Testing the Wide-Sense Stationarity of Bandpass Signals for Underwater Acoustic Communications

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***Abstract*—The Underwater Acoustic Communication (UAC) systems work in communication channels characterized by a large variety of multipath propagation conditions that can additionally change over time. Designing a reliable communication system requires knowledge of the transmission parameters of the channel. There is a need for the development of adaptive signaling schemes that would dynamically optimize the signal parameters of the physical layer of communication protocols. Wireless channels are often assumed to be wide sense stationary (WSS) with uncorrelated scattering (US). The WSSUS assumption allows for simultaneous modeling of time dispersion leading to the frequency selectivity of the channel, and time variability resulting in frequency dispersion. However, in the case of many underwater communication channels, especially when system terminals are in movement, the channel can be modeled as WSSUS only in restricted periods of time, and in limited frequency ranges. The paper presents a method for determining the time interval, wherein the WSS assumption is fulfilled, on the basis of a complex envelope of the received bandpass signal.**

***Index Terms*—underwater communications, underwater channel meassurements, WSSUS, nonstationary impulse response**

I. Introduction

The large variability of communication properties of underwater acoustic channels, and especially the strongly varying instantaneous conditions in shallow waters, is a tough challenge for the designers of UAC systems. Non-adaptive data transmission systems are forced to work with settings assuming the worst possible conditions; this strongly limits its effective bandwidth, range, speed and efficiency. Due to the technical capabilities of acoustic wave generation and reception, and to unfavorable underwater acoustic channel properties, such as: attenuation, multipath propagation, time dispersion, Doppler effect and time-variability, the data transmission rates achieved in UAC systems are much lower than in radio communications. Due to the wide range in the transmission properties of UAC channels, there are, in the world, only a few standards used to define very slow communication. In deep-water channels, transmission rates of up to 100 kbps can be achieved, while the same research centers offer much slower standards in shallow water channels, where reliable communication at a speed of 40-80 bps is a significant achievement. Such differences in UAC systems performances is due, inter alia, to the large temporal uncertainty of underwater channel transmission characteristics [1]. There is a need for developing adaptive signaling schemes that would dynamically optimize the signal parameters of the physical layer of communication protocols, especially those designed for ad-hoc underwater sensor networks [2].

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The underwater acoustic channels show some resemblance to radio channels; accumulating, however, most of the adverse properties. In wireless communications, the physical layer of the transmission protocol is matched to the conditions of the radio link. The procedure is known as link adaptation, or adaptive coding and modulation (ACM). Adaptive modulation systems improve the rate of transmission, and bit error rates, by exploiting the channel state information (CSI) that is present at the transmitter. An adaptive system uses both the long-term knowledge about the channel statistical characteristics (statistical CSI), and measured instantaneous impulse responses (instantaneous CSI). The impulse response $h(t, \tau)$ is defined in the window of observation time t, and delay τ. It is the basis for calculation of the transmission parameters of the channel, namely multipath delay spread τ_M, Doppler spread ν_M, coherence time T_C, and coherence bandwidth B_C. Such analysis is possible if the impulse response of the channel can be described as a WSSUS stochastic process, and this is true for most wireless channels [3].

As it is shown in [4], UAC channels are hardly ever WSSUS. The second-order statistics of non-WSSUS impulse response are nonstationary, and thus the transmission parameters can be determined only in a restricted period of time, and in a limited frequency range [5]. Determining time interval, wherein the second-order statistics of impulse response are approximately independent of time is crucial for the performance of communication systems working in time-varying conditions. Only within this time interval it is possible to calculate transmission parameters $\{\tau_M, T_C, \nu_M, B_C\}$. The transmitted signal can then be designed so that the modulation symbol is short enough to avoid the interference due to the channel variability, and long enough to avoid the influence of intersymbol interference due to multipath propagation. These requirements can be expressed with: $\tau_M < T_S < T_C$, where T_S is symbol duration [3]. Thus, the signal corresponding to the modulation symbol can also be described as a WSS process, and its power spectrum can be defined via the Fourier Transform, according to the Wiener-Khinchin Theorem. This is specially important for signal detection performed in frequency domain, as is the case with communication systems using the Orthogonal Frequency Division Multiplexing (OFDM), and the Frequency Hopping Spread Spectrum (FHSS) techniques [1].

Numerous UAC systems use a matched filtration technique that requires the received signal to be a WSS process [7]; otherwise their performance is not optimal, and complex adaptive methods are needed, that increase the system latency. Moreover, the matched filtration technique is commonly used in sonar system; thus their performance also strongly depends on stationarity of second order statistics of received signal. The FFT-based matched filtration algorithm is often used due to its computational efficiency [8][9]. Signal $y(n)$ at the output of matched filter is calculated as:

$$y(n) = \mathscr{F}^{-1}\{X(k) \cdot S^*(k)\} \tag{1}$$

where $X(k) = \mathscr{F}\{x(n)\}$ and $S(k) = \mathscr{F}\{s(n)\}$ are Fourier transforms of the signal received and transmitted, respectively. However, the FFT algorithm is not suitable to analyze nonstationary signals. In case of received echo signal being a non-WSS process, a Short Time Fourier Transform (STFT) should be applied. The STFT algorithm divides a nonstationary signal into a sequence of time segments wherein the signal may be considered as WSS process. Then, the Fourier transform is applied to each of the segments of the signal. The accuracy of spectral power density estimation with the use of the STFT strongly depends on the choice of segment length, thus its application requires *a priori* information about the so-called stationary time, at which the signal can be considered as WSS process [10].

As it is shown in [8] the range of sonar systems with continuous wave frequency modulation (CWFM) strongly depends on time structure of transmitted signal. Long range sonar uses long period of sounding signal and long observation time in the receiver. On the other hand, the received signal can be nonstationary due to multipath, short-term fluctuations in the environment, scattering from a rough boundary, and dispersion phenomena. This leads to reduction in the processing gain known as coherence loss [11]. Thus, the time structure of sounding pulse should be chosen as to minimize the influence of these interferences on the received signal and to ensure the second order statistics of the received signal being time-invariant.

The knowledge of stationary time can be exploited to improve the performance of sonar systems with pulse compression technique. Such a system uses linear frequency modulated waveform (LFM) or hyperbolic frequency modulated waveform (HFM) as the probe signal [12][13]. The disadvantage of LFM signal is that it suffers severe distortion when it reflects from a moving object, while the HFM system performance is Doppler-invariant. However, if target velocity is not a constant, similar to the Doppler effect on LFM waveform, the received HFM waveform will mismatch with the matched filter and the output is again degraded. Thus, it is worthwhile to ensure that the duration of the sounding pulse is shorter than the rate of Doppler shift changes. Otherwise complex filtering techniques are needed in the receiver to improve the compression of the pulse [14].

The measurement of time interval, wherein the underwater acoustic channel can be considered as wide-sense stationary, can improve the performance of underwater acoustic systems, both communication and sonar systems. In [5] and [6] methods of determining the stationary time on the basis of impulse response analysis, are proposed. This approach, however, requires time-consuming impulse response measurements, which can be difficult, if not impossible, to perform in real-time adaptive underwater systems. This article presents a method for calculating stationary time based on the data transmission signal.

II. Analysis of Impulse Response

Underwater acoustic communication channel can be characterized by a time-variant impulse response $h(t, \tau)$, defined in window of observation time t and delay τ [5]. The standard procedure for determining transmission parameters of a channel requires calculation of the two-dimensional scattering function:

$$S(\nu, \tau) = \int_{-\infty}^{\infty} R_h(\Delta t, \tau) e^{-j2\pi(\nu \Delta t)} d\Delta t, \tag{2}$$

where ν is Doppler shift, and $R_h(\Delta t, \tau)$ is stationary autocorrelation function of the impulse response of the channel:

$$R_h(\Delta t, \tau) = E\{h^*(t, \tau) h(t + \Delta t, \tau)\} \tag{3}$$

In case of non-WSSUS impulse response, the autocorrelation function $R_h(\Delta t, \tau)$ is no longer stationary, and the scattering function can be defined only for specific time t. In [6] a local sense stationary uncorrelated scattering (LSSUS) assumption is proposed for characterizing the time-varying radiocommunication channel. The author propose to find a time interval J_i, at which the autocorrelation of impulse response $h(t, \tau)$ is WSS but non-WSS at all other time intervals $J_k \neq J_i$. In [5] stationary time is defined as maximum observation time interval T_D, wherein the autocorrelation function $R_h(\Delta t, \tau)$ is a stationary process, and the mean value of $h(t, \tau)$ is constant. With this approach, the input data for stationary time analysis is an impulse response $h(t, \tau)$ that must be measured beforehand.

The measurement is usually done with the use of bandpass pseudo-random binary sequence (PRBS) or linear frequency modulation (LFM) signals, and the impulse response is obtained by matched filtration technique (Eq.1). Even with the use of computationally efficient FFT, this requires performing at least $3 \cdot 5 \cdot N log_2 N$ operations on the baseband equivalents of received and transmitted signals of length N to calculate a single estimate of $h(t, \tau)$ [15]. To obtain statistically significant information about the underwater channel, numerous impulse responses have to be gathered. This makes the statistical analysis of $h(t, \tau)$ a valuable tool for off-line processing, but in case of real-time system it is of limited value.

III. Testing the Wide-Sense Stationarity of Bandpass Stochastic Process

The signal transmitted in a UAC system can be considered as a stochastic process $x(t)$ described with its autocorrelation function (ACF). The process $x(t)$ is called wide-sense stationary when its mean value is constant

$$[x(t)] = m_x(t) = m_x(t + \Delta t) \tag{4}$$

and its autocorrelation function $R_x(t_1, t_2)$ depends only on the difference $\Delta t = t_1 - t_2$:

$$R_x(t_1, t_2) = E[x(t_1)x^*(t_2)] = R_x(\Delta t) \tag{5}$$

Let the $r(t)$ be a bandpass signal, e.g. a:

$$r(t) = Re[y(t)e^{j2\pi f_0 t}] \tag{6}$$

where $y(t)$ is complex envelope, and f_0 is the carrier frequency.

The relation between bandpass time process, and its complex envelope, has been described in [16]. It is shown, that $y(t)$ must be a zero-mean process in order that $E[r(t)]$ be independent of t. Moreover, the autocorrelation function (ACF) of bandpass signal $r(t)$ is related to the complex-value $y(t)$ in the following manner:

$$\begin{aligned} R_{rr}(t + \Delta t, t) &= E[r(t + \Delta t)r(t)] \\ &= \frac{1}{2}\mathrm{Re}[R_{yy}(\Delta t) \cdot e^{j2\pi f_0 \Delta t}] \\ &+ \frac{1}{2}\mathrm{Re}[R_{yy*}(\Delta t) \cdot e^{j4\pi f_0 t + j2\pi f_0 \Delta t}] \end{aligned} \tag{7}$$

where:

$$R_{yy}(\Delta t) = E[y(t + \Delta t)y^*(t)] \tag{8}$$

$$R_{yy^*}(\Delta t) = E[y(t + \Delta t)y(t)] \tag{9}$$

If $r(t)$ is WSS, then the $R_{yy^*}(\Delta t)$ should be equal to zero in order that the t-dependent term in Eq. (7) vanishes. The process $y(t) = I(t) + jQ(t)$ is a complex-value, thus $R_{yy^*}(\Delta t)$ can be denoted as:

$$\begin{aligned} R_{yy*}(\Delta t) &= R_{II}(\Delta t) - R_{QQ}(\Delta t) \\ &+ j[R_{IQ}(\Delta t) + R_{QI}(\Delta t)] \end{aligned} \tag{10}$$

where the corresponding correlation functions are defined as:

$$\begin{aligned} R^{II}(\Delta t) &= E[I(t + \Delta t)I(t)] \\ R^{QQ}(\Delta t) &= E[Q(t + \Delta t)Q(t)] \\ R^{IQ}(\Delta t) &= E[I(t + \Delta t)Q(t)] \\ R^{QI}(\Delta t) &= E[Q(t + \Delta t)I(t)] \end{aligned}$$

Thus, the condition for wide-sense stationarity of bandpass signal $r(t)$ requires that:

$$R_{II}(\Delta t) = R_{QQ}(\Delta t) \tag{11}$$

$$R_{IQ}(\Delta t) = -R_{QI}(\Delta t) = -R_{IQ}(-\Delta t) \tag{12}$$

The above dependencies can be exploited to assess whether the signal $r(t)$ is a WSS process. Two algorithms for this classification are proposed.

A. Algorithm I - similarity of autocorrelation and cross-correlation functions

Bandpass stochastic process $r(t)$ can be classified as WSS or non-WSS based on the mean value m_y of the baseband equivalent process $y(t)$, the similarity of autocorrelation functions R_{II} and R_{QQ} (Eq. 11), and the odd-feature of cross-correlation function R_{IQ} (Eq. 12). The similarity of two autocorrelation functions $R_{II}(\Delta t)$ and $R_{QQ}(\Delta t)$, can be expressed with the value of the coefficient c_{IQ}, calculated as the maximum value of cross-correlation of $R_{II}(\Delta t)$ and $R_{QQ}(\Delta t)$:

$$c_{IQ} = \max_{\Delta t'}[E[R_{II}(\Delta t + \Delta t'), R^*_{QQ}(\Delta t)]] \tag{13}$$

The odd-feature of cross-correlation function $R_{IQ}(\Delta t)$ can be expressed with the value of coefficient c_{IQQI}, in an analogous way:

$$c_{IQQI} = \max_{\Delta t'}[E[R_{IQ}(\Delta t + \Delta t'), -R^*_{QI}(\Delta t)]] \tag{14}$$

Two proposed correlation coefficients c_{IQ} and c_{IQQI}, along with the mean value m_y of signal $y(t)$, can be used to construct the WSS indicator:

$$I_{WSS} = \sqrt{\frac{c_{IQ} \cdot c_{IQQI}}{1 + m_y}} \tag{15}$$

It can be assumed that the received signal $r(t)$ represents the stochastic WSS process, if I_{WSS} is less than or equal to 0.5 (as its maximum value is 1). Fig. 1. shows the block diagram of the procedure for calculating I_{WSS}.

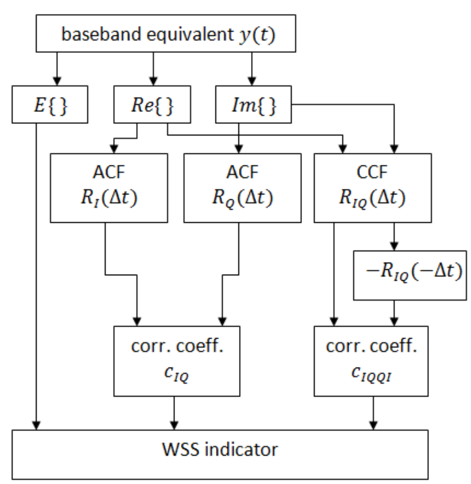

Fig. 1. Block diagram of the procedure for calculating I_{WSS} on the basis of correlation functions similarity.

B. Algorithm II - Symmetry of Power Spectrum

Stationarity of the second-order statistics of the bandpass signal $r(t)$ can be assessed also in the frequency domain, on the basis of the power spectral density (PSD) $S_y(f)$ of the baseband process $y(t)$. The autocorrelation function $R_{yy}(\Delta t)$ of process $y(t)$ can be denoted as:

$$\begin{aligned} R_{yy}(\Delta t) &= R_{II}(\Delta t) + R_{QQ}(\Delta t) \\ &+ j[R_{QI}(\Delta t) - R_{IQ}(\Delta t)] \end{aligned} \tag{16}$$

The stochastic analysis of $y(t)$ is performed in a discrete-time domain, thus the PSD can be calculated with the use of Discrete Fourier Transform (DFT). Let the $R_{yy}(n)$ be a discrete representation of $R_{yy}(\Delta t)$. The k-th element of the Fourier series representation $X(k)$ of $R_{yy}(n)$ is calculated as:

$$\begin{aligned} X(k) &= \sum_{n=0}^{N}\left((R_{II}(n) + R_{QQ}(n))\cos 2\pi\frac{nk}{N}\right) \\ &+ \sum_{n=0}^{N}\left((R_{QI}(n) - R_{IQ}(n))\sin 2\pi\frac{nk}{N}\right) \\ &+ j\sum_{n=0}^{N}\left(-(R_{II}(n) + R_{QQ}(n))\sin 2\pi\frac{nk}{N}\right) \\ &+ j\sum_{n=0}^{N}\left((R_{QI}(n) - R_{IQ}(n))\cos 2\pi\frac{nk}{N}\right) \end{aligned}$$

It can be easily shown, that the corresponding element of the power spectrum, which is equal to $S_y(k) = X(k)X^*(k)$, can be written as:

$$\begin{aligned} S_y(k) &= \sum_{n=0}^{N}\sum_{m=0}^{N}(A(n)A(m))\cos 2\pi\frac{(n-m)k}{N} \\ &+ \sum_{n=0}^{N}\sum_{m=0}^{N}(B(n)B(m))\cos 2\pi\frac{(n-m)k}{N} \\ &+ \sum_{n=0}^{N}\sum_{m=0}^{N}(A(m)B(n))\sin 2\pi\frac{(n-m)k}{N} \\ &- \sum_{n=0}^{N}\sum_{m=0}^{N}(A(n)B(m))\sin 2\pi\frac{(n-m)k}{N} \end{aligned}$$

where $A(n) = R_{II}(n) + R_{QQ}(n)$ and $B(n) = R_{QI}(n) - R_{IQ}(n)$. The part of the equation associated with $\cos 2\pi\frac{(n-m)k}{N}$ is a symmetric function of k. Asymmetry is introduced by the $\sin 2\pi\frac{(n-m)k}{N}$ term. Let the $y(n)$ be the baseband equivalent of a WSS process. Thus, $R_{II}(n) = R_{QQ}(n)$ and $R_{IQ}(n) = -R_{QI}(n)$, and the power spectrum density can be written as, after substituting back $A(n) = R_{II}(n) + R_{QQ}(n)$ and $B(n) = R_{QI}(n) - R_{IQ}(n)$:

$$\begin{aligned} S_y(k) &= \sum_{n=0}^{N}\sum_{m=0}^{N}4R_{II}(n)R_{II}(m)\cos 2\pi\frac{(n-m)k}{N} \\ &+ \sum_{n=0}^{N}\sum_{m=0}^{N}4R_{QI}(n)R_{QI}(m)\cos 2\pi\frac{(n-m)k}{N} \\ &+ \sum_{n=0}^{N}\sum_{m=0}^{N}4R_{II}(m)R_{QI}(n)\sin 2\pi\frac{(n-m)k}{N} \\ &- \sum_{n=0}^{N}\sum_{m=0}^{N}4R_{II}(n)R_{QI}(m)\sin 2\pi\frac{(n-m)k}{N} \end{aligned}$$

The sine term vanishes, and the PSD becomes symmetric:

$$\begin{aligned} S_y(k) &= \sum_{n=0}^{N}\sum_{m=0}^{N}4R_{II}(n)R_{II}(m)\cos 2\pi\frac{(n-m)k}{N} \\ &+ \sum_{n=0}^{N}\sum_{m=0}^{N}4R_{QI}(n)R_{QI}(m)\cos 2\pi\frac{(n-m)k}{N} \end{aligned}$$

Hence, it can be assumed that the symmetric power spectrum corresponds to a stochastic process for which Eq. 11-12 are satisfied.

The symmetry of $S_y(f)$ in the frequency domain f can be expressed with the cross-correlation coefficient c_{SS} defined as:

$$c_{SS} = \max_{\Delta f}[E[S_y(f + \Delta f), S_y(-f)] \tag{17}$$

The WSS indicator can be constructed of the cross-correlation

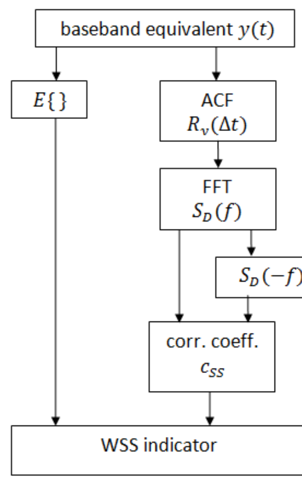

Fig. 2. Block diagram of the procedure for calculating I_{WSS} on the basis of PSD symmetry analysis

coefficient c_{SS} and the mean value m_y of process $y(t)$:

$$I_{WSS} = \frac{c_{SS}}{1 + m_y} \tag{18}$$

If the WSS indicator I_{WSS} is greater than the treshold value D, the power spectrum $S(f)$ corresponds to the complex envelope of the WSS process $r(t)$. Fig. 2. shows the block diagram of the procedure for calculating I_{WSS} on the basis of PSD symmetry analysis.

C. Stationary time

To provide adaptive underwater acoustic communications, the transmitter has to shape the transmitted wave in a way which minimizes the bit-error-rate at the receivers side. The I_{WSS} indicator can be exploited in the UAC system receiver, to calculate a maximum observation time interval, namely stationary time T_D, for which the received signal $r(t)$ can be treated as a stochastic WSS process.

$$T_D = \max \Delta t|_{I_{WSS} \geq D} \tag{19}$$

Knowledge about the stationary time T_D is crucial for designing the transmission signal in a bandpass communication system. It can be calculated continuously on the basis of the received signal $r(t)$, and sent back to the transmitter once a certain amount of time has passed, or whenever a significant change of T_D occurs. After updating T_D in the transmitter, the UAC system can be re-tuned to the transmission and reception of the signal of the updated transmission parameters.

IV. Simulations

The proposed algorithms for determining stationary time have been verified in preliminary simulation tests. Pseudo-random binary sequence $s(n)$ was transmitted in nonstationary underwater channel that was simulated using impulse responses gathered during the underwater experiment in the Baltic Sea, described in [4]. A module of sample complex-value impulse response $h(t, \tau)$ is shown in Fig. 3. Each of

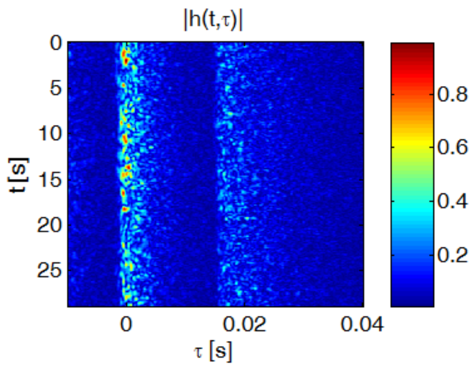

Fig. 3. Module of the impulse response measured during the underwater experiment [4]

the impulse responses was interpolated to produce as many realizations, as the transmitted signal samples. The received signal $y(n)$ was calculated as:

$$y(n) = \sum_{k=0}^{K-1} s(k)h(n; n-k), 0 \leq n < N \qquad (20)$$

where N is a number of transmitted signal samples. Each of received signal samples $y(n)$ is obtained by convolution of input signal samples $s(n)$ and consecutive impulse response realisation. Thus, each output sample $y(n)$ is produced using a different impulse response.

Fig. 4 shows the results of received signal $y(n)$ analysis. WSS indicators were calculated using the two proposed algorithms. The third WSS indicator is a result of impulse response analysis, as it is described in [5]. The WSS indicators are presented as functions of time interval T_D, at which signal $y(n)$ and impulse response $h(t, \tau)$ are analysed. It is clearly seen, that increasing the time interval T_D causes each of the WSS indicators to decrease, which means that the stochastic process loses its wide-sense stationarity feature.

It is necessary to calibrate the treshold value D of WSS indicator, which defines stationary time T_D (Eq. 19). For impulse response analysis the threshold D was set at value

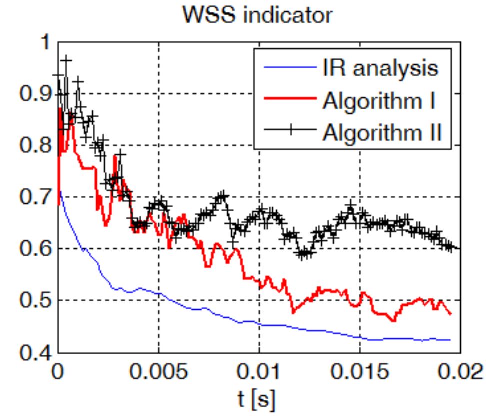

Fig. 4. WSS indicators calculated for the received signal and the impulse response during the simmulation experiment

of 0.5 [5]. In case of WSS indicators calculated with the two proposed algorithms on the basis of received signal, this value should be higher. It is highly probable that the threshold should depend on correlation and spectral properties of the measurement signal, which should be further investigated.

V. Conclusion

UAC system designers tend to transmit as much information as possible, per unit of time, at an as-low-as–possible error rate. However, the bit rate achieved in UAC systems is much lower than for wire or radio-communication systems. This is due to the disadvantageous properties of the UAC channels. There is a need to develop and implement data transmission algorithms matching the UAC system physical layer parameters to dynamically varying channel conditions, in order to ensure efficient performance in any geographical conditions; in both the deep ocean channels and in tough shallow inland water channels. The estimation of stationary time, wherein the second-order statistics of the channel are approximately independent of time, is crucial for the performance of communication systems working in time-varying conditions. Moreover, stationary time parameter can be exploited in sonar systems to configure digital signal processing algrithms to ensure accurate target position and velocity estimation. It is possible to determine this stationary time parameter on the basis of the complex envelope of the bandpass signal analysis performed in the receiver, thus bypassing the need for calculating the impulse response to determine the statistical properties of the channel. The proposed method can be implemented in real-time adaptive underwater systems.

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