

# Gradient\_Checking

September 25, 2023

## 1 Gradient Checking

Welcome to the final assignment for this week! In this assignment you'll be implementing gradient checking.

By the end of this notebook, you'll be able to:

Implement gradient checking to verify the accuracy of your backprop implementation.

### 1.1 Important Note on Submission to the AutoGrader

Before submitting your assignment to the AutoGrader, please make sure you are not doing the following:

1. You have not added any *extra* `print` statement(s) in the assignment.
2. You have not added any *extra* code cell(s) in the assignment.
3. You have not changed any of the function parameters.
4. You are not using any global variables inside your graded exercises. Unless specifically instructed to do so, please refrain from it and use the local variables instead.
5. You are not changing the assignment code where it is not required, like creating *extra* variables.

If you do any of the following, you will get something like, **Grader Error: Grader feedback not found** (or similarly unexpected) error upon submitting your assignment. Before asking for help/debugging the errors in your assignment, check for these first. If this is the case, and you don't remember the changes you have made, you can get a fresh copy of the assignment by following these [instructions](#).

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## ## 1 - Packages

```
[1]: import numpy as np
    from testCases import *
    from public_tests import *
    from gc_utils import sigmoid, relu, dictionary_to_vector, vector_to_dictionary, \
    ↪gradients_to_vector

    %load_ext autoreload
    %autoreload 2
```

## ## 2 - Problem Statement

You are part of a team working to make mobile payments available globally, and are asked to build a deep learning model to detect fraud—whenever someone makes a payment, you want to see if the payment might be fraudulent, such as if the user’s account has been taken over by a hacker.

You already know that backpropagation is quite challenging to implement, and sometimes has bugs. Because this is a mission-critical application, your company’s CEO wants to be really certain that your implementation of backpropagation is correct. Your CEO says, “Give me proof that your backpropagation is actually working!” To give this reassurance, you are going to use “gradient checking.”

Let’s do it!

## 3 - How does Gradient Checking work? Backpropagation computes the gradients  $\frac{\partial J}{\partial \theta}$ , where  $\theta$  denotes the parameters of the model.  $J$  is computed using forward propagation and your loss function.

Because forward propagation is relatively easy to implement, you’re confident you got that right, and so you’re almost 100% sure that you’re computing the cost  $J$  correctly. Thus, you can use your code for computing  $J$  to verify the code for computing  $\frac{\partial J}{\partial \theta}$ .

Let’s look back at the definition of a derivative (or gradient):

$$\frac{\partial J}{\partial \theta} = \lim_{\varepsilon \rightarrow 0} \frac{J(\theta + \varepsilon) - J(\theta - \varepsilon)}{2\varepsilon} \quad (1)$$

If you’re not familiar with the “ $\lim_{\varepsilon \rightarrow 0}$ ” notation, it’s just a way of saying “when  $\varepsilon$  is really, really small.”

You know the following:

$\frac{\partial J}{\partial \theta}$  is what you want to make sure you’re computing correctly. You can compute  $J(\theta + \varepsilon)$  and  $J(\theta - \varepsilon)$  (in the case that  $\theta$  is a real number), since you’re confident your implementation for  $J$  is correct. Let’s use equation (1) and a small value for  $\varepsilon$  to convince your CEO that your code for computing  $\frac{\partial J}{\partial \theta}$  is correct!

## ## 4 - 1-Dimensional Gradient Checking

Consider a 1D linear function  $J(\theta) = \theta x$ . The model contains only a single real-valued parameter  $\theta$ , and takes  $x$  as input.

You will implement code to compute  $J(\cdot)$  and its derivative  $\frac{\partial J}{\partial \theta}$ . You will then use gradient checking to make sure your derivative computation for  $J$  is correct.

Figure 1: 1D linear model

The diagram above shows the key computation steps: First start with  $x$ , then evaluate the function  $J(x)$  (“forward propagation”). Then compute the derivative  $\frac{\partial J}{\partial \theta}$  (“backward propagation”).

### Exercise 1 - forward\_propagation

Implement forward propagation. For this simple function compute  $J(\cdot)$

```
[2]: # GRADED FUNCTION: forward_propagation

def forward_propagation(x, theta):
    """
    Implement the linear forward propagation (compute J) presented in Figure 1.
    ↪ (J(theta) = theta * x)

    Arguments:
    x -- a real-valued input
    theta -- our parameter, a real number as well

    Returns:
    J -- the value of function J, computed using the formula J(theta) = theta *
    ↪ x
    """

    # (approx. 1 line)
    # J =
    # YOUR CODE STARTS HERE
    J = theta * x

    # YOUR CODE ENDS HERE

    return J
```

```
[3]: x, theta = 2, 4
J = forward_propagation(x, theta)
print ("J = " + str(J))
forward_propagation_test(forward_propagation)
```

J = 8

All tests passed.

### Exercise 2 - backward\_propagation

Now, implement the backward propagation step (derivative computation) of Figure 1. That is, compute the derivative of  $J(\theta) = \theta x$  with respect to  $\theta$ . To save you from doing the calculus, you should get  $d\theta = \frac{\partial J}{\partial \theta} = x$ .

```
[4]: # GRADED FUNCTION: backward_propagation

def backward_propagation(x, theta):
    """
    Computes the derivative of J with respect to theta (see Figure 1).

    Arguments:
    x -- a real-valued input
    theta -- our parameter, a real number as well

    Returns:
    dtheta -- the gradient of the cost with respect to theta
    """

    # (approx. 1 line)
    # dtheta =
    # YOUR CODE STARTS HERE
    dtheta = x

    # YOUR CODE ENDS HERE

    return dtheta
```

```
[5]: x, theta = 3, 4
dtheta = backward_propagation(x, theta)
print ("dtheta = " + str(dtheta))
backward_propagation_test(backward_propagation)
```

```
dtheta = 3
All tests passed.
```

### Expected output:

```
dtheta = 3
All tests passed.
```

### Exercise 3 - gradient\_check

To show that the `backward_propagation()` function is correctly computing the gradient  $\frac{\partial J}{\partial \theta}$ , let's implement gradient checking.

**Instructions:** - First compute “gradapprox” using the formula above (1) and a small value of  $\epsilon$ . Here are the Steps to follow: 1.  $\theta^+ = \theta + \epsilon$  2.  $\theta^- = \theta - \epsilon$  3.  $J^+ = J(\theta^+)$  4.  $J^- = J(\theta^-)$  5.  $gradapprox = \frac{J^+ - J^-}{2\epsilon}$  - Then compute the gradient using backward propagation, and store the result in a variable “grad” - Finally, compute the relative difference between “gradapprox” and the “grad” using the following formula:

$$difference = \frac{\| grad - gradapprox \|_2}{\| grad \|_2 + \| gradapprox \|_2} \quad (2)$$

You will need 3 Steps to compute this formula: - 1'. compute the numerator using `np.linalg.norm(...)`  
 - 2'. compute the denominator. You will need to call `np.linalg.norm(...)` twice. - 3'. divide them. -  
 If this difference is small (say less than  $10^{-7}$ ), you can be quite confident that you have computed your gradient correctly. Otherwise, there may be a mistake in the gradient computation.

```
[6]: # GRADED FUNCTION: gradient_check

def gradient_check(x, theta, epsilon=1e-7, print_msg=False):
    """
    Implement the gradient checking presented in Figure 1.

    Arguments:
    x -- a float input
    theta -- our parameter, a float as well
    epsilon -- tiny shift to the input to compute approximated gradient with
    ↪ formula(1)

    Returns:
    difference -- difference (2) between the approximated gradient and the
    ↪ backward propagation gradient. Float output
    """

    # Compute gradapprox using right side of formula (1). epsilon is small
    ↪ enough, you don't need to worry about the limit.
    # (approx. 5 lines)
    # theta_plus =                                # Step 1
    # theta_minus =                               # Step 2
    # J_plus =                                     # Step 3
    # J_minus =                                   # Step 4
    # gradapprox =                                # Step 5
    # YOUR CODE STARTS HERE
    theta_plus = theta + epsilon
    theta_minus = theta - epsilon
    J_plus = forward_propagation(theta_plus, theta)
    J_minus = forward_propagation(theta_minus, theta)
    gradapprox = (J_plus - J_minus)/(2*epsilon)

    # YOUR CODE ENDS HERE

    # Check if gradapprox is close enough to the output of
    ↪ backward_propagation()
    # (approx. 1 line) DO NOT USE "grad = gradapprox"
    # grad =
    # YOUR CODE STARTS HERE
    grad = backward_propagation(theta, epsilon)

    # YOUR CODE ENDS HERE
```

```

#(approx. 3 lines)
# numerator =                                     # Step 1'
# denominator =                                   # Step 2'
# difference =                                     # Step 3'
# YOUR CODE STARTS HERE
numerator = np.linalg.norm(grad-gradapprox)
denominator = np.linalg.norm(grad) + np.linalg.norm(gradapprox)
difference = numerator/denominator

# YOUR CODE ENDS HERE
if print_msg:
    if difference > 2e-7:
        print ("\033[93m" + "There is a mistake in the backward propagation!"
→ difference = " + str(difference) + "\033[0m")
    else:
        print ("\033[92m" + "Your backward propagation works perfectly fine!"
→ difference = " + str(difference) + "\033[0m")

return difference

```

```

[7]: x, theta = 3, 4
difference = gradient_check(x, theta, print_msg=True)

```

Your backward propagation works perfectly fine! difference =  
2.919335883291695e-10

### Expected output:

Your backward propagation works perfectly fine!  
difference = 7.814075313343006e-11

Congrats, the difference is smaller than the  $2 * 10^{-7}$  threshold. So you can have high confidence that you've correctly computed the gradient in `backward_propagation()`.

Now, in the more general case, your cost function  $J$  has more than a single 1D input. When you are training a neural network,  $\theta$  actually consists of multiple matrices  $W^{[l]}$  and biases  $b^{[l]}$ ! It is important to know how to do a gradient check with higher-dimensional inputs. Let's do it!

### ## 5 - N-Dimensional Gradient Checking

The following figure describes the forward and backward propagation of your fraud detection model.

Figure 2: Deep neural network. LINEAR -> RELU -> LINEAR -> RELU -> LINEAR -> SIGMOID

Let's look at your implementations for `forward_propagation` and `backward_propagation`.

Below is the code provided for `forward_propagation_n`. Note here that `n` in the name implies it is for `n`-dimensions

```
[8]: def forward_propagation_n(X, Y, parameters):
    """
    Implements the forward propagation (and computes the cost) presented in
    ↪Figure 3.

    Arguments:
    X -- training set for m examples
    Y -- labels for m examples
    parameters -- python dictionary containing your parameters "W1", "b1",
    ↪"W2", "b2", "W3", "b3":
        W1 -- weight matrix of shape (5, 4)
        b1 -- bias vector of shape (5, 1)
        W2 -- weight matrix of shape (3, 5)
        b2 -- bias vector of shape (3, 1)
        W3 -- weight matrix of shape (1, 3)
        b3 -- bias vector of shape (1, 1)

    Returns:
    cost -- the cost function (logistic cost for m examples)
    cache -- a tuple with the intermediate values (Z1, A1, W1, b1, Z2, A2, W2,
    ↪b2, Z3, A3, W3, b3)

    """

    # retrieve parameters
    m = X.shape[1]
    W1 = parameters["W1"]
    b1 = parameters["b1"]
    W2 = parameters["W2"]
    b2 = parameters["b2"]
    W3 = parameters["W3"]
    b3 = parameters["b3"]

    # LINEAR -> RELU -> LINEAR -> RELU -> LINEAR -> SIGMOID
    Z1 = np.dot(W1, X) + b1
    A1 = relu(Z1)
    Z2 = np.dot(W2, A1) + b2
    A2 = relu(Z2)
    Z3 = np.dot(W3, A2) + b3
    A3 = sigmoid(Z3)

    # Cost
    log_probs = np.multiply(-np.log(A3), Y) + np.multiply(-np.log(1 - A3), 1 - Y)
    cost = 1. / m * np.sum(log_probs)

    cache = (Z1, A1, W1, b1, Z2, A2, W2, b2, Z3, A3, W3, b3)
```

```
return cost, cache
```

Now, let's look at the code for backward propagation.

Below is the code provided for `backward_propagation_n`. Note here that `n` in the name implies it is for `n`-dimensions

```
[9]: def backward_propagation_n(X, Y, cache):
    """
    Implement the backward propagation presented in figure 2.

    Arguments:
    X -- input datapoint, of shape (input size, 1)
    Y -- true "label"
    cache -- cache output from forward_propagation_n()

    Returns:
    gradients -- A dictionary with the gradients of the cost with respect to
    ↪ each parameter, activation and pre-activation variables.
    """

    m = X.shape[1]
    (Z1, A1, W1, b1, Z2, A2, W2, b2, Z3, A3, W3, b3) = cache

    dZ3 = A3 - Y
    dW3 = 1. / m * np.dot(dZ3, A2.T)
    db3 = 1. / m * np.sum(dZ3, axis=1, keepdims=True)

    dA2 = np.dot(W3.T, dZ3)
    dZ2 = np.multiply(dA2, np.int64(A2 > 0))
    dW2 = 1. / m * np.dot(dZ2, A1.T) * 2
    db2 = 1. / m * np.sum(dZ2, axis=1, keepdims=True)

    dA1 = np.dot(W2.T, dZ2)
    dZ1 = np.multiply(dA1, np.int64(A1 > 0))
    dW1 = 1. / m * np.dot(dZ1, X.T)
    db1 = 4. / m * np.sum(dZ1, axis=1, keepdims=True)

    gradients = {"dZ3": dZ3, "dW3": dW3, "db3": db3,
                 "dA2": dA2, "dZ2": dZ2, "dW2": dW2, "db2": db2,
                 "dA1": dA1, "dZ1": dZ1, "dW1": dW1, "db1": db1}

    return gradients
```

If you had just implemented these functions, you might not have high confidence whether they work correctly. So let's implement gradient checking to help verify the performance.

**How does gradient checking work?.**



As in Section 3 and 4, you want to compare “gradapprox” to the gradient computed by backpropagation. The formula is still:

$$\frac{\partial J}{\partial \theta} = \lim_{\varepsilon \rightarrow 0} \frac{J(\theta + \varepsilon) - J(\theta - \varepsilon)}{2\varepsilon} \quad (1)$$

However,  $\theta$  is not a scalar anymore. It is a dictionary called “parameters”. The function “dictionary\_to\_vector()” has been implemented for you. It converts the “parameters” dictionary into a vector called “values”, obtained by reshaping all parameters (W1, b1, W2, b2, W3, b3) into vectors and concatenating them.

The inverse function is “vector\_to\_dictionary” which outputs back the “parameters” dictionary.

Figure 2: dictionary\_to\_vector() and vector\_to\_dictionary(). You will need these functions in gradient\_check\_n()

The “gradients” dictionary has also been converted into a vector “grad” using gradients\_to\_vector(), so you don’t need to worry about that.

Now, for every single parameter in your vector, you will apply the same procedure as for the gradient\_check exercise. You will store each gradient approximation in a vector **gradapprox**. If the check goes as expected, each value in this approximation must match the real gradient values stored in the **grad** vector.

Note that **grad** is calculated using the function **gradients\_to\_vector**, which uses the gradients outputs of the **backward\_propagation\_n** function.

### Exercise 4 - gradient\_check\_n

Implement the function below.

**Instructions:** Here is pseudo-code that will help you implement the gradient check.

For each i in num\_parameters: - To compute J\_plus[i]: 1. Set  $\theta^+$  to np.copy(parameters\_values) 2. Set  $\theta_i^+$  to  $\theta_i^+ + \varepsilon$  3. Calculate  $J_i^+$  using forward\_propagation\_n(x, y, vector\_to\_dictionary( $\theta^+$ )).

- To compute J\_minus[i]: do the same thing with  $\theta^-$  - Compute  $gradapprox[i] = \frac{J_i^+ - J_i^-}{2\varepsilon}$

Thus, you get a vector gradapprox, where gradapprox[i] is an approximation of the gradient with respect to parameter\_values[i]. You can now compare this gradapprox vector to the gradients vector from backpropagation. Just like for the 1D case (Steps 1’, 2’, 3’), compute:

$$difference = \frac{\|grad - gradapprox\|_2}{\|grad\|_2 + \|gradapprox\|_2} \quad (3)$$

**Note:** Use np.linalg.norm to get the norms

```
[32]: # GRADED FUNCTION: gradient_check_n

def gradient_check_n(parameters, gradients, X, Y, epsilon=1e-7,
    print_msg=False):
    """
```

Checks if backward\_propagation\_n computes correctly the gradient of the cost output by forward\_propagation\_n

*Arguments:*

- parameters* -- python dictionary containing your parameters "W1", "b1", "W2", "b2", "W3", "b3"
- grad* -- output of backward\_propagation\_n, contains gradients of the cost with respect to the parameters
- X* -- input datapoint, of shape (input size, number of examples)
- Y* -- true "label"
- epsilon* -- tiny shift to the input to compute approximated gradient with formula(1)

*Returns:*

- difference* -- difference (2) between the approximated gradient and the backward propagation gradient

```

"""

# Set-up variables
parameters_values, _ = dictionary_to_vector(parameters)

grad = gradients_to_vector(gradients)
num_parameters = parameters_values.shape[0]
J_plus = np.zeros((num_parameters, 1))
J_minus = np.zeros((num_parameters, 1))
gradapprox = np.zeros((num_parameters, 1))

# Compute gradapprox
for i in range(num_parameters):

    # Compute J_plus[i]. Inputs: "parameters_values, epsilon". Output = "J_plus[i]".
    # "_" is used because the function you have outputs two parameters but we only care about the first one
    #(approx. 3 lines)
    # theta_plus =                                # Step 1
    # theta_plus[i] =                             # Step 2
    # J_plus[i], _ =                               # Step 3
    # YOUR CODE STARTS HERE
    theta_plus = np.copy(parameters_values)
    theta_plus[i] = theta_plus[i] + epsilon
    J_plus[i], _ = forward_propagation_n(X, Y, vector_to_dictionary(theta_plus))

    # YOUR CODE ENDS HERE

```

```

    # Compute J_minus[i]. Inputs: "parameters_values, epsilon". Output = ␣
    ↪ "J_minus[i]".
    # (approx. 3 lines)
    # theta_minus = # Step 1
    # theta_minus[i] = # Step 2
    # J_minus[i], _ = # Step 3
    # YOUR CODE STARTS HERE
    theta_minus = np.copy(parameters_values)
    theta_minus[i] = theta_minus[i] - epsilon
    J_minus[i], _ = forward_propagation_n(X, Y, ␣
    ↪ vector_to_dictionary(theta_minus))

    # YOUR CODE ENDS HERE

    # Compute gradapprox[i]
    # (approx. 1 line)
    # gradapprox[i] =
    # YOUR CODE STARTS HERE
    gradapprox[i] = (J_plus[i] - J_minus[i]) / (2 * epsilon)

    # YOUR CODE ENDS HERE

    # Compare gradapprox to backward propagation gradients by computing ␣
    ↪ difference.
    # (approx. 3 line)
    # numerator = # Step 1'
    # denominator = # Step 2'
    # difference = # Step 3'
    # YOUR CODE STARTS HERE
    numerator = np.linalg.norm(grad - gradapprox)
    denominator = np.linalg.norm(grad) + np.linalg.norm(gradapprox)
    difference = numerator / denominator

    # YOUR CODE ENDS HERE
    if print_msg:
        if difference > 2e-7:
            print ("\033[93m" + "There is a mistake in the backward propagation!"
            ↪ difference = " + str(difference) + "\033[0m")
        else:
            print ("\033[92m" + "Your backward propagation works perfectly fine!"
            ↪ difference = " + str(difference) + "\033[0m")

    return difference

```

```
[33]: X, Y, parameters = gradient_check_n_test_case()

cost, cache = forward_propagation_n(X, Y, parameters)
gradients = backward_propagation_n(X, Y, cache)
difference = gradient_check_n(parameters, gradients, X, Y, 1e-7, True)
expected_values = [0.2850931567761623, 1.1890913024229996e-07]
assert not(type(difference) == np.ndarray), "You are not using np.linalg.norm_
↳for numerator or denominator"
assert np.any(np.isclose(difference, expected_values)), "Wrong value. It is not_
↳one of the expected values"
```

0.2850931567761623

There is a mistake in the backward propagation! difference =

0.2850931567761623

### Expected output:

There is a mistake in the backward propagation!

difference = 0.2850931567761623

It seems that there were errors in the `backward_propagation_n` code! Good thing you've implemented the gradient check. Go back to `backward_propagation_n` and try to find/correct the errors (*Hint: check  $dW2$  and  $db1$* ). Rerun the gradient check when you think you've fixed it. Remember, you'll need to re-execute the cell defining `backward_propagation_n()` if you modify the code.

Can you get gradient check to declare your derivative computation correct? Even though this part of the assignment isn't graded, you should try to find the bug and re-run gradient check until you're convinced backprop is now correctly implemented.

**Notes** - Gradient Checking is slow! Approximating the gradient with  $\frac{\partial J}{\partial \theta} \approx \frac{J(\theta+\epsilon) - J(\theta-\epsilon)}{2\epsilon}$  is computationally costly. For this reason, we don't run gradient checking at every iteration during training. Just a few times to check if the gradient is correct. - Gradient Checking, at least as we've presented it, doesn't work with dropout. You would usually run the gradient check algorithm without dropout to make sure your backprop is correct, then add dropout.

Congrats! Now you can be confident that your deep learning model for fraud detection is working correctly! You can even use this to convince your CEO. :)

**What you should remember from this notebook:** - Gradient checking verifies closeness between the gradients from backpropagation and the numerical approximation of the gradient (computed using forward propagation). - Gradient checking is slow, so you don't want to run it in every iteration of training. You would usually run it only to make sure your code is correct, then turn it off and use backprop for the actual learning process.

[ ]: