# A Lattice Theoretic Look: A Negated Approach to Adjectival (Intersective, Neutrosophic and Private) Phrases

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***Abstract*—The aim of this paper is to provide a contribution to Natural Logic and Neutrosophic Theory. This paper considers lattice structures built on noun phrases. Firstly, we present some new negations of intersective adjectival phrases and their set-theoretic semantics such as *non-red non-cars* and *red non-cars*. Secondly, a lattice structure is built on positive and negative nouns and their positive and negative intersective adjectival phrases. Thirdly, a richer lattice is obtained from previous one by adding neutrosophic prefixes *neut* and *anti* to intersective adjectival phrases. Finally, the richest lattice is constructed via extending the previous lattice structures by private adjectives (fake, counterfeit). We call these lattice classes *Neutrosophic Linguistic Lattices* (*NLL*).**

**Keywords: Logic of natural languages; neutrosophy; pre-orders, orders and lattices; adjectives; noun phrases; negation**

## I. Introduction

One of the basic subfields of the foundations of mathematics and mathematical logic, lattice theory, is a powerful tool of many areas such as linguistics, chemistry, physics, and information science. Especially, with a set theoretical view, lattice applications of mathematical models in linguistics are a common occurrence.

Fundamentally, Natural Logic [1], [2] is a human reasoning discipline that explores inference patterns and logics in natural language. Those patterns and logics are constructed on relations between syntax and semantics of sentences and phrases. In order to explore and identify the entailment relations among sentences by mathematical structures, it is first necessary to determine the relations between words and clauses themselves. We would like to find new connections between natural logic and neutrosophic by discovering the phrases and neutrosophic clauses. In this sense, we will associate phrases and negated phrases to neutrosophic concepts.

Recently, a theory called Neutrosophy, introduced by Smarandache [4], [6], [5] has widespread mathematics, philosophy and applied sciences coverage. Mathematically, it offers a system which is an extension of intuitionistic fuzzy system. Neutrosophy considers an entity, “$A$” in relation to its opposite, “$anti - A$” and that which is not $A$, “$non - A$”, and that which is neither “$A$” nor “$anti - A$”, denoted by “$neut - A$”.

Up to section 3.3, we will obtain various negated versions of phrases (intersective adjectival) because Neutrosophy considers opposite property of concepts and we would like to associate the phrases and Neutrosophic phrases. We will present the first *NLL* in section 3.3. Notice that all models and interpretations of phrases will be finite throughout the paper.

## II. Negating Intersective Adjectival Phrases

Phrases such as “red cars” can be interpreted the intersection of the set of *red things* with the set of *cars* and get the set of “red cars”. In the sense of model-theoretic semantics, the interpretation of a phrase such as *red cars* would be the intersection of the interpretation of *cars* with a set of *red individuals* (the region $b$ in Figure 1). Such adjectives are called intersective adjectives or intersecting adjectives. As to negational interpretation, Keenan and Faltz told that “similarly, intersective adjectives, like common nouns, are negatable by non-: non-Albanian (cf. non-student) ”in their book [7]. In this sense, *non-red cars* would interpret the intersection of the of *non-red things* and the set of *cars*. Negating intersective adjectives without nouns (red things) would be complements of the set of *red things*, in other words, *non-red things*. We mean by *non-red things* are which the things are which are *not red*. Remark that *non-red things* does not guarantee that *those individuals* have to have a colour property or something else. It is changeable under incorporating situations but we will might say something about it in another paper. On the other hand, negating nouns (cars) would be complements of the set of *cars*, in other words, *non-cars*. We mean by non-cars that the things are which are not cars. Adhering to the spirit of intersective adjectivity, we can explore new meanings and their interpretations from negated intersective adjectival phrases by intersecting negated (or not) adjectives with negated (or not) nouns. As was in the book, *non-red cars* is the intersection

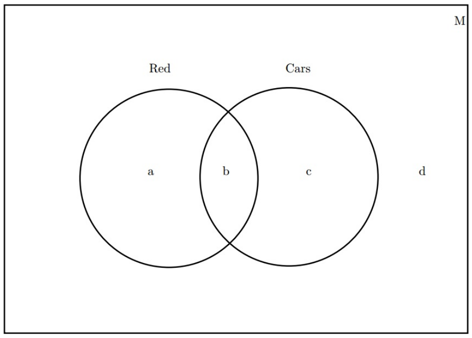


Fig. 1: An example of $cars$ and $red$ in a discourse universe

the set of things that are not red with cars. In other words, *non-red cars* are the cars but not red (the region $c$ in Figure 1). Another candidate for the negated case, *non-red non-cars* refers to intersect the set of non-red things (things that are not red) with non-cars (the region $d$ in Figure 1). The last one, *red non-cars* has meaning that is the set of intersection of the set of red things and the set of non-cars (the region $a$ in Figure 1). $red\bar{x}$ is called *noun level partially semantic complement*. $\overline{red}x$ is called *adjective level partially semantic complement*. $\overline{red}\,\bar{x}$ is called *full phrasal semantic complement*. In summary, we obtain *non-red cars*, *red non-cars* and *non-red non-cars* from *red cars* we already had.

The intersective theory and conjunctives suits well into boolean semantics [7], [8] which proposes very close relationship between *and* and *or* in natural language, as conjunction and disjunction in propositional and predicate logics that have been applied to natural language semantics. In these logics, the relationship between conjunction and disjunction corresponds to the relationship between the set-theoretic notions of intersection and union [9], [10]. On the other hand, correlative conjunctions might help to interpret negated intersective adjectival phrases within boolean semantics because the conjunctions are paired conjunctions (neither/nor, either/or, both/and,) that link words, phrases, and clauses. We might reassessment those negated intersective adjectival phrases in perspective of correlative conjunctions. "*neither A nor B* " and "*both non-A and non-B* can be used interchangeably where *A* is an intersective adjective and *B* is a noun. Therefore, we say "*neither red (things) nor pencils* "and "*both non-red (things) and non-pencils* "equivalent sentences. An evidence for the interchangeability comes from equivalent statements in propositional logic, that is, $\neg(R \vee C)$ is logically equivalent to $\neg R \wedge \neg C$ [11]. Other negated statements would be $\neg R \wedge C$ and $R \wedge \neg C$. Semantically, $\neg R \wedge \neg C$ is *full phrasal semantic complement* of $R \vee C$, and also $\neg R \wedge C$ and $R \wedge \neg C$ are partially semantic complements of $R \vee C$.

We will explore full and partially semantic complements of several adjectival phrases. We will generally negate the phrases and nouns by adding prefix "*non*", "*anti*" and "*neut*". We will use interpretation function [[ ]] from set of phrases ($Ph$) to power set of universe ($\mathcal{P}(M)$) (set of individuals) to express phrases with understanding of a set-theoretic viewpoint. Hence, $[[p]] \subseteq M$ for every $p \in Ph$. For an adjective $a$ (negated or not ) and a plural noun $n$ (negated or not ) , $a\ n$ will be interpreted as $[[a]] \cap [[n]]$. If $n$ is a positive plural noun, $non - n$ will be interpreted as $[[non - n]] = [[\bar{n}]] = M \setminus [[n]]$. Similarly, if $a$ is a positive adjective, $non - a$ will be interpreted as $[[non - a]] = [[\bar{a}]] = M \setminus [[a]]$. While we will add *non* to both nouns and adjectives as prefix, "*anti*" and "*neut*" will be added in front of only adjectives. Some adjectives themselves have negational meaning such as *fake*. Semantics of phrases with *anti*, *neut* and *fake* will be mentioned in next sections.

## III. Lattice Theoretic Look

We will give some fundamental definitions before we start to construct lattice structures from those adjectival phrases.

A lattice is an algebraic structure that consists of a partially ordered set in which every two elements have a unique supremum (a least upper bound or join) and a unique infimum (a greatest lower bound or meet) [12]. The most classical example is on sets by interpreting set intersection as meet and union as join. For any set $A$, the power set of $A$ can be ordered via subset inclusion to obtain a lattice bounded by $A$ and the empty set. We will give two new definitions in subsection 3.2 to start constructing lattice structures.

*Remark 3.1:* We will use the letter *a* and *red* for intersective adjectives, and the letter *x*, *n* and *cars* for common plural nouns in the name of abbreviation and space saving throughout the paper.

### A. Individuals

Each element of $[[ax]]$ and $[[\bar{a}x]]$ is a distinct individual and belongs to $[[x]]$. It is already known that $[[a\,x]] \cap [[\bar{a}\,x]] = \emptyset$ and $[[a\,x]] \overset{\star}{\bigcup} [[\bar{a}\,x]] = [[x]]$. It means that no common elements exist in $[[a\,x]]$ and $[[\bar{a}\,x]]$. Hence, every element of those sets can be considered as individual objects such as Larry, John, Meg,.. etc. Uchida and Cassimatis [13] already gave a lattice structure on power set of all of individuals (a domain or a universe).

### B. Lattice $\mathcal{L}_{IA}$

Intersective adjectives (red) provide some properties for nouns (cars). Excluding (complementing) a property from an intersective adjective phrase also provide another property for nouns. In this direction, "*red*" is an property for a noun, "$non - red$" is another property for the noun as well. $red$ and $non - red$ have discrete meaning and sets as can be seen in Figure 1. Naturally, every set of restricted objects with a property (red cars) is a subset of those objects without the properties (cars). $[[red\ x]]$ and $[[\overline{red}\ x]]$ are always subsets of $[[x]]$. Neither $[[red\ x]] \leq_\star [[\overline{red}\ x]]$ nor $[[\overline{red}\ x]] \leq_\star [[red\ x]]$ since $[[red\ x]] \cap [[\overline{red}\ x]]$ by assuming $[[\overline{red}\ x]] \neq \emptyset$ and $[[red\ x]] \neq \emptyset$. Without loss of generality, for negative (complement) of the noun $x$ and the

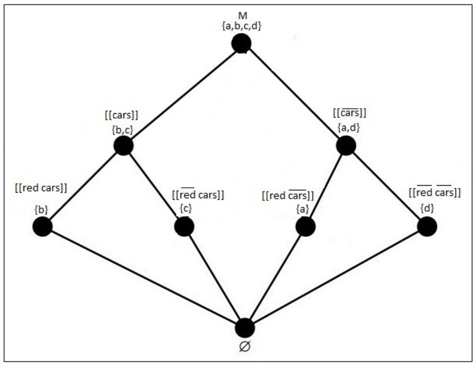


Fig. 2: Lattice on $cars$ and $red$

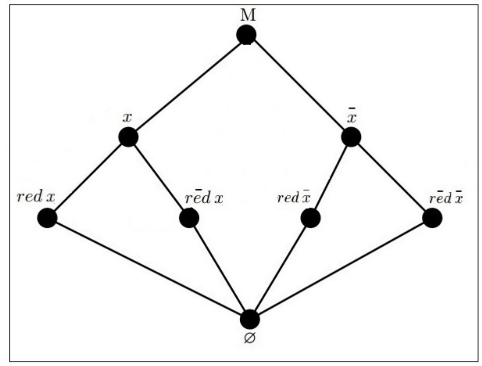


Fig. 3: Hasse Diagram of lattice of $\mathcal{L}_{\mathcal{IA}} = (L, \emptyset, \bigcap, \overset{\star}{\bigcup})$

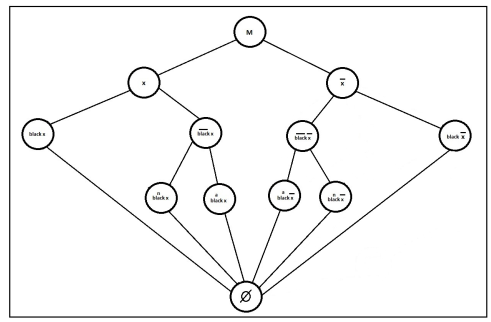


Fig. 4: The Lattice $\mathcal{L}_{IA}^{N}$

intersective adjective $red$ (positive and negative) are $\bar{x}, red\,\bar{x}$ and $\overline{red}\,\bar{x}$. $[[red\ \bar{x}]]$ and $[[\overline{red}\ \bar{x}]]$ are always subsets of $[[\bar{x}]]$. Neither $[[red\ \bar{x}]] \leq_\star [[\overline{red}\ \bar{x}]]$ nor $[[\overline{red}\ \bar{x}] \leq_\star [[red\ \bar{x}]]$ since $[[red\ \bar{x}]] \cap [[\overline{red}\ \bar{x}]]$ by assuming $[[\overline{red}\ \bar{x}]] \neq \emptyset$ and $[[red\ \bar{x}]] \neq \emptyset$. On the other hand, $[[x]] \cap [[\bar{x}]] = \emptyset$ and $[[x]]\overset{\star}{\bigcup}[[\bar{x}]] = M$ ($M$ is the universe of discourse) and also $[[red\ x]]$, $[[\overline{red}\ x]]$, $[[red\ \bar{x}]]$ and $[[\overline{red}\ \bar{x}]]$ are by two discrete. We do not allow $[[red\ x]]\overset{\star}{\bigcup}[[\overline{red}\ x]]$ and $[[red\ x]]\overset{\star}{\bigcup}[[\overline{red}\ \bar{x}]]$ and $[[\overline{red}\ x]]\overset{\star}{\bigcup}[[red\ \bar{x}]]$ and $[[\overline{red}\ x]]\overset{\star}{\bigcup}[[\overline{red}\ \bar{x}]]$ to take places in the lattice in Figure 2 because we try to build the lattice from phrases only in our language. To do this, we define a set operation $\overset{\star}{\bigcup}$ and an order relation $\leq_\star$ as the follows:

*Definition 3.2:* We define a binary set operator $\overset{\star}{\bigcup}$ for our languages as the follow: Let $S$ be a set of sets and $A,\ B \in S$. $A \overset{\star}{\bigcup} B = \ C \ :\Leftrightarrow C$ is the smallest set which includes both $A$ and $B$, and also $C \in S$.

*Definition 3.3:* We define a partial order $\leq_\star$ on sets as the follow:

$$A \leq_\star B \text{ if } B = A \overset{\star}{\bigcup} B$$
$$A \leq_\star B \text{ if } A = A \bigcap B$$

*Example 3.4:* Let $A = \{1,2\},\ B = \{2,3\},\ C = \{1,2,4\},\ D = \{1,2,3,4\}$ and $S = \{A, B, C, D\}$.

$A \overset{\star}{\bigcup} A = A,\ A \overset{\star}{\bigcup} C = C,\ A \overset{\star}{\bigcup} B = D,\ B \overset{\star}{\bigcup} C = D,\ C \overset{\star}{\bigcup} D = D.$

$C \leq_\star C,\ A \leq_\star C,\ A \leq_\star D,\ B \leq_\star D,\ C \leq_\star D$

Notice that $\leq_\star$ is a reflexive, transitive relation (pre-order) and $\overset{\star}{\bigcup}$ is a reflexive, symmetric relation.

Figure 3 illustrates a diagram on $cars$ and $red$. The diagram does not contain sets $\{b,d\}, \{a,b\}, \{a,c\}$ and $\{c,d\}$ because the sets do not represent linguistically any phrases in the language. Because of this reason, $\{a\}\overset{\star}{\bigcup}\{c\}$ and $\{a\}\overset{\star}{\bigcup}\{b\}$ and $\{d\}\overset{\star}{\bigcup}\{c\}$ and $\{d\}\overset{\star}{\bigcup}\{b\}$ are $\{a,b,c,d\} = M$. This structure builds a lattice up by $\overset{\star}{\bigcup}$ and $\bigcap$ that is the classical set intersection operation.

$\mathcal{L}_{\mathcal{IA}} = (L,\ \emptyset,\ \bigcap, \overset{\star}{\bigcup})$ is a lattice where $L = \{M,\ x,\ \bar{x},\ red\ x,\ red\ \bar{x},\ \overline{red}\ x,\ \overline{red}\ \bar{x}\}$. Remark that $\mathcal{L}_{\mathcal{IA}} = (L,\ \emptyset,\ \bigcap, \overset{\star}{\bigcup}) = (L,\ \emptyset,\ \ \leq_\star)$. We call this lattice briefly $\mathcal{L}_{\mathcal{IA}}$.

### *C. Lattice $\mathcal{L}_{IA}^{N}$*

In this section, we present first *NLL*. Let $A$ be the color $white$. Then, $non{-}A = \{black, red, yellow, blue, ...\}$, $anti{-}A$ points at $black$, and $neut - A = \{red, yellow, blue, ...\}$. In our interpretation base, $anti - black\ cars$ ($\overset{a}{black}\ cars$) is a specific set of $cars$ which is a subset of set $non - black\ cars$ ($\overline{black}\ cars$). $neut - black\ cars$ ($\overset{n}{black}\ cars$) is a subset of $\overline{black}\ cars$ which is obtained by excluding sets $black\ cars$ and $\overset{a}{black}\ cars$ from $\overline{black}\ cars$. Similarly, $anti - black\,cars$ ($\overset{a}{black}\ \overline{cars}$) is a specific set of $\overline{cars}$ which is a subset of set $non - black\,non - cars$ ($\overline{black}\ \overline{cars}$). $neut - black\ \overline{cars}$ ($\overset{n}{black}\ \overline{cars}$) is a subset of $\overline{black}\ \overline{cars}$ which is obtained by excluding sets of $black\ \overline{cars}$ and $\overset{a}{black}\ \overline{cars}$ from $\overline{black}\ \overline{cars}$. The new structure represents an extended lattice equipped with $\leq_\star$ as can be seen in Figure 4. We call this lattice $\mathcal{L}_{IA}^{N}$.

### *D. Lattice $\mathcal{L}_{IA}^{N}(F)$*

Another *NLL* is an extended version of $\mathcal{L}_{IA}^{N}$ by private adjectives. Those adjectives have negative effects on nouns such *fake* and *counterfeit*. The adjectives are representative elements

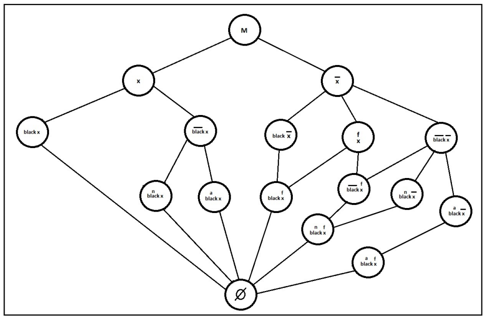

Fig. 5: The lattice $\mathcal{L}^N_{IA}(F)$

of, called *private*, a special class of adjectives [14], [15], [16]. Chatzikyriakidis and Luo treated transition from the adjectival phrase to noun as $Private\ Adj(N) \Rightarrow \neg N$ in inferential base. Furthermore, they gave an equivalence "$real\_gun(g)$ iff $\neg\ fake\_gun(g)$" where $[|g\ is\ a\ real\ gun|] = real\ \_\ gun(g)$ and $[|f\ is\ not\ a\ real\ gun|] = \neg\ real\_\ gun(f)$ in order to constitute a modern type-theoretical setting. In light of these facts, *fake car is not a car (real)* and plural form: *fake cars are not cars*. Hence, set of fake cars is a subset of set of non-cars in our treatment.

On the one hand, compositions with private adjectives and intersective adjectival phrases do not effect the intersective adjectives negatively but nouns as usual. Then, interpretation of "*fake red cars*" would be intersection of set of *red things* and set of $non-cars$.

Applying "*non*" to private adjectival phrases, $non-fake\ cars$ are cars (real), $[[non-fake\ cars]] = [[cars]]$ whereas $[[fake\ cars]] \subseteq [[non-cars]]$. $non-fake\ cars$ will be not given a place in the lattice. Remark that phrase "non-fake non-cars" is ambiguous since $fake$ is not a intersective adjective. We will not consider this phrase in our lattice.

$\overset{f}{x}$ is incomparable both $\overline{black}\ x$ and $\overline{black}\ \bar{x}$ except $\bar{x}$ as can be seen in Figure 5. So, we can not determine that set of $fake\ cars$ is a subset or superset of a set of any adjectival phrases. But we know that $[[fake\ cars]] \subseteq [[non-cars]]$. Then, we can see easily $[[fake\ black\ cars]]] \subseteq [[blacks\ non-cars]]$ by using $[[fake\ cars]] \bigcap [[black\ things]] \subseteq [[\overline{cars}]] \bigcap [[black\ things]]$.

Without loss of generality, set of $fake\ black\ cars$ is a subset of set $black\ non-cars$ and also set of $fake\ non-black\ cars$ is a subset of set $non-black\ non-cars$. Continuing with *neut* and *anti*, set of $fake\ neut\ black\ cars$ is a subset of set of $neut\ black\ non-cars$ and also $fake\ anti\ black\ cars$ is a subset of set of $anti\ black\ non-cars$. Those phrases build the lattice $\mathcal{L}^N_{IA}(F)$ in Figure 5.

Notice that when $M$ and empty set are removed from lattices will construct, the structures lose property of lattice. The structures will be hold neither join nor meet semi-lattice property as well. On the other hand, set of $\{\overset{n}{black}\overset{f}{x},\ \overline{black}\overset{f}{x},\ \overset{n}{black}\bar{x},\ \overline{black}\bar{x}\}$ equipped with $\leq_\star$ is the only one sub-lattice of $\mathcal{L}^N_{IA}(F)$ without using $M$ and empty set.

## IV. Conclusion and Future Work

In this paper, we have proposed some new negated versions of set and model theoretical semantics of intersective adjectival phrases (plural). After we first have obtained the lattice structure $\mathcal{L}_{IA}$, two lattices $\mathcal{L}^N_{IA}$ and $\mathcal{L}^N_{IA}(F)$ have been built from the proposed phrases by adding 'neut', 'anti' and 'fake' step by step.

It might be interesting that lattices in this paper can be extended with incorporating coordinates such as *light red cars* and *red cars*. One might work on algebraic properties as filters and ideals of the lattices considering the languages. Some decidable logics might be investigated by extending syllogistic logics with the phrases. Another possible work in future, this idea can be extended to complex neutrosophic set, bipolar neutrosophic set, interval neutrosophic set [17], [18], [19], [20].

We hope that linguists, computer scientists and logicians might be interested in results in this paper and the results will help with other results in several areas.

## V. Thanks

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