The Impact of Diversity on the Quality of Collective Prediction

Van Du Nguyen
Faculty of Computer Science and Management, Wroclaw University of Science and Technology
Wroclaw, Poland
van.du.nguyen@pwr.edu.pl

Ngoc Thanh Nguyen
Faculty of Computer Science and Management, Wroclaw University of Science and Technology
Wroclaw, Poland
Ngoc-Thanh.Nguyen@pwr.edu.pl

Abstract— Nowadays, there appears to be sufficient evidence that a collective of uninformed individuals can outperform single individuals (even individual experts) in solving some difficult judgment and prediction problems. In some situations, the predictions given by collective members on the outcome of a future event can be modified (such as by updating or removing predictions) to reach a higher consistency level (less diversity) in a collective. The main concern of this paper is to investigate the impact of diversity on the quality of collective prediction by taking into account such modifications within a collective. Diversity is understood as the variety of individual predictions on the same problem. The simulation experiments (with different collective cardinalities and levels of diversity reduction) have revealed that such modifications are not useful in improving the quality of collective prediction. Instead, they have emphasized the important role of diversity in leading to a better collective prediction. That is the more diverse the collective, the better the collective prediction.

Keywords— intelligent collective, wisdom of crowds, integration computing

I. INTRODUCTION

In recent years, research on Collective Intelligence has shown that a collective of uninformed individuals can outperform single individuals (even individual experts) in solving some difficult judgment and prediction problems, especially with the kind of problems that are complex in themselves and based on insufficient or higher uncertain information [1, 2]. This capability is due to the fact that the knowledge of a collective is often larger than the sum of individual knowledge [3, 4]. For example, if member A knows that $\{a > b\}$ and member B knows that $\{b > c\}$, then together we will know that $\{a > b, b > c, a > c\}$. However, it is not true that all collectives can outperform their individuals. According to [1, 2], the underlying criteria for which a collective to be accurate are *diversity*, *independence*, *decentralization* and *aggregation*. Diversity can be understood as the variety of individual backgrounds or opinions on the problem that needs to be solved. Intuitively, a collective seems not to be intelligent if all members have the same background and the same opinion on the given problem. In [5], it has been recommended that using of heterogeneous collectives, which involve members of different backgrounds is an efficient way to make the collectives to be accurate. Independence means that an individual opinion must be made independently of others in the collective and it has been proved helpful in avoiding the phenomenon of the so-called correlated errors among individual opinions [1, 2, 6-9]. Meanwhile, decentralization means that collective members are able to specialize and draw on local knowledge [1]. It allows members to act freely and independently of one another. The last criterion, aggregation, is understood as a suitable mechanism that aims at determining the collective prediction.

The concept of collective used here is understood as a set of predictions given by collective members (such as humans or agent systems). Notably that, these members can interact or not in solving a given problem. In this paper, we concentrate on the kind of problems whose proper values exist independently of the predictions given by collective members such as the problem of predicting the outcome of a future event or forecasting the temperature of a region on a future day. In addition, these proper values are not known when members are asked for giving their predictions. Therefore, the members' predictions to some degree reflect the proper values. We named this kind as *objective case* to differentiate it from the kind of problems whose proper values are dependent on members' opinions (called *subjective case*) as in some voting systems. A collective of predictions is a set of predictions given by collective members on the same problem. In addition, these members can have different backgrounds, different knowledge bases; therefore, their predictions on the given problem can be different from each other. Collective prediction is a prediction determined on the basis of members' predictions and can be considered as a representative for the collective as a whole.

Suppose that a number of people are asked for giving predictions on the temperature on a future day in a region. After individual predictions are given, the members whose predictions are furthest from the collective one will be encouraged to revise their predictions to reach a higher consistency level. In some situations, these extreme predictions can be treated as outliers and removed. A raising question is which collective will give a better collective prediction? Intuitively speaking, the collective whose individual predictions are more consistent will lead to a better collective prediction. It is because the decision determined on the basis of more consistent decisions will have a higher

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reliability. In this paper, however, we will show a paradox that such modifications that cause the collective to be more consistent do not lead to better collective predictions.

Refer to [5, 10-13], a collective composing of members that are more diverse can outperform that of less diverse members. It is because they will add new information, new perspectives, etc. on the problem that needs to be solved. Moreover, the authors in [1, 6] have found that the diversity of members' predictions also leads to a better collective prediction. This kind of diversity is helpful in avoiding the phenomenon of the so-called correlated errors [14-16]. The main contribution of the paper is to investigate the impact of the latter kind of diversity on the quality of collective prediction by taking into account some modifications within a collective to make it becomes more consistent (less diverse). The diversity of a collective is measured using the functions proposed in [17]. Notice that, these functions are based on the differences between individual predictions in a collective besides the difference between the collective prediction and members' predictions. In addition, the measures of the quality are not only based on the difference between the collective prediction and the proper value (*Diff*); but also based on the win ration that presents the number of times in which the collective prediction is superior to the individual ones (*WR*) and the quotient between the collective error and individual errors (*QIC*).

The remainder of the paper is structured as follows. Section II introduces some related works. Some basic notions related to collectives, collective of predictions, the measures of diversity and the quality of collective prediction are presented in Section III. The research model and the simulation results are provided and analyzed in section IV. Finally, some conclusions and future works are pointed out in Section V.

II. Related Works

The research in the literature so far has revealed that a collective of uninformed individuals can be superior to single individuals (even individuals experts) on some difficult judgment and prediction problems [1, 2]. Even though, it has been proved that the collective prediction of uninformed individuals outperforms those generated by traditional forecasting approaches [18, 19]. It can be said that the longest experiment on the efficiency of a collective is conducted by Galton in 1907 with of 800 people [20]. They tried to estimate the weight of an ox. Surprisingly, the average of their estimates was only 9 pounds off the actual weight.

After the publication of the influential book entitled *"The Wisdom of Crowds"* [1], this research problem has been widely investigated and attracted the attention of many researchers. The statistical analysis from the game show *"Who wants to be a millionaire?"* showed that 91% the answers given by the audience were on the target in compared to 65% those of experts were on target. From this fact, the author has put forward a hypothesis *"A collective is more intelligent than a single individual."* This hypothesis has been proved by means of statistical analysis of several experiments in [1]. One of the most cited experiments is the problem of guessing the number of beans in a jar. The average of guesses is 871 while the actual number of beans in the jar is 850. Then the author has concluded that the most important criteria for which a collective to be intelligent are *diversity, independence, decentralization* and *aggregation*. They can be considered as the characteristics of intelligent collectives. The research so far has shown that a collective involving diverse members (which are randomly chosen from the population of problem solvers) will be superior to the higher-ability members in problem-solving [10]. In [5], the authors have shown the effectiveness of using heterogeneous collective for improving the performance of a collective. In addition, the diversity of gender in the boardroom has a positive impact on the firm performance [11]. Refer to Robert et al. [12], if collectives involve diverse members, then the increase of collective cardinalities will lead to better collective performances. However, it also exists the case in which less diverse collectives can be superior.

Beyond the diversity in the composition of collective members, the diversity of members' opinions has been proved positively associated with the quality of collective prediction. It is due to the fact that this kind of diversity is helpful in avoiding the phenomenon of the so-called correlated errors [6, 21, 22]. In general, a common approach to foster the diversity level of a collective is to expand its cardinality. In [23], the authors have conducted experiments with 500 members. They are 45% women and 55% men; age from 21 to 59; education 18% high school education, 72% college, and 9% graduate school education; and 67% members were office workers. They are asked for giving their forecasts about the temperature of a city, guessing the weights of specific amounts of coffee, milk, gasoline, air, and gold. The collective cardinalities used are 10, 20, 50, 100, and 200. The experimental results have indicated that the larger collectives we use, the higher accuracy we get. Other experiments conducted by Cui et al. [24] on the problem of predicting the demand for 38 summer trips. In these experiments, 52 students were randomly assigned to different collectives with different cardinalities (two collectives of 8 and two collectives of 18). Again, the large collectives are more accurate the small ones.

In the previous work [17, 25] we have formally proved the following statements: the collective prediction is better than the worst one in a collective; in case all predictions in a collective to the same degree reflect the proper value, the collective prediction is the best one. In [26, 27], the simulation results have revealed that the large collective cardinality has the positive impact on the quality of its prediction.

In addition to these research problems, this study presents another approach to determine the impact of diversity on the quality of collective prediction by taking into consideration some modifications (such as updating or removing prediction in a collective) within a collective. The main aim of these modifications is to reach a higher level of consistency in a collective. To the best of our knowledge, this research problem has not been widely investigated in the literature.

III. BACKGROUND

A. Collective of Predictions

As mentioned above, the concept of collective used here is understood as a set of predictions given by collective members (such as humans or agent systems). Notably that, these members can interact or not in solving a given problem. Moreover, it is not requisite that they must have the same viewpoints or opinions on the given problem. This characteristic is often known as diversity, one of the most important criteria for which a collective to be intelligent [1, 2]. Let U be a set of objects representing the potential values referring to a concrete problem in the real world. By symbol $\prod(U)$ we denote a set of all non-empty finite subsets with repetitions of U. By $X \in \prod(U)$ we denote a collective of predictions given by collective members on the same problem and X has the following form:

$$X = \{x_1, x_2, ..., x_n\}$$

where n is the collective cardinality representing the number of predictions in a collective.

B. Collective Prediction Determination

Collective prediction is often determined on the basis of members' predictions in a collective and can be considered as the representative for the collective as a whole. The two most popular criteria that can be used to determine the collective prediction of a collective are 1-Optimality and 2-Optimality (O_1 and O_2 respectively). In this paper, criterion O_2 is used. It has been stated that the sum of squared differences between the collective prediction and predictions in a collective is minimal. That is,

$$d^2\left(x^*, X\right) = min_{y \in U}\, d^2\left(y, X\right)$$

where x^* presents the collective prediction of collective X and

$$d^2\left(x^*, X\right) = \sum_{i=1}^{n} d^2\left(x^*, x_i\right).$$

In case of satisfying criterion O_2, the collective prediction has the following form:

$$x^* = \frac{1}{n}\left(\sum_{i=1}^{n} x_{i1}, \sum_{i=1}^{n} x_{i2}, ..., \sum_{i=1}^{n} x_{im}\right)$$

C. Diversity Measures

In this paper, the functions defined in [17] are used for measuring the diversity of a collective. A formal definition of the function that serves for measuring the diversity of a collective is presented below [17]:

Definition 1. *The diversity of a collective is defined by function c as follows:*

$$c: \prod(U) \rightarrow [0,1]$$

where $\prod(U)$ *is a set of non-empty subsets (with repetitions) of universe U.*

The first function is defined by taken into account the average of the differences between predictions in a collective. A collective, which its predictions are close together, will have the small diversity value.

$$c_3\left(X\right) = \begin{cases} \dfrac{1}{n\left(1-n\right)} \displaystyle\sum_{i=1}^{n} d\left(x_i, X\right), for\ n > 1 \\ o, \qquad\qquad otherwise \end{cases}$$

where $d\left(x_i, X\right) = \sum_{j=1}^{n} d\left(x_i, x_j\right)$, $d\left(x_i, x_j\right)$ is a function measuring the difference between x_i and x_j.

The next function is based on the minimal average of differences between an element of universe U and the predictions in a collective. It is defined as follows:

$$c_5\left(X\right) = d_{min}\left(X\right) = \frac{1}{n} min\, D\left(X\right)$$

where $D\left(X\right) = \left\{d\left(x^*, X\right) : x^* \in U\right\}$ is the sum of differences from an element x^* to the predictions in collective X. That is:

$$d\left(x^*, X\right) = \sum_{i=1}^{n} d\left(x^*, x_i\right)$$

Element x^*, called *collective prediction*, is considered as a representative of the predictions in collective X.

D. Quality of Collective Prediction

1) Based on the Difference between the Collective Prediction and the Proper Value.

The common measure the quality of collective prediction is based on the difference between the collective prediction and the proper value [17, 25]. In this case, the best collective prediction is the closest one to the proper value. The definition is as follows:

$$Diff\left(X\right) = 1 - d\left(r, x^*\right)$$

where $d\left(r, x^*\right) \in [0,1]$.

2) Based on Win Ratio.

This measure is based on the number of times in which the collective prediction is superior to the members' predictions [28, 29]. Its definition is as follows:

$$WR\left(X\right) = \frac{1}{n} \sum_{i=1}^{n} f\left(r, x^*, x_i\right)$$

where $f\left(r, x^*, x_i\right) = \begin{cases} 1, if\ d\left(r, x^*\right) < d\left(r, x_i\right) \\ 0, otherwise \end{cases}$

Notice that, a win ratio (WR) of 75% means that 3-to-4 times the collective prediction outperforms the predictions in a collective.

3) Based on Collective Error and Individual Errors

The last measure takes into account both the collective error and individual errors (in terms of squared value). This is

O_2-based measure, while the two previous ones are usable for both criterion O_1 or O_2.

$$QIC(X) = 1 - \frac{d^2(r, x^*)}{d^2(r, X)/n}$$

It follows from [6] that the collective error does not exceed the individual errors. Then, it should be true for $QIC(X) \in [0,1]$.

IV. THE IMPACT OF MODIFYING PREDICTIONS ON THE QUALITY OF COLLECTIVE PREDICTION

A. Research Model

This main concern of the paper is to investigate the impact of diversity on the quality of collective prediction by taking into account some modifications within a collective to obtain a higher consistency level. The research problem can be stated as follows: a number of people are asked for giving their predictions about the outcome of a future event. After all predictions are given, the members whose predictions are furthest from the collective prediction will be encouraged to revise their predictions. In some situations, these extreme predictions can be considered as outliers and removed. Let X be a collective involving n predictions and Y be the collective after modifying its predictions by updating or removing. Let $c(X)$, $c(Y)$ be the diversity levels of collectives X and Y, respectively. The general research model is described in Fig.1.

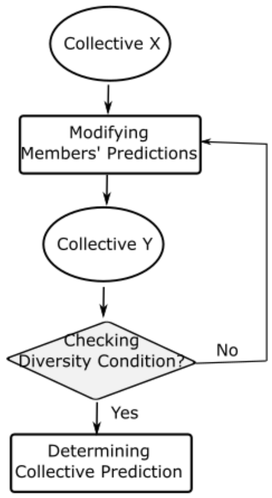

Fig. 1. The general research model

The procedure for modifying members' predictions repeats until the diversity reduction reaches a predefined threshold. The collective after modification must be satisfied: $c(X) \geq c(Y)$. The diversity condition is described as follows:

$$\left(\left(c(X) \geq c(Y)\right) \wedge \left(\frac{\left|c(X) - c(Y)\right|}{c(X)} \right) \right) \geq \partial$$

In this paper, we experiment with different thresholds of this reduction (such as 5%, 10%, 20%, and 30%). The impact of removing predictions on the diversity of a collective is described in the following theorem.

Theorem 1. For two collectives as follows:

$$X = \{x_1, x_2, \ldots, x_n\}, Y = \{X \setminus \{y\}\}$$

Let x^*, y^* be the collective predictions of X and Y, respectively. Then the following dependencies should be true:

a) If $c_3(Y) \leq c_3(X)$, then $d(y, X) \geq \frac{1}{n}\sum_{i=1}^{n} d(x_i, X)$

b) If $c_5(Y) \leq c_5(X)$, then $d(y, x^*) \geq \frac{1}{n}\sum_{i=1}^{n} d(x^*, X)$

Proof. The truth of these dependencies can be proved by taking into account the definitions of functions c_3, c_5 and the characteristics of a collective prediction determined based on criteria O_1 and O_2.

Suppose that the difference between the proper value and any prediction in a collective does not exceed a predefined threshold (α). In this paper, the value of α used is 500 and the proper value used is 1000. For such assumption, U is a set of integers bound between 500 and 1500. In addition, we experimented with different collective cardinalities such as 59, 109, and 209. With a predefined collective cardinality (n), first, we generate a collective involving n predictions belonging to U. Then the modifying predictions process will be invoked to make the collective to be more consistent. This process is repeated until the predefined threshold of diversity reduction is reached. Notably that, for each value of collective cardinality, we run 100-repetition experiments. The Manhattan distance is used for measuring the difference between two predictions in a collective. In addition, the collective prediction of a collective is determined by using criterion O_2.

B. Simulation Results and Their Evaluation

As mentioned earlier, the simulation experiments were carried out to find out the impact of diversity on the quality of collective prediction. Notably that, the values of *Diff, WR,* and *QIC* are the average of 100 corresponding values. In the following, *Or* represents the case in which the collectives will be modified and *Per5* represents the case in which the collectives modified such that the diversity reduction values are least 5% (similar for *Per10*, *Per20,* and *Per30*).

In the case of updating predictions, the simulation results reveal that the quality of collective prediction tends to decrease as the diversity value of a collective decreases (see Table I). Notably that, the strategy proposed in [30] is used for updating the furthest predictions from the collective prediction.

TABLE I. UPDATING CASE WITH COLLECTIVE CARDINALITY OF 109

	Function c_3			Function c_5		
	Diff	***WR***	***QIC***	***Diff***	***WR***	***QIC***

Or	0.975	0.953	0.988	0.976	0.954	0.990
Per5	0.959	0.887	0.973	0.963	0.899	0.978
Per10	0.946	0.831	0.951	0.953	0.852	0.964
Per20	0.925	0.739	0.893	0.933	0.756	0.918
Per30	0.909	0.660	0.817	0.920	0.669	0.859

The simulation results have shown that updating predictions, which cause collectives to be more consistent, does not lead to better collective predictions. Consider Figs. 2, 3 for more visual representation of the impact of diversity on the quality of collective prediction.

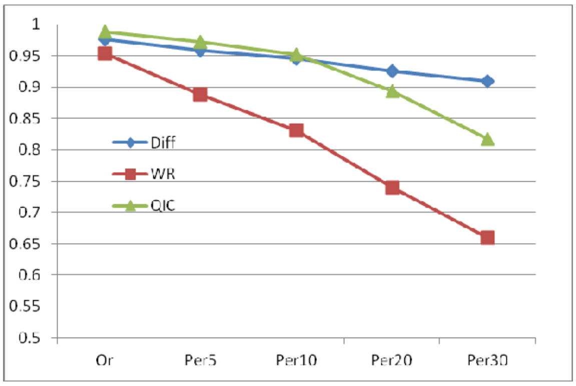

Fig. 2. Results of updating predictions and collective cardinality of 109 (function c_3)

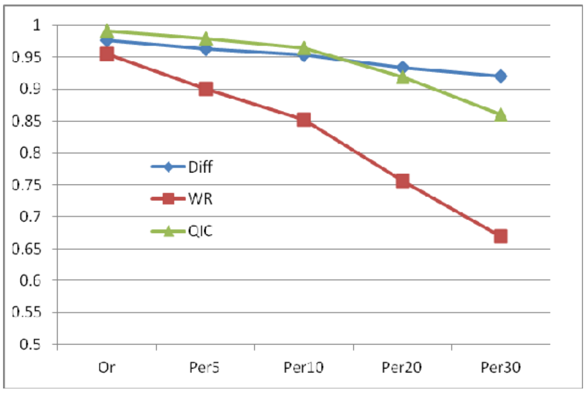

Fig. 3. Results of updating predictions and collective cardinality of 109 (function c_5)

In the case of eliminating predictions, we also found the similar finding. That is the quality of collective prediction tends to decrease when the diversity value of collective decrease (see Table II).

TABLE II. REMOVING CASE WITH COLLECTIVE CARDINALITY OF 109

	Function c_3			Function c_5		
	Diff	***WR***	***QIC***	***Diff***	***WR***	***QIC***
Or	0.978	0.959	0.991	0.976	0.952	0.989
Per5	0.952	0.900	0.964	0.954	0.906	0.967
Per10	0.932	0.851	0.928	0.937	0.864	0.935
Per20	0.901	0.762	0.829	0.904	0.769	0.837
Per30	0.870	0.650	0.697	0.876	0.678	0.721

The simulation results have shown that eliminating predictions that cause collectives to be more consistent does not lead to better collective predictions (see Figs. 4, 5).

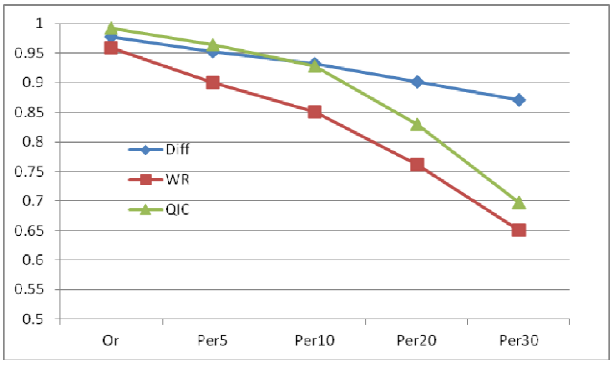

Fig. 4. Results of removing predictions and the collective cardinality of 109 (function c_3)

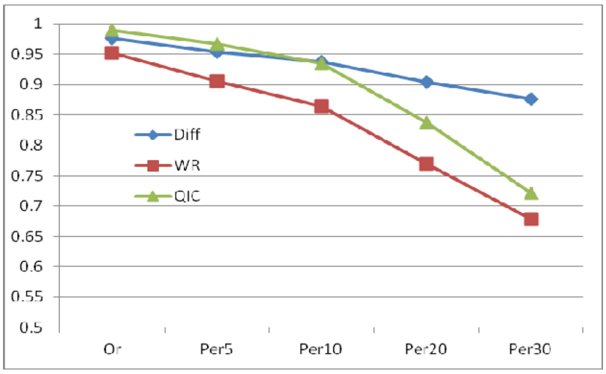

Fig. 5. Results of removing predictions and the collective cardinality of 109 (function c_5)

As can be seen from Fig. 2 to Fig. 5, we can state that the modifications (such as by updating or removing predictions) causing the diversity level of a collective to be decreased will not lead to a better collective prediction. In such situations, the quality of collective prediction decreases when the predictions in the collective become more consistent (less diverse). Moreover, the values of *WR* and *QIC* are decreased much more than those of *Diff*. It is because the measure of *Diff* only takes into account the collective prediction, while the others are based on both collective predictions and members' prediction. In addition, when the predictions in a collective are close to each other, it is hard for the collective prediction to outperform the members' predictions. The collective error, in this case, seems to be close to the average of individual errors.

A raising question is whether these differences are statistically significant? Since our data do not come from a normal distribution (according to the Shapiro-Wilk test), we choose Mann–Whitney U test to verify the difference between the collective predictions before and after updating predictions. According to the test results, this difference is statistically significant. Concretely, the *p-values* are much less than 0.05 (the significance level). Similarly, we also found the significant difference between the collective predictions of collectives before and after removing predictions that cause collectives to be more consistent ($p< 2.2e-16$). From these findings, we can draw a conclusion that the diversity is positively correlated with the quality of collective prediction (in terms of *Diff, WR,* and *QIC*).

V. Conclusions and Future Works

In some situations, the predictions given by collective members can be modified (such as by updating or removing) to reach a higher consistency level in a collective. This paper investigated to determine how such modifications affect the quality of collective prediction. The measures of the quality are based on the difference between the collective prediction and the proper value; the number of times that the collective prediction is superior to the individual ones; the quotient between the collective error and individual errors. The simulation results have indicated that updating (or removing) predictions that cause the collectives to be more consistent will not lead to better collective predictions. These findings again emphasize the important role of diversity in making a collective to be more accurate. That is the more diverse the collective, the better the quality of collective prediction.

As future work, we intend to work out a mathematical model for intelligent collectives that based on the criteria introduced by Surowiecki in [1]. For this aim, we will investigate to build a collective in which its members satisfy diversity in backgrounds but less influence on each other. In addition, methods for evaluating the independence level of a collective will be worked out. Moreover, the problem of processing large collectives [31], and transferring knowledge among collective members [32] should also be considered.

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