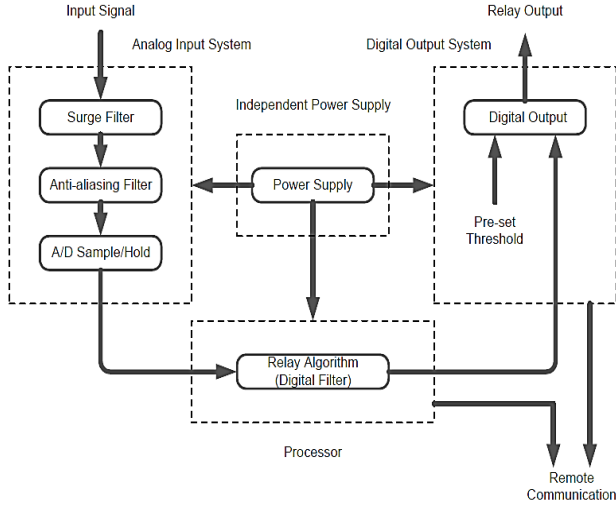


Fig. 1: Block diagram of typical numerical relay [4].

The input signals, which come from CTs or VTs, are always analog signals which need to be converted into digital signals before being processed.

### A. Analog Input system

Converting analog signals to sequences of numerical values passes through below steps.

- **Surge filter:** The large inrush in the input analog signals will be suppressed using surge filter for the safety of the digital relay.
- **Anti-Aliasing Filter:** To avoid possible errors in reconstructing the input signal, Anti-aliasing filter is used. According to the Nyquist criterion.
- **Analog/Digital Sample and Hold:** to convert the input signal from analog to digital. To scan the whole signal, a data window of limited length is applied to acquire information on part of the signal. Within the section of the signal that is scanned by the data window, a limited number of points of the waveform are sampled. While the window moving forward, more samples are obtained at different snapshots of time. The sampling window length, numbers of samples in the window, as well as the shape of the sampling window are dependent on the relay algorithm.

### B. Relay Algorithm

The relay algorithm stored in the processor is the core of the digital relay. It determines the way to reconstruct the input signal based on the digital samples from the A/D converter. The algorithm is designed to remove unwanted components the

input signal may contain as much as possible such as harmonics, interharmonics and dc. The algorithm functions as a digital filter to extract the fundamental component of the input signal. The relay operation is carried out based on the fundamental component. With different algorithm principles, the shape of the data window varies. The length of the data window is dependent on the required decision speed of the algorithm and on the required tolerance against disturbances.

### C. Digital Output System

Clearing fault represents the goal of protection system, so relay outputs vary from tripping local breakers, tripping remote end breakers, messages to regional and control centers, and data saving for relay engineers.

## III. PHASOR ESTIMATION TECHNIQUES

### A. Discrete Fourier transform (DFT)

Any signal is a combination of periodic components represents the concept of the Fourier algorithm, the input signal to be sampled can be written as:

$$y(t) = \sum_{n=1}^N Y_n s_n(t) + \varepsilon(t) \quad (1)$$

$s_n(t)$  are pre-selected as follow:

$$\begin{aligned} \left. \begin{aligned} s_1(t) &= \cos(\omega t) \\ s_2(t) &= \sin(\omega t) \end{aligned} \right\} && \text{Fundamental component} \\ \left. \begin{aligned} s_3(t) &= \cos(2\omega t) \\ s_4(t) &= \sin(2\omega t) \end{aligned} \right\} && \text{Second Harmonics} \\ \left. \begin{aligned} s_{N-1}(t) &= \cos\left(\frac{N}{2}\omega t\right) \\ s_N(t) &= \sin\left(\frac{N}{2}\omega t\right) \end{aligned} \right\} && \frac{N}{2} \text{ th Harmonics} \end{aligned}$$

$N/2$  is the highest harmonic order contained in the signal, assuming  $N$  is an even number;  $\varepsilon(t)$  stands for the noise in the measurement. The choice of  $s_n(t)$  above is in accordance with the form of the Discrete Fourier Transform. The component at a certain harmonic order  $n$  is split into two orthogonal terms,  $\sin(n\omega t)$  and  $\cos(n\omega t)$ . Equation (1) can be expressed in matrix form as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} s_1(\Delta t) & s_2(\Delta t) & \dots & s_N(\Delta t) \\ s_1(2\Delta t) & s_2(2\Delta t) & \dots & s_N(2\Delta t) \\ \vdots & \vdots & \ddots & \vdots \\ s_1(K\Delta t) & s_2(K\Delta t) & \dots & s_N(K\Delta t) \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$

Or

$$y = S Y + \varepsilon \quad (2)$$

Where  $K$  is the number of samples in one sampling window, and  $\Delta t$  represents the time interval between two neighboring sample points. To estimate all  $N$  parameters,  $K \geq N$  is required. If the error vector  $\varepsilon$  is assumed to have zero mean, and a covariance matrix.

$$E(\varepsilon \varepsilon^T) = W \quad (3)$$

Then the solution to Equation (3) yields

$$Y = (S^T W^{-1} S)^{-1} S^T W^{-1} y \quad (4)$$

With the assumption that the errors are uncorrelated and independent from sample to sample and have a constant covariance,  $W$  is a multiple of the unit matrix. Therefore the least square solution (when  $\epsilon^T W^{-1} \epsilon$  is minimized) is

$$Y = (S^T S)^{-1} S^T y \quad (5)$$

Substituting the orthogonal expressions of sine and cosine terms in Equation (5), the  $ij^{\text{th}}$  entry of the matrix  $S^T S$  is

$$(S^T S)_{ij} = \sum_{k=1}^K S_i(k\Delta t) S_j(k\Delta t) = \begin{cases} \frac{K}{2}, & i = j \\ 0, & i \neq j \end{cases} \quad (6)$$

The fundamental frequency components are given by

$$Y_c = \frac{2}{K} \sum_{k=1}^K y_k \cos(k\theta) \quad (7)$$

$$Y_s = \frac{2}{K} \sum_{k=1}^K y_k \sin(k\theta) \quad (8)$$

Where  $\theta = 2\pi/K$ . the magnitude of the fundamental component can be calculated by

$$|Y| = \sqrt{(Y_c)^2 + (Y_s)^2} \quad (9)$$

#### B. Least Square Algorithm (LSQ)

The principles of Least Squares algorithm are quite similar to the one for Fourier algorithm, except that there is no requirement of dc offset removal from the input signal before signal processing.

The input signal in such a case can be written as:

$$y(t) = Y_0 e^{-t/\tau} + \sum_{n=1}^N Y_n s_n(t) + \epsilon(t) \quad (10)$$

Where  $Y_0$  the magnitude of the decaying dc component at  $t=0$

$\tau$  the time constant of the decaying dc component

The other parameters are the same as in Equation (1).

Because of the presence of the dc component in the signal, the matrix  $S$  also contains the dc component. The equation in frequency domain is then like:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} s_0(\Delta t) & s_1(\Delta t) & s_2(\Delta t) & \dots & s_N(\Delta t) \\ s_0(2\Delta t) & s_1(2\Delta t) & s_2(2\Delta t) & \dots & s_N(2\Delta t) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_0(K\Delta t) & s_1(K\Delta t) & s_2(K\Delta t) & \dots & s_N(K\Delta t) \end{bmatrix} \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_k \end{bmatrix}$$

Where  $s_0(K\Delta t) = e^{-K\Delta t/\tau}$ ,  $k = 1, 2, \dots, K$

Similarly as in the case of Discrete Fourier algorithm, the following equation holds:

$$Y = (S^T S)^{-1} S^T y \quad (11)$$

However, due to the presence of the dc component, the matrix  $S^T S$  will no longer be a diagonal matrix as in the case of Discrete Fourier algorithm. The matrix  $(S^T S)^{-1}$  will be a full matrix under any condition. Let

$$A = (S^T S)^{-1} S^T \quad (12)$$

The fundamental frequency components are estimated by

$$Y_c = \sum_{k=1}^K y_k A(2, k) \quad (13)$$

$$Y_s = \sum_{k=1}^K y_k A(3, k) \quad (14)$$

The magnitude of the fundamental component of the signal can be calculated by Equation (9) as showed before.

#### IV. FREQUENCY TRACKING TECHNIQUES

Keeping the phasor estimation correct even the frequency of system deviates from its nominal represents the aim of frequency tracking.

##### A. Frequency Tracking Algorithm

Flow chart in Fig. 2 below shows the Algorithm of Frequency Tracking.

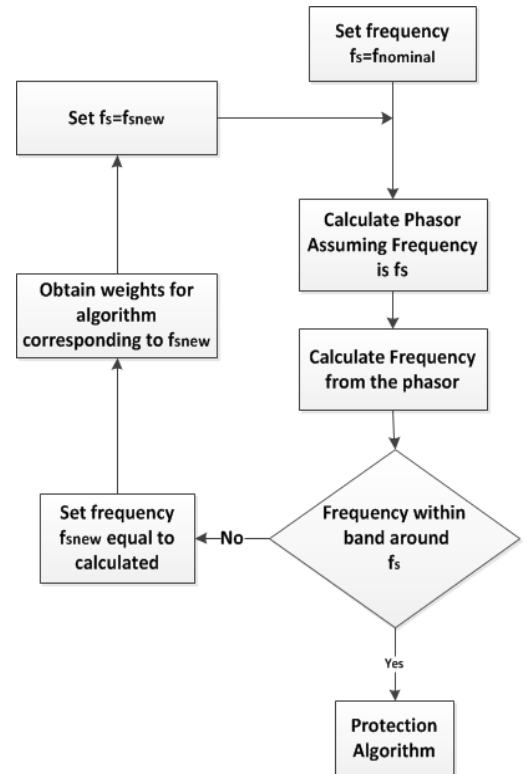


Fig. 2: Process of calculating off-nominal frequency by tracking [22].

In this algorithm, nominal frequency uses to calculate phasors, the estimated phasors will be used to find frequency. If calculated frequency error with in the accepted range; the processes will continue and accept with low error phasor. If not, the phasor estimation must repeat to find more accurate estimation.

### B. Proposed Frequency Tracking Algorithm

Fig. 3 below presents flow chart of proposed Algorithm for frequency tracking.

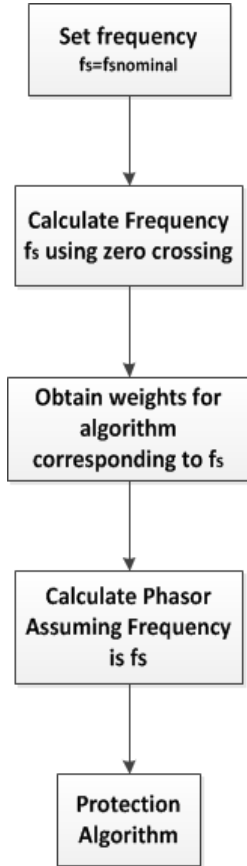


Fig. 3: Proposed Frequency Tracking

On this algorithm, zero crossing method is used to calculate frequency before any phasor estimation process. Zero-crossing technique can be implemented either in hardware or software on numerical relay. The time between two consecutive zero crossing is used to find time of one cycle and frequency. The fundamental frequency is estimated on this method by:

$$f = \frac{M-1}{2} \frac{1}{t_M - t_1} \quad (15)$$

Where

$M$  is the number of zero crossings.  
 $t_M$  time of the  $m$ th zero crossing

$t_1$  is the time of the first zero crossing  
 $f$  is the estimated frequency

Using time stamps of the samples instead of the sampling frequency represents the safest solution for the zero crossing method to avoid error on estimation. The calculated frequency using zero crossing method will be used to estimate phasors then protection algorithm. In Fig.3, the reason of calculating frequency using separate method before any phasor estimation is that to increase the accuracy of estimation procedures that will be done later on relay. While on traditional frequency tracking as shown on Fig. 2, the accepted error range on estimated frequency will effect on the accuracy of later phasor estimation and relay decision making.

### V. SIMULATION RESULTS

MATLAB programming environment is used to program the algorithms for testing performances of them. Equation (16) represents the fundamental voltage test signal with off-nominal frequencies:

$$v = 300\sqrt{2} \sin(2\pi f_1 t + \frac{\pi}{6}) \quad (16)$$

The nominal frequency for test signal is 50 Hz. Tests at off-nominal frequencies are performed using LSQ with traditional Tracking, DFT without any tracking, and finally LSQ with Proposed Tracking Algorithm. Fig (4-5) show the results obtained from simulations at 48 Hz.

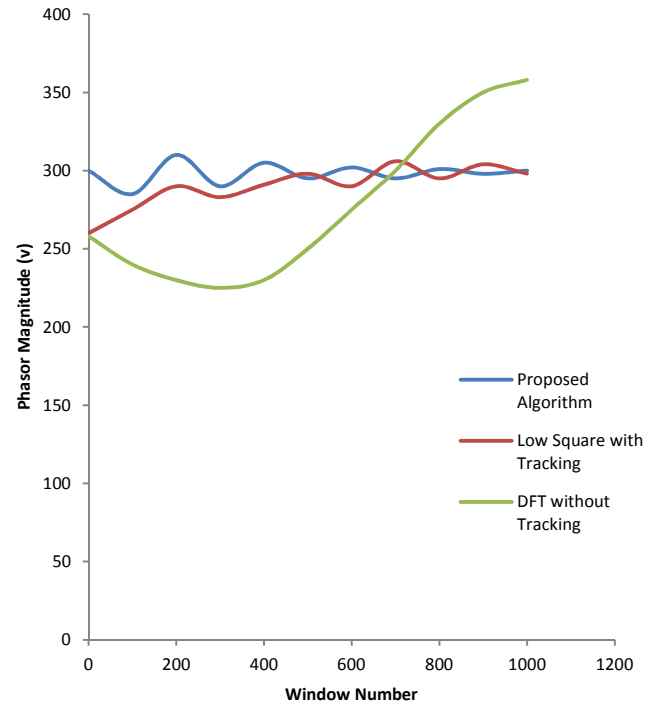


Fig. 4: Plot of Estimated Phasor Magnitude by LSQ with traditional Tracking, DFT, LSQ with Proposed Algorithm when Frequency of Signal is 48 Hz

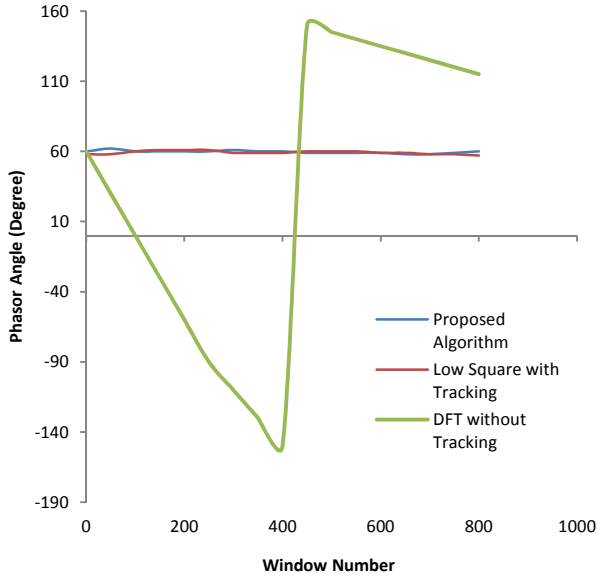


Fig.5: Plot of Estimated Phasor Angle LSQ with traditional Tracking, DFT, LSQ with Proposed Algorithm when Frequency of Signal is 48 Hz

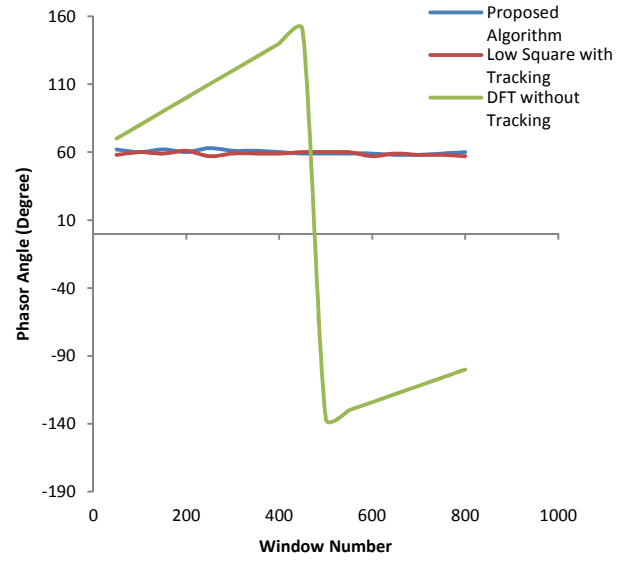


Fig.7: Plot of Estimated Phasor Angle LSQ with traditional Tracking, DFT, LSQ with Proposed Algorithm when Frequency of Signal is 52 Hz

The estimation accuracy using LSQ with proposed algorithm is better than other algorithms is shown on Fig. 4. At low sliding window numbers, results of LSQ with proposed tracking algorithm are more stable than LSQ with traditional tracking, while performance of DFT without any frequency tracking is affected extremely by changing of nominal frequency. Also, Fig. 5 shows the good accuracy of proposed algorithm comparing with other algorithm on estimating phasor angle.

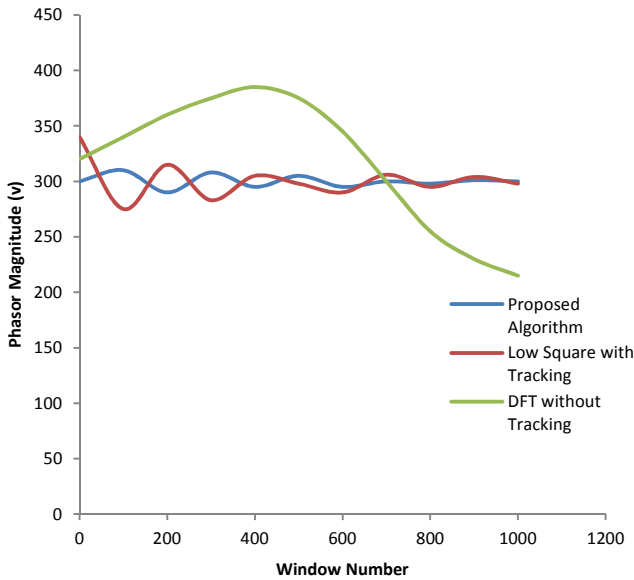


Figure 6: Plot of Estimated Phasor Magnitude by LSQ with traditional Tracking, DFT, LSQ with Proposed Algorithm when Frequency of Signal is 52 Hz

Tests on Algorithms under evaluation during 52 Hz show also that the LSQ with proposed algorithm is more accurate and more stable at low window number than other algorithms. Increasing accuracy of LSQ with both proposed and traditional frequency tracking during high window number seems in all tests results.

## VI. CONCLUSION

This paper discusses the effects of frequency deviation as one of power quality factors on the performance of digital protection relays. Frequency Tracking is presented as practical improvement on performance of protection relays during frequency deviation cases. Proposed frequency tracking algorithm is done by doing some modification on traditional one. Results of simulations show that the proposed Algorithm is more accurate and less error compared with other two techniques during over and less than nominal frequency cases which increase the reliability of relays. The disadvantage of proposed algorithm is the need for more processing time than other techniques when the error on estimated frequency is at accepted range only.

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