Difference of Squares

Formula: $$a^2 - b^2 = (a - b)(a + b)$$

Difference and Sum of Cubes

Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

S *O* *AP*

Sum of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

S *O* *AP*

***S**ame*

***O**pposite*

***A**lways*

***P**ositive*

$$\square^{2}$$
$$\times$$
$$\square^{2}$$

Examples: Difference of squares

1. $m^2 - n^2$
2. $x^2 - 49$
3. $36 - y^2$
4. $64x^2 - y^2$
5. $4x^2 - 9y^2$
6. $16m^2 - 49n^2$

Examples: Sum and Difference of cubes

1. $x^3 - 8$
2. $x^3 + 64$
3. $27 - 8y^3$
4. $64x^3 - 27$
5. $27x^3 - 125$
6. $64m^3 + 216$

Same

Opposite

Always

Positive

$\square^2$

$\times$

$\square^2$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Remark: Notice that the equation must be set equal to zero first!

THE DISCRIMINANT

Definition: For the quadratic equation $ax^2 + bx + c = 0$, the discriminant is $b^2 - 4ac$.

If $b^2 - 4ac > 0$ and is a perfect square, then $ax^2 + bx + c = 0$ has two real and rational solutions.

($\textit{to solve the equation we can use quadratic formula or algebrially}$)

If $b^2 - 4ac > 0$ but is not a perfect square, then $ax^2 + bx + c = 0$ has two irrational solutions.

($\textit{to solve the equation we need to use quadratic formula}$)

If $b^2 - 4ac = 0$, then $ax^2 + bx + c = 0$ has one real and rational solution.

($\textit{to solve the equation we can use quadratic formula or algebrially}$)

If $b^2 - 4ac < 0$, then $ax^2 + bx + c = 0$ has no real solution but it has two complex solutions (involving the imaginary number).

($\textit{to solve the equation we need to use quadratic formula}$)

Directions: Solve each equation by factoring.

1.

$$x^2 + 9x = 0$$

$$x\,(x + 9) = 0$$

The Zero Product Property

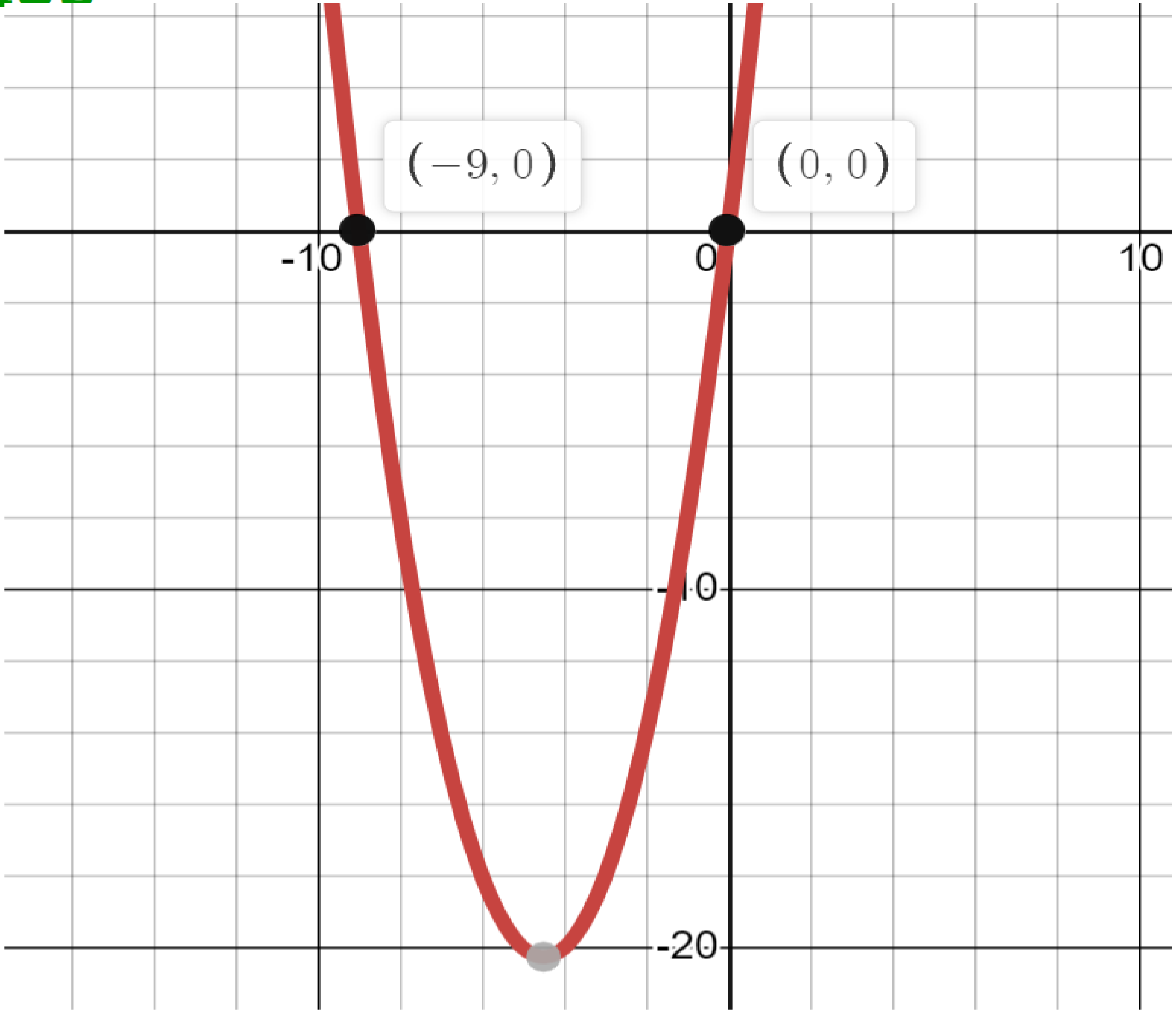
$$x = 0 \; or \; (x + 9) = 0$$

If ab=0 , then either a=0 or b=0 (or both).

Therefore, x=0 or x= -9 are the solutions.

Check Using Desmos

$x^2 + 9x$



Zeros: $-9; 0$

Factors flip: $(x+9); x$

Factored form: $x(x+9)$

You try!

2

$$4x^2 - 8x = 0$$

$$4x\ (x - 2) = 0$$

$$4x = 0 \quad or \ (x - 2) = 0$$

Therefore, x =0 and x = 2.

3

$$9x^2 - 25 = 0$$

$$(3x)^2 - 5^2 = 0$$

$$(3x-5)(3x+5) = 0$$

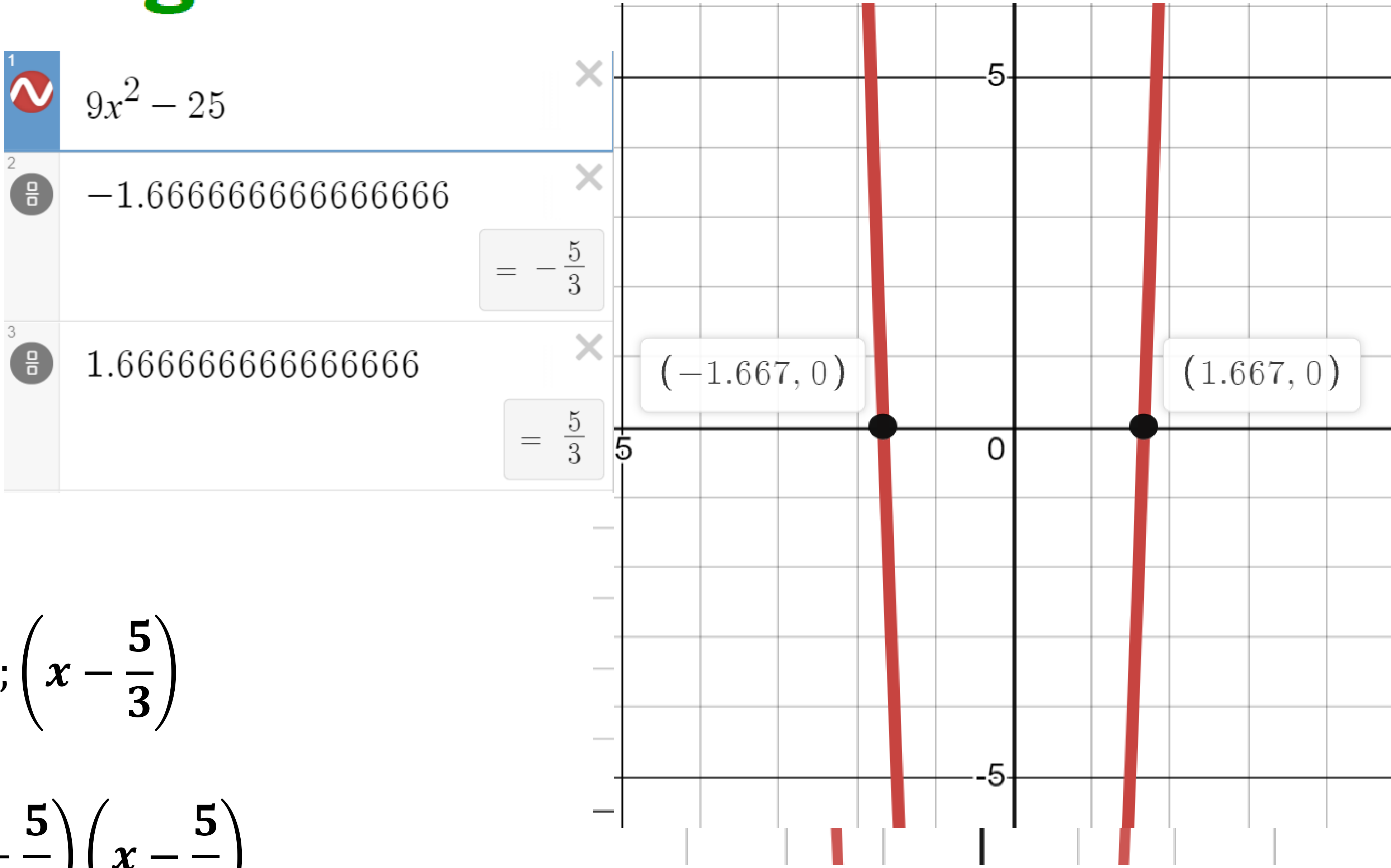
$$(3x-5) = 0 \quad or \quad (3x+5) = 0$$

$$Therefore, x = -\frac{5}{3} \; and \; x = \frac{5}{3}.$$

Remember!

$$a^2 - b^2 = (a-b)(a+b)$$

Check Using Desmos



Zeros: $-\frac{5}{3};\frac{5}{3}$

Factors flip: $\left(x+\frac{5}{3}\right);\left(x-\frac{5}{3}\right)$

Factored form: $\left(x+\frac{5}{3}\right)\left(x-\frac{5}{3}\right)$

You try!

Remember!

$a^2 - b^2 = (a - b)(a + b)$

4 $81x^2 - 1 = 0$

$(9x)^2 - 1^2 = 0$

$(9x - 1)(9x + 1) = 0$

$(9x - 1) = 0 \quad or \quad (9x + 1) = 0$

$Therefore, x = -\frac{1}{9} \; and \; x = \frac{1}{9}.$

5 $x^2 - 10x + 21 = 0$

$a = 1, \qquad b = -10, \qquad c = 21$

Check the dicriminat: $D = b^2 - 4ac$

$D = (-10)^2 - 4(1)(21) = 16$

$D = b^2 - 4ac > 0$ and is a perfect square

(*to solve the equation we can use quadratic formula or algebrially*)

Method − 1: Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{16}}{2(1)}$$

$$x = \frac{10+4}{2(1)} \quad or \quad x = \frac{10-4}{2(1)}$$

$$x = 3 \; and \; x = 7$$

$Method\ -2: Algebrically$

$$x^2 - 10x + 21 = 0$$

$a = 1,\quad b = -10,\quad c = 21$

$$ac = 21$$

$22 = 1$	21	-1	-21	= -22
$10 = 3$	7	-3	-7	$= -10$

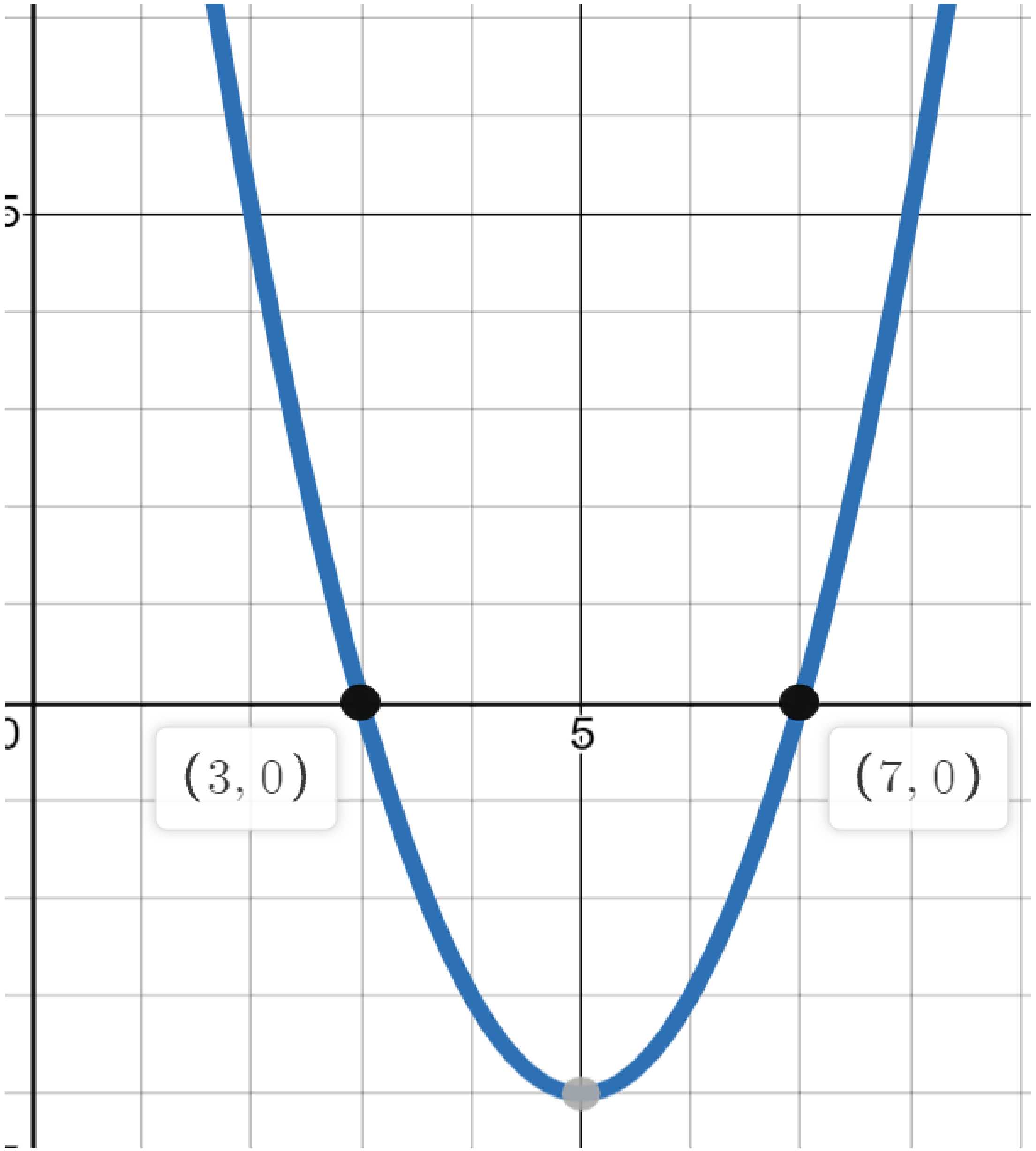
$$(x-3)(x-7) = 0$$

$$(x-3) = 0 \quad or \quad (x-7) = 0$$

$$Therefore, x = 3\ and\ x = 7.$$

Check Using Desmos

$x^2 - 10x + 21$



Zeros: $3; 7$

Factors flip: $(x-3); (x-7)$

Factored form: $(x-3)\,(x-7)$

6 $x^2 + 2x - 80 = 0$

$a = 1, \quad b = 2, \quad c = -80$

Check the dicriminat: $D = b^2 - 4ac$

$D = 2^2 - 4(1)(-80) = 324$

$D = b^2 - 4ac > 0$ and is a perfect square

(*to solve the equation we can use quadratic formula or algebrially*)

Method – 1: Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{324}}{2(1)}$$

$$x = \frac{-2+18}{2(1)} \quad or \quad x = \frac{-2-18}{2(1)}$$

$$x = -10 \ and \ x = 8$$

Method – 2: Algebrically

$$x^2 + 2x - 80 = 0$$

$a = 1,$ $b = 2,$ $c = -80$

$$ac = -80$$

1	-80	-1	80
2	-40	-2	40
4	-20	-4	20
5	-16	-5	16
8	-10	-8	10

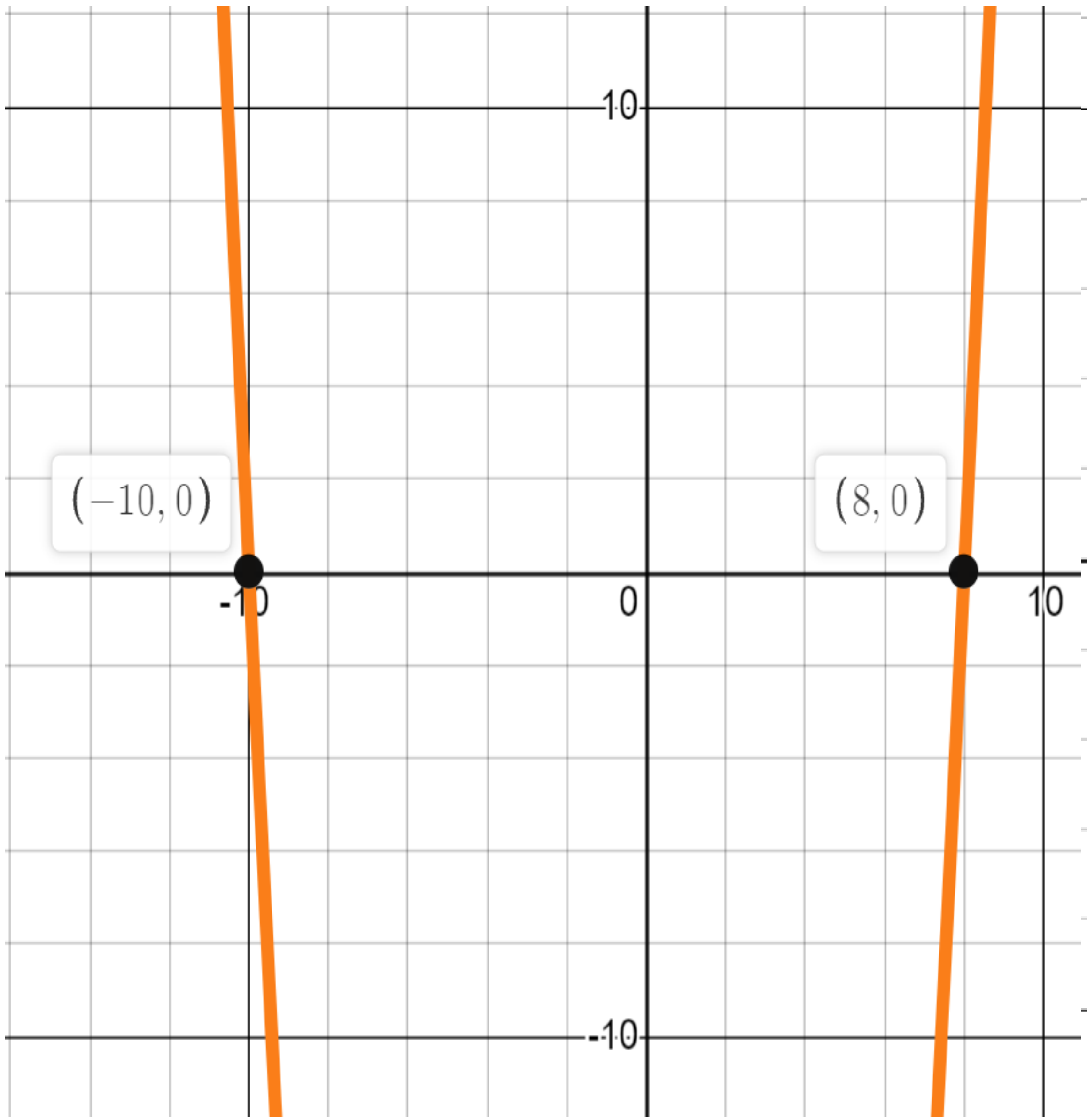
$$(x-8)(x+10) = 0$$

$$(x-8) = 0 \quad or \quad (x+10) = 0$$

$$Therefore, x = -10 \; and \; x = 8.$$

Check Using Desmos

1 $x^2 + 2x - 80$



Zeros: $-10; 8$

Factors flip: $(x + 10); (x - 8)$

Factored form: $(x + 10)\ (x + 8)$

You try!

7 $x^2 + 9x + 20 = 0$

$a = 1, \quad b = 9, \quad c = 20$

$Check\ the\ dicriminat:\ D = b^2 - 4ac$

$D = 9^2 - 4(1)(20) = 1$

$D = b^2 - 4ac > 0$ and is a perfect square

$(to\ solve\ the\ equation\ we\ can\ use\ quadratic\ formula\ or\ algebrially)$

$Method\ -1: Using\ quadratic\ formula$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(9) \pm \sqrt{1}}{2(1)}$$

$$x = \frac{-9+1}{2(1)}\ or\ \ x = \frac{-9-1}{2(1)}$$

$$x = -5\ and\ x = 4$$

You try!

Method − 2: Algebrically

$$x^2 + 9x + 20 = 0$$

$a = 1,$ $\quad b = 9,$ $\quad c = 20$

$$ac = 20$$

1	20	−1	−20
2	10	−2	−10
4	5	−4	−5

$$(x + 4)(x + 5) = 0$$

$$(x + 4) = 0 \quad or \; (x + 5) = 0$$

$$Therefore, x = -5 \; and \; x = -4.$$

You try!

8 $x^2 + 6x - 72 = 0$

$a = 1, \quad b = 6, \quad c = -72$

$Check\ the\ dicriminat$: $D = b^2 - 4ac$

$D = 6^2 - 4(1)(-72) = 324$

$D = b^2 - 4ac > 0$ and is a perfect square

$(to\ solve\ the\ equation\ we\ can\ use\ quadratic\ formula\ or\ algebrially)$

$Method\ -1: Using\ quadratic\ formula$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{324}}{2(1)}$$

$$x = \frac{-6+18}{2(1)}\ or\ x = \frac{-6-18}{2(1)}$$

$$x = -12\ and\ x = 6$$

You try!

$$x^2 + 6x - 72 = 0$$

$a = 1,\qquad b = 6,\qquad c = -72$

$$(x-6)(x+12) = 0$$

$$(x-6) = 0 \ \ or\ (x+12) = 0$$

$$Therefore, x = \ -12\ or\ x = 6$$

$$ac = -72$$

1	-72	-1	72
2	-36	-2	36
3	-24	-3	24
4	-18	-4	18
6	-12	-6	12
8	-9	-8	9

9

$$2x^2 + 21x + 10 = 0$$

$$a = 2, \qquad b = 21, \qquad c = 10$$

Check the dicriminat: $D = b^2 - 4ac$

$D = 21^2 - 4(2)(10) = 361$

$D = b^2 - 4ac > 0$ and is a perfect square

(*to solve the equation we can use quadratic formula or algebrially*)

Method − 1: Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(21) \pm \sqrt{361}}{2(2)}$$

$$x = \frac{-21+19}{4} \quad or \quad x = \frac{-21-19}{4}$$

$$x = -10 \text{ and } x = -\frac{1}{2}$$

9

$$2x^2 + 21x + 10 = 0$$

$a = 2,$ $b = 21,$ $c = 10$

$ac = 20$

1	20	-1	-20
2	10	-2	-10
4	5	-4	-5

$$(x+1)(x+20) = 0$$

Divide by a

$$\left(x+\frac{1}{2}\right)\left(x+\frac{20}{2}\right) = 0$$

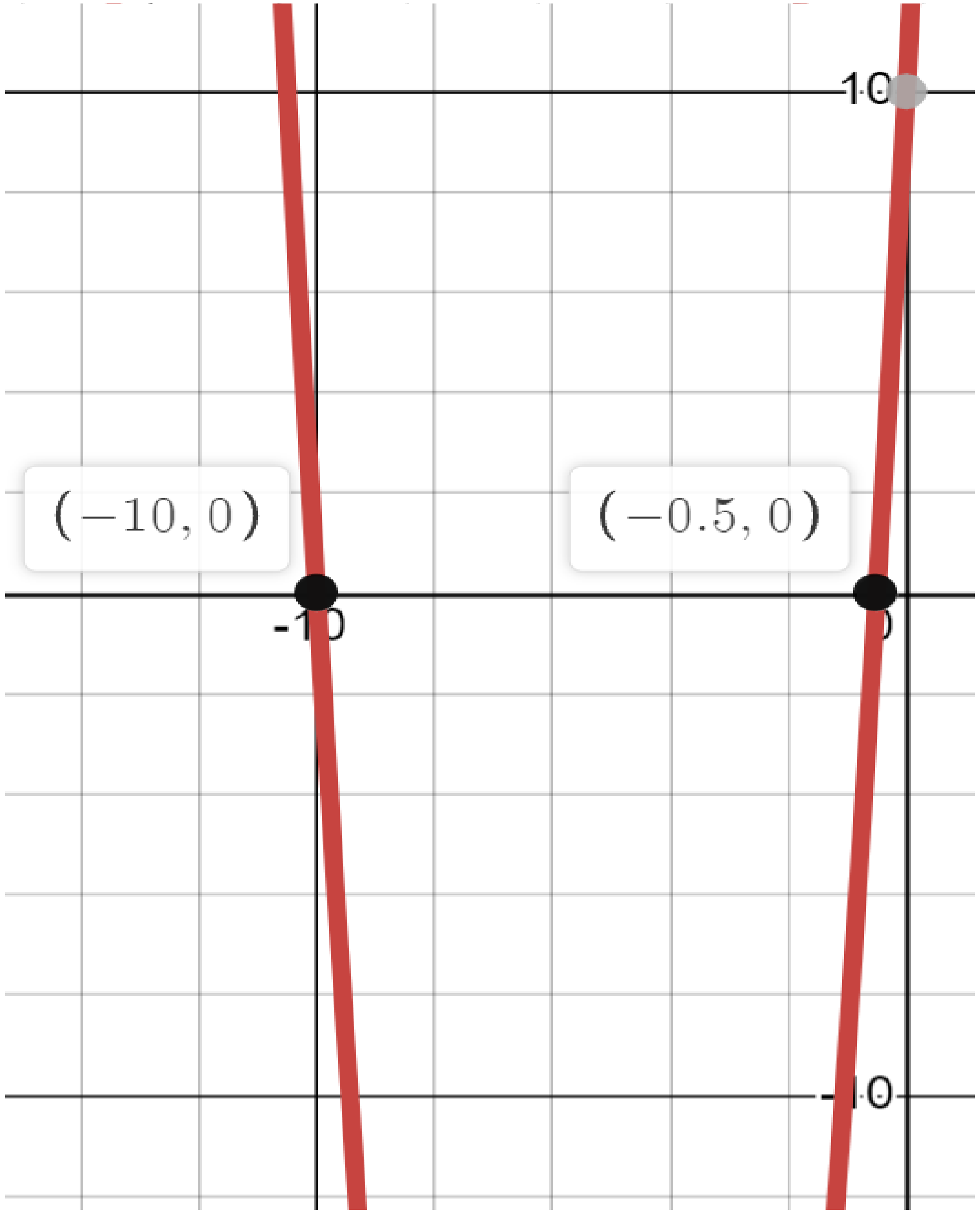
$$\left(x+\frac{1}{2}\right)(x+10) = 0 \longrightarrow \left(x+\frac{1}{2}\right) = 0 \; or \; (x+10) = 0$$

$$Therefore, x = -10 \; and \; x = -\frac{1}{2}.$$

Check Using Desmos

1

$$2x^2 + 21x + 10$$



Zeros: $-\mathbf{10}; -\frac{\mathbf{1}}{\mathbf{2}}$

Factors flip: $(\boldsymbol{x} + \mathbf{10}); (\mathbf{2}\boldsymbol{x} + \mathbf{1})$

Factored form: $(\boldsymbol{x} + \mathbf{10})\ (\mathbf{2}\boldsymbol{x} + \mathbf{1})$

10 $3x^2 + 10x - 8 = 0$

$a = 3, \qquad b = 10, \qquad c = -8$

Check the dicriminat: $D = b^2 - 4ac$

$D = 10^2 - 4(3)(-8) = 196$

$D = b^2 - 4ac > 0$ and is a perfect square

(*to solve the equation we can use quadratic formula or algebrially*)

Method − 1: Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(10) \pm \sqrt{196}}{2(3)}$$

$$x = \frac{-10+14}{6} \quad or \quad x = \frac{-10-14}{6}$$

$$x = -4 \text{ and } x = \frac{2}{3}$$

$$3x^2 + 10x - 8 = 0$$

$a = 3,$ $b = 10,$ $c = -8$

$$ac = -24$$

-1	24	1	-24
-2	12	2	-12
-3	8	3	-8
-4	6	4	-6

$$(x - 2)(x + 12) = 0$$

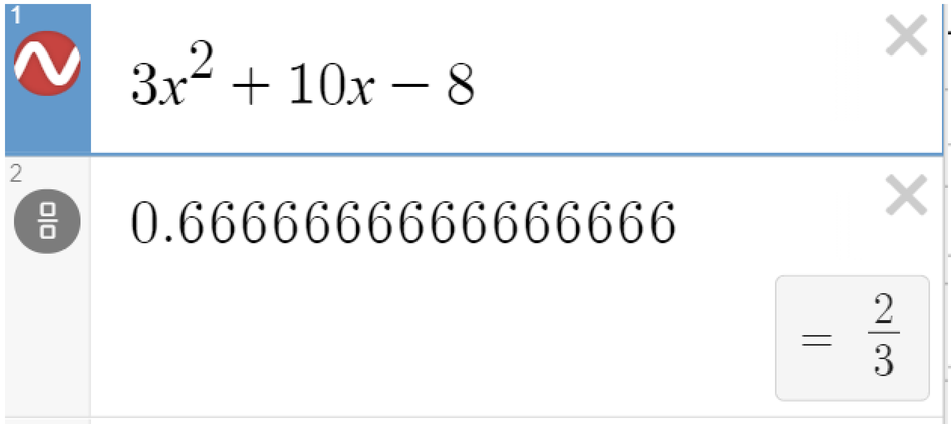
Divide by a

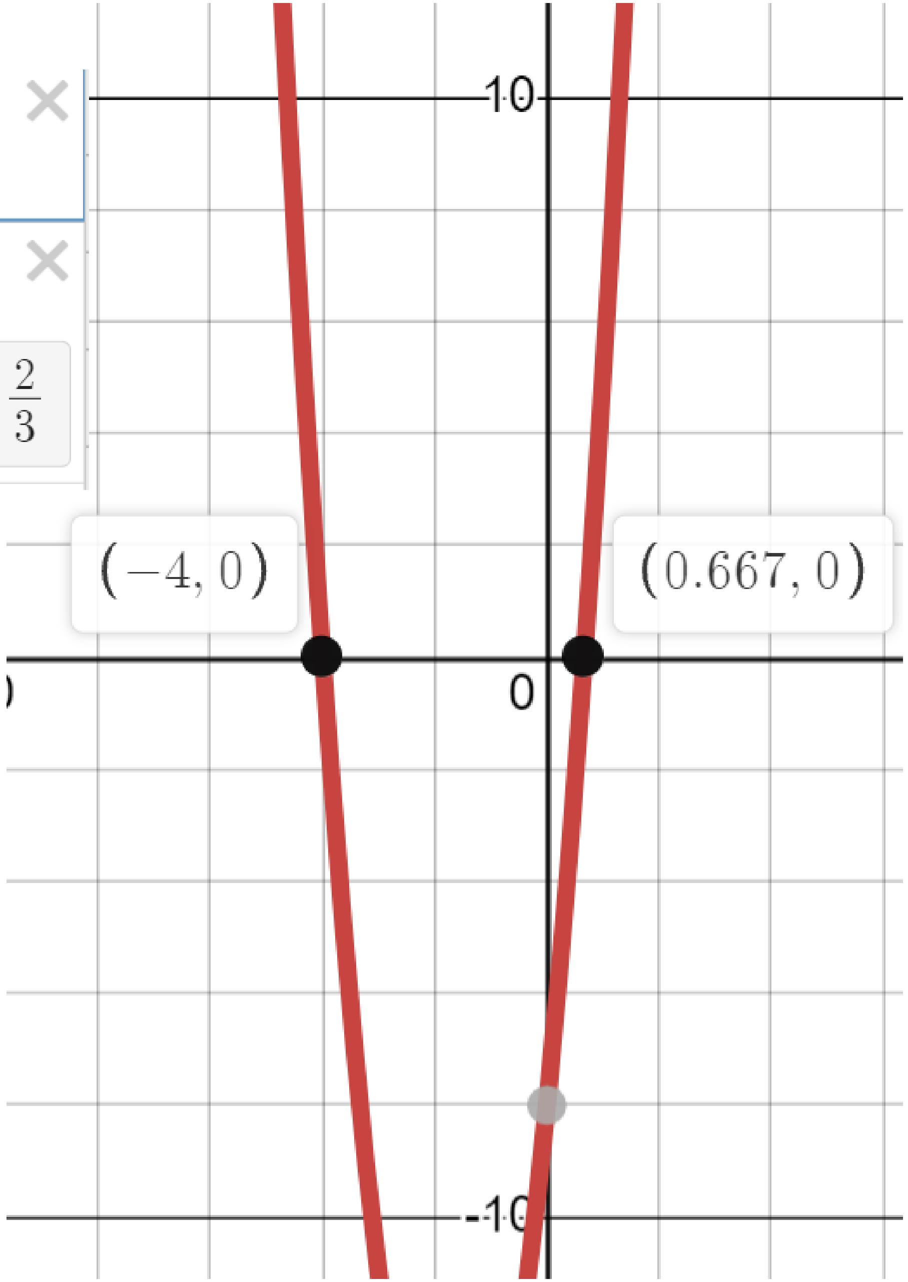
$$\left(x - \frac{2}{3}\right)\left(x + \frac{12}{3}\right) = 0$$

$$\left(x - \frac{2}{3}\right)(x + 4) = 0 \longrightarrow \left(x - \frac{2}{3}\right) = 0 \text{ or } (x + 4) = 0$$

$$\text{Therefore, } x = -4 \text{ and } x = \frac{2}{3}$$

Check Using Desmos





Zeros: $-4; \frac{2}{3}$

Factors flip: $(x + 4); (3x - 2)$

Factored form: $(x + 4)\,(3x - 2)$

11

$$4x^2 - 12x + 9 = 0$$

You try!

$$a = 4, \qquad b = -12, \qquad c = 9$$

Check the dicriminat: $D = b^2 - 4ac$

$D = (-12)^2 - 4(4)(9) = 0$

$D = b^2 - 4ac = 0$ and is a perfect square

($\textit{to solve the equation we can use quadratic formula or algebrially}$)

Method − 1: Using quadratic formula

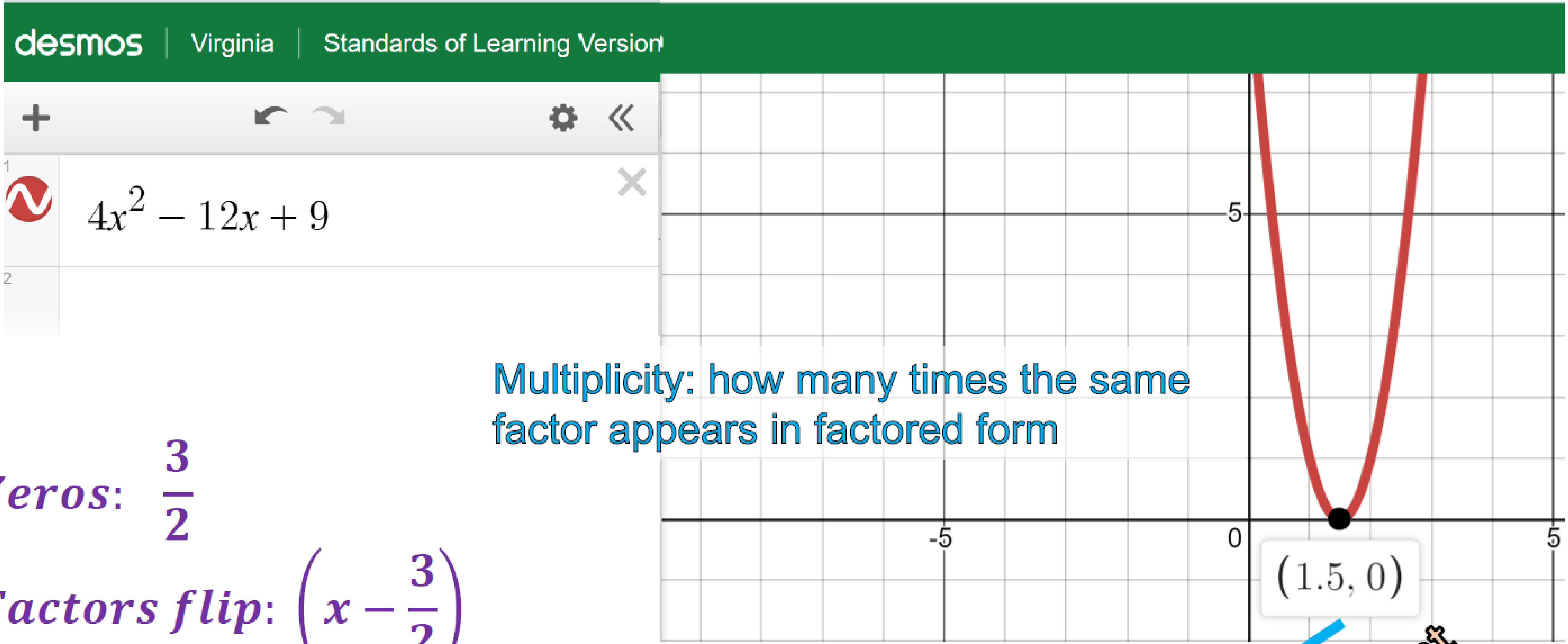
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{0}}{2(4)}$$

$$x = \frac{12}{8}$$

$$x = \frac{3}{2}$$

Check Using Desmos



Multiplicity: how many times the same factor appears in factored form

Zeros: $\frac{3}{2}$

Factors flip: $\left(x - \frac{3}{2}\right)$

Factored form: $\left(x - \frac{3}{2}\right)^2$

Double root
Multiplicity of 2

Exponent

12 $4x^2 + 5x - 3 = 0$

You try!

$a = 4, \quad b = 5, \quad c = -3$

Check the dicriminat: $D = b^2 - 4ac$

$D = (5)^2 - 4(4)(-3) = 73$

$D = b^2 - 4ac > 0$ but is not a perfect square number

(*to solve the equation we use only quadratic formula*)

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(5) \pm \sqrt{73}}{2(4)}$$

$$x = \frac{-5 + \sqrt{73}}{8} \quad \text{or} \quad x = \frac{-5 - \sqrt{73}}{8}$$

You try!

$$4x^2 - 12x + 9 = 0$$

$a = 4,$ $b = -12,$ $c = 9$

$ac = 36$

1	36	-1	-36
2	18	-2	-18
3	12	-3	-12
4	9	-4	-9
6	6	-6	-6

$$(x - 6)\,(x - 6) = 0$$

Divide by a

$$\left(x - \frac{6}{4}\right)\left(x - \frac{6}{4}\right) = 0$$

$$\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right) = 0 \longrightarrow \left(x - \frac{3}{2}\right) = 0 \ \ or \ \ \left(x - \frac{3}{2}\right) = 0$$

$$Therefore,\ x = \frac{3}{2}$$

13 $3x^2 + 4x + 8 = 0$

You try!

$a = 3, \quad b = 4, \quad c = 8$

Check the dicriminat: $D = b^2 - 4ac$

$D = (4)^2 - 4(3)(8) = -80$

$D = b^2 - 4ac < 0$ it has no real solution, but it has two imaginary solutions

(*to solve the equation we use only quadratic formula*)

Using quadratic formula

$\sqrt{80} = \sqrt{5.16} = 4\sqrt{5}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{-80}}{2(3)}$$

$$x = \frac{-4 + i\sqrt{80}}{6} \quad \text{or } x = \frac{-4 - i\sqrt{80}}{6}$$

$$x = \frac{-4 + 4i\sqrt{5}}{6} \quad \text{or } x = \frac{-4 - 4i\sqrt{5}}{6}$$

$$x = \frac{-2 + 2i\sqrt{5}}{3} \quad \text{or } x = \frac{-2 - 2i\sqrt{5}}{3}$$

Example:

A parallelogram has a base that is 4 units shorter than its height, h. If the height is increased by 5 units and the base is increased by 3 units, write the expression for the new area, A, of the parallelogram in square units.

$A.\ A = (h-1)(h+5)$

$B.\ A = (h+3)(h+4)$

C. $A = (h-1)(h+4)$

$D.\ A = (h+3)(h+5)$

$b = h - 4$

The height is increased by 5 units: (h+5)

The base is increased by 3 units: (h-4)+3=(h-1)

Therefore: A=(h+5)(h-1)

Example:

Emily tossed a soccer ball straight up into the air. The height of the soccer ball at different times after it was thrown is shown in the table below.

Time(seconds)	Height(feet)
1	25
2	36
3	32
4	10

The height h(t) of the ball as a function of time can be described by:

$$\boldsymbol{h(t) = -10t^2 + 40t - 5}$$

At which of the following times was the ball located exactly at a height of 30 feet?

A. 3.2 seconds *B. 2.7 seconds* *C. 2.1 second* *D. 1.6 seconds*