Atanassov's Intuitionistic Fuzzy Risk Estimation of the Ship System Failures Based on the Expert Judgments

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Abstract—**In the safety engineering, risk estimation is in practice confronted with difficulties connected with shortage of data. In such cases, we have to rely on subjective estimations made by persons with practical knowledge in the field of interest, i.e. experts. However, in some realistic situations, the decision makers might be reluctant or unable to assign the crisp values to the evaluation judgments due to his/her limited knowledge. In other words, there is a certain degree of hesitancy in human cognition and his/her judgment. The probabilistic approach turns out to be insufficient in modeling the subjective uncertainty. Taking advantages of the Atanassov's intuitionistic fuzzy sets in dealing with ambiguity and uncertainty into account, they can be used to handle with the subjective evaluations of experts, who may have insufficient knowledge of the problem domain or uncertainty in assigning the evaluation values to the objects considered. This paper addresses the Atanassov's intuitionistic fuzzy methods in the risk estimation of the ship system failures based on the expert judgments.**

Keywords—Atanassov's intuitionistic fuzzy set; risk estimation; expert judgment; ship system failure.

I. INTRODUCTION

In sea transport, ship system failures, especially the failure of key systems, may cause accidents even disaster to the ship and crew on board. For example, loss of the propulsion function by a ship is one of the most serious categories of hazardous events. In specific external conditions, it may lead to a loss of ship and environmental pollution. The consequences of propulsion loss by a ship are events classified by the International Maritime Organization as casualties or incidents [3]. Therefore, the risk estimation of ship system failures based on failure mode and effect analysis (FMEA) is necessary for making appropriate decisions concerning inspection and maintenance, which in turn will make shipping more safe and reliable.

Among the risk assessment methods, FMEA is the most popular, widely used engineering technique in many industries, which can be used to identify and eliminate known or potential failure modes to enhance reliability and safety of complex systems. It is also intended to provide information for making risk management decisions. The traditional FMEA determines the risk priorities of failure modes using the so-called risk priority numbers (RPNs), which is defined as simple product of probabilities of the occurrence (O), severity (S) and detection (D) of the failure mode. Determination of these probabilities is in practice confronted with difficulties connected with shortage of data. In such cases, we have to rely on subjective estimations made by persons with practical knowledge in the field of interest, i.e. experts. However, in great majority they have not experienced events where such losses occur. So, their practical knowledge may contain ambiguousness and uncertainty in some extent. The experts, on the other hand, prefer to formulate their opinions in the linguistic categories.

Hence, the traditional FMEA seems to be inadequate to explicitly capture the important assessments for deriving the priorities in these situations. Wang, Chin, Poon and Yang [14] have showed drawbacks and given significant criticisms for the traditional FMEA. To overcome this issue, the fuzzy theory was introduced to the traditional FMEA, making it more flexible in dealing with the real situation. The authors proposed in [14] a new fuzzy FMEA which allows the risk factors and their relative weights to be evaluated in linguistic forms. Abdelgawad and Fayek [1] used fuzzy logic to address the limitations of traditional FMEA, and the results confirmed the capability of fuzzy FMEA to address several drawbacks of the traditional FMEA application. Laarhoven and Pedrycz [8] introduced the fuzzy analytical hierarchy process (FAHP), where each pairwise comparison judgment is represented as a triangle fuzzy number with a membership function. The membership function denotes the degree to which elements considered belong to the set. The fuzzy set theory has been also introduced to the decision-making problems [4, 7] and preference relations [6].

Since the membership function of a fuzzy set is only single-valued function, it cannot be used to express the support and objection evidences simultaneously in many practical situations. The decision makers may not be able to express their evaluations or preferences accurately due to the fact that they may not grasp sufficient knowledge on the domain considered, or they are unable discriminate explicitly the degree to which an alternative is better than others. In other words, there is a certain degree of hesitation. In order to describe such situations and to model human's perception and cognition more comprehensively, Atanassov [2] extended Zadeh's fuzzy set to the intuitionistic fuzzy set (IFS), which is characterized by membership degree, non-membership degree, and hesitancy

978-1-5090-5795-5/17/$31.00 ©2017 IEEE

degree, which sum up to one. Afterwards, the IFSs have attracted increasingly scholars' attention and has been applied to many different fields, such as decision making [15], [16], intuitionistic fuzzy (IF) preference relations [17], IF cognitive maps [10], medical diagnosis [5], fault diagnosis [13] and pattern recognition [9], [14]. Xu and Liao [18] extended the classical AHP and the FAHP to the IF circumstances and developed the IF-AHP procedure for handling comprehensive multi-criteria decision-making problems.

In this paper, a method for estimating the risk caused by ship system failures is proposed. The estimation is fully based on the expert judgments. Expert is assumed to be well acquainted with the subject he is expected to formulate his judgment on. Expert should also be capable of formulating his judgment. This is connected with level of his education and the language used in the elicitation process, particularly as regards the parameters the expert is expected to estimate. This may be the language of numerical or linguistic values. Numerical values are better but are more difficult to articulate - also errors in judgments are more likely. However, in some realistic situations, the decision makers might be reluctant or unable to assign the crisp evaluation values to the judgments due to his/her limited knowledge. In such cases, we have to rely on subjective estimations made by persons with practical knowledge in the field of interest, i.e. experts. However, in great majority they have not experienced events where such losses occur. So, their practical knowledge may contain ambiguousness and uncertainty in some extent. The experts, on the other hand, prefer to formulate their opinions in the linguistic categories. This paper presents a method of the subjective estimation of propulsion risk by a seagoing ship, based on the expert judgments. It is adjusted to the knowledge of experts from ships' machinery crews and to their capability of expressing that knowledge. The method presented has been developed with an intention of using it in the decision-making procedures in risk prediction during the seagoing ship operation.

II. BASIC CONCEPTS

A. Intuitionistic Fuzzy Sets

In 1983 Atanassov generalized the concept of fuzzy sets given by Zadeh [19] by using membership function and non-membership function for any elements of the universe of discourse. An Atanassov's Intuitionistic Fuzzy Set (IFS) is described by [2]:

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}, \tag{1}$$

where $\mu_A(x)$ denotes a degree of membership and $\nu_A(x)$ denotes a degree of non-membership of x to A, $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ such that

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X. \tag{2}$$

To measure hesitancy of membership of an element to a intuitionistic fuzzy set, Atanassov introduced a third function given by:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \tag{3}$$

which is also called the intuitionistic fuzzy index or the hesitation margin of x to A. It is obvious that $0 \le \pi_A(x) \le 1, \forall x \in X$.

If $\pi_A(x) = 0, \forall x \in X$, then $\mu_A(x) + \nu_A(x) = 1$ and the intuitionistic fuzzy set A is reduced to an ordinary fuzzy set.

The concept of a complement of an IFS A, denoted by A^c is defined as:

$$A^c = \{(x, \nu_A(x), \mu_A(x), \pi_A(x)) | x \in X\}. \tag{4}$$

B. Intuitionistic Fuzzy Operators

The operations of addition $\oplus$ and multiplication $\otimes$ on intuitionistic fuzzy values (IFVs) were defined by Atanassov [2] as follows. Let $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ be IFVs, then the following operators were defined:

$$A \oplus B = (\mu_A + \mu_B - \mu_A\mu_B, \nu_A\nu_B), \tag{5}$$

$$A \otimes B = (\mu_A\mu_B, \nu_A + \nu_B - \nu_A\nu_B), \tag{6}$$

$$\lambda A = \left(1 - (1 - \mu_A)^\lambda, {\nu_A}^\lambda\right), (\lambda > 0), \tag{7}$$

$$A^\lambda = \left({\mu_A}^\lambda, 1 - (1 - \upsilon_A)^\lambda\right), (\lambda > 0). \tag{8}$$

The operations (5) – (8) are used to aggregate local criteria for solving MCDM problems in the intuitionistic fuzzy setting. Let $A_1, \ldots, A_m$ be IFVs representing the values of local criteria and $w_1, \ldots, w_m$; $\sum_{j=1}^{m} \lambda_j = 1$ be their weights. Then intuitionistic fuzzy weighted arithmetic mean (IFWA) can be obtained using operations (5) and (7) as follows:

$$\begin{aligned} \mathrm{IFWA}_w(A_1, \ldots, A_m) &= w_1A_1 \oplus \ldots \oplus w_mA_m \\ &= \langle 1 - \textstyle\prod_{j=1}^{m}(1 - \mu_A)^{w_j},\ \prod_{j=1}^{m}(\nu_A)^{w_j} \rangle. \end{aligned} \tag{9}$$

Intuitionistic fuzzy weighted geometric mean (IFWG) can be obtained using operations (6) and (8) as follows:

$$\begin{aligned} \mathrm{IFWG}_w(A_1, \ldots, A_m) &= w_1A_1 \otimes \ldots \otimes w_mA_m \\ &= \langle \textstyle\prod_{j=1}^{m}(\mu_A)^{w_j}, 1 - \prod_{j=1}^{m}(1 - \nu_A)^{w_j} \rangle. \end{aligned} \tag{10}$$

These aggregation operators provide IFVs, are idempotent and currently are most popular in the solution of decision-making problems in the intuitionistic fuzzy setting. An important problem is the comparison of IFVs to choose the best alternative when the final scores of alternatives are presented by IFVs. The specific methods were developed to compare IFVs. Chen and Tan [4] proposed to use the so-called score function S (or net membership). Hong and Choi [7] in addition to the above score function introduced the so-called accuracy function H and showed that the relation between functions S and H is similar to the relation between mean and variance in statistics. Szmidt, Kacprzyk and Bujnowski [11] proposed a knowledge measure of IFV, taking into account all its parameters, i.e. membership, non-membership and hesitancy degrees. Since these methods are rather of heuristic nature, there are different definitions of score function proposed in the literature.

C. Score Function for IFVS

In order to rank the IFVs, we utilize the membership knowledge measure $K_F(\alpha)$ proposed in [9], which is intuitively appealing and simple in computation. Let $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ be an IFV in finite universe of discourse X. The score function of $\alpha \in X$ is defined as:

$$S(\alpha) = \begin{cases} K_F(\alpha) & \text{for } \mu_\alpha \ge \nu_\alpha \\ -K_F(\alpha) & \text{for } \mu_\alpha < \nu_\alpha \end{cases}, \tag{11}$$

$$\text{where } K_F(\alpha) = \frac{1}{\sqrt{2}}\sqrt{{\mu_\alpha}^2 + {\nu_\alpha}^2 + (\mu_\alpha + \nu_\alpha)^2}. \tag{12}$$

The score function $-1 \geq S(\alpha) \geq 1$ measures amount of knowledge conveyed by linguistic evaluation presented in form of IFVs, whereas the plus sign is assigned to the positive information for $\mu_\alpha \geq \nu_\alpha$ and minus one to the negative information for $\mu_\alpha < \nu_\alpha$. Naturally, that the positive information is preferred to than negative one, so the larger value of $S(\alpha)$, the higher rank of IFV α.

III. METHODOLOGY OF THE INTUITIONISTIC FUZZY FMEA

A. Intuitionistic Fuzzy Risk Factors

Usually, the risk factors O, S and D are evaluated by experts in linguistic terms. The linguistic terms and their related intuitionistic fuzzy numbers are shown in Tables I–III, respectively. For example, experts revealed their opinions on the occurrence probability of the ship system failures in the form of linguistic values chosen from the given linguistic set (Table I): very high (VH), high (H), moderate (M), low (L) and very low (VL). We take into account period of practical experience of experts as a factor of their hesitancy degree in judgments as follows:

$$\pi_j = 1/2^{x_j} \tag{13}$$

where x_j denotes the expert's practical experience in years. The more experience, the less uncertainty he/she has. The rating of failure mode F_i made by the expert E_j on the risk factor O is represented by $R_{ij}^o = \langle \mu_{ij}^o, \nu_{ij}^o \rangle$, where μ_{ij}^o is the membership degree of the failure mode to the risk factor O, related to the linguistic rating. The non-membership degree is determined as:

$$\nu_{ij}^o = 1 - \mu_{ij}^o - \pi_{ij}^o, 0 \leq \nu_{ij}^o \leq 1. \tag{14}$$

Hence, the intuitionistic fuzzy numbers related to the linguistic values of the given set should be suitable to hesitancy degree (knowledge level) of the experts.

Suppose there are n failure modes $F_i, (i = 1, \ldots, n)$ of the ship propulsion system, and m experts $E_j, (j = 1, \ldots, m)$. Let $R_{ij}^o = \langle \mu_{ij}^o, \nu_{ij}^o \rangle$, $R_{ij}^s = \langle \mu_{ij}^s, \nu_{ij}^s \rangle$ and $R_{ij}^d = \langle \mu_{ij}^d, \nu_{ij}^d \rangle$ be the IF ratings of F_i on the risk factors O, S and D; ω_o, ω_s and ω_d be the fuzzy weights of the three risk factors, $\lambda_j, (j = 1, \ldots, m)$ be the relative importance weights of the experts, $\sum_{j=1}^m \lambda_j = 1$.

Using the intuitionistic fuzzy weighted averaging (IFWA) operator (9) we aggregate the IF ratings of F_i on the risk factors O, S and D, respectively:

$$R_i^o = \mathrm{IFWA}_\lambda(R_{i1}^o, R_{i2}^o, \ldots, R_{im}^o) = \lambda_1 R_{i1}^o \oplus \ldots \oplus \lambda_m R_{im}^o = \langle 1 - \prod_{j=1}^m (1 - \mu_{ij}^o)^{\lambda_j}, \ \prod_{j=1}^m (\nu_{ij}^o)^{\lambda_j} \rangle = \langle \mu_i^o, \nu_i^o \rangle, \tag{15}$$

$$R_i^s = \mathrm{IFWA}_\lambda(R_{i1}^s, R_{i2}^s, \ldots, R_{im}^s) = \lambda_1 R_{i1}^s \oplus \ldots \oplus \lambda_m R_{im}^s = \langle 1 - \prod_{j=1}^m (1 - \mu_{ij}^s)^{\lambda_j}, \ \prod_{j=1}^m (\nu_{ij}^s)^{\lambda_j} \rangle = \langle \mu_i^s, \nu_i^s \rangle, \tag{16}$$

$$R_i^d = \mathrm{IFWA}_\lambda(R_{i1}^d, R_{i2}^d, \ldots, R_{im}^d) = \lambda_1 R_{i1}^d \oplus \ldots \oplus \lambda_m R_{im}^d = \langle 1 - \prod_{j=1}^m (1 - \mu_{ij}^d)^{\lambda_j}, \ \prod_{j=1}^m (\nu_{ij}^d)^{\lambda_j} \rangle = \langle \mu_i^d, \nu_i^d \rangle. \tag{17}$$

B. Intuitionistic Fuzzy Risk Priority Number (IFRPN)

The traditional FMEA defines RPNs as the simple product of O, S and D without considering their relative importance weights, whereas the IFRPN is defined as the fuzzy weighted geometric mean of the three risk factors O, S and D. This overcomes the drawback that the three risk factors are treated equally. IFRPN of the failure mode F_i can be aggregated using the intuitionistic fuzzy weighted geometric (IFWG) operator (10) as follows:

$$\mathrm{IFRPN}_i = \omega_o R_i^o \otimes \omega_s R_i^s \otimes \omega_d R_i^d = \langle (\mu_i^o)^{\omega_o} . (\mu_i^s)^{\omega_s} . (\mu_i^d)^{\omega_d}, \ 1 - (1 - \nu_i^o)^{\omega_o} . (1 - \nu_i^s)^{\omega_s} . (1 - \nu_i^d)^{\omega_d} \rangle = \langle \mu_i, \nu_i \rangle. \tag{18}$$

TABLE I. FUZZY RATINGS FOR PROBABILITY OF FAILURE OCCURRENCE (WITH $\pi_j = 0.1$).

Rating	Probability of occurrence	Intuitionistic fuzzy number
Very high (VH)	Failure is almost inevitable	<0.8, 0.1>
High (H)	Repeated failures	<0.6, 0.3>
Moderate (M)	Occasional failures	<0.45, 0.45>
Low (L)	Relatively few failures	<0.3, 0.6>
Remote (R)	Failure is unlikely	<0.1, 0.8>

TABLE II. FUZZY RATINGS FOR PROBABILITY OF FAILURE SEVERITY (WITH $\pi_j = 0.1$).

Rating	Severity of occurrence of a failure	Intuitionistic fuzzy number
Very serious casualty (C1)	Loss of the ship, loss of human life and/or heavy marine environment pollution.	<0.8, 0.1>
Serious casualty (C2)	Injuries or human health deterioration, ship grounding, touching a submarine object, contact with a solid object, lost seaworthiness due to defects, necessity of towing or assistance from the shore and/or marine environment pollution.	<0.6, 0.3>
Incident I (I1)	Prolonged hazard to the ship, people and environment which can cause a sea accident. After repair by the ship crew, the ship propulsion function is not fully restored (lower propulsion system operational parameters).	<0.45, 0.45>
Incident II (I2)	As in I1, but after repair the ship propulsion function is fully restored.	<0.3, 0.6>
Incident III (I3)	Temporary hazard to the ship, people and environment which can cause a sea accident. No repair needed.	<0.1, 0.8>

TABLE III. FUZZY RATINGS FOR PROBABILITY OF FAILURE DETECTION (WITH $\pi_j = 0.1$).

Rating	Probability of occurrence	Intuitionistic fuzzy number
Very remote (VR)	Very remote chance	<0.1, 0.8>
Very low (VL)	Very low chance	<0.2, 0.7>
Low (L)	Low chance	<0.3, 0.6>
Moderately low (ML)	Moderately low chance	<0.4, 0.5>
Moderate (M)	Moderate chance	<0.45, 0.45>
Moderately high (MH)	Moderately high chance	<0.5, 0.4>
High (H)	High chance	<0.6, 0.3>
Very high (VH)	Very high chance	<0.7, 0.2>
Almost certain (AC)	Almost certainty	<0.8, 0.1>

Using (11), the score functions of the IFRPNs of failure modes F_i can be calculated. The increasing order of the score functions represents the risk priority of potential causes. For the ship system failure analysis, failure mode with the biggest score function should be given the top priority.

IV. Intuitionistic Fuzzy Risk Estimation Of The Ship System Failures

To demonstrate the applicability of the proposed method, an example about tanker system failure from a global tanker ship management company is adopted from [20]. Assume that a FMEA team consisting of five experts identifies 17 potential system failure modes on tankers (Table IV) and needs to prioritize them in terms of their failure risks so that high risky failure modes can be corrected with top priorities. Experts evaluate the risk factors of failure modes as probability of their occurrence, severity and detect ability using the linguistic terms defined in Tables I–III. The five experts are assigned with the following relative weights: 0.15, 0.25, 0.25, 0.20 and 0.15. The weights of the risk factors O, S and D are assumed to be 0.40, 0.35 and 0.25. The relative weights of risk factors can be decided by experts, considering both historical data and factors which are more concerned about. For example, if the consequence of a failure is more important, the weight of its severity may be assigned with a higher value than that of others. Based on the above information, intuitionistic fuzzy RPNs of the 17 failure modes can be calculated. The score functions of the obtained IFRPNs indicate the priority order of ship system failure modes.

V. Results and Discussion

Table IV shows the results of comparing the proposed method with fuzzy method [20] for the given example. The rankings of the tanker system failure modes by both approaches are almost the same, i.e. the riskiest failure is F_{12} (main engine) and the least risky one is F_5 (Cargo system). The ranking of other failure modes is also consistent, e.g. the five most risky failures and three least risky ones. There are some differences in the middle rankings between approaches due to different used methods. For example, the rank of F_1 (Auxiliary engine) is seventh by the IFRPN method, while it is eighth by the fuzzy method. Meanwhile, the rank of F_3 (Boiler) is eighth by the IFRPN method, while it is seventh by the fuzzy method. Additionally, the proposed method showed that with the increasing hesitation margin ($\pi_j = 0.1$ related to about 3 yrs of practical experience) the consistency of the obtained results is deteriorating.

As can be seen from Table IV, F_{12} (Main engine) is apparently the failure mode with the maximum overall risk and should be given the top priority, followed by F_{15} (Navigation system), F_6 (Deck Machinery), F_{13} (Monitoring system) and F_{16} (Piping system). The ranking can be used for the decision-making of managers, arranging the inspection and maintenance of the equipment properly, which can optimise the maintenance resources and avoid the risk.

VI. Conclusions

In this paper, the IF method has been proposed for the risk estimation of the ship system failures, which is based exclusively on the judgments elicited by experts - experienced marine engineers. The obtained results show that the proposed method is powerful and useful in dealing with imprecise and uncertain data, which are available in the such cases. Combining IFS and FMEA methods allows incorporating the hesitancy and limited knowledge of expert judgments. Compared with the traditional FMEA, the proposed method seems more useful and effective for risk evaluation. Compared with the fuzzy FMEA, the proposed method shows more practical and flexible in describing the real-life problems. The proposed method is particularly useful in the expert investigations. It is worth noticing that subjective investigation results may (but not necessarily) be charged with greater error than objective results acquired in real operational process. Therefore, the further researches should be focused on validation of the proposed method by the objective results.

TABLE IV. RESULTS OF COMPARING PROPOSED IF METHOD WITH FUZZY METHOD [20].

No.	Failure mode (F_i)	Ranking by FRPN	Ranking by IFRPN ($\pi_j = 0$)	Ranking by IFRPN ($\pi_j = 0.05$)	Ranking by IFRPN ($\pi_j = 0.1$)
1	Auxiliary engine	8	7	9	4
2	Auxiliary machinery	6	9	7	9
3	Boiler	7	8	10	5
4	Cargo pump	14	14	13	11
5	Cargo system	17	17	17	17
6	Deck machinery	3	3	3	2
7	Electrical system	10	10	8	10
8	Emergency system	12	13	14	14
9	Hull part	15	15	15	15
10	Hydraulic system	13	12	12	13
11	Inert gas system	11	11	11	12
12	Main engine	1	1	1	1
13	Monitoring system	4	4	4	3
14	Mooring	9	6	6	7
15	Navigation system	2	2	2	8
16	Piping system	5	5	5	6
17	Steering Gear	16	16	16	16

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