

Basic Data Structures: Trees

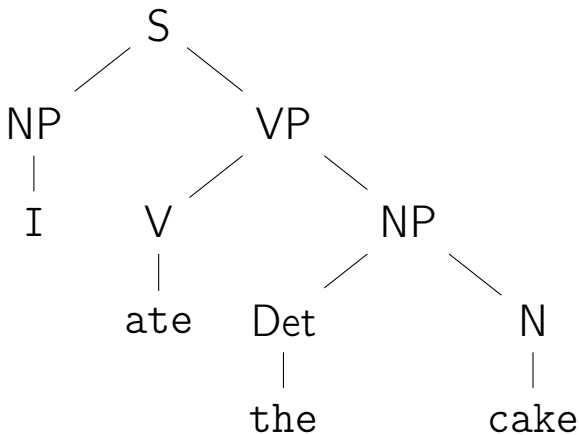
Neil Rhodes

Department of Computer Science and Engineering
University of California, San Diego

Data Structures
Data Structures and Algorithms

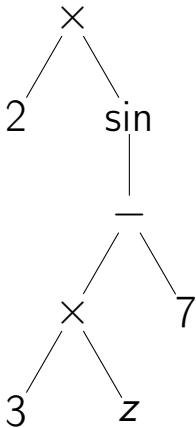
Syntax Tree for a Sentence

I ate the cake

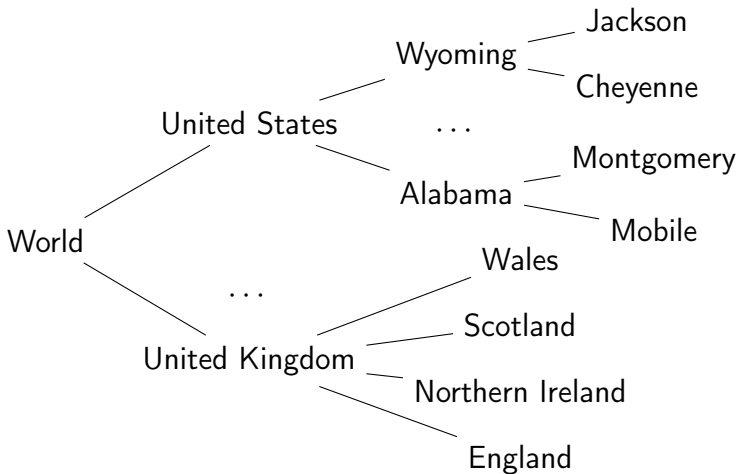


Syntax tree for an Expression

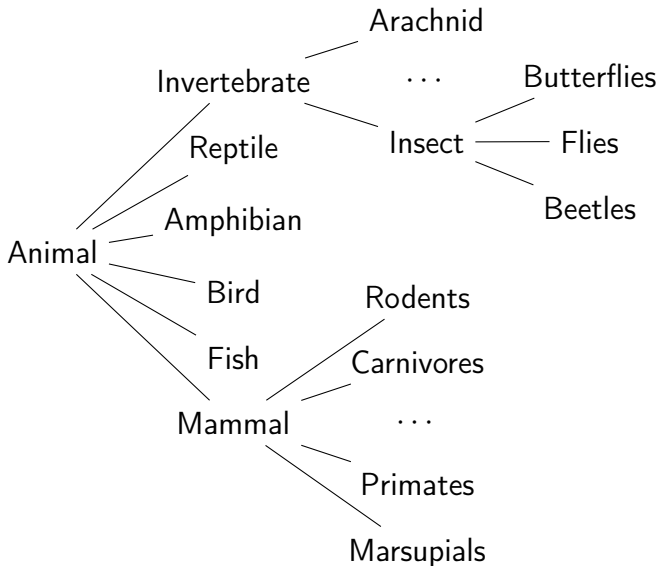
$$2 \sin(3z - 7)$$



Geography Hierarchy



Animal Kingdom (partial)

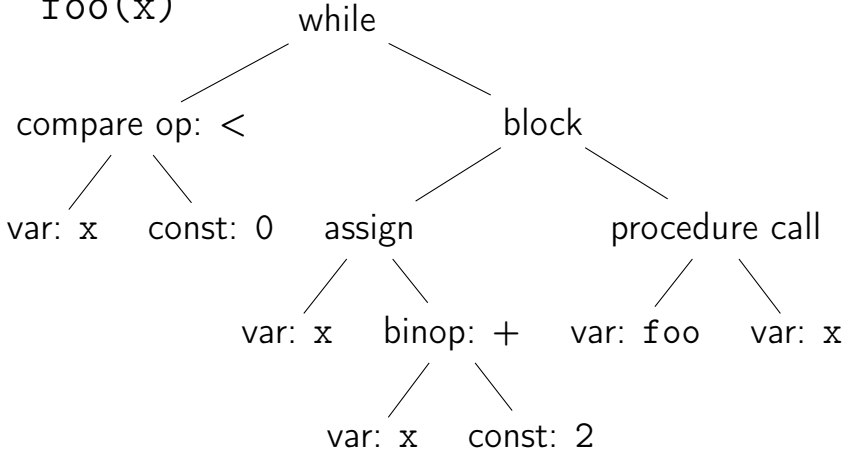


Abstract Syntax Tree for Code

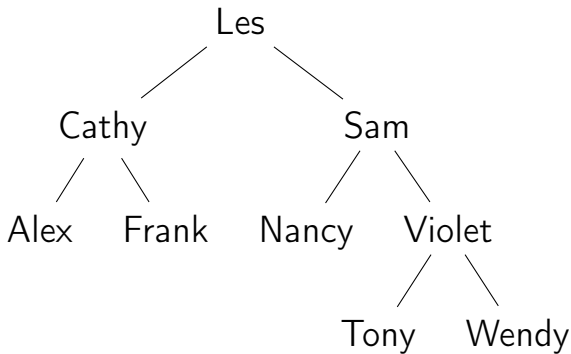
```
while x < 0:
```

```
    x = x + 2
```

```
    foo(x)
```



Binary Search Tree



Definition

A **Tree** is:

- empty, or
- a node with:
 - a key, and
 - a list of child trees.

Simple Tree

Empty tree:

Tree with one node:

Fred

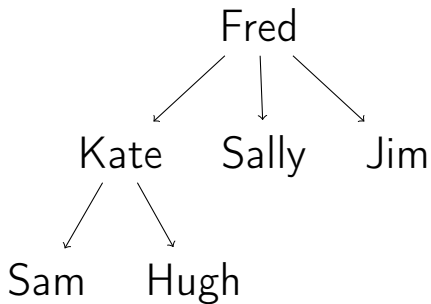
Tree with two nodes:

Fred

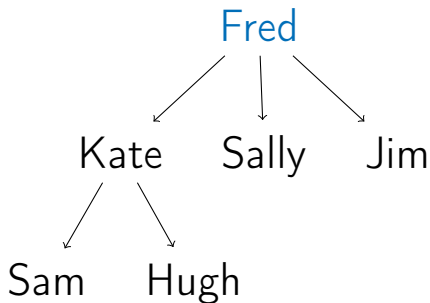
|

Sally

Terminology

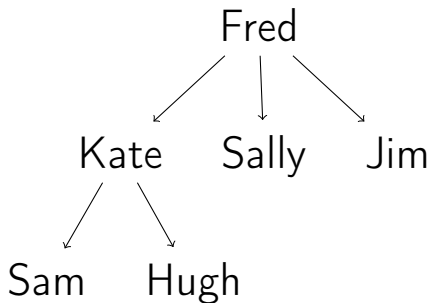


Terminology



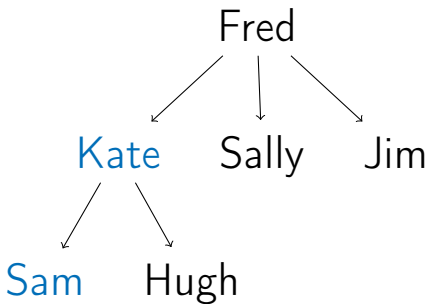
Root:
top node in the tree

Terminology



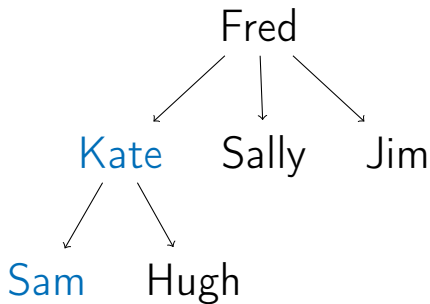
A child has a line down directly
from a *parent*

Terminology



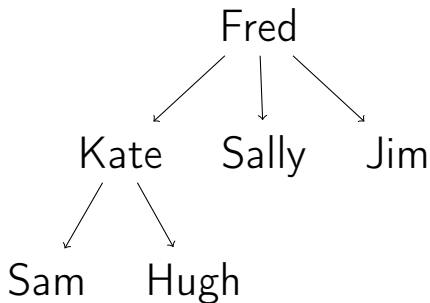
Kate is a *parent* of Sam

Terminology



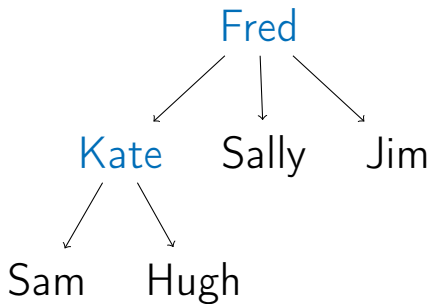
Sam is a *child* of Kate

Terminology



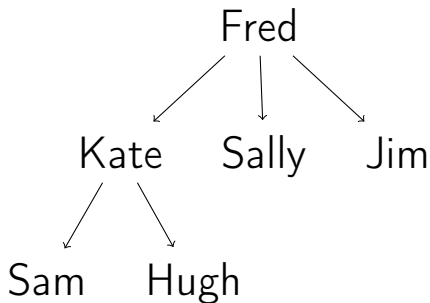
Ancestor:
parent, or parent of parent, etc.

Terminology



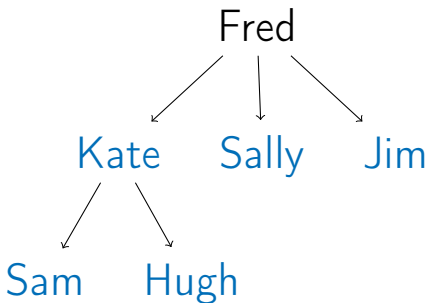
Ancestors of Sam

Terminology



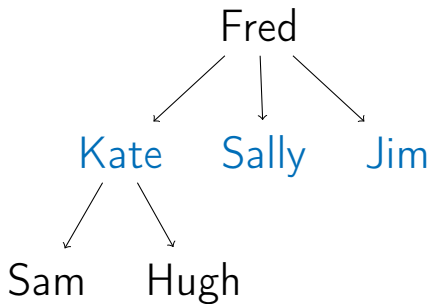
Descendant:
child, or child of child, etc.

Terminology



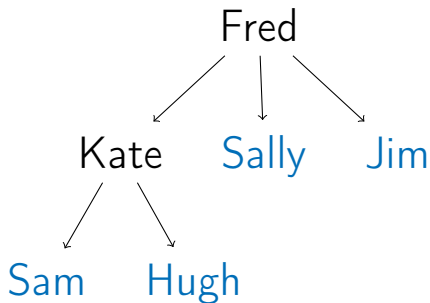
Descendants of Fred

Terminology



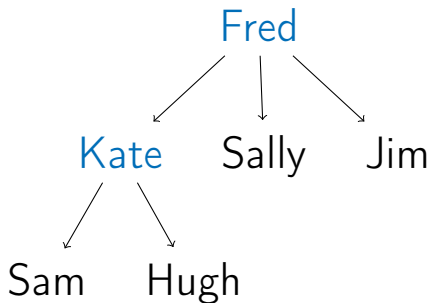
Sibling:
sharing the same parent

Terminology



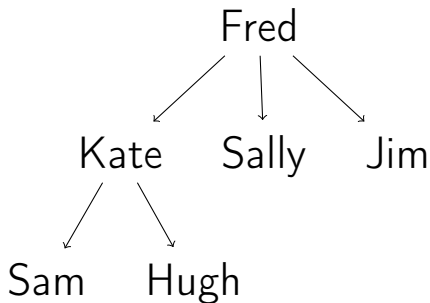
Leaf:
node with no children

Terminology



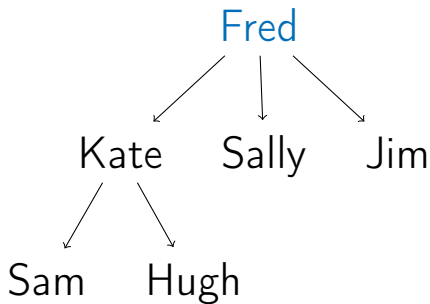
**Interior node
(non-leaf)**

Terminology



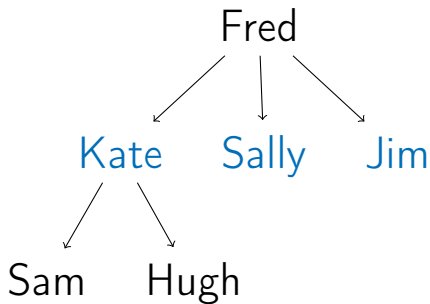
Level: $1 + \text{num edges between root and node}$

Terminology



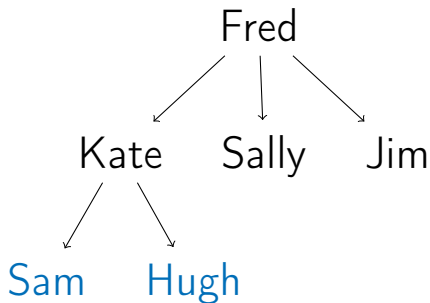
Level 1

Terminology



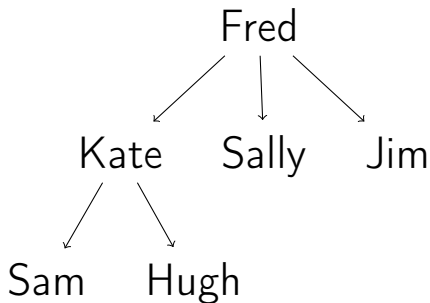
Level 2

Terminology



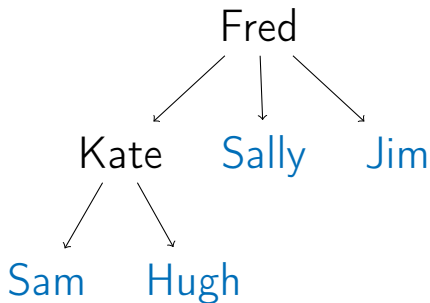
Level 3

Terminology



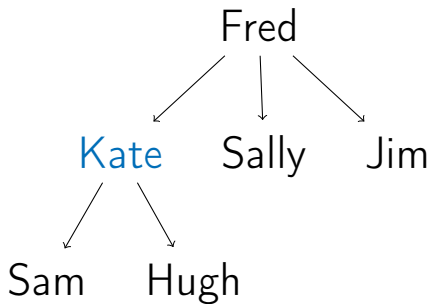
Height: maximum depth of subtree
node and farthest leaf

Terminology



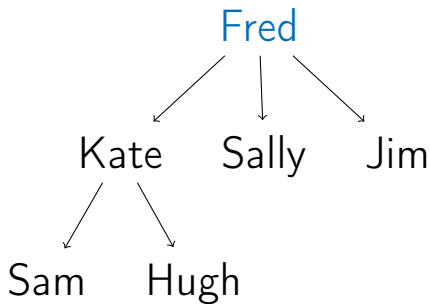
Height 1

Terminology



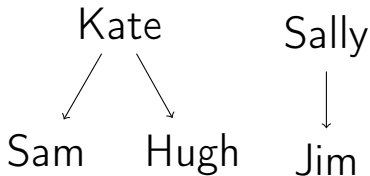
Height 2

Terminology



Height 3

Terminology



Forest:
collection of trees

Node contains:

- key
- children: list of children nodes
- (optional) parent

For binary tree, node contains:

- key
- left
- right
- (optional) parent

Height(*tree*)

```
if tree = nil:  
    return 0  
return 1 + Max(Height(tree.left),  
               Height(tree.right))
```

Size(*tree*)

```
if tree = nil  
    return 0  
return 1 + Size(tree.left) +  
           Size(tree.right)
```

Walking a Tree

Often we want to visit the nodes of a tree in a particular order.

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For example, print the nodes of the tree.

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- Depth-first: We completely traverse one sub-tree before exploring a sibling sub-tree.

Walking a Tree

Often we want to visit the nodes of a tree in a particular order.

For example, print the nodes of the tree.

- Depth-first: We completely traverse one sub-tree before exploring a sibling sub-tree.
- Breadth-first: We traverse all nodes at one level before progressing to the next level.

Depth-first

InOrderTraversal(*tree*)

```
if tree = nil:
```

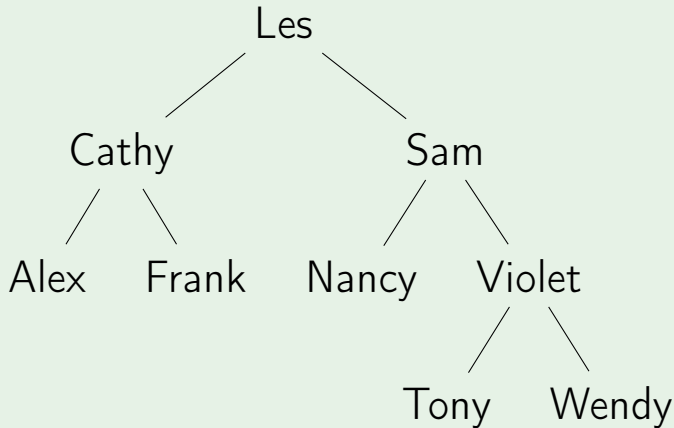
```
    return
```

```
InOrderTraversal(tree.left)
```

```
Print(tree.key)
```

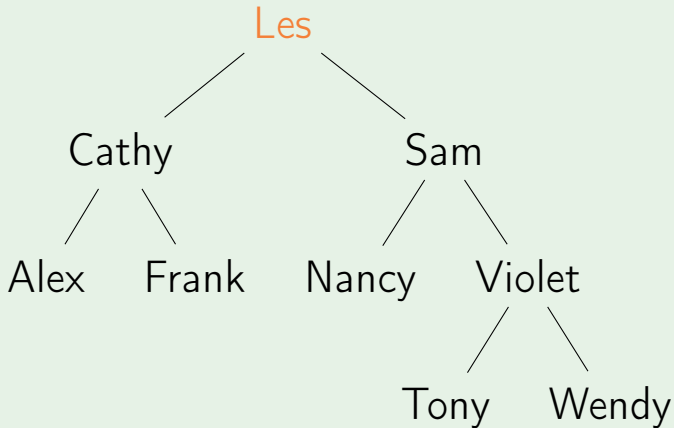
```
InOrderTraversal(tree.right)
```

InOrderTraversal



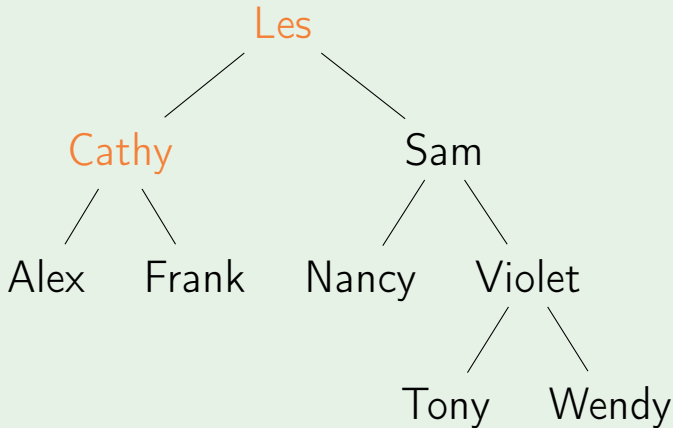
Output:

InOrderTraversal



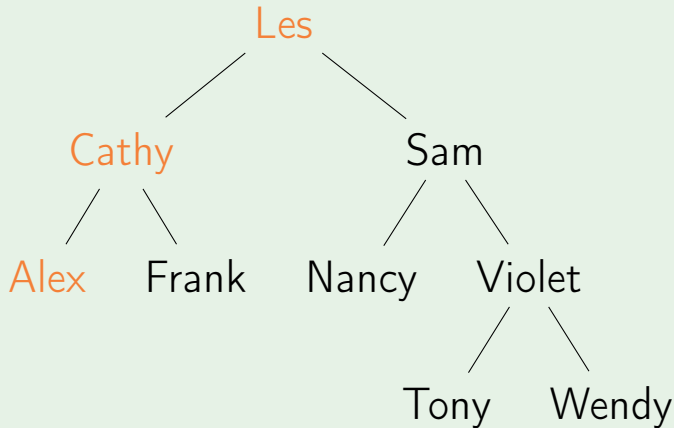
Output:

InOrderTraversal



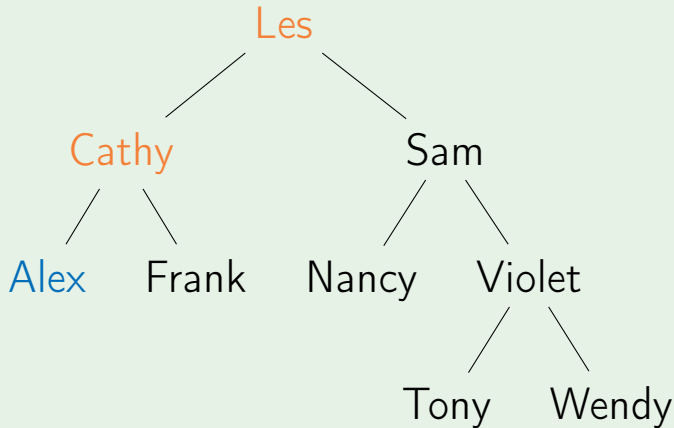
Output:

InOrderTraversal



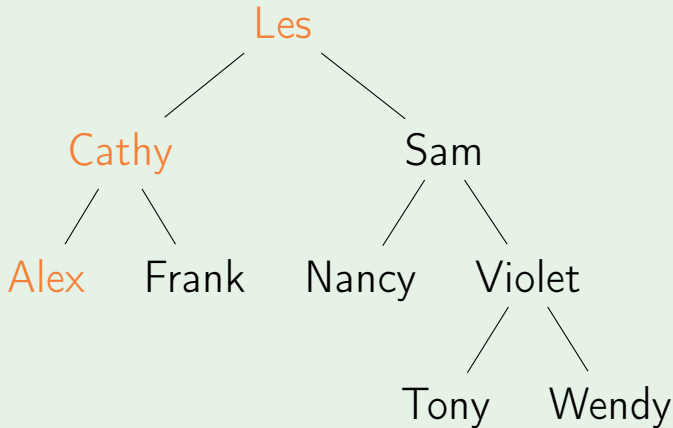
Output:

InOrderTraversal



Output: Alex

InOrderTraversal



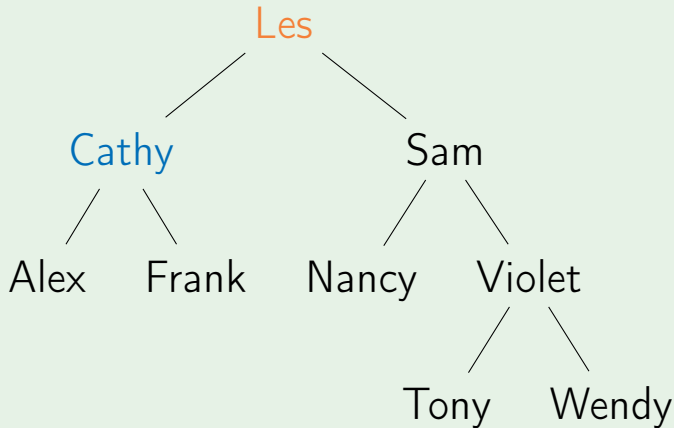
Output: Alex

InOrderTraversal



Output: Alex

InOrderTraversal



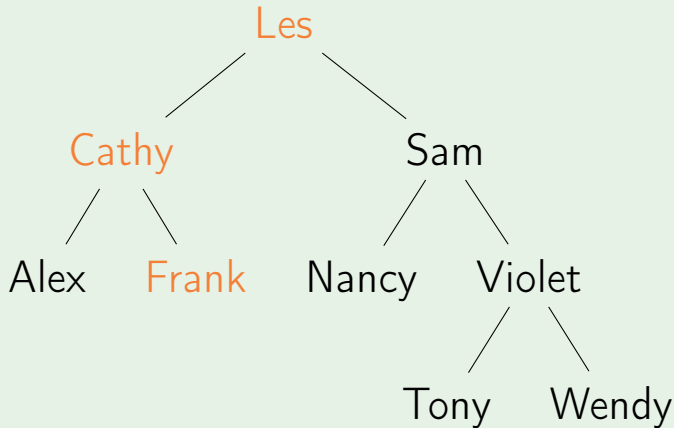
Output: Alex Cathy

InOrderTraversal



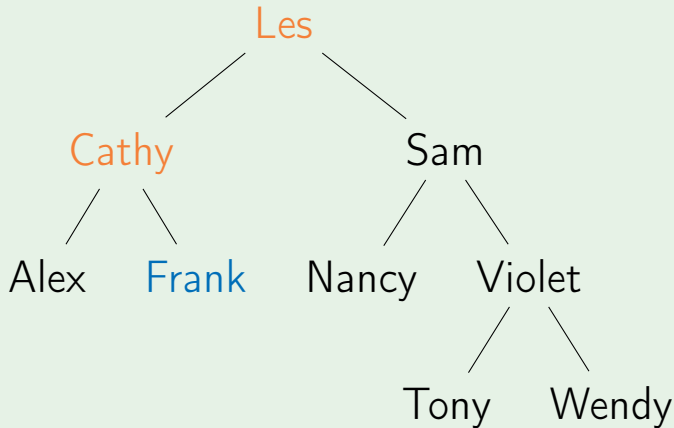
Output: Alex Cathy

InOrderTraversal



Output: Alex Cathy

InOrderTraversal



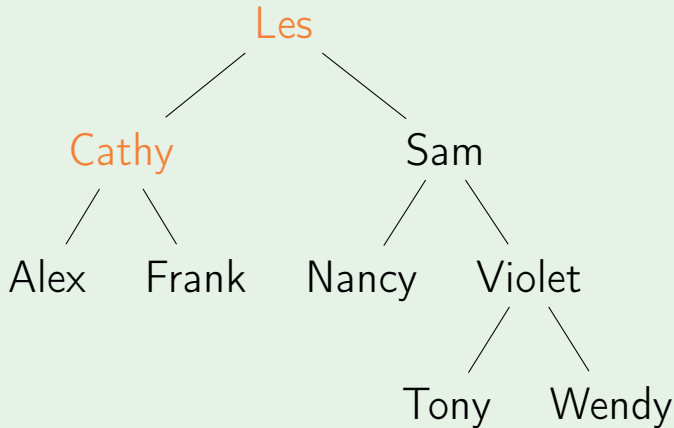
Output: Alex Cathy Frank

InOrderTraversal



Output: Alex Cathy Frank

InOrderTraversal



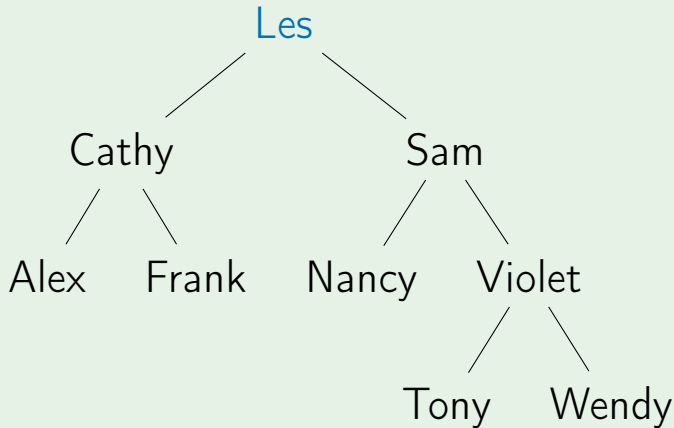
Output: Alex Cathy Frank

InOrderTraversal



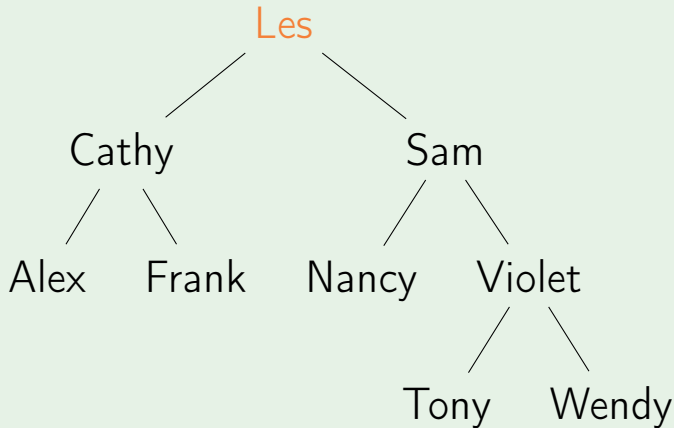
Output: Alex Cathy Frank

InOrderTraversal



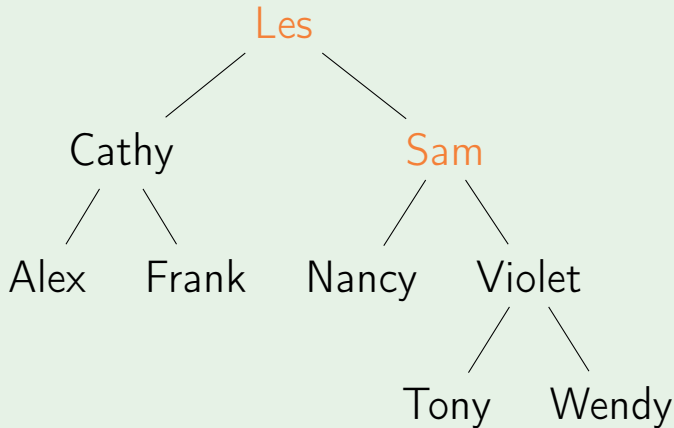
Output: Alex Cathy Frank Les

InOrderTraversal



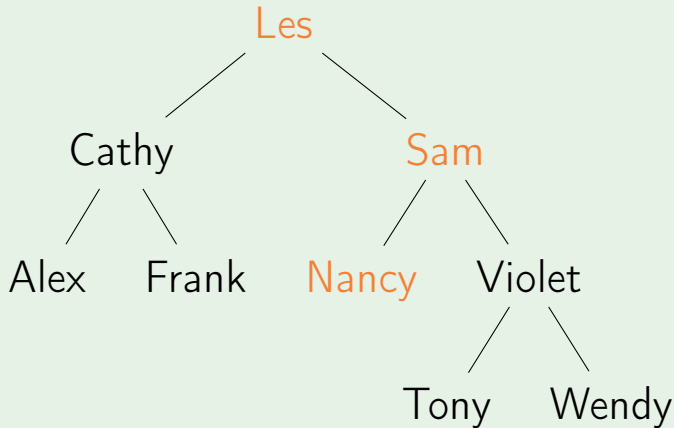
Output: Alex Cathy Frank Les

InOrderTraversal



Output: Alex Cathy Frank Les

InOrderTraversal



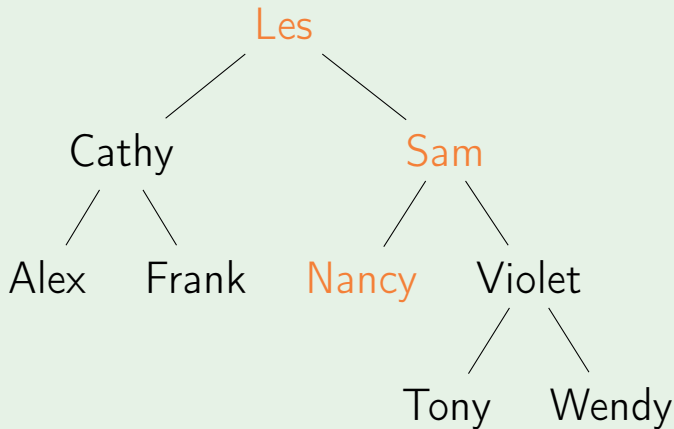
Output: Alex Cathy Frank Les

InOrderTraversal



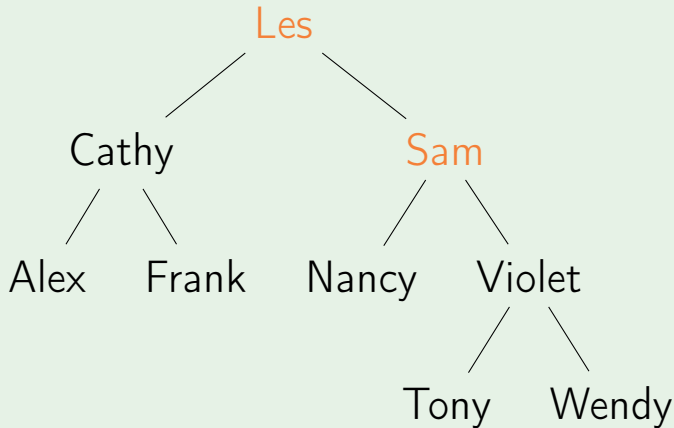
Output: Alex Cathy Frank Les Nancy

InOrderTraversal



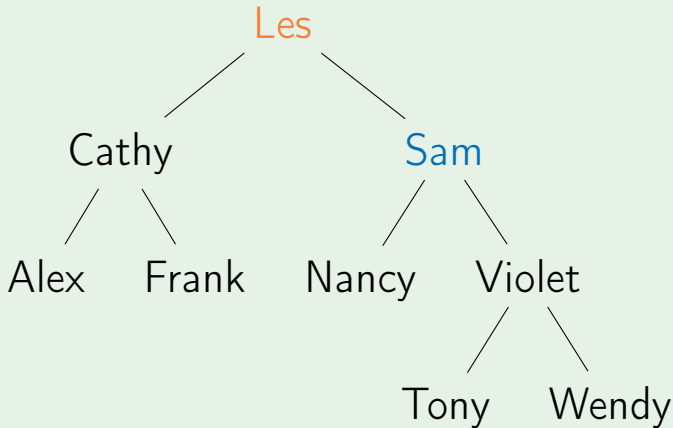
Output: Alex Cathy Frank Les Nancy

InOrderTraversal



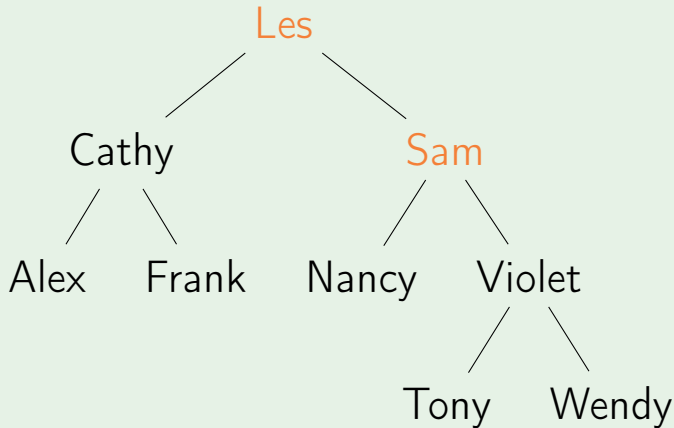
Output: Alex Cathy Frank Les Nancy

InOrderTraversal



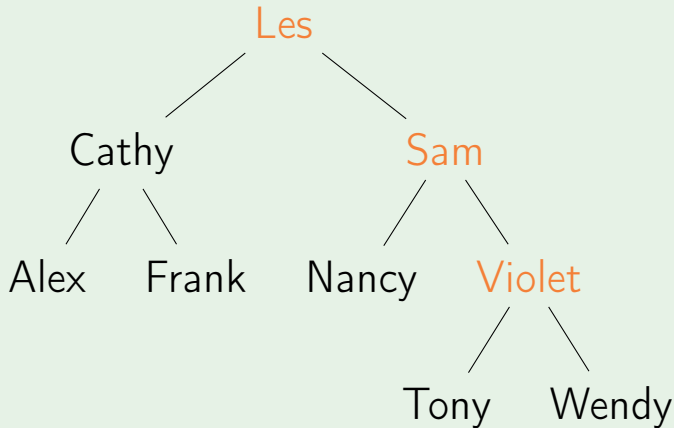
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



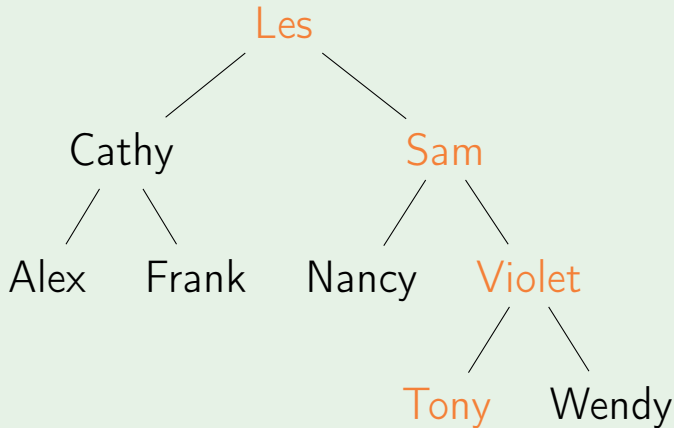
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



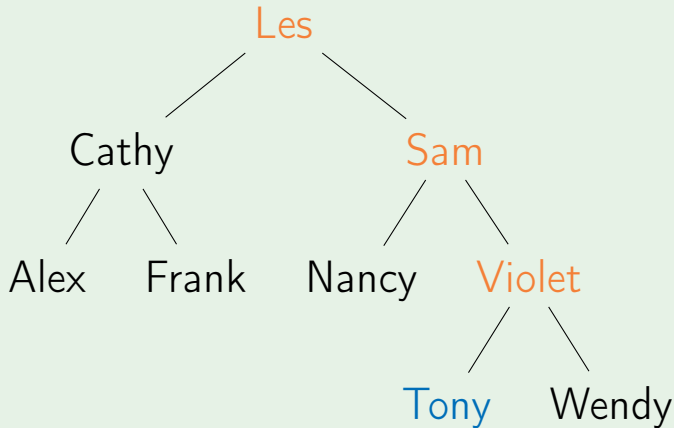
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



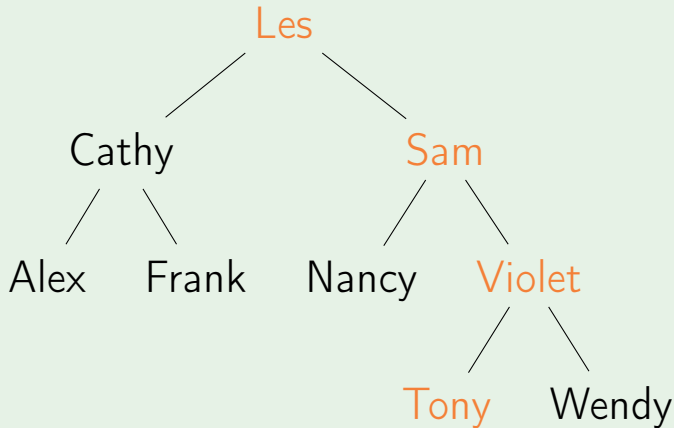
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



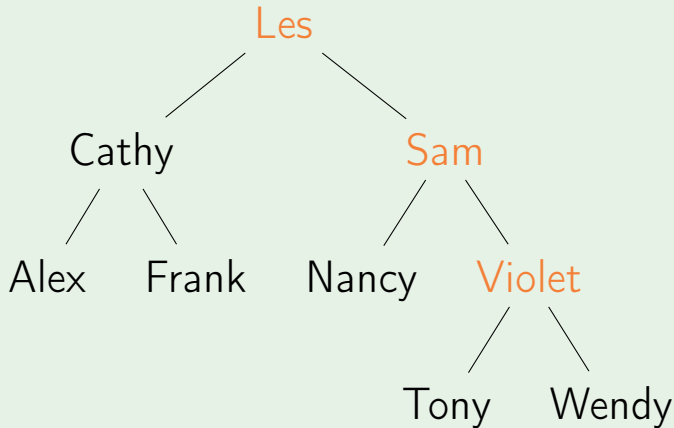
Output: Alex Cathy Frank Les Nancy Sam
Tony

InOrderTraversal



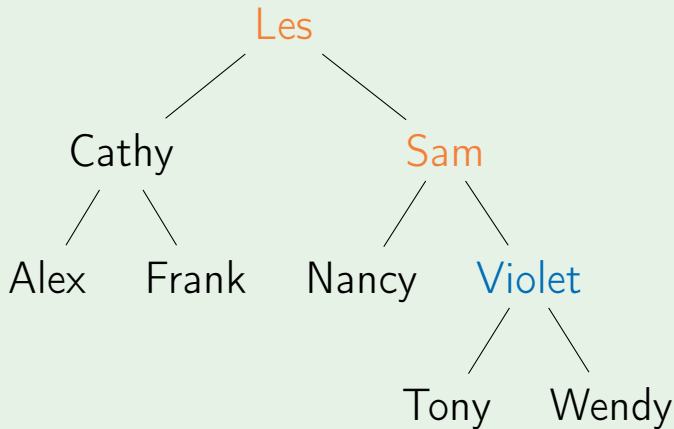
Output: Alex Cathy Frank Les Nancy Sam
Tony

InOrderTraversal



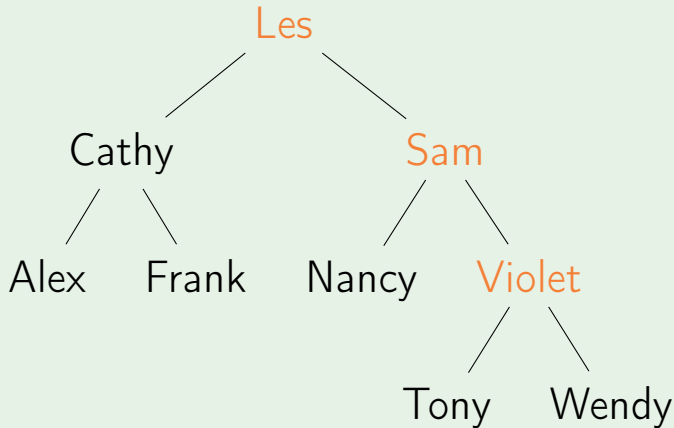
Output: Alex Cathy Frank Les Nancy Sam
Tony

InOrderTraversal



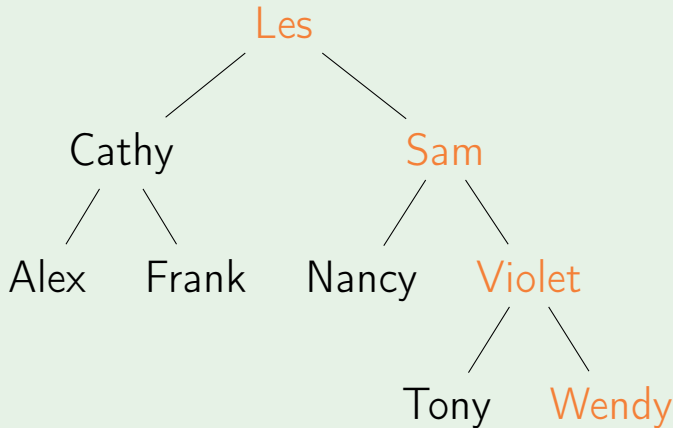
Output: Alex Cathy Frank Les Nancy Sam
Tony Violet

InOrderTraversal



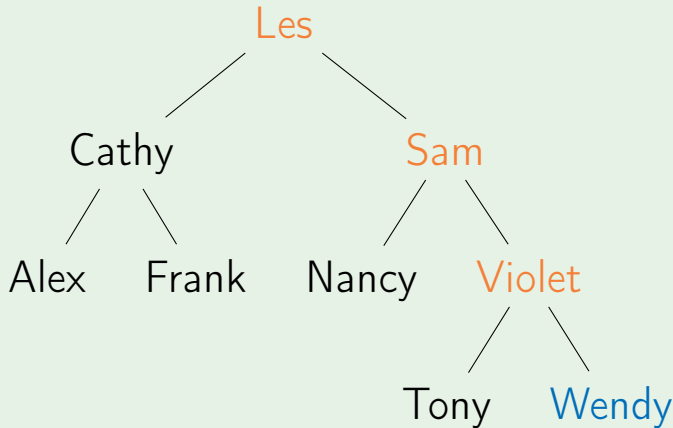
Output: Alex Cathy Frank Les Nancy Sam
Tony Violet

InOrderTraversal



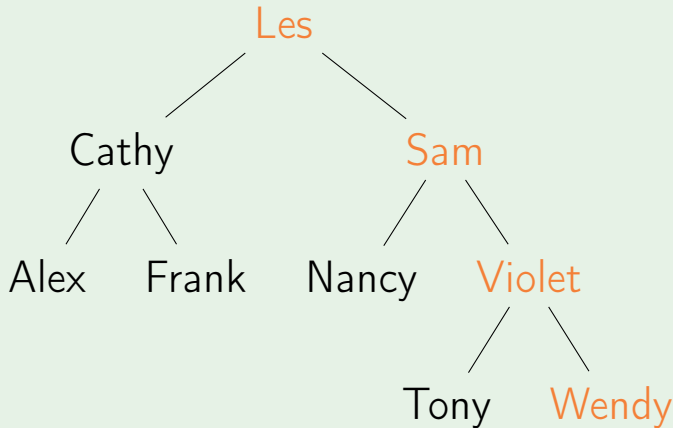
Output: Alex Cathy Frank Les Nancy Sam
Tony Violet

InOrderTraversal



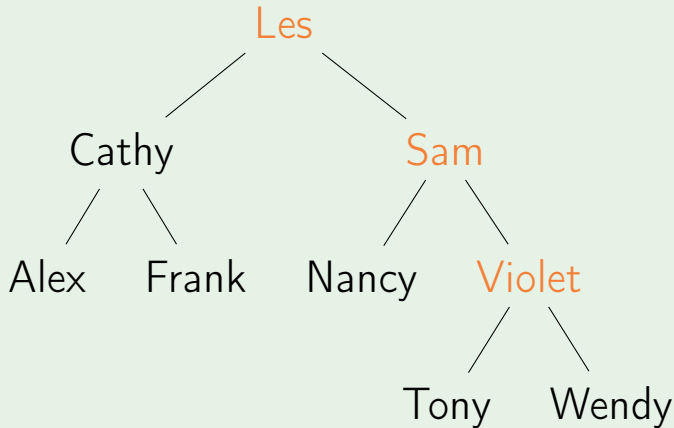
Output: Alex Cathy Frank Les Nancy Sam
Tony Violet Wendy

InOrderTraversal



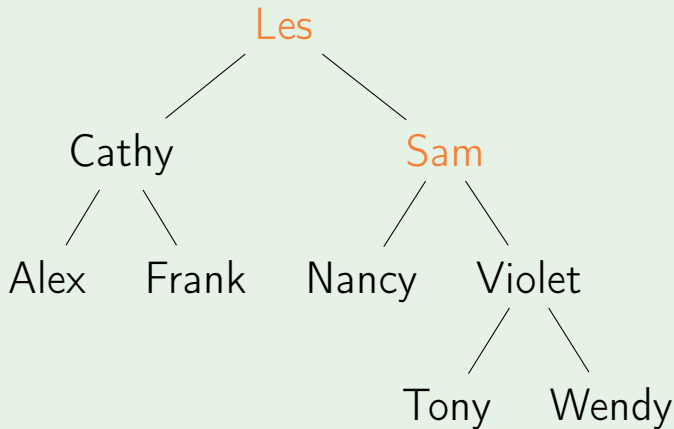
Output: Alex Cathy Frank Les Nancy Sam
Tony Violet Wendy

InOrderTraversal



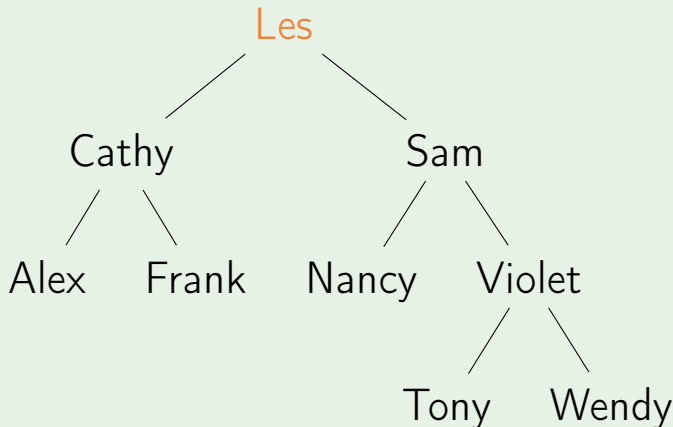
Output: Alex Cathy Frank Les Nancy Sam
Tony Violet Wendy

InOrderTraversal



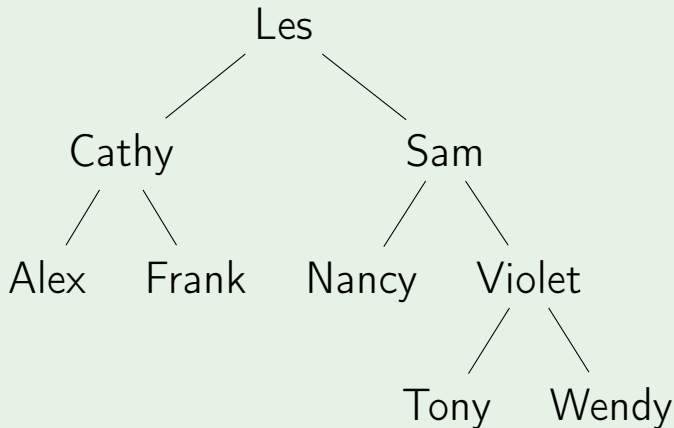
Output: Alex Cathy Frank Les Nancy Sam
Tony Violet Wendy

InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam
Tony Violet Wendy

InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam
Tony Violet Wendy

Depth-first

```
PreOrderTraversal(tree)
```

```
if tree = nil:
```

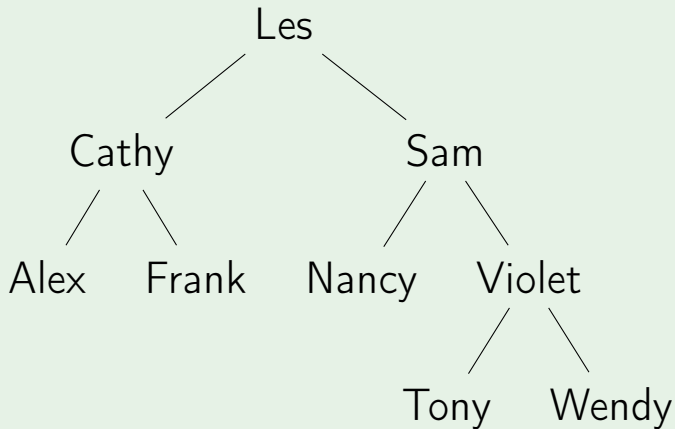
```
    return
```

```
Print(tree.key)
```

```
PreOrderTraversal(tree.left)
```

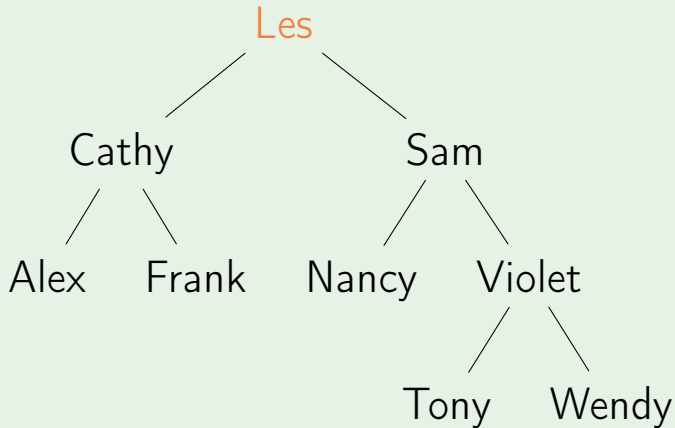
```
PreOrderTraversal(tree.right)
```

PreOrderTraversal



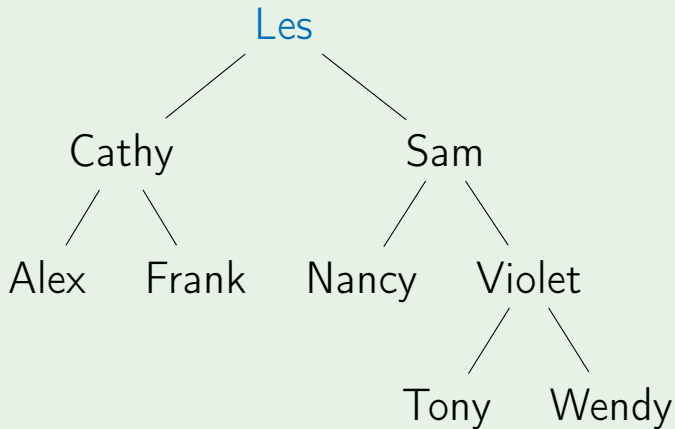
Output:

PreOrderTraversal



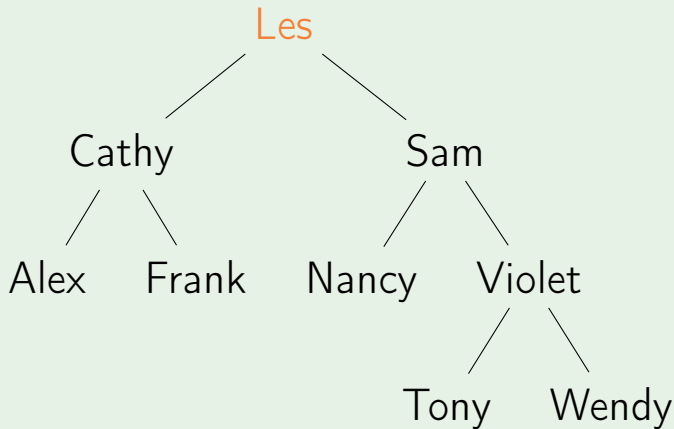
Output:

PreOrderTraversal



Output: Les

PreOrderTraversal



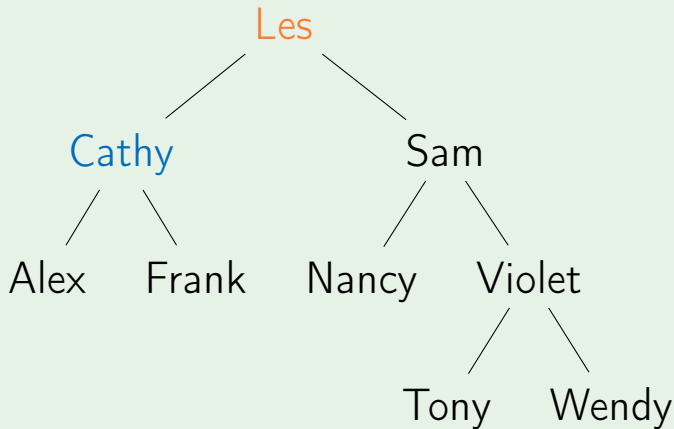
Output: Les

PreOrderTraversal



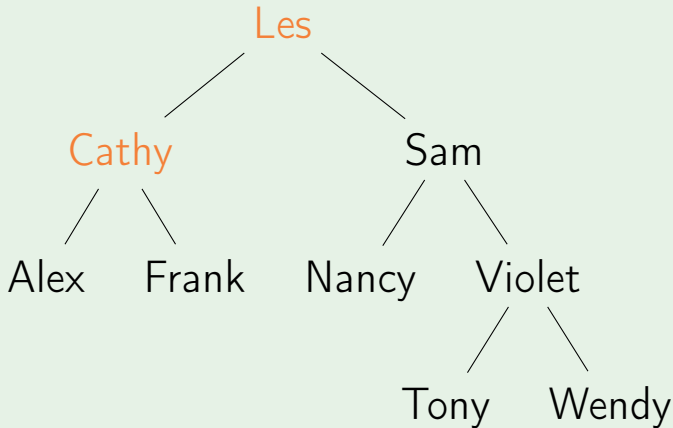
Output: Les

PreOrderTraversal



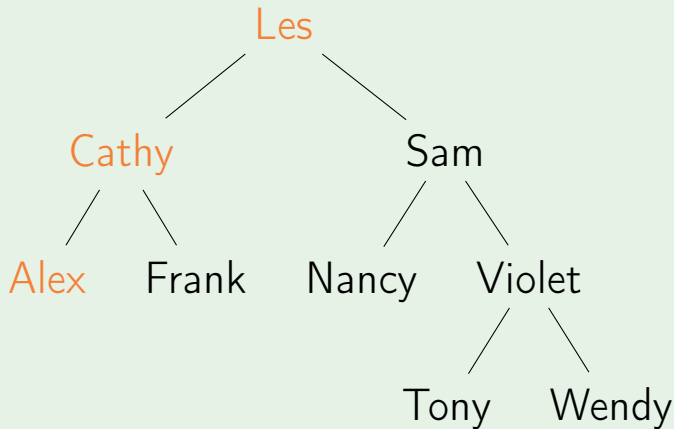
Output: Les Cathy

PreOrderTraversal



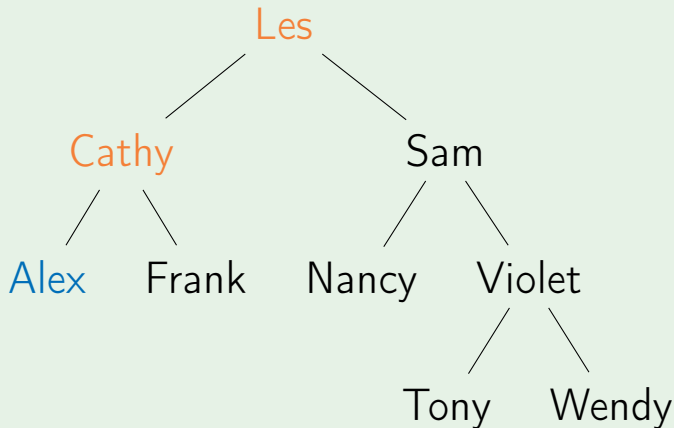
Output: Les Cathy

PreOrderTraversal



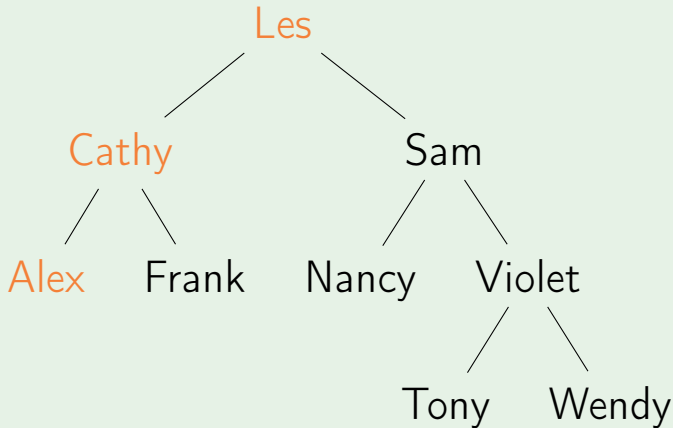
Output: Les Cathy

PreOrderTraversal



Output: Les Cathy Alex

PreOrderTraversal



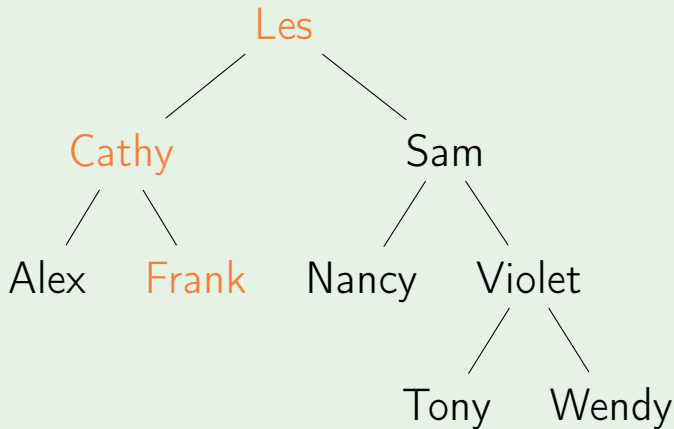
Output: Les Cathy Alex

PreOrderTraversal



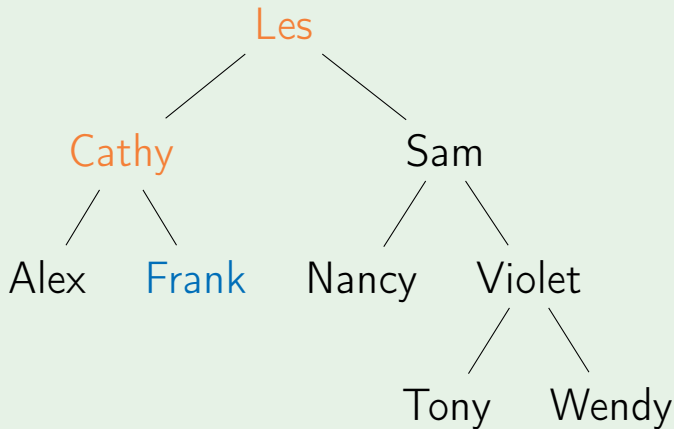
Output: Les Cathy Alex

PreOrderTraversal



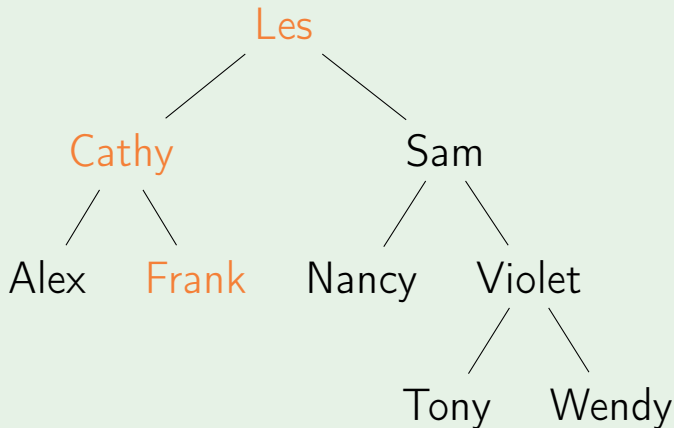
Output: Les Cathy Alex

PreOrderTraversal



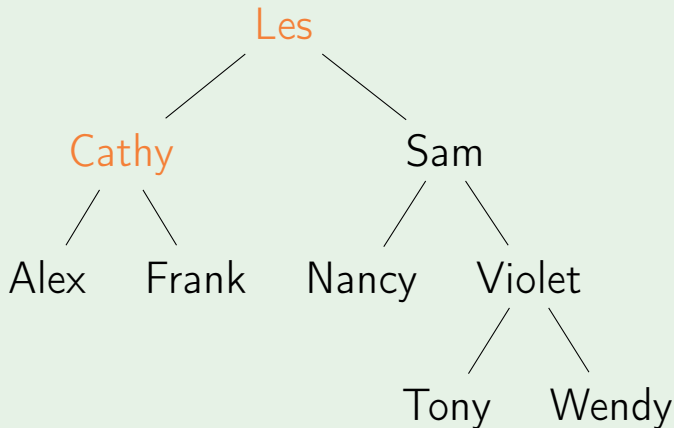
Output: Les Cathy Alex Frank

PreOrderTraversal



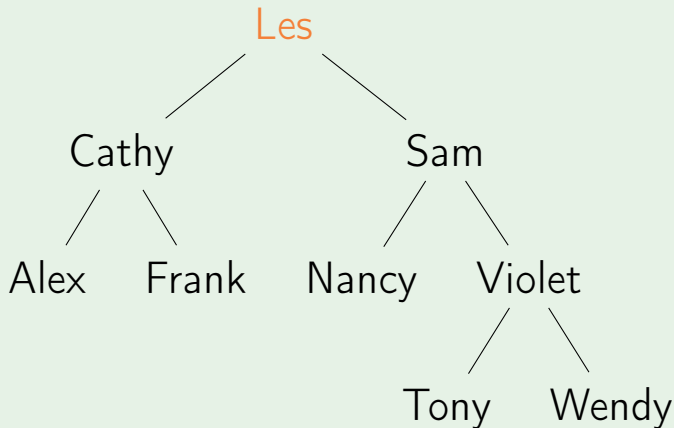
Output: Les Cathy Alex Frank

PreOrderTraversal



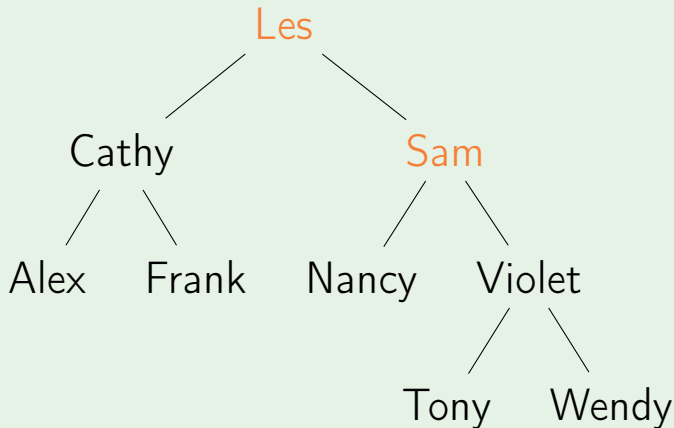
Output: Les Cathy Alex Frank

PreOrderTraversal



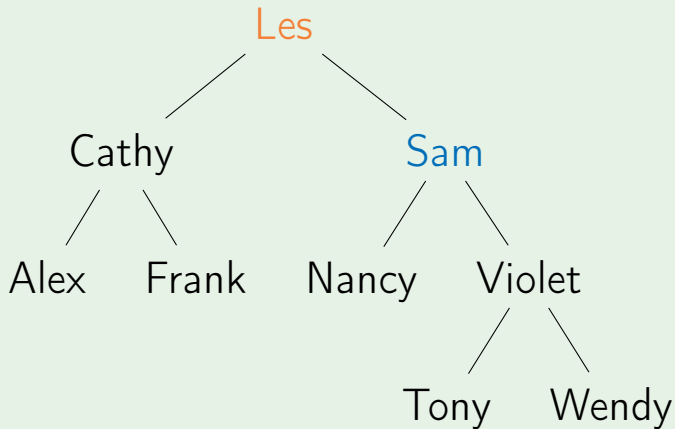
Output: Les Cathy Alex Frank

PreOrderTraversal



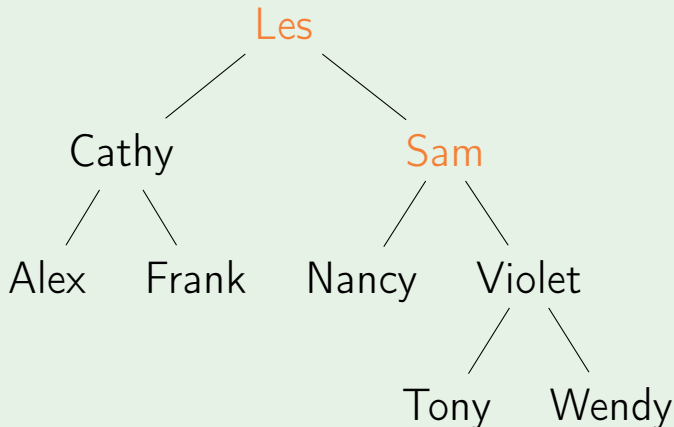
Output: Les Cathy Alex Frank

PreOrderTraversal



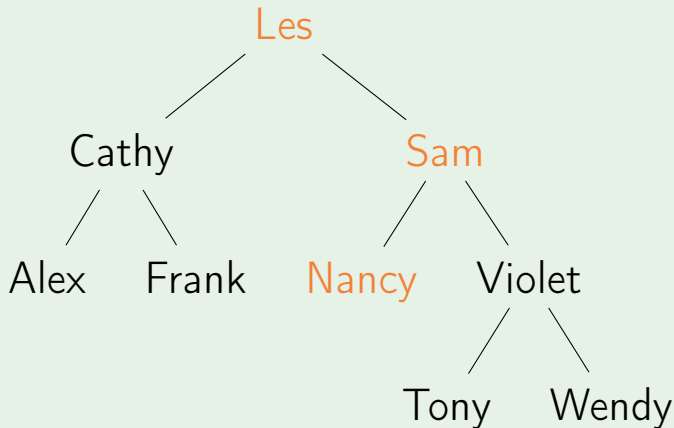
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



Output: Les Cathy Alex Frank Sam

PreOrderTraversal



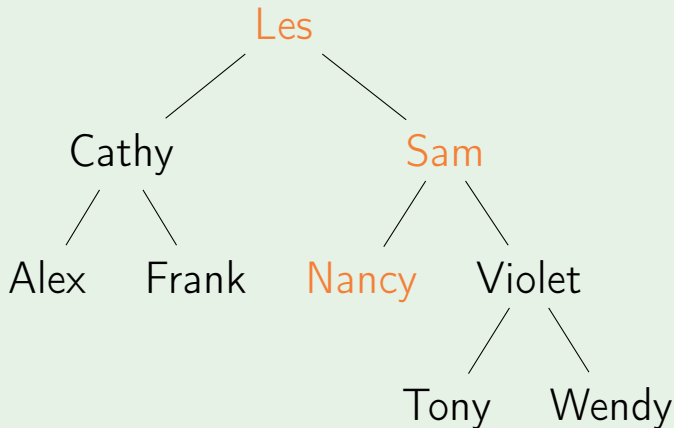
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



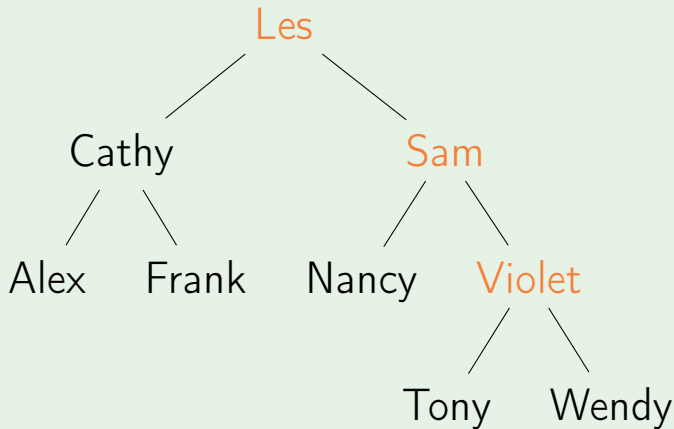
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



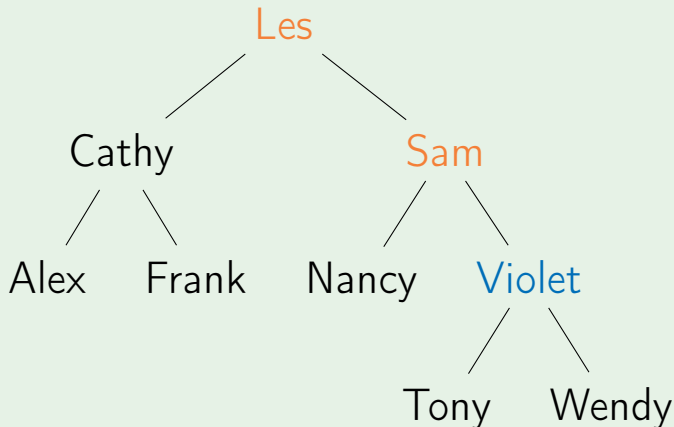
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



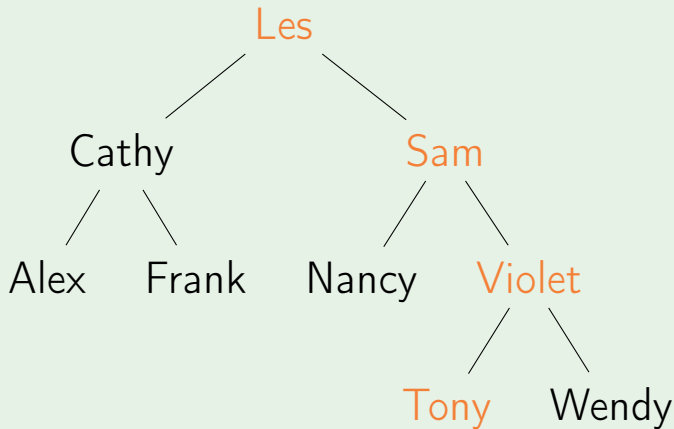
Output: Les Cathy Alex Frank Sam Nancy
Violet

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy
Violet

PreOrderTraversal



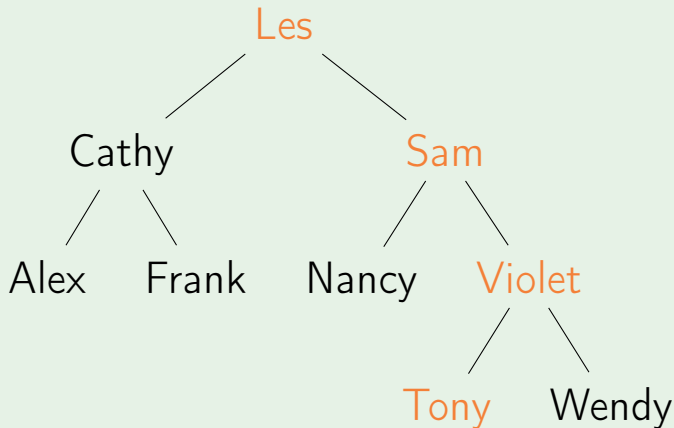
Output: Les Cathy Alex Frank Sam Nancy
Violet

PreOrderTraversal



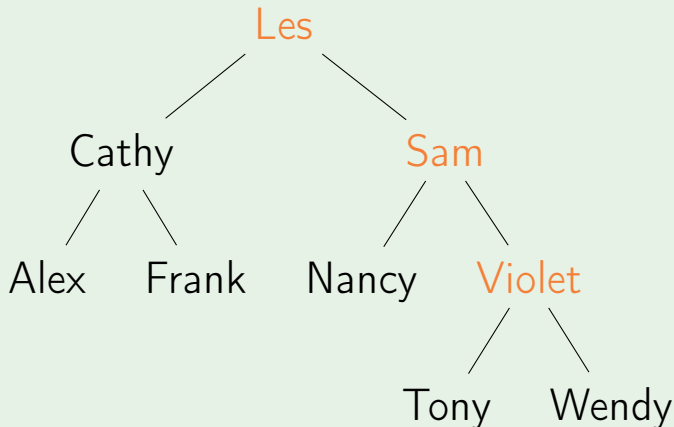
Output: Les Cathy Alex Frank Sam Nancy
Violet Tony

PreOrderTraversal



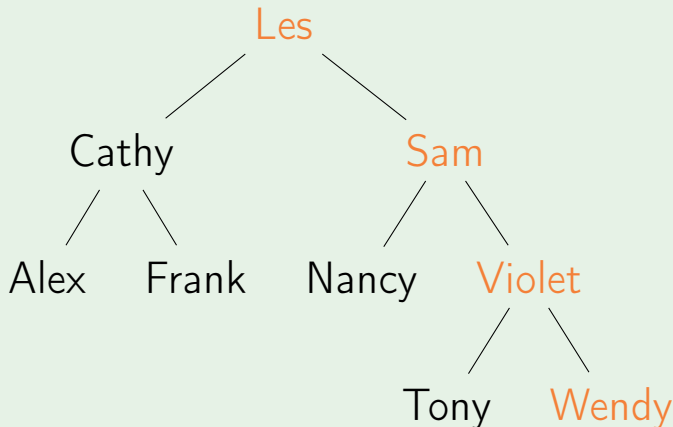
Output: Les Cathy Alex Frank Sam Nancy
Violet Tony

PreOrderTraversal



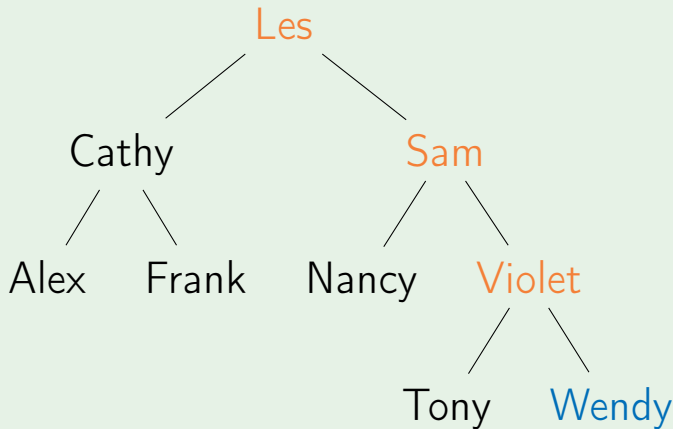
Output: Les Cathy Alex Frank Sam Nancy
Violet Tony

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy
Violet Tony

PreOrderTraversal



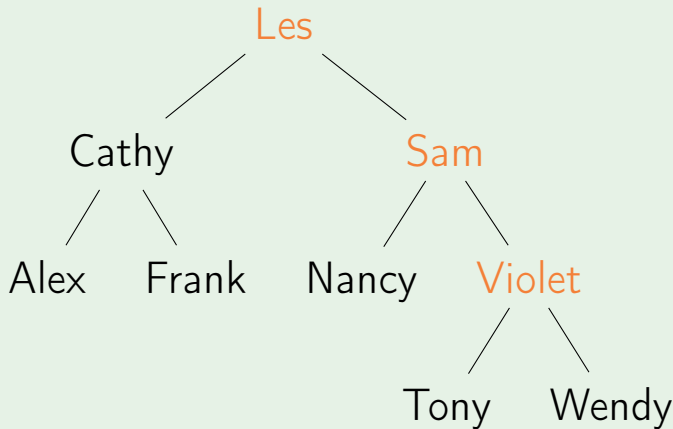
Output: Les Cathy Alex Frank Sam Nancy
Violet Tony Wendy

PreOrderTraversal



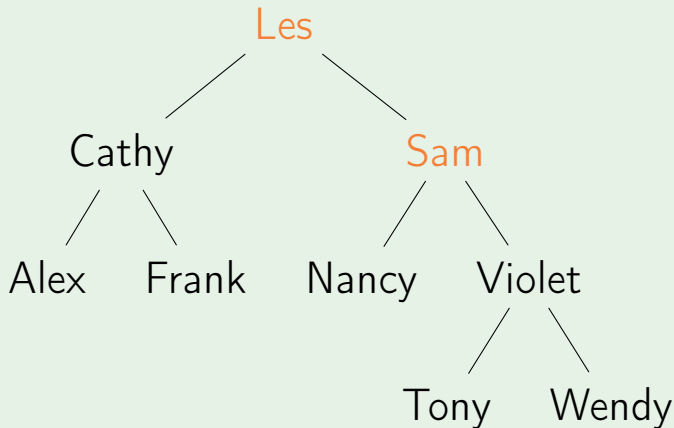
Output: Les Cathy Alex Frank Sam Nancy
Violet Tony Wendy

PreOrderTraversal



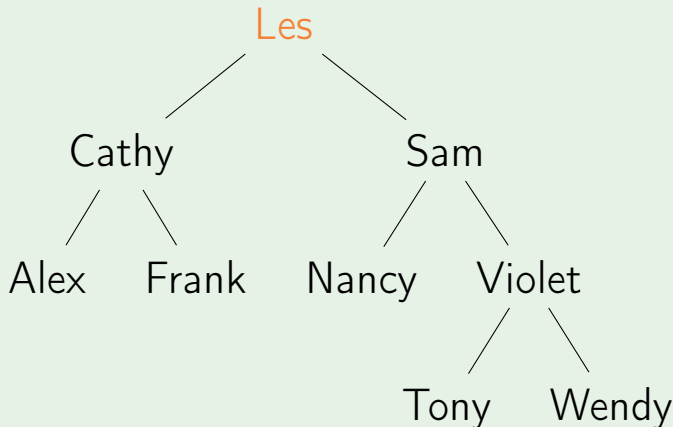
Output: Les Cathy Alex Frank Sam Nancy
Violet Tony Wendy

PreOrderTraversal



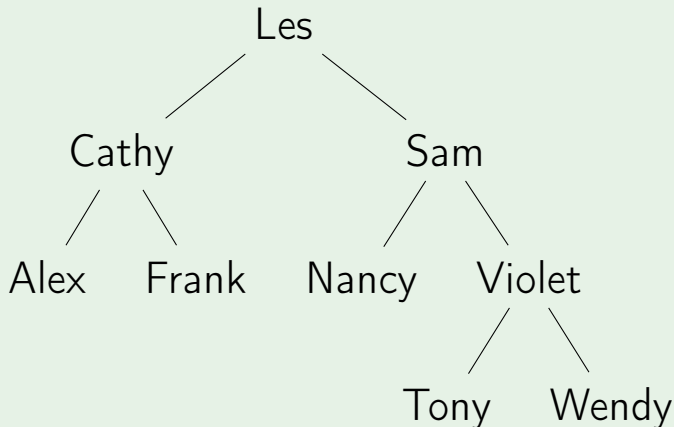
Output: Les Cathy Alex Frank Sam Nancy
Violet Tony Wendy

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy
Violet Tony Wendy

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy
Violet Tony Wendy

Depth-first

PostOrderTraversal(*tree*)

```
if tree = nil:
```

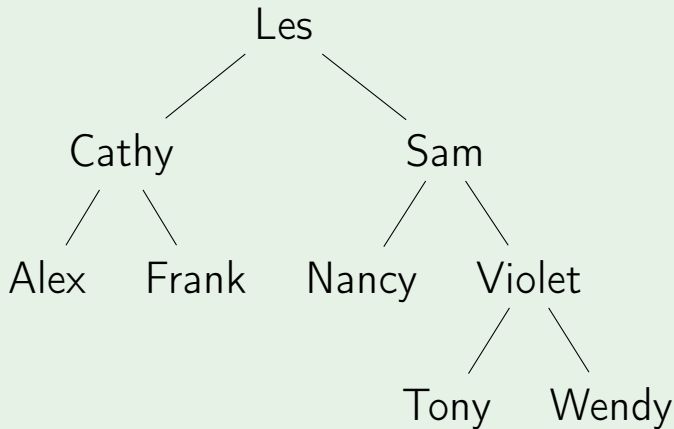
```
    return
```

```
PostOrderTraversal(tree.left)
```

```
PostOrderTraversal(tree.right)
```

```
Print(tree.key)
```

PostOrderTraversal



Output:

PostOrderTraversal



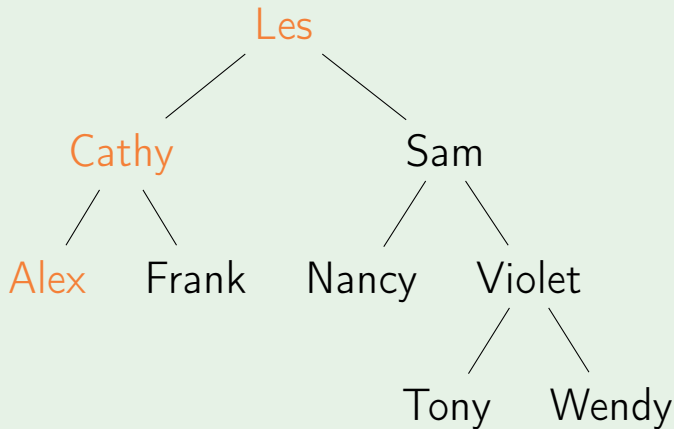
Output:

PostOrderTraversal



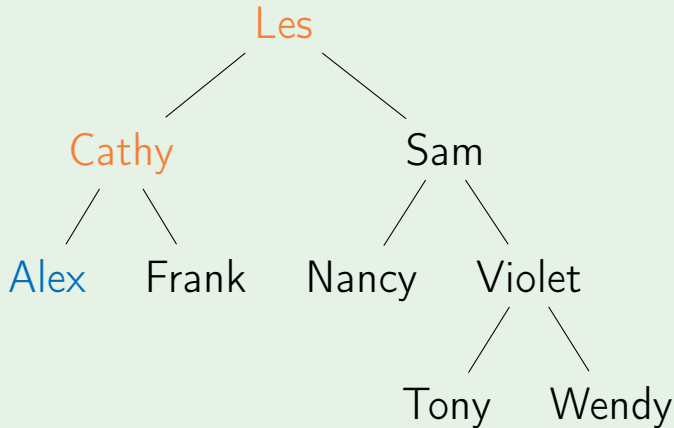
Output:

PostOrderTraversal



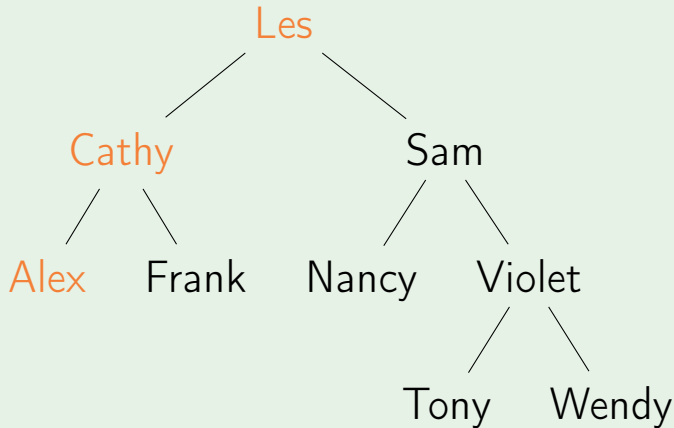
Output:

PostOrderTraversal



Output: Alex

PostOrderTraversal



Output: Alex

PostOrderTraversal



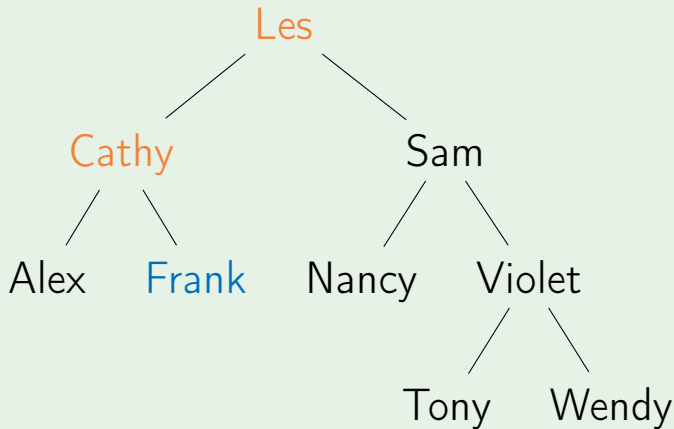
Output: Alex

PostOrderTraversal



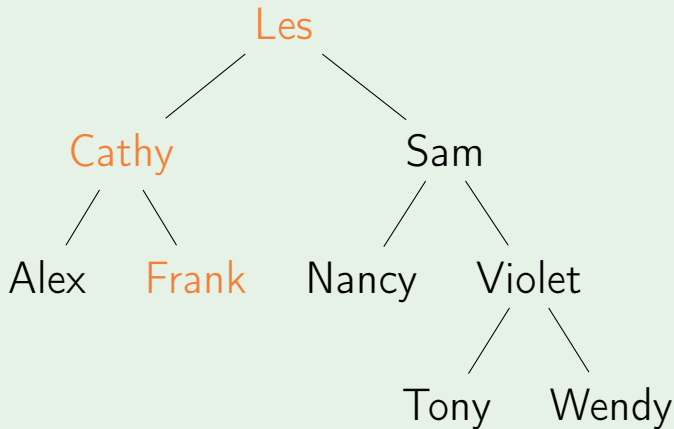
Output: Alex

PostOrderTraversal



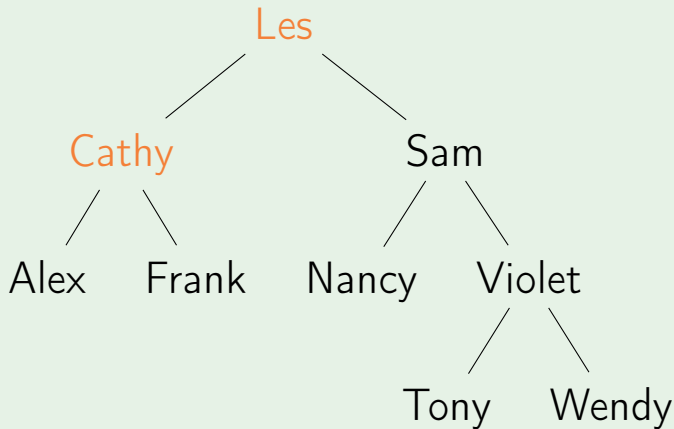
Output: Alex Frank

PostOrderTraversal



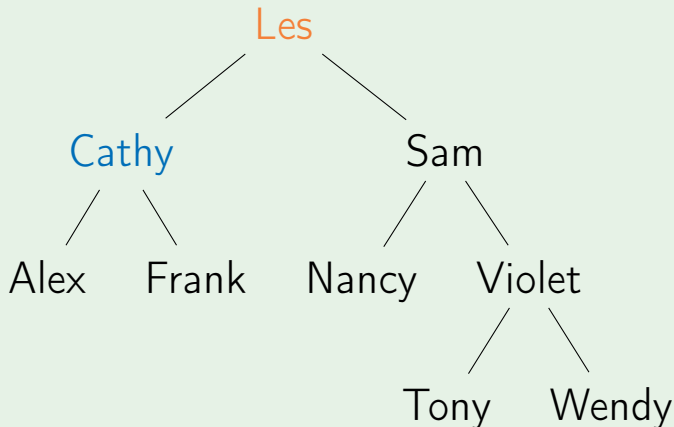
Output: Alex Frank

PostOrderTraversal



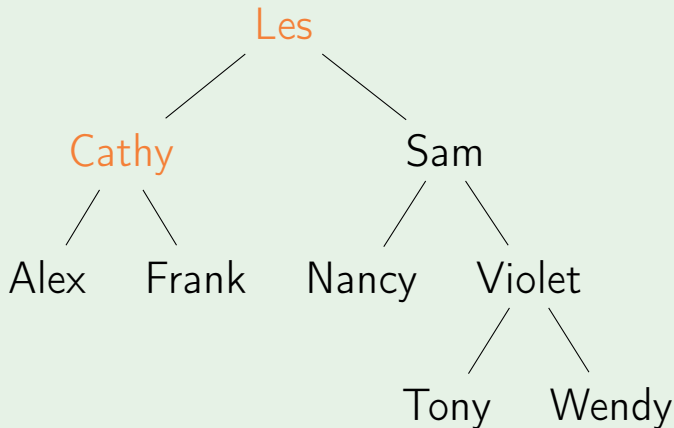
Output: Alex Frank

PostOrderTraversal



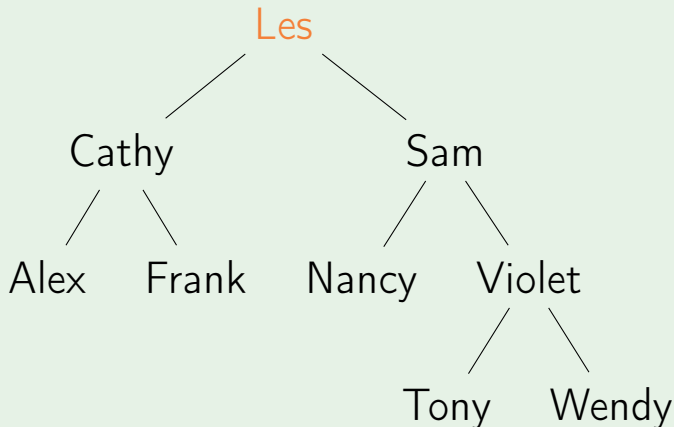
Output: Alex Frank Cathy

PostOrderTraversal



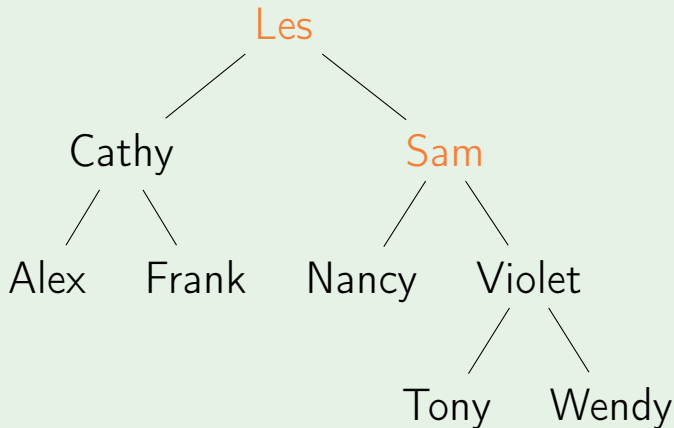
Output: Alex Frank Cathy

PostOrderTraversal



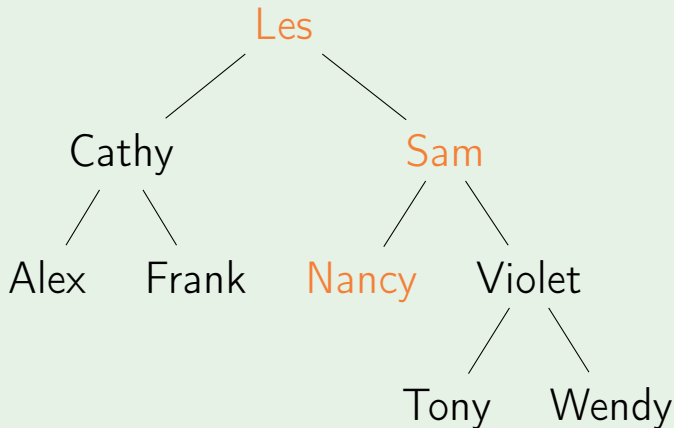
Output: Alex Frank Cathy

PostOrderTraversal



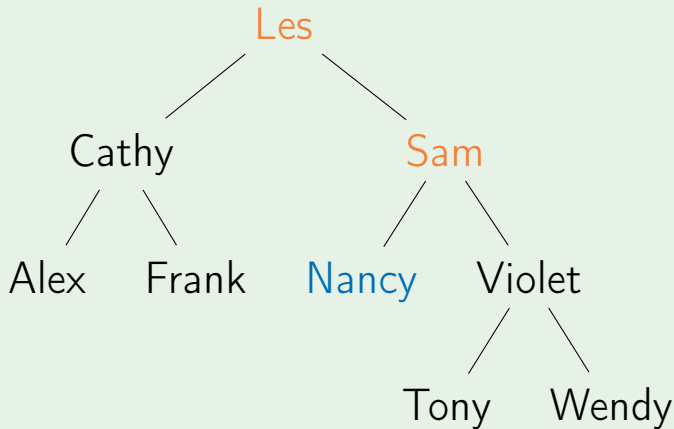
Output: Alex Frank Cathy

PostOrderTraversal



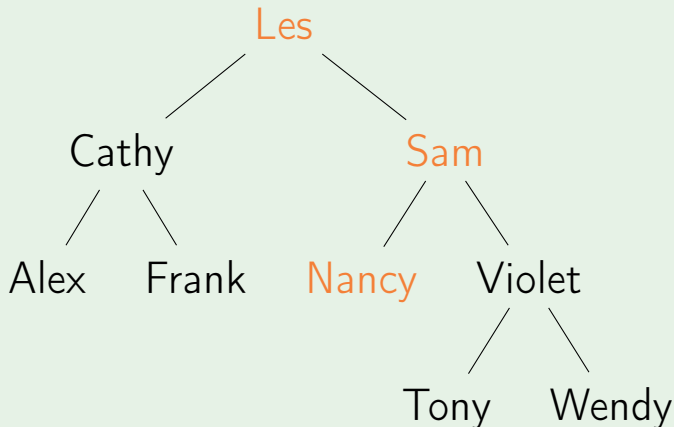
Output: Alex Frank Cathy

PostOrderTraversal



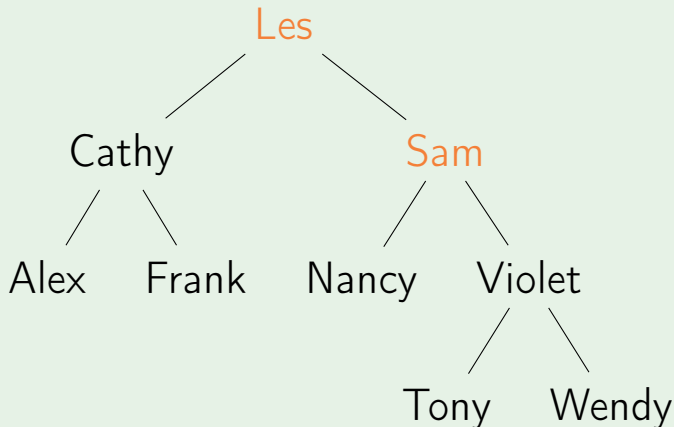
Output: Alex Frank Cathy Nancy

PostOrderTraversal



Output: Alex Frank Cathy Nancy

PostOrderTraversal



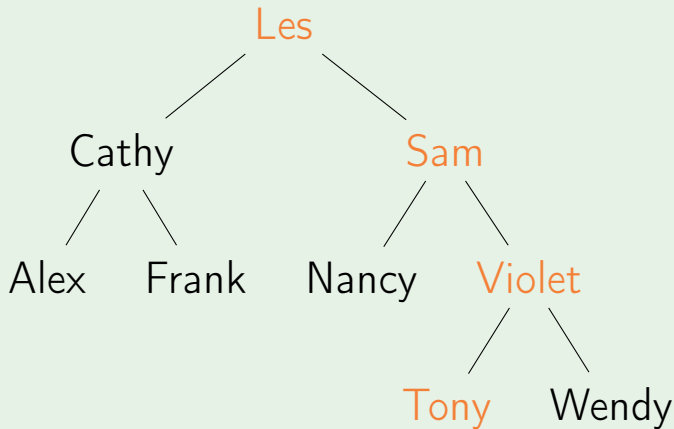
Output: Alex Frank Cathy Nancy

PostOrderTraversal



Output: Alex Frank Cathy Nancy

PostOrderTraversal



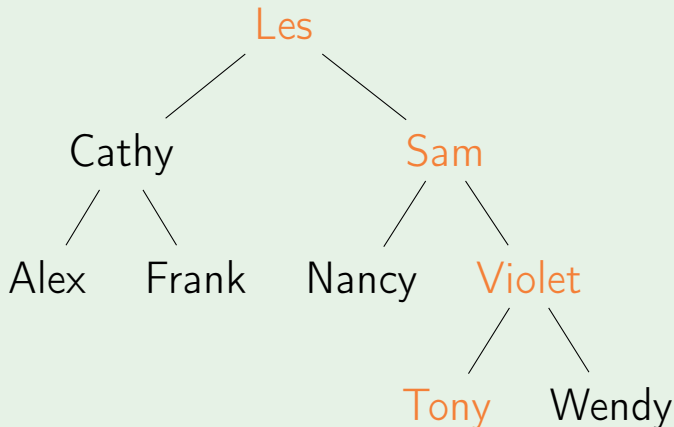
Output: Alex Frank Cathy Nancy

PostOrderTraversal



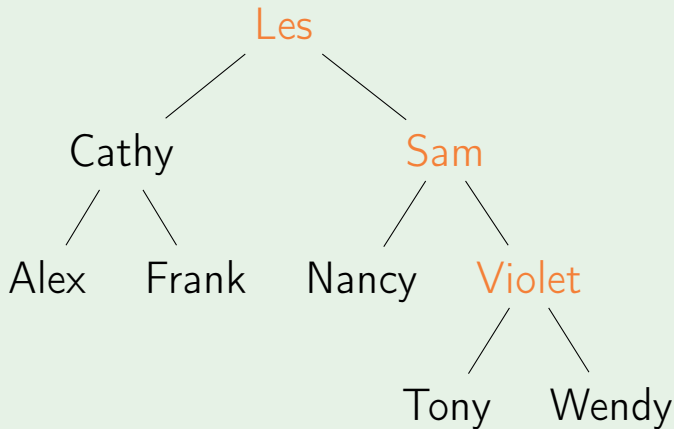
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



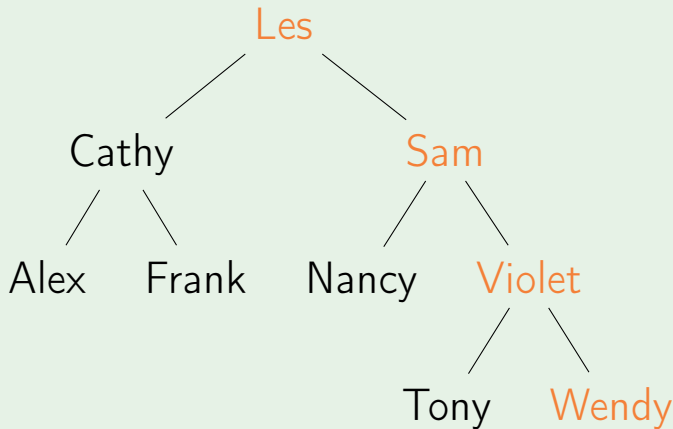
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



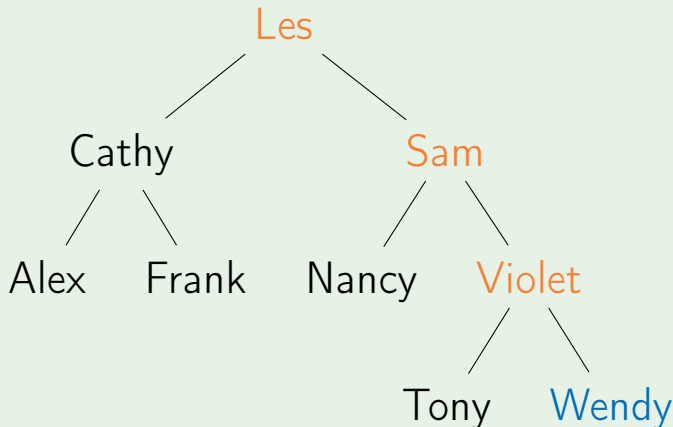
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



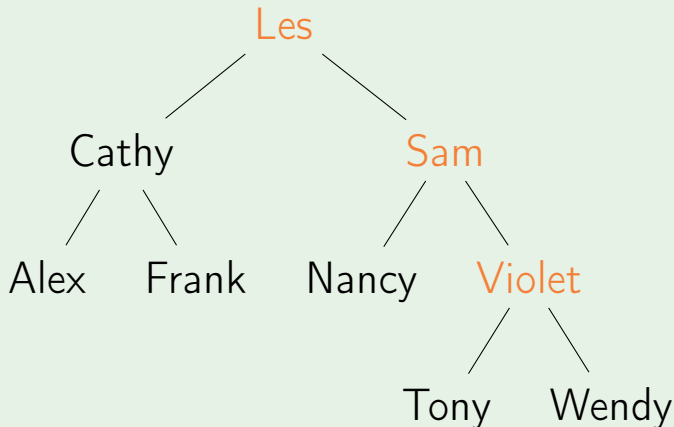
Output: Alex Frank Cathy Nancy Tony
Wendy

PostOrderTraversal



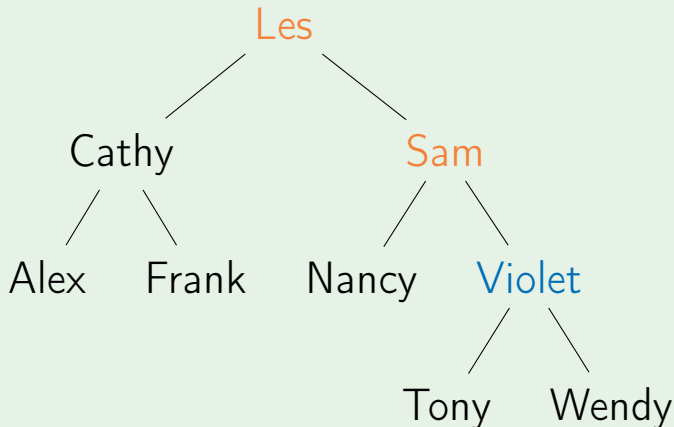
Output: Alex Frank Cathy Nancy Tony
Wendy

PostOrderTraversal



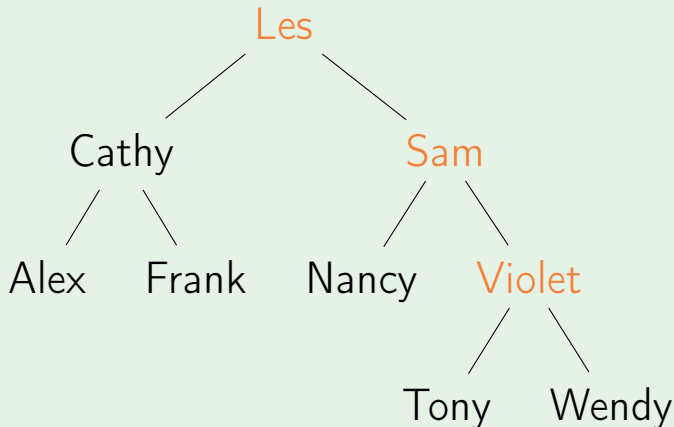
Output: Alex Frank Cathy Nancy Tony
Wendy

PostOrderTraversal



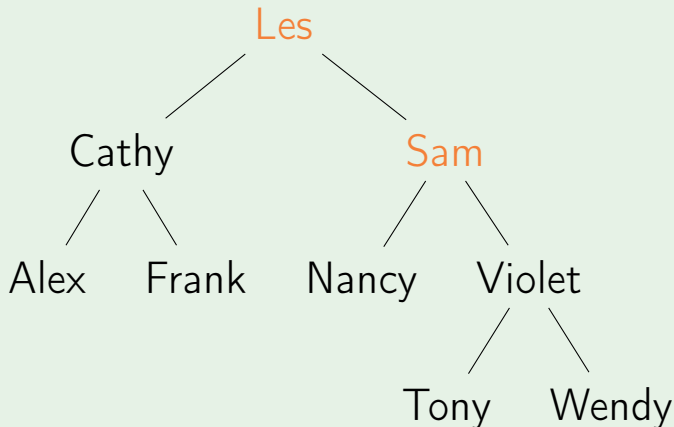
Output: Alex Frank Cathy Nancy Tony
Wendy Violet

PostOrderTraversal



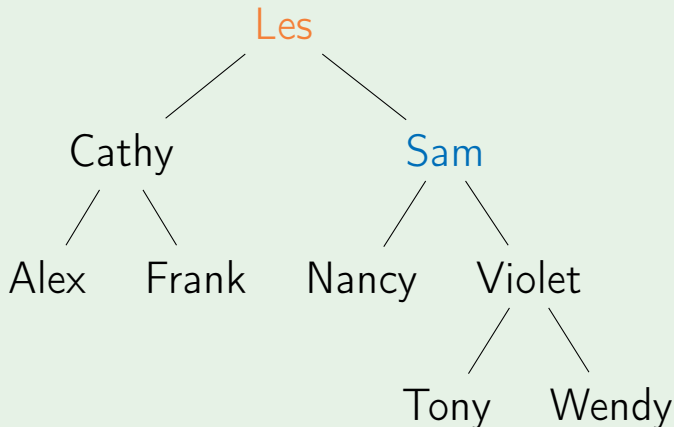
Output: Alex Frank Cathy Nancy Tony
Wendy Violet

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony
Wendy Violet

PostOrderTraversal



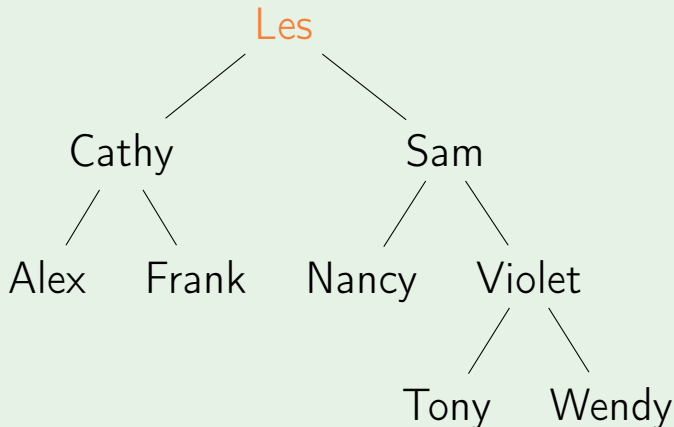
Output: Alex Frank Cathy Nancy Tony
Wendy Violet Sam

PostOrderTraversal



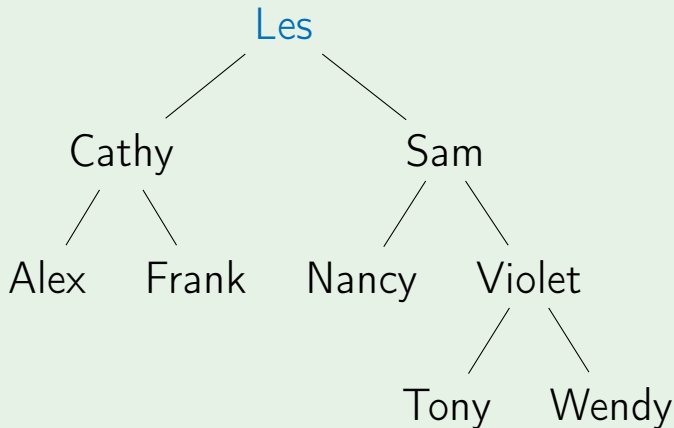
Output: Alex Frank Cathy Nancy Tony
Wendy Violet Sam

PostOrderTraversal



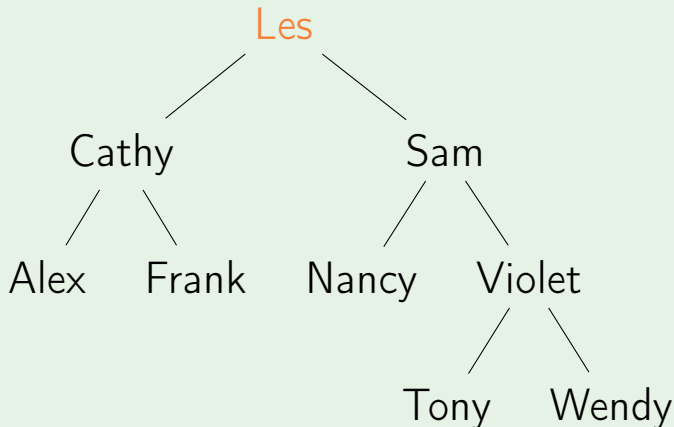
Output: Alex Frank Cathy Nancy Tony
Wendy Violet Sam

PostOrderTraversal



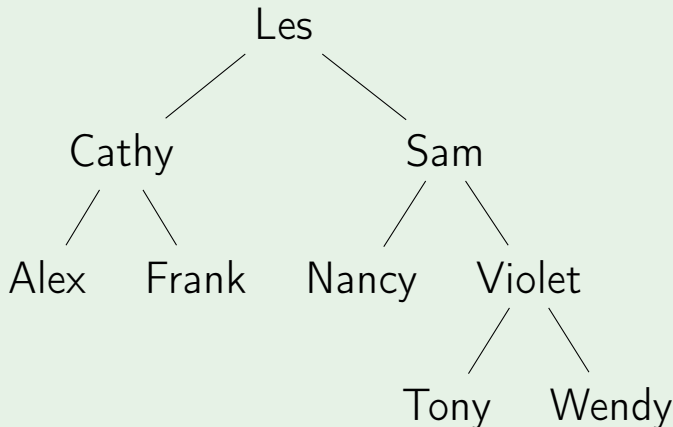
Output: Alex Frank Cathy Nancy Tony
Wendy Violet Sam Les

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony
Wendy Violet Sam Les

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony
Wendy Violet Sam Les

Breadth-first

LevelTraversal(*tree*)

```
if tree = nil:    return
```

```
    Queue q
```

```
    q.Enqueue(tree)
```

Breadth-first

LevelTraversal(*tree*)

```
if tree = nil:    return  
  
Queue q  
q.Enqueue(tree)  
while not q.Empty():  
    node  $\leftarrow$  q.Dequeue()
```

Breadth-first

LevelTraversal(*tree*)

```
if tree = nil:    return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node ← q.Dequeue()
    Print(node)
```

Breadth-first

LevelTraversal(*tree*)

```
if tree = nil:    return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node  $\leftarrow$  q.Dequeue()
    Print(node)
    if node.left  $\neq$  nil:
        q.Enqueue(node.left)
```

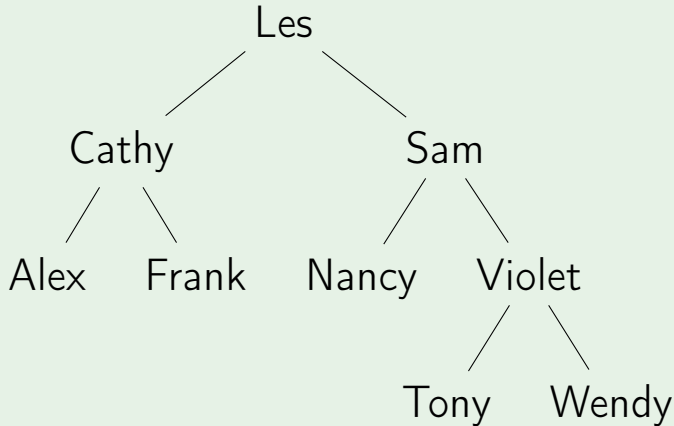
Breadth-first

LevelTraversal(*tree*)

```
if tree = nil:    return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node  $\leftarrow$  q.Dequeue()
    Print(node)
    if node.left  $\neq$  nil:
        q.Enqueue(node.left)
    if node.right  $\neq$  nil:
        q.Enqueue(node.right)
```

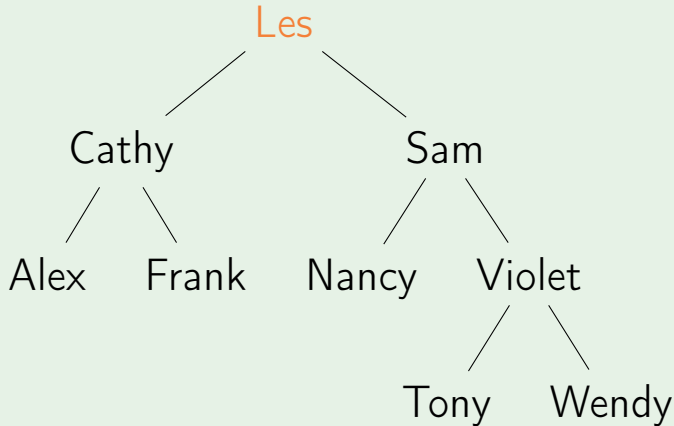
LevelTraversal



Output:

Queue: Les

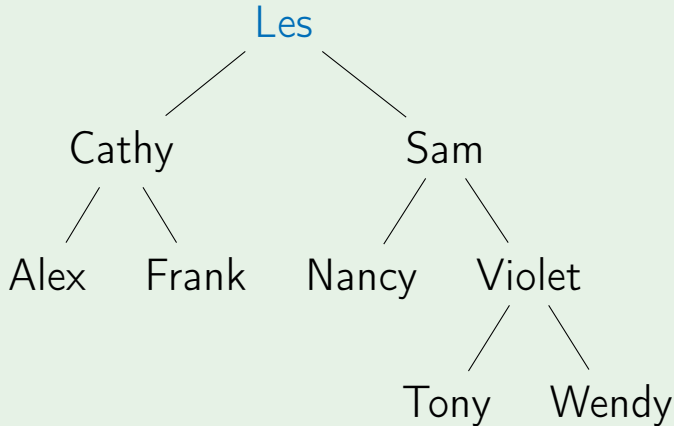
LevelTraversal



Output:

Queue:

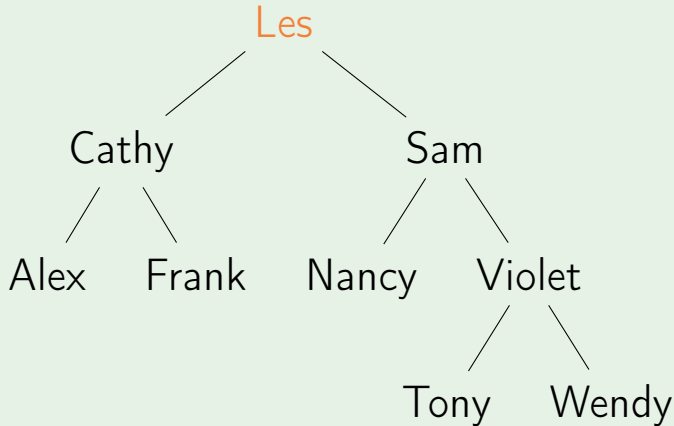
LevelTraversal



Output: Les

Queue:

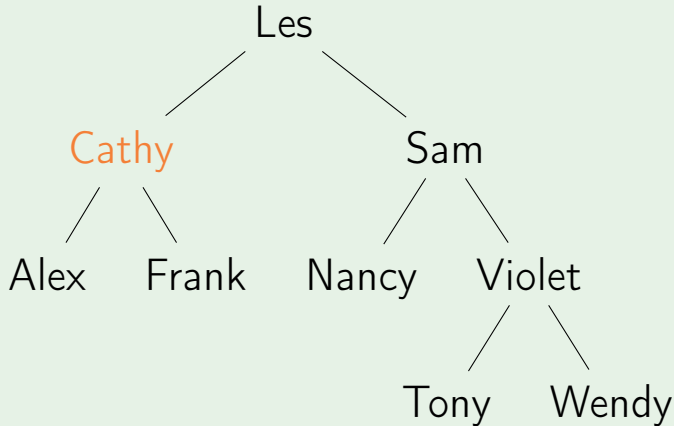
LevelTraversal



Output: Les

Queue: Cathy, Sam

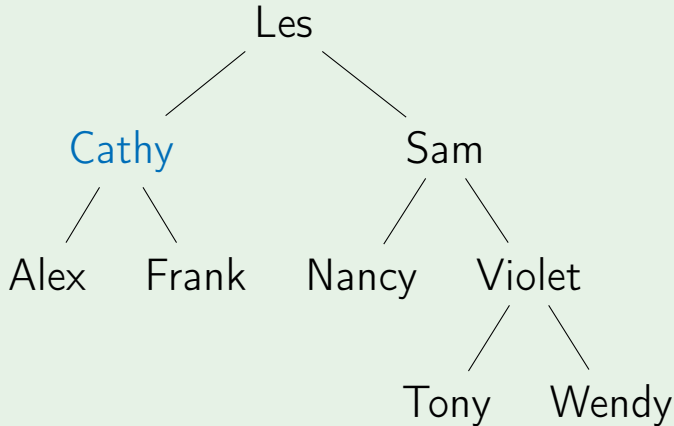
LevelTraversal



Output: Les

Queue: Sam

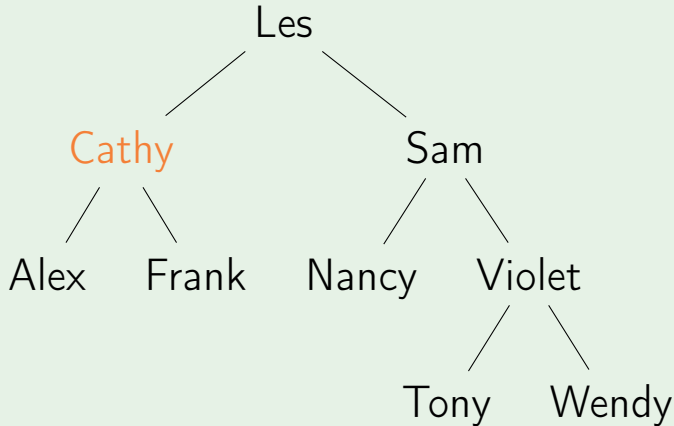
LevelTraversal



Output: Les Cathy

Queue: Sam

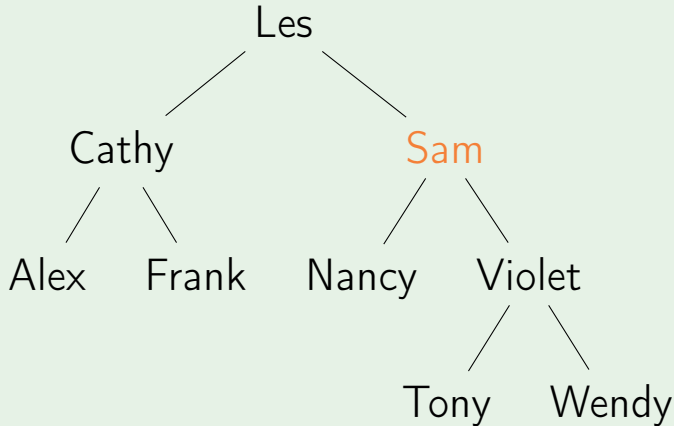
LevelTraversal



Output: Les Cathy

Queue: Sam,Alex, Frank

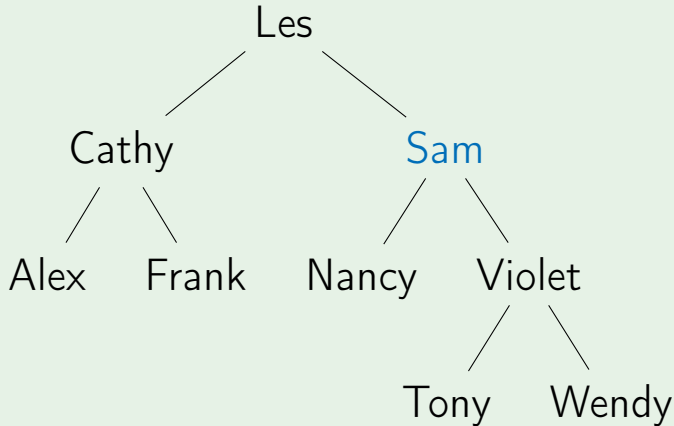
LevelTraversal



Output: Les Cathy

Queue: Alex, Frank

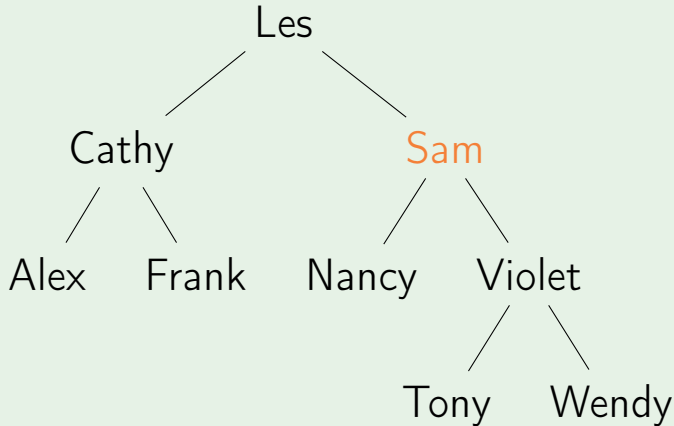
LevelTraversal



Output: Les Cathy Sam

Queue: Alex, Frank

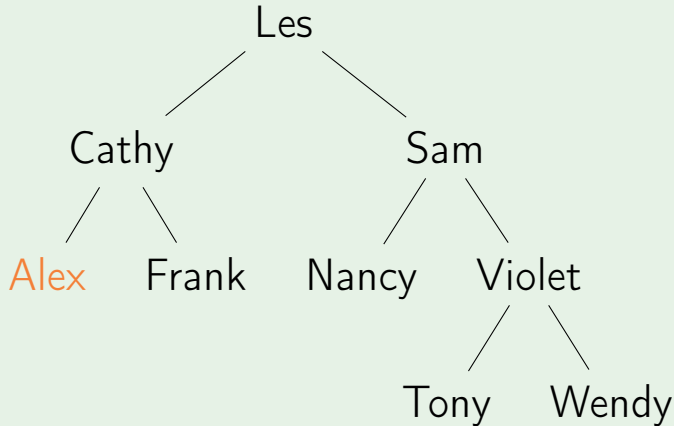
LevelTraversal



Output: Les Cathy Sam

Queue: Alex, Frank, Nancy, Violet

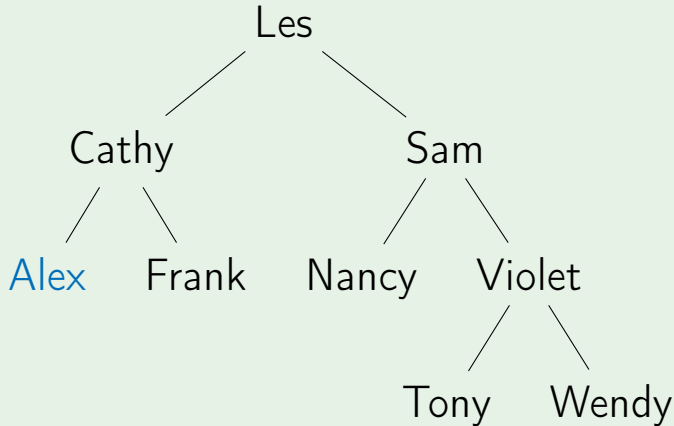
LevelTraversal



Output: Les Cathy Sam

Queue: Frank, Nancy, Violet

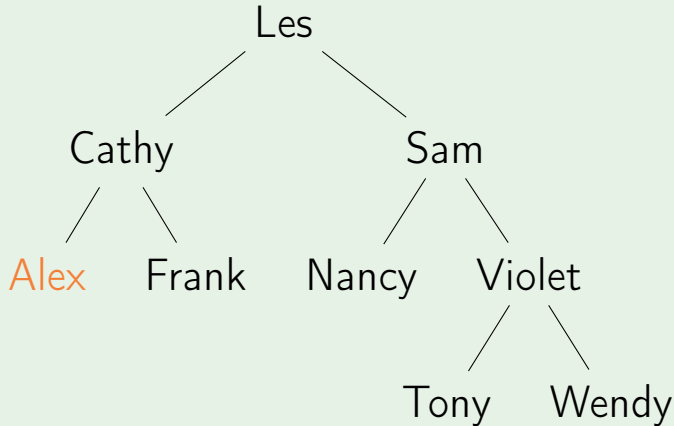
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Frank, Nancy, Violet

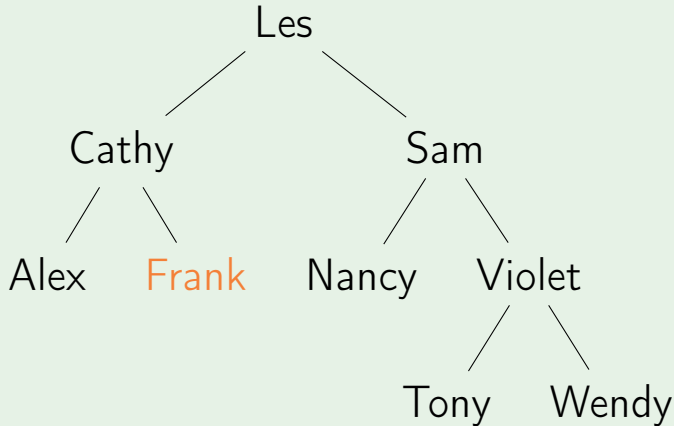
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Frank, Nancy, Violet

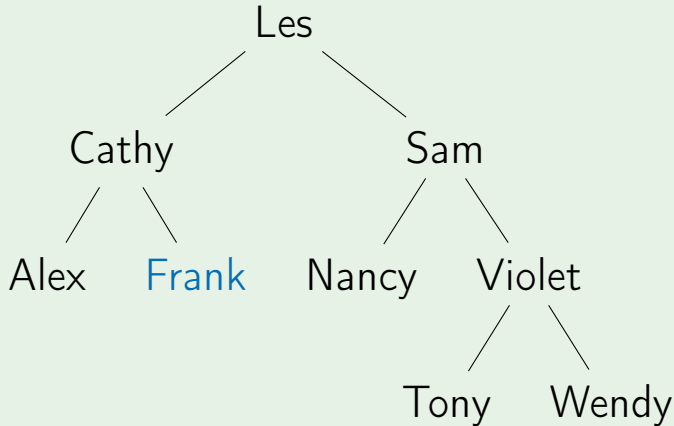
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Nancy, Violet

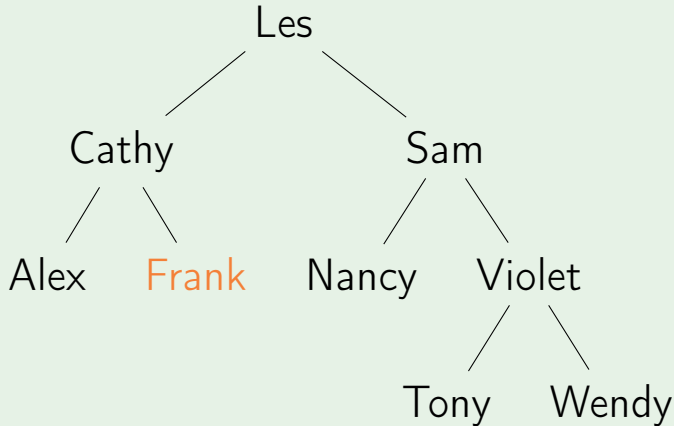
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Nancy, Violet

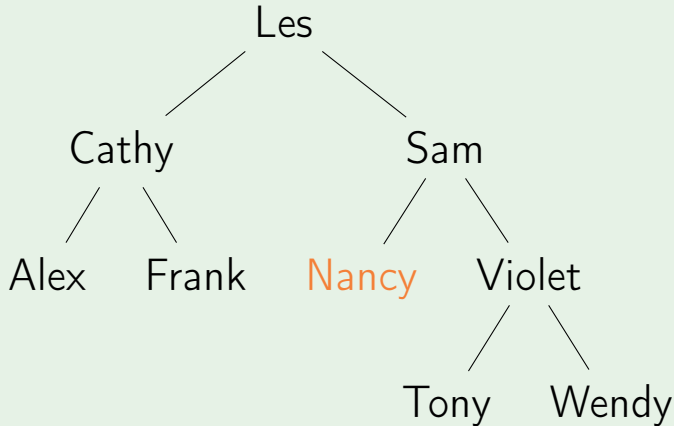
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Nancy, Violet

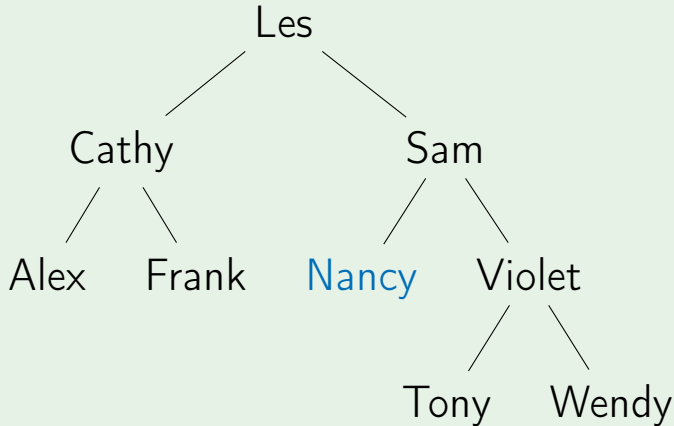
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Violet

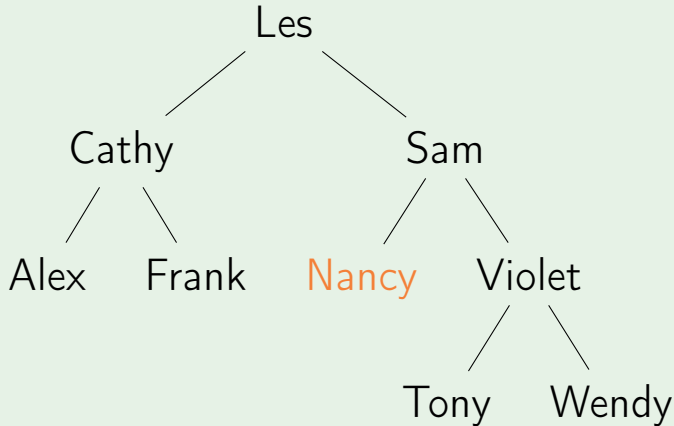
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue: Violet

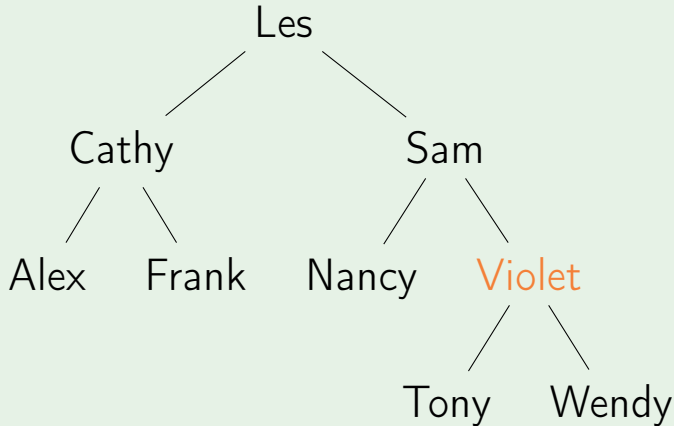
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue: Violet

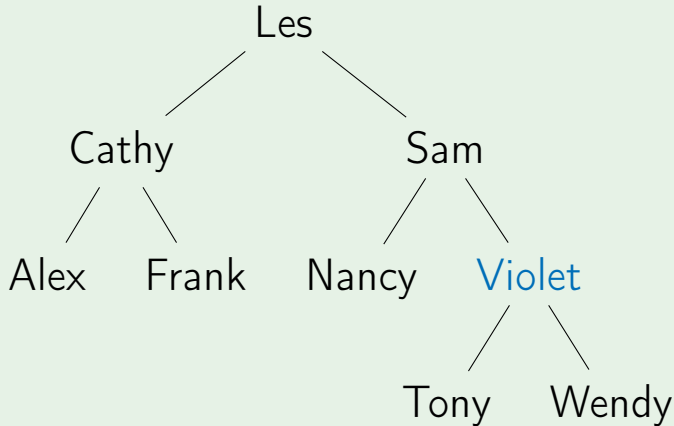
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue:

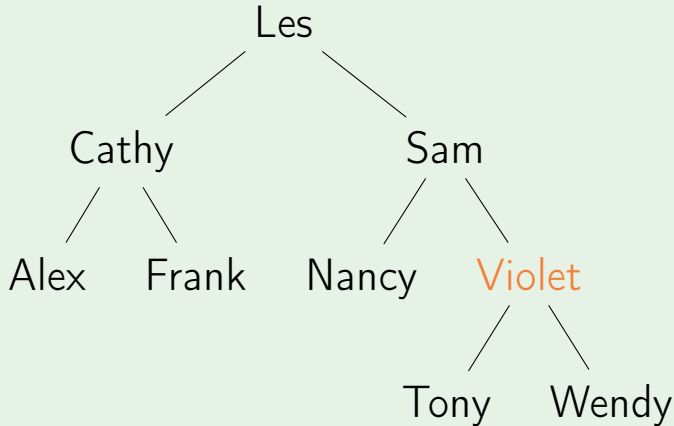
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet

Queue:

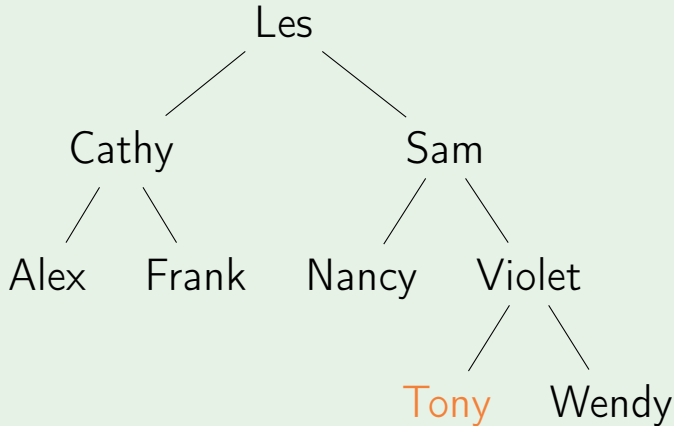
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet

Queue: Tony Wendy

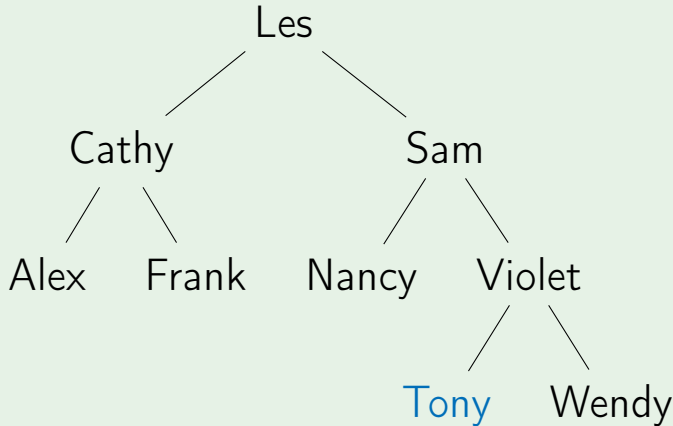
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet

Queue: Wendy

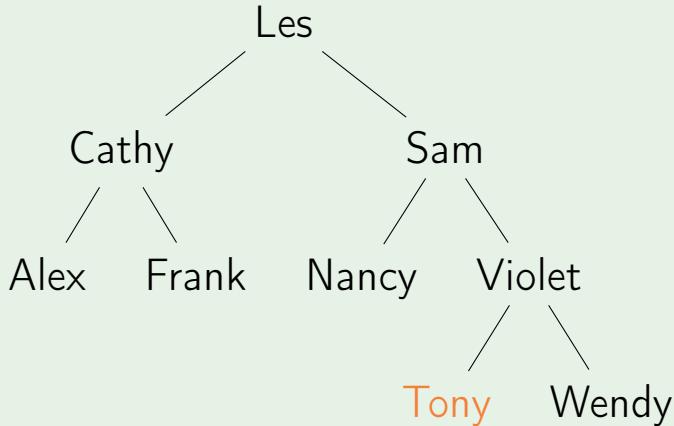
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet Tony

Queue: Wendy

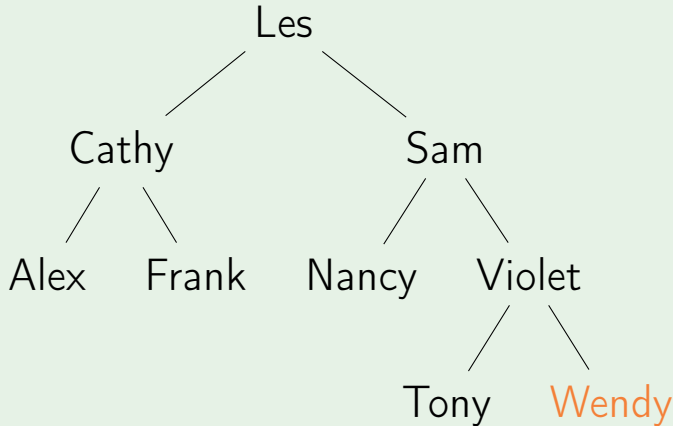
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet Tony

Queue: Wendy

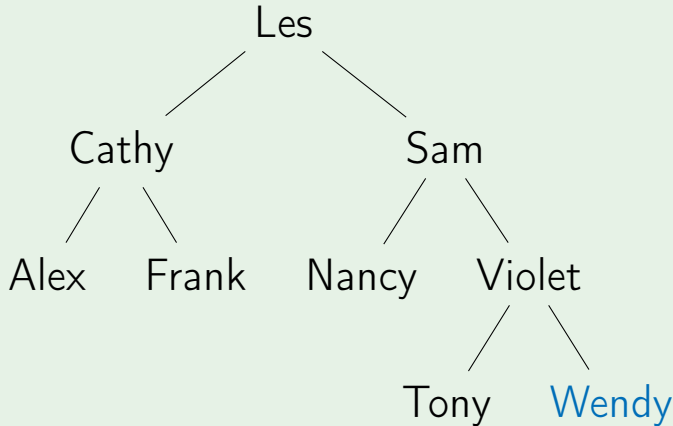
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet Tony

Queue:

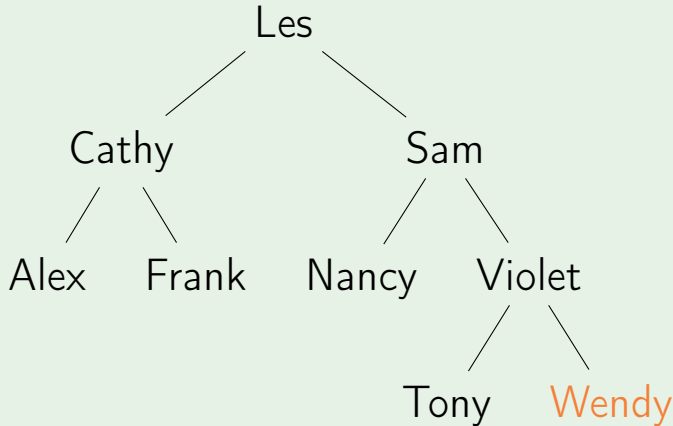
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet Tony Wendy

Queue:

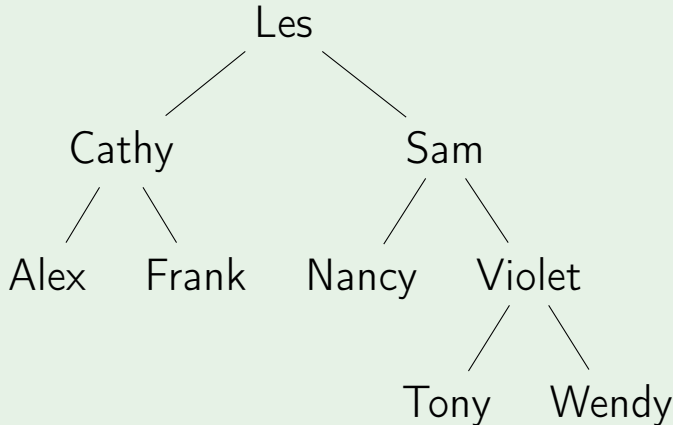
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy
Violet Tony Wendy

Queue:

LevelTraversal



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- In Computer Science, trees grow down!

For Tree-traversal quiz

