Implementation of equation 3

Effective transconductance analysis

LAN topology overview

To the write, their is the LNA Topology and It has three cascode stages.

<u>Stage A</u>; Input LNA stage,

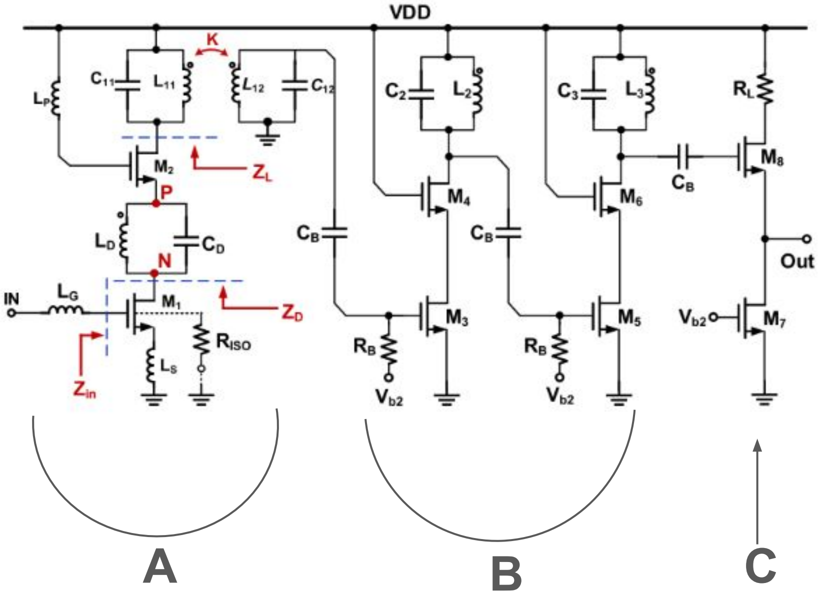
- M1 + Ls,Lg: provides main transconductance & input matching
- Cascode M2 : boosts output resistance & improve isolation
- Tank Load: extends BW and flattens gain

<u>Stage B</u>; Gain boost stage (M3-M6)

- Additional cascode pairs
- Each gain stage is tuned to a slightly different resonant freq & together produced a flat gain

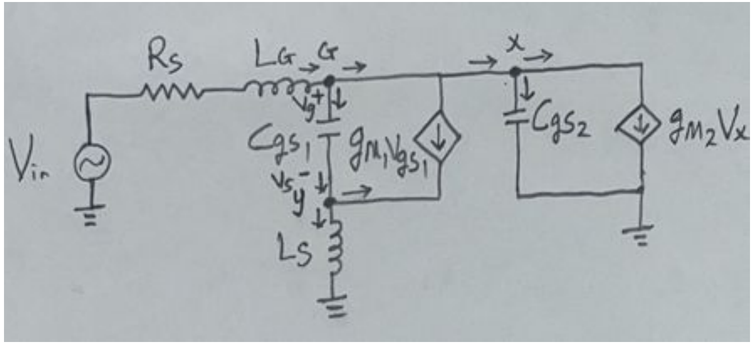
<u>Stage C</u>; Output buffer for impedance matching(M7-M8)

- Provides ~ 50 Ohm output matching without adding significant noise.



Derivation of Eqn.(3)

- To the right is the small signal model to derive equation 3.
- Working in the phasor domain (jw) with voltages Vg, Vx, and Vy.



Setups

- Rs = source impedance 50 ohm
- Output is grounded
- To characterized only transistor stack,load replaced with an AC short to ground (Removed LD, CD, LP)

Vgs1 = Vg − Vs

Final goal , $G_m = \frac{i_{\text{out}}}{v_{\text{in}}},$

KCL @ G:

$$\frac{v_g - v_{\text{in}}}{R_S + j\omega L_G} + j\omega C_{gs1}(v_g - v_s) = 0$$

$$v_{\text{in}} = v_g + (R_S + j\omega L_G)\, j\omega C_{gs1}\,(v_g - v_s)$$
$$= v_g + (R_S + j\omega L_G)\, j\omega C_{gs1}\, v_{gs1}.$$

KCL @ y:

$$\frac{v_s}{j\omega L_S} + j\omega C_{gs1}(v_s - v_g) - g_{m1} v_{gs1} = 0$$

$$v_s = j\omega L_S\big(j\omega C_{gs1} + g_{m1}\big)\, v_{gs1}$$

$$v_g = v_s + v_{gs1} = \Big[1 + j\omega L_S\big(j\omega C_{gs1} + g_{m1}\big)\Big]\, v_{gs1}$$

KCL @ x:

$$g_{m1} v_{gs1} = (j\omega C_{gs2} + g_{m2})\, v_x$$

$$v_x = \frac{g_{m1}}{j\omega C_{gs2} + g_{m2}}\, v_{gs1}, \qquad i_{\text{out}} = g_{m2} v_x = \frac{g_{m1} g_{m2}}{j\omega C_{gs2} + g_{m2}}\, v_{gs1}$$

Derivation of Eqn.(3) continued …

Relate Vgs1 to Vin

$$\begin{aligned} v_{\text{in}} &= \Big[1 + j\omega L_S(j\omega C_{gs1} + g_{m1})\Big] v_{gs1} + (R_S + j\omega L_G)\, j\omega C_{gs1}\, v_{gs1} \\ &= \Big[1 + j\omega\big(g_{m1}L_S + C_{gs1}R_S\big) \; - \; \omega^2 C_{gs1}(L_S + L_G)\Big]\, v_{gs1}. \end{aligned}$$

Hence, $$\frac{v_{gs1}}{v_{\text{in}}} = \frac{1}{1 + j\omega\big(g_{m1}L_S + C_{gs1}R_S\big) - \omega^2 C_{gs1}(L_S + L_G)}$$

Regroup the M1 denominator as $j\omega\big[C_{gs1}(R_S + j\omega(L_S + L_G)) + g_{m1}L_S\big]$

Then, the full expression becomes;

$$G_m(j\omega) = \frac{g_{m2}}{j\omega C_{gs2} + g_{m2}} \cdot \frac{g_{m1}}{j\omega C_{gs1}\big(R_S + j\omega(L_S + L_G)\big) + j\omega g_{m1} L_S}$$

Poles and Zeros

Transconductance: $$G_m(j\omega) = \underbrace{\frac{g_{m2}}{j\omega C_{gs2} + g_{m2}}}_{\text{cascode factor}} \cdot \underbrace{\frac{g_{m1}}{j\omega C_{gs1}\big(R_S + j\omega(L_S + L_G)\big) + j\omega g_{m1} L_S}}_{\text{M1+input network}}$$

<u>Two Pols</u>: root of $A(j\omega)^2 + B(j\omega) + 1 = 0$

Numerical values ; $$g_{m1} = 0.22 \text{ S}, \quad g_{m2} = 0.10 \text{ S},$$
$$C_{gs1} = 0.24 \text{ pF}, \quad C_{gs2} = 0.24 \text{ pF (assumed)},$$
$$L_S = 0.25 \text{ nH}, \quad L_G = 0.85 \text{ nH}, \quad R_S = 50\ \Omega.$$

<u>M1 + input network</u>

Compute the coefficient : - $A = C_{gs1}(L_S + L_G) = 0.24 \text{ pF} \cdot (1.10 \text{ nH}) = 2.64 \times 10^{-22}$

$$B = g_{m1} L_S + C_{gs1} R_S = (0.22)(0.25 \text{ nH}) + (0.24 \text{ pF})(50) = 6.7 \times 10^{-11}$$

Quadratic poles (convert to HZ with f =ω/2π) ; $$f_{p1} \approx \frac{1}{2\pi} \cdot \frac{B - \sqrt{B^2 - 4A}}{2A} = 2.53 \text{ GHz}$$
$$f_{p2} \approx \frac{1}{2\pi} \cdot \frac{B + \sqrt{B^2 - 4A}}{2A} = 37.86 \text{ GHz}$$

Poles and Zeros continued …

<u>Cascode Pole</u> : $$f_{p,\mathrm{CG}} = \frac{1}{2\pi} \cdot \frac{g_{m2}}{C_{gs2}} = \frac{0.10}{2\pi \cdot 0.24\,\mathrm{pF}} = 66.31\ \mathrm{GHz}$$

<u>No Zeros</u> : Since, both the numerators are constant, gm1 and gm2.

Effective Transconductance

$$G_{m1} = \left(\frac{g_{m1}}{j\omega C_{gs1}(j\omega L_S + j\omega L_G + R_S) + j\omega g_{m1} L_S} \right) \cdot \left(\frac{g_{m2}}{j\omega C_{gs2} + g_{m2}} \right)$$

- Shows how Gm1 decreases as frq. Increases due to reactive term- Cgs1,LS,LG , and Cgs2
- **Cgs2** :- assumed ≈0.24 pF(Size matched to M1) , sensitivity sweep confirms only high frequency roll-off shifts
- **Gm1** is complex : - has magnitude (gain) and phase

Cascode Factor Equation

$$\underbrace{\frac{g_{m2}}{j\omega C_{gs2} + g_{m2}}}_{\text{cascode factor}}$$

The Low-frequency vs High-frequency behaviour

- At low freq :- jwCgs2 << gm2 , cascode Factor ≈ 1
- At high freq : - jwCgs2 >> gm2 , cascode factor ~ 1/ (jwCgs2) so it drops off
- The corner Condition -3 dB point when : -

$$\omega C_{gs2} = g_{m2} \longrightarrow f_c = \frac{g_{m2}}{2\pi C_{gs2}} = \frac{0.10}{1.5079644 \times 10^{-12}} \approx 6.63 \times 10^{10}\ Hz \approx 66.3\ GHz.$$

Take the magnitude of the Cascode Factor and substitute gm2 = 0.10 S, Cgs2 = 0.24 pF

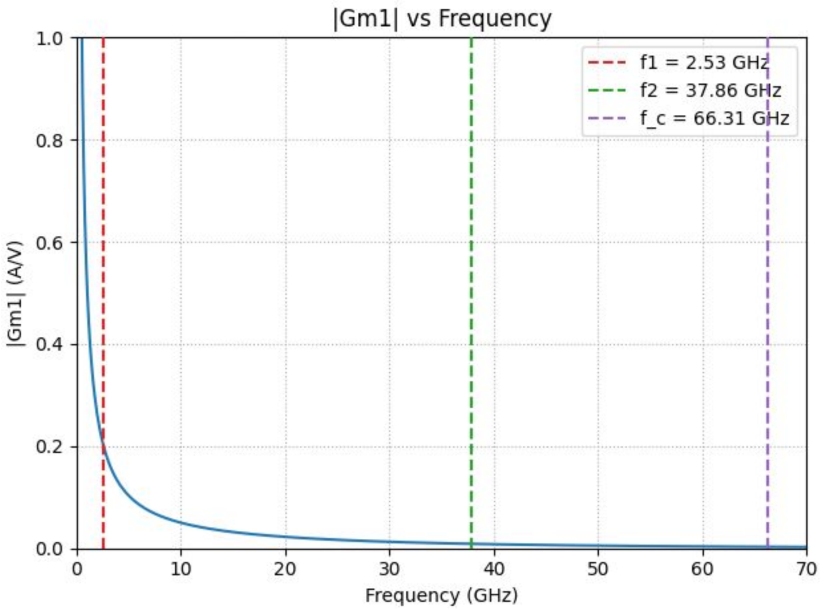
$$\longrightarrow \frac{g_{m2}}{\sqrt{g_{m2}^2 + g_{m2}^2}} = \frac{1}{\sqrt{2}} \approx 0.707\ \ (-3.01\ dB).$$

- Magnitude drops to 0.707 (-3 dB) at fc

|Gm1| vs Freq. plot

Why |Gm|

- because the complex Gm itself hard to visualize
- Magnitude shows the strength of the transistor vs freq.
- |Gm1| tells me how much output current i get from a small input voltage and the load then converts that current into voltage and that what’s i finally measure as an amplifier gain.



Plot shows

- The first stage alone can not keep Gm1 flat at high freq., parasitic caps & inductors limits it
- The paper added transformer tank load with dual resonance to compensate for this natural roll-off & flatten the over all gain
- Larger Cgs2 can cause earlier roll-off
- Shows that M1 + M2 pair loses strength as freq increases.

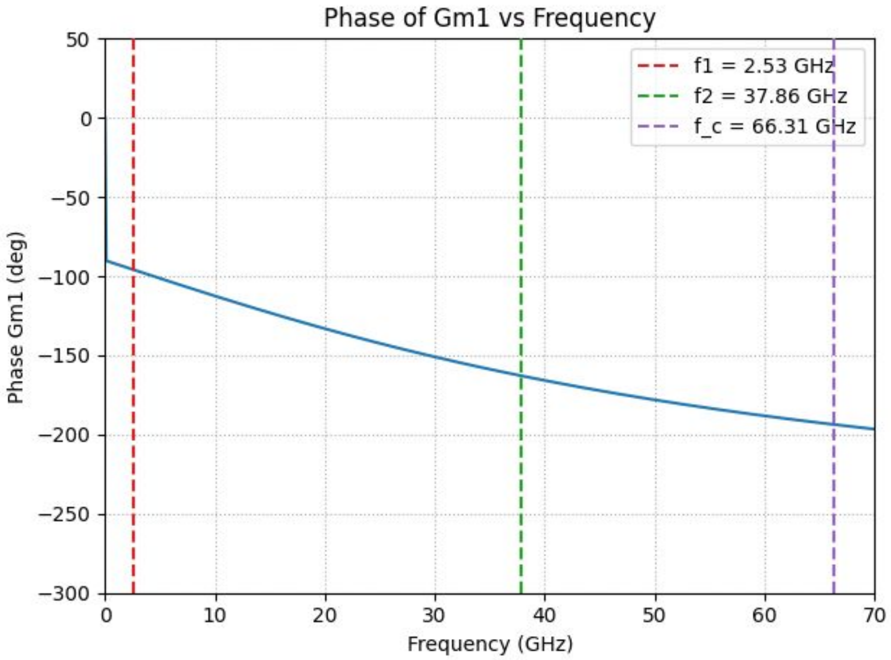
Phase of Gm1 vs Frequency Plot

Why Look at Phase

- Shows the relationship b/n input V and output current
- Shows timing delay between Vin and output current
- Important for stability, group delay, and overall amplifier behaviour when cascading stage

Plot shows

- At f1: phase ≈ –98° (output current is lagging the input voltage by about 98°) almost quadrature
- At f2: Phase ≈ -153° more lag
- At fc: Phase ≈ -189°
- Trend : smooth but lag increases with frequency
- Its stable no sharp jump



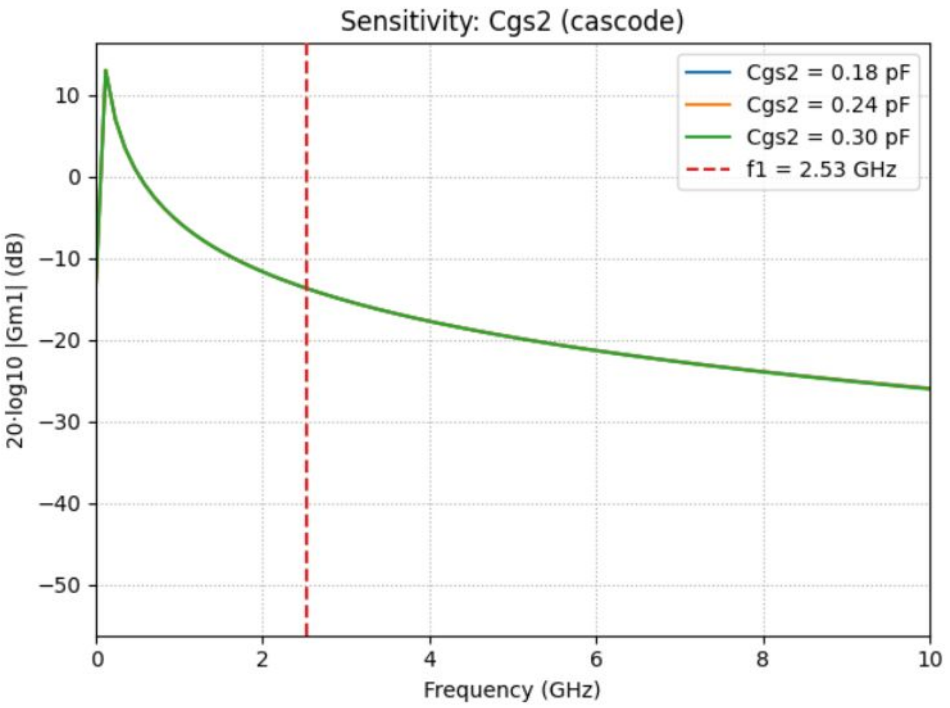
Sensitivity Sweep Cgs2 Plots

Why Cgs2 is important

- It sets high-frequency pole(corner frq.) of the cascode device, beyond gain cant stay flat anymore
- Large Cgs2 - lower pole frequency- earlier gain roll-off
- Small Cgs2 - higher pole frequency - wider bandwidth
- Controlled by the size of M2

Plot shows

- Cgs2 = 0.18,0.24,0.30 pF curves are overlapped
- Zoomed plot, to show the behaviour in lower freq.



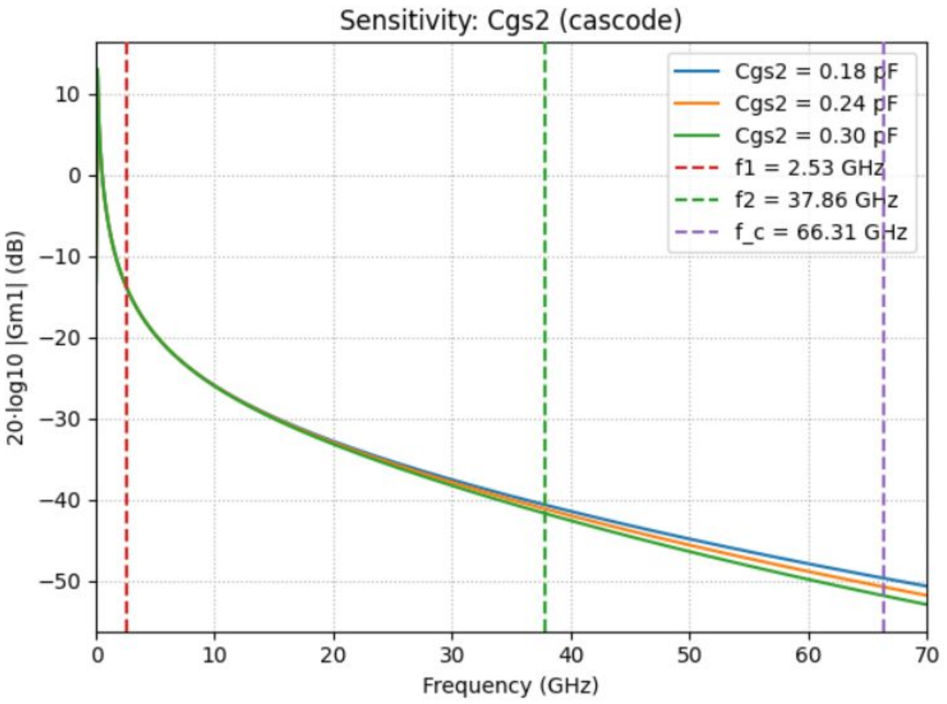
Sensitivity Sweep Cgs2 with High frequency Plots

Plot shows

- Cgs2 = 0.18,0.24,0.30 pF curves are separated with high frequency
- For the band 4 - 8 GHz, the corner frq. Is way up even with Cgs2 = 0.24 pF so the roll-off wont hurt the band

Trade-offs

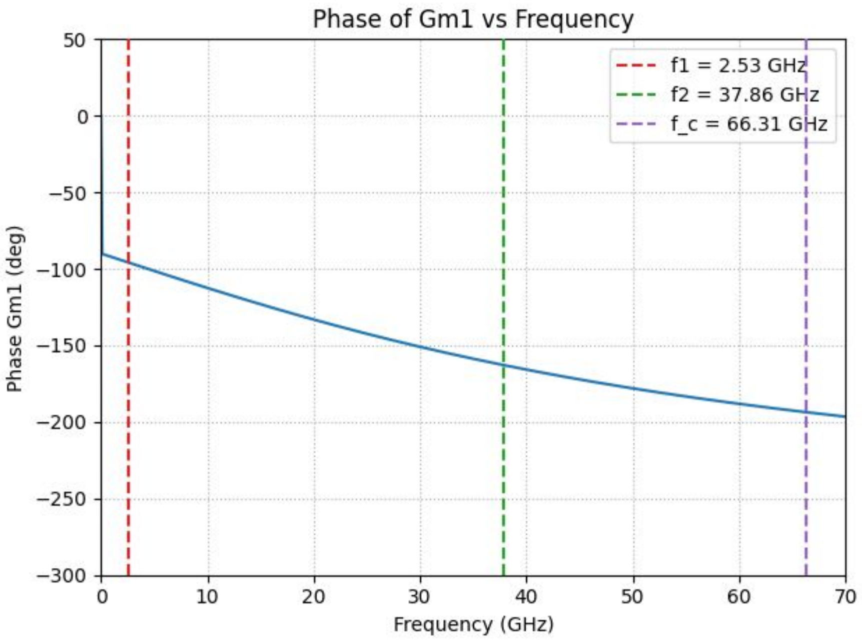
- Large M2 - unnecessary capacitance hurts BW
- Small M2 - poor isolation from M1's drain leads to worse NF
- Recommendation:- the good spot is choosing M2 just big enough to give good cascode isolation while keeping Cgs2 modest so the pole/corner frq stays far above the design BW



The effect of high frequency pole

Plot shows

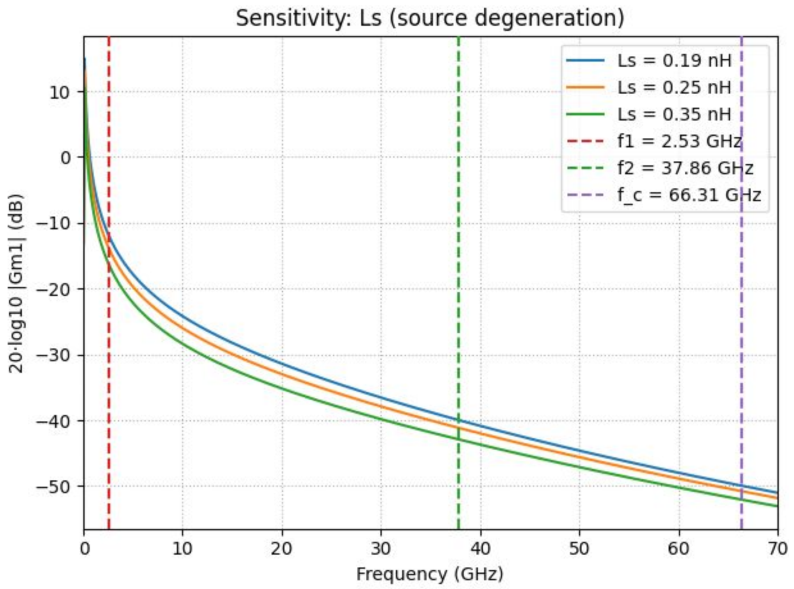
- Higer frq. the cascode device can't keep up
- Shows -90 degree shift
- Shows the output current lags (comes later) & shrinks as freq. increases
- Bellow the Band of interest 4 - 8 GHz , the cascode passes the signal almost normal

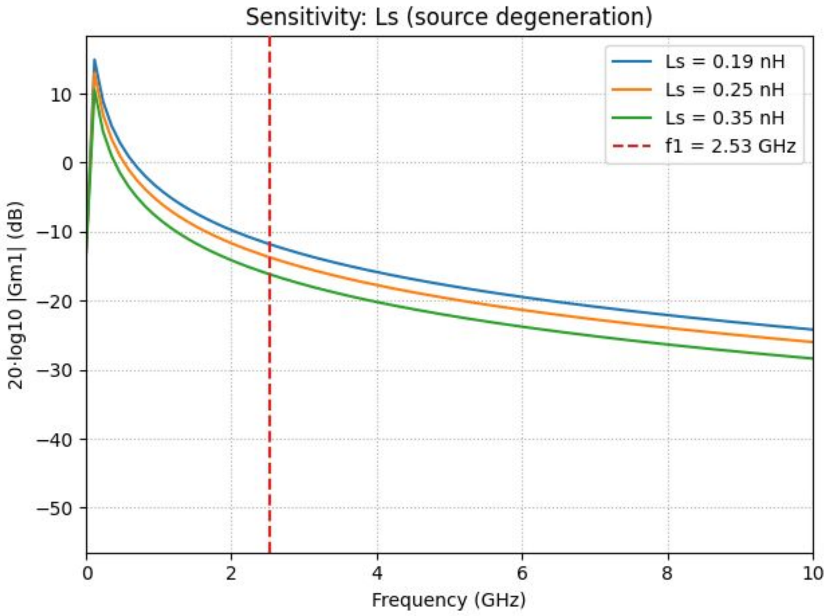


Sensitivity to Ls Sweep

Plot shows

- For Ls sweep gain drops as frequency increases
- Smaller Ls gives higher gain
- Larger Ls lower gain





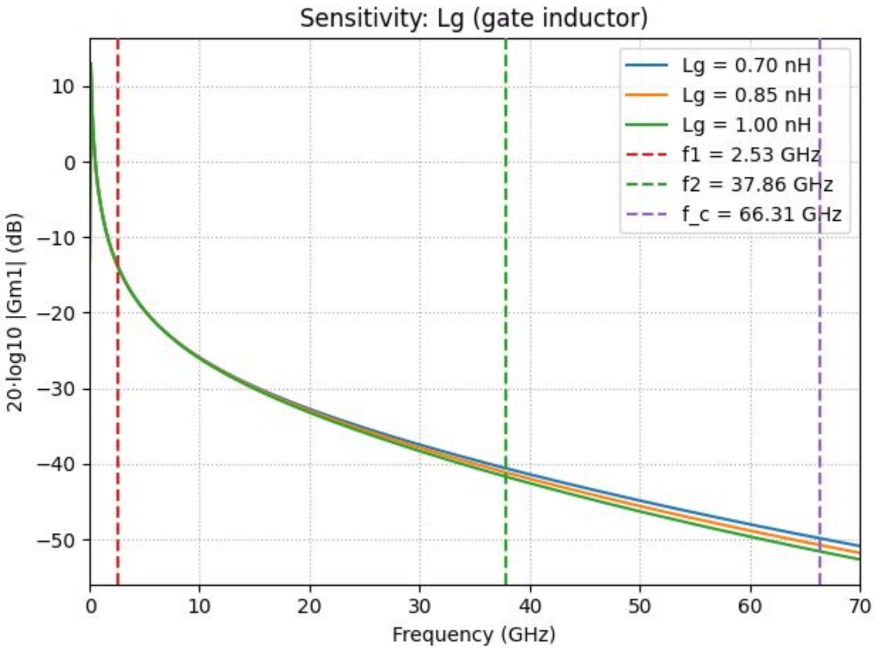
With high frequency

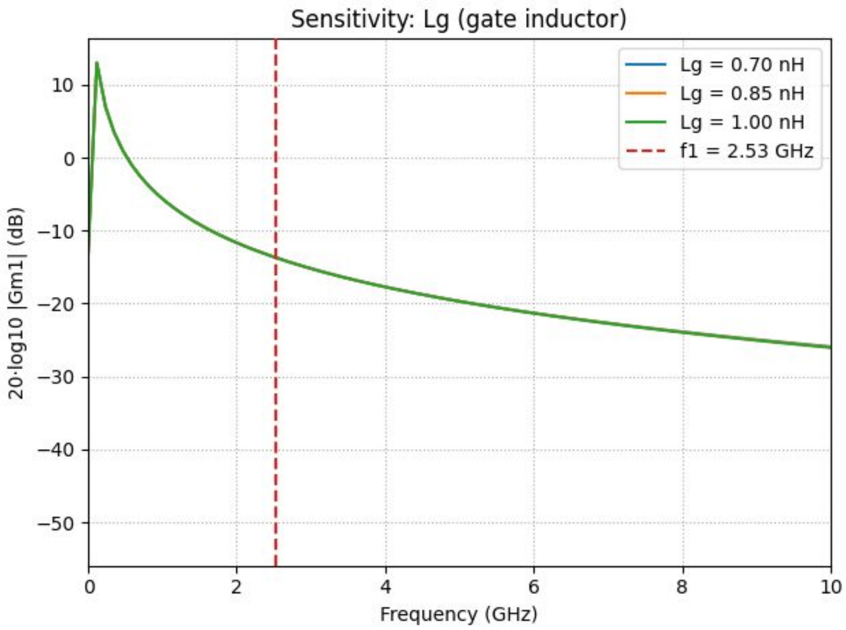
- The first stage gain drops compare to the desired design frequency 4 - 8 GHz

Sensitivity to Lg Sweep

Plot shows

- For Lg sweep gain drops as frequency increases
- 0 - 10 GHz, seems that they are perfectly overlapped





With high frequency

- The curve start separate
- lower Lg- higher gain
- Higher Lg - lower gain

Equation(4) Implementation

Resonant frequency and First stage gain

Resonant Frequencies - Transformer LC Tank

Let the node voltages across the tanks be $v_1(t), v_2(t)$ and the inductor branch currents be $i_1(t), i_2(t)$. For sinusoidal steady state with $s = j\omega$:

$$v_1 = sL_{11}\, i_1 + sM\, i_2, \qquad (1)$$
$$v_2 = sM\, i_1 + sL_{12}\, i_2.$$

Because the capacitors are in parallel with the inductors at each node, KCL gives

$$i_1 + C_{11} s\, v_1 = 0, \qquad i_2 + C_{12} s\, v_2 = 0. \qquad (2)$$

Eliminate i_1, i_2 from (1) using (2):

$$\begin{bmatrix} 1 - s^2 L_{11} C_{11} & -s^2 M C_{11} \\ -s^2 M C_{12} & 1 - s^2 L_{12} C_{12} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbf{0}. \qquad (3)$$

Nontrivial solutions require $\det(\cdot) = 0$:

$$(1 - s^2 L_{11} C_{11})(1 - s^2 L_{12} C_{12}) - s^4 M^2 C_{11} C_{12} = 0. \qquad (4)$$

Let $x = s^2 = \omega^2$ and divide (4) by $L_{11}C_{11}L_{12}C_{12}$:

$$x^2(1 - k^2) - x\left(\frac{1}{L_{11}C_{11}} + \frac{1}{L_{12}C_{12}}\right) + \frac{1}{L_{11}C_{11}L_{12}C_{12}} = 0, \qquad (5)$$

using $M^2 = k^2 L_{11} L_{12}$.

Write $\eta = \dfrac{L_{12}C_{12}}{L_{11}C_{11}}$ and multiply (5) by $L_{12}C_{12}$:

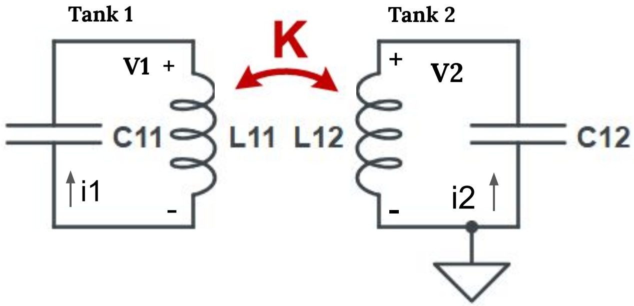
$$x^2(1 - k^2) L_{12} C_{12} - x\,(1 + \eta) + \frac{1}{L_{11}C_{11}} = 0. \qquad (6)$$

Solve the quadratic in $x = \omega^2$:

$$\omega_{1,2}^2 = \frac{(1 + \eta) \mp \sqrt{(1 - \eta)^2 + 4\eta k^2}}{2\, L_{12} C_{12}\, (1 - k^2)}. \qquad (7)$$

Converting to frequency $f = \omega/(2\pi)$ gives

$$f_{1,2}^2 = \frac{1 + \eta \mp \sqrt{1 + \eta^2 - 2\eta(1 - 2k^2)}}{8\pi^2\, L_{12} C_{12}\, (1 - k^2)} \qquad (8)$$



- Primary Tank (L11 // C11)
- Secondary Tank (L12 // C12)
- Coupling factor k ∈ [0,1) by Mutial inductance M = k√L11L12
- Asymmetry parameter η = (L12C12)∕(L11C11)

Resonant Frequencies - Transformer LC Tank

- The two resonant frequencies of the transformer-coupled LC tank are given by;

$$f_{1,2}^2 = \frac{1+\eta \mp \sqrt{1+\eta^2-2\eta(1-2k^2)}}{8\pi^2 L_{12}C_{12}(1-k^2)}, \qquad \eta = \frac{L_{12}C_{12}}{L_{11}C_{11}},\ k = coupling$$

- No coupling (k=0) and equal tanks (η=1) - Two independent LC tank
  - The formula collapses to

$$f_1 = \frac{1}{2\pi\sqrt{L_{12}C_{12}}}, \quad f_2 = \frac{1}{2\pi\sqrt{L_{11}C_{11}}}$$

- With coupling K > 0 - mode splitting into

$$f_1^2 = \frac{1}{4\pi^2 L_{12}C_{12}(1+k)}, \quad f_2^2 = \frac{1}{4\pi^2 L_{12}C_{12}(1-k)}$$

  - In-phase mode f1 - Lower frequency (k+1)
  - Out-of-phase mode f2 - Higher frequency (k-1)

Numerical values

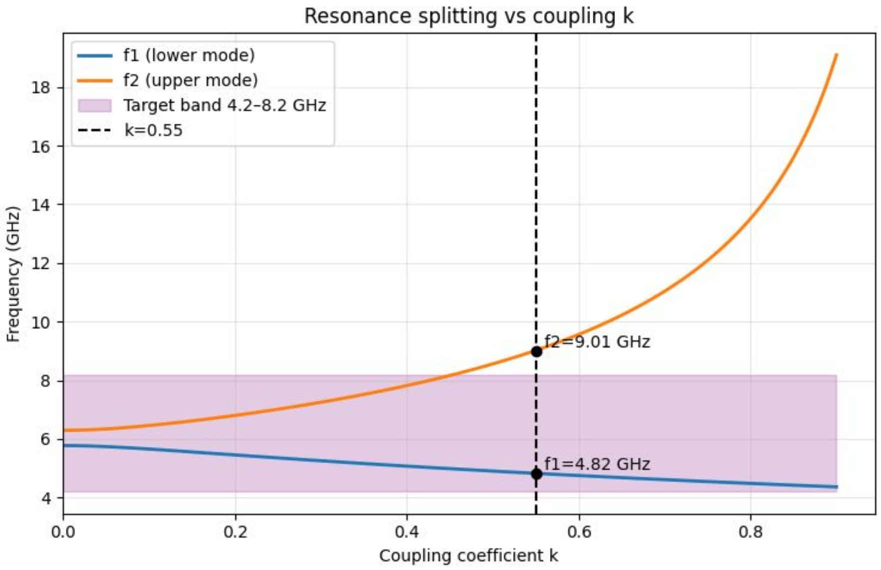
- Using: C11=C12= 0.4 pF, L11= 1.9 nH, L12=1.6 nH, k ~ = 0.55 and η= 0.8421. f1 & f2 becomes 4.82 & 9.01 GHz respectively.
- To match the paper exactly (f1 = 4.2 GHz, f2 = 8.2 GHz), both capacitors were scaled to C11=C12 ≈ 0.518 pF

M. Babaie and R. B. Staszewski, “A Class-F CMOS Oscillator,” IEEE Journal of Solid-State Circuits, vol. 48, no. 12, pp. 3120–3133, Dec. 2013 — see Eq. (6) on p. 3123 and the discussion below Fig. 3.

Equation(4) Implementation - resonant frequency

Plot shows:

- **Blue curve (f1, lower mode):** In phase resonance frequency
  - As k increases, f1 shifts from ~6 GHz to ~4 GHz
- **Orange curve(f2, upper mode):** out of phase resonant frq.
  - As k increases, f2 shifts upward from ~ 6 GHz to ~19 GHz
- **Splitting behavior :** Two resonant frequencies move apart as coupling increases
  - At k = 0, the two modes are equal only if eta = 1(i.e., L11C11=L12C12).Otherwise they start separated
  - At k = 0.55, they are separated significantly
- **Target band (gray region 4.2 - 8.2 GHz):**
  - Target operating band of the LNA (the band we care about)
- **Coupling Coefficient K**
  - Shows how strongly the two inductors in the transformer tank are magnetically coupled
  - K = 0 - no coupling
  - Large k - stronger coupling, more separation between f1 & f2

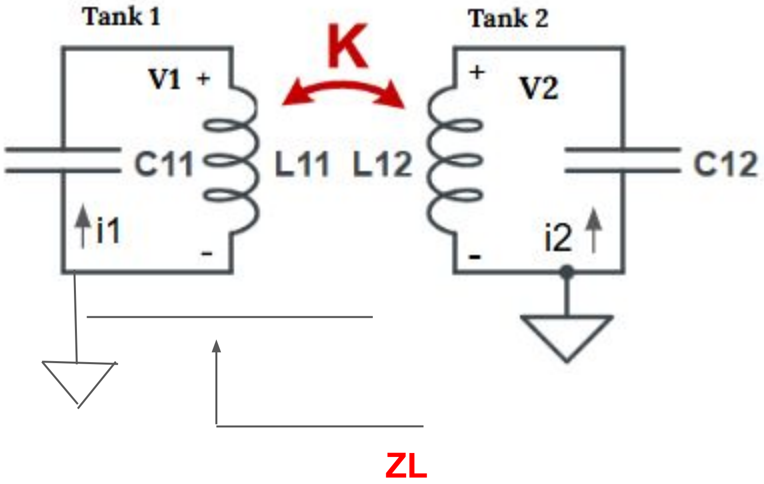


Parameter used:
L11=1.9nH,L12=1.6nH,C11=C12=0.4pF,η=0.842,k=0.55

Computed from Eq. (4) — M. Babaie & R. Staszewski (2013)

Derivation of ZL

Set Up



- Tank 1 = (L11 // C11)
- Tank 2 = (L12 // C12)
- Coupling factor k ∈ [0,1) by M = k√L11L12
- Asymmetry parameter η = (L12C12)⁄(L11C11)

Derivation of ZL

Let the two parallel tanks be coupled via mutual inductance

$$M = k\sqrt{L_{11}L_{12}}, \qquad \Delta_L = L_{11}L_{12} - M^2 = L_{11}L_{12}(1-k^2).$$

Coupled inductor relation in frequency domain with node voltage v1,v2 and inductor branch currents i1 and i2

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = j\omega \underbrace{\begin{bmatrix} L_{11} & M \\ M & L_{12} \end{bmatrix}}_{\mathbf{L}} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{j\omega}\mathbf{L}^{-1}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix},$$

where the inverse of the 2×2 inductance matrix is

$$\mathbf{L}^{-1} = \frac{1}{\Delta_L}\begin{bmatrix} L_{12} & -M \\ -M & L_{11} \end{bmatrix}.$$

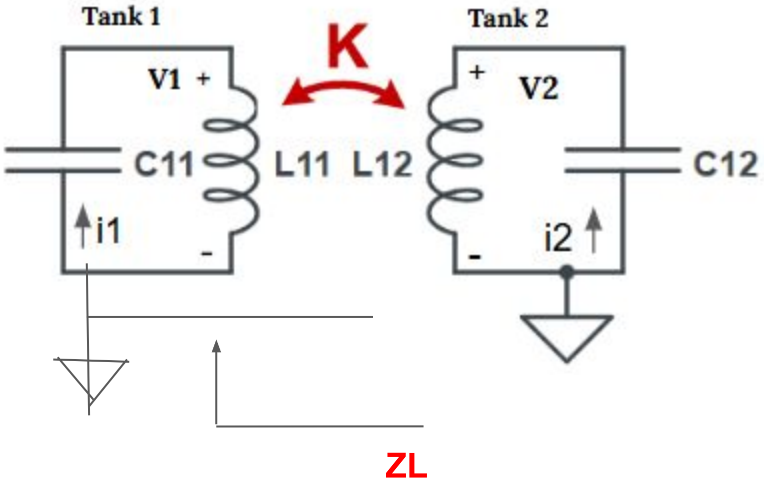
Shunt Capacitor currents

$$i_{C1} = j\omega C_{11} v_1, \qquad i_{C2} = j\omega C_{12} v_2.$$

Since $Y = j\omega L^{-1}$ for an inductor branch,

$$\mathbf{Y}_L(\omega) = \frac{1}{j\omega\,\Delta_L}\begin{bmatrix} L_{12} & -M \\ -M & L_{11} \end{bmatrix}.$$

Add shunts C and Rp to ground shunts at each node add to the diagonal.



- Tank 1 = (L11 // C11)
- Tank 2 = (L12 // C12)
- Coupling factor k ∈ [0,1) by M = k√L11L12
- Asymmetry parameter η = (L12C12)∕(L11C11)

Reference Note: All relations originate from the transformer-coupled LC-tank analysis by M. Babaie and R. B. Staszewski, “A Class-F CMOS Oscillator,” IEEE Journal of Solid-State Circuits, vol. 48, no. 12, pp. 3120–3133, Dec. 2013 (see Eq. 6 and Fig. 3). Additional nodal-admittance derivation steps follow T. H. Lee (2004) and D. M. Pozar (2012) for the 1-port impedance reduction used in the derivation and Python simulation

Derivation of ZL continued . . .

$$\mathbf{Y}_{\text{sh}}(\omega) = \begin{bmatrix} G_1 + j\omega C_{11} & 0 \\ 0 & G_2 + j\omega C_{12} \end{bmatrix}$$

Thus the total two–node nodal admittance is

$$\boldsymbol{Y}(\omega) = \mathbf{Y}_{\text{sh}}(\omega) + \mathbf{Y}_L(\omega),$$

with entries

$$Y_{11}(\omega) = G_1 + j\omega C_{11} + \frac{L_{12}}{j\omega\,\Delta_L},$$

$$Y_{22}(\omega) = G_2 + j\omega C_{12} + \frac{L_{11}}{j\omega\,\Delta_L},$$

$$Y_{12}(\omega) = Y_{21}(\omega) = -\frac{M}{j\omega\,\Delta_L}.$$

For compactness the reactive parts can be written as;

$$X_1(\omega) = \omega C_{11} - \frac{L_{12}}{\omega\Delta_L}, \qquad X_2(\omega) = \omega C_{12} - \frac{L_{11}}{\omega\Delta_L},$$

so that $Y_{11} = G_1 + jX_1$, $Y_{22} = G_2 + jX_2$, and $Y_{12} = +j\,\dfrac{M}{\omega\Delta_L}$

From Tank 1 and 2 circuit, nodal-admittance relationship for a two-node network is:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Node 2 is floating, the input admittance is the Schur complement of Y22:

$$\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2, \\ I_2 = Y_{21}V_1 + Y_{22}V_2 = 0. \end{cases}$$

From the this equation (since $I_2 = 0$), solve for V_2:

$$V_2 = -\,Y_{22}^{-1}Y_{21}V_1.$$

Substituting this into I_1 gives

$$I_1 = Y_{11}V_1 + Y_{12}(-Y_{22}^{-1}Y_{21}V_1) = \left(Y_{11} - Y_{12}Y_{22}^{-1}Y_{21}\right)V_1.$$

Therefore,

$$Y_{\text{in}} = Y_{11} - Y_{12}Y_{22}^{-1}Y_{21}$$

F. Zhang, The Schur Complement and Its Applications. Springer, 2005.

Derivation of ZL continued . . .

The block Y22 in the matrix Y . It represents the effective admittance seen at Node 1 after “eliminating” Node 2 mathematically.

$$Y_{\text{in}}(\omega) = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}} = \frac{Y_{11}Y_{22} - Y_{12}^2}{Y_{22}}.$$

Therefore

$$Z_L(\omega) = \frac{1}{Y_{\text{in}}(\omega)} = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}^2}.$$

Expand the denominator and substitute

First note

$$Y_{12}^2 = \left(j\,\frac{M}{\omega\Delta_L}\right)^2 = -\frac{M^2}{\omega^2\Delta_L^2}.$$

Next,

$$Y_{11}Y_{22} = (G_1 + jX_1)(G_2 + jX_2) = G_1G_2 + j(G_1X_2 + G_2X_1) - X_1X_2.$$

Thus

$$Y_{11}Y_{22} - Y_{12}^2 = \left(G_1G_2 - X_1X_2 + \frac{M^2}{\omega^2\Delta_L^2}\right) + j\,(G_1X_2 + G_2X_1).$$

Since $Y_{22} = G_2 + jX_2$, the final explicit result is

$$Z_L(\omega) = \frac{G_2 + jX_2}{\left(G_1G_2 - X_1X_2 + \frac{M^2}{\omega^2\Delta_L^2}\right) + j\,(G_1X_2 + G_2X_1)}$$

With,

$$X_1(\omega) = \omega C_{11} - \frac{L_{12}}{\omega\Delta_L},$$

$$M = k\sqrt{L_{11}L_{12}},$$

$$G_1 = \frac{1}{R_{p1}},$$

$$X_2(\omega) = \omega C_{12} - \frac{L_{11}}{\omega\Delta_L},$$

$$\Delta_L = L_{11}L_{12} - M^2 = L_{11}L_{12}(1 - k^2),$$

$$G_2 = \frac{1}{R_{p2}}.$$

Derivation of ZL continued . . .

Substitute everything back again to get the final ZL

$$Z_L(\omega) = \frac{G_2 + j\left(\omega C_{12} - \frac{L_{11}}{\omega \Delta_L}\right)}{\left(G_1 G_2 - \left(\omega C_{11} - \frac{L_{12}}{\omega \Delta_L}\right)\left(\omega C_{12} - \frac{L_{11}}{\omega \Delta_L}\right) + \frac{M^2}{\omega^2 \Delta_L^2}\right) + j\left(G_1\left(\omega C_{12} - \frac{L_{11}}{\omega \Delta_L}\right) + G_2\left(\omega C_{11} - \frac{L_{12}}{\omega \Delta_L}\right)\right)}$$

Integrating Quality factor Q1 and Q2

For high-Q inductors, the series and parallel conversion is:

$$R_p(\omega) \approx \omega L Q \quad \Longrightarrow \quad G(\omega) = \frac{1}{R_p(\omega)} \approx \frac{1}{\omega L Q}.$$

Then,

$$G_1(\omega) \approx \frac{1}{\omega L_{11} Q_1}, \qquad G_2(\omega) \approx \frac{1}{\omega L_{12} Q_2},$$

With $G_1 = G_2 = 0$, parallel resonance occurs when $Y_{11}Y_{22} - Y_{12}^2 = 0$, leading to

$$\omega_{1,2}^2 = \frac{1 + \eta \mp \sqrt{1 + \eta^2 - 2\eta(1 - 2k^2)}}{2\, L_{12} C_{12} (1 - k^2)}, \qquad \eta \equiv \frac{L_{12} C_{12}}{L_{11} C_{11}}, \quad f_{1,2} = \frac{\omega_{1,2}}{2\pi}.$$

$$Y_{11}Y_{22} - Y_{12}^2 = \left(G_1 G_2 - X_1 X_2 + \frac{M^2}{\omega^2 \Delta_L^2}\right) + j\,(G_1 X_2 + G_2 X_1).$$

M. Babaie and R. B. Staszewski, “A class-F CMOS oscillator,” IEEE J. Solid-State Circuits, vol. 48, no. 12, pp. 3120–3133, Dec. 2013.

Derivation of ZL continued . . .

Zeros and poles: $$Z_L(\omega) = \frac{G_2 + j\left(\omega C_{12} - \frac{L_{11}}{\omega \Delta_L}\right) \quad = N(s)}{\left(G_1 G_2 - \left(\omega C_{11} - \frac{L_{12}}{\omega \Delta_L}\right)\left(\omega C_{12} - \frac{L_{11}}{\omega \Delta_L}\right) + \frac{M^2}{\omega^2 \Delta_L^2}\right) + j\left(G_1\left(\omega C_{12} - \frac{L_{11}}{\omega \Delta_L}\right) + G_2\left(\omega C_{11} - \frac{L_{12}}{\omega \Delta_L}\right)\right)} \quad = D(s)$$

Zeros

$$Y_{22}(s) = \bar{G}_2 + s\,C_{12} + \frac{L_{11}}{\Delta_L}\,\frac{1}{s}, \qquad \Delta_L \equiv L_{11}L_{12} - M^2, \quad M = k\sqrt{L_{11}L_{12}}.$$

$$N(s) = s^2 Y_{22}(s) = C_{12}s^3 + \bar{G}_2 s^2 + \frac{L_{11}}{\Delta_L}\,s.$$

Therefore the zeros of $Z_L(s)$ are the roots of $N(s) = 0$:

$$s = 0\,, \qquad C_{12}s^2 + \bar{G}_2 s + \frac{L_{11}}{\Delta_L} = 0\,.$$

Solving the quadratic gives the two finite zeros

$$s_{z1,2} = \frac{-\bar{G}_2 \pm \sqrt{\bar{G}_2^2 - 4C_{12}\frac{L_{11}}{\Delta_L}}}{2C_{12}} = -8.0219 \times 10^9 \;\pm\; j\,4.1566 \times 10^{10}\ \text{rad/s}\ .$$

$$z_0 = 0, \quad z_{1,2} = -8.022 \times 10^9 \pm \mathrm{j}\,4.1566 \times 10^{10}\ \text{rad/s}, \quad f_z = \frac{|\operatorname{Im} z|}{2\pi} = 6.616\ \text{GHz}.$$

$$\omega_{\text{ref}} = \frac{\omega_1 + \omega_2}{2} \quad \Rightarrow \quad f_{\text{ref}} = \frac{f_1 + f_2}{2}.$$

With band ends $f_1 = 4.20\,\text{GHz}$, $f_2 = 8.20\,\text{GHz}$:

$f_{\text{ref}} = \frac{4.2+8.2}{2} = 6.20\,\text{GHz}.$

Poles

$$Y_{11}(s) = \bar{G}_1 + s\,C_{11} + \frac{L_{12}}{\Delta_L}\frac{1}{s},$$
$$Y_{22}(s) = \bar{G}_2 + s\,C_{12} + \frac{L_{11}}{\Delta_L}\frac{1}{s}, \qquad \Delta_L = L_{11}L_{12} - M^2,$$
$$Y_{12}(s) = -\frac{M}{\Delta_L}\frac{1}{s},$$

Substitute constants ($L_{11} = 1.9\,\mathbf{nH},\ L_{12} = 1.6\,\mathbf{nH},\ C_{11} = C_{12} = 0.5\,\mathbf{pF},\ k = 0.55$).

$$\Delta_L = (1-k^2)L_{11}L_{12} = 2.12 \times 10^{-18}\,\mathrm{H}^2, \quad \frac{L_{11}}{\Delta_L} = 8.9606 \times 10^8\ \mathrm{s}^{-2},$$

$$\bar{G}_1 = \frac{1}{2\pi f_{\mathrm{ref}} L_{11} Q_1} = \mathbf{1.6476 \times 10^{-3}}\ \mathrm{S}\ (Q_1 = 8.2), \quad \bar{G}_2 = \frac{1}{2\pi f_{\mathrm{ref}} L_{12} Q_2} = \mathbf{8.0219 \times 10^{-3}}\ \mathrm{S}\ (Q_2 = 2).$$

Poles from the denominator $D(s) = s^2(Y_{11}Y_{22} - Y_{12}^2)$.

$$D(s) = (C_{11}C_{12})s^4 + (\bar{G}_1 C_{12} + \bar{G}_2 C_{11})s^3 + \left(\bar{G}_1\bar{G}_2 + \frac{C_{11}L_{11} + C_{12}L_{12}}{\Delta_L}\right)s^2 + \frac{\bar{G}_1 L_{11} + \bar{G}_2 L_{12}}{\Delta_L}s + \frac{1}{\Delta_L}.$$

Coeffs: $a_4 = 2.50 \times 10^{-25}$, $a_3 = 4.8348 \times 10^{-15}$, $a_2 = 8.3853 \times 10^{-4}$, $a_1 = 7.5295 \times 10^{6}$,
$a_0 = 4.7161 \times 10^{17}$.
Roots $D(s) = 0$:

$$p_{1,2} = -5.2378 \times 10^9 \ \pm\ j\,4.9821 \times 10^{10}\ \mathrm{rad/s}\,, \qquad p_{3,4} = -4.4317 \times 10^9 \ \pm\ j\,2.7056 \times 10^{10}\ \mathrm{rad/s}\,.$$

Imaginary parts $\Rightarrow$ $f_1 = 7.929$ GHz, $f_2 = 4.306$ GHz.

|ZL| vs Frequency plot

Key Observations:

- The plot shows the magnitude of the load impedance |ZL|, drive from the coupled-tank network:

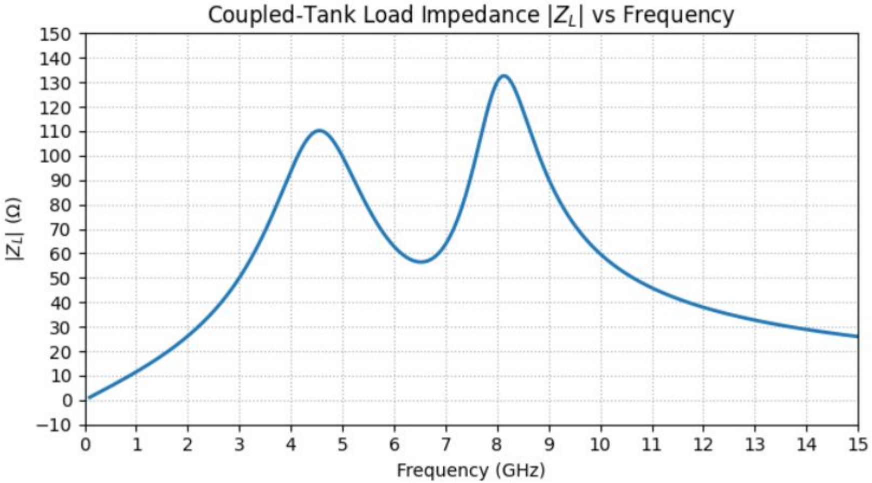
$$Z_L(f) = \frac{1}{Y_{11} - \frac{Y_{12}^2}{Y_{22}}}$$

- Two resonance peak appears:

  – f1 ≈ 4.2 GHz — lower-mode resonance.

  – f2 ≈ 8.2 GHz — upper-mode resonance.

- The valley around 6–7 GHz arises from coupling-induced mode splitting between even and odd tank modes

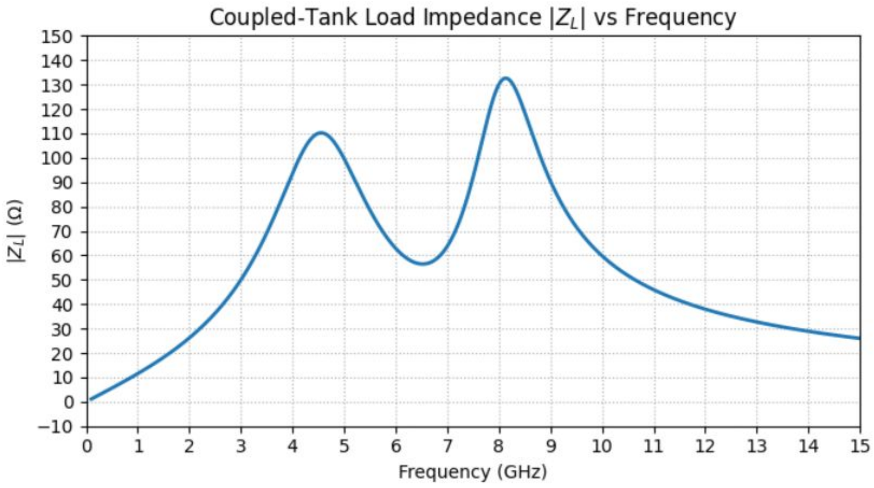


Parameter Setup:
• L11 = 1.9 nH, L12 = 1.6 nH, C11 = C12 = 0.5pF, k = 0.55
• Q1 = 8.2, Q2 = 2.0

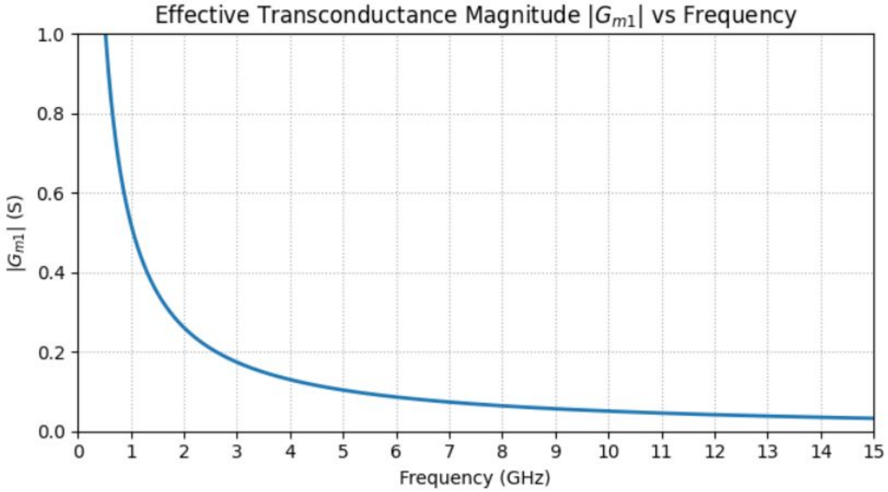
|ZL| vs Frequency plot

What Changes What:

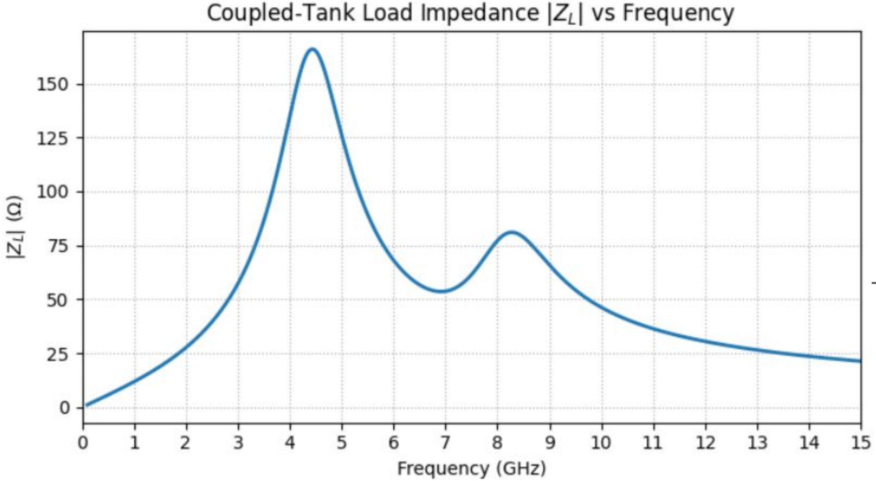
- Increase k → widens the separation between f1 and f2 (stronger mode split).
- Decrease k → merges the peaks into a single resonance.
- Increase Q1 or Q2 → raises corresponding peak height and sharpness.
- Increase C12 or L12 → shifts both peaks to lower frequencies.
- Increase losses (lower Q) → flattens peaks and widens bandwidth.



Parameter Setup:
- L11 = 1.9 nH, L12 = 1.6 nH, C11 = C12 = 0.4 pF, k = 0.55
- Q1 = 8.2, Q2 = 2.0

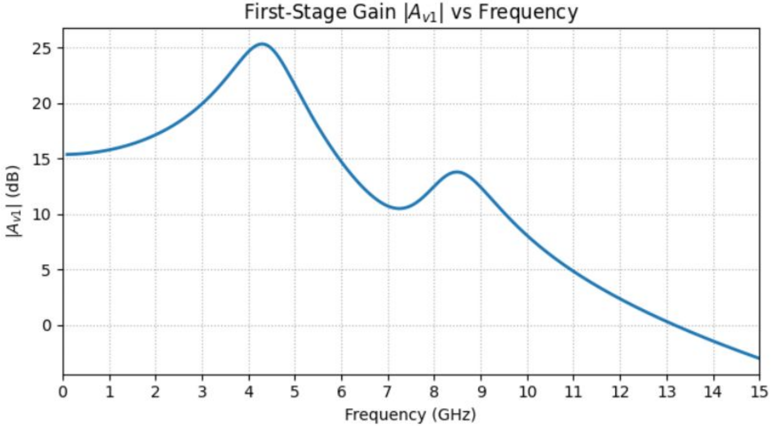


X

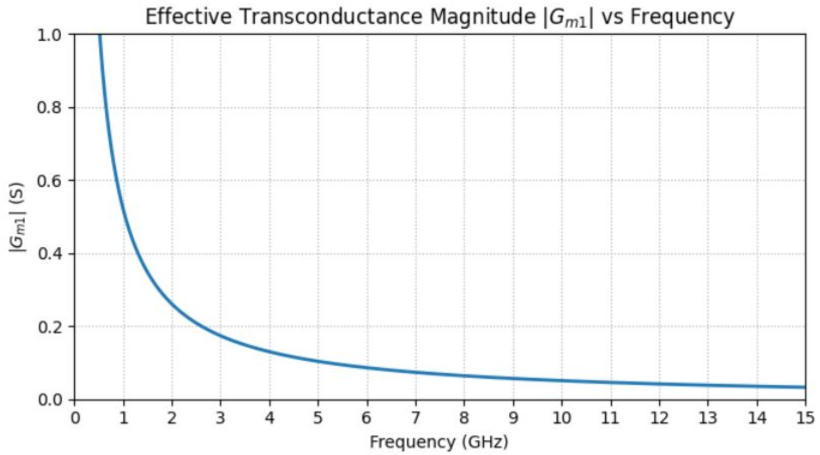


=

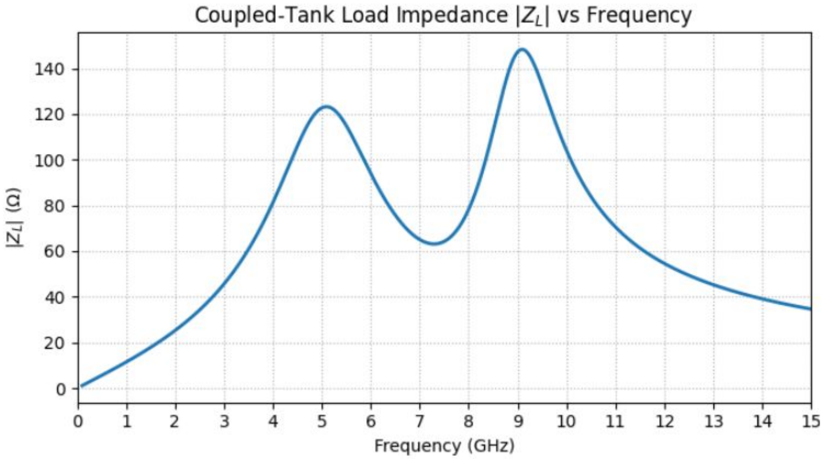
With Q1=8.2,Q1=2



When C11 and C12 are not equal
K = 0.58

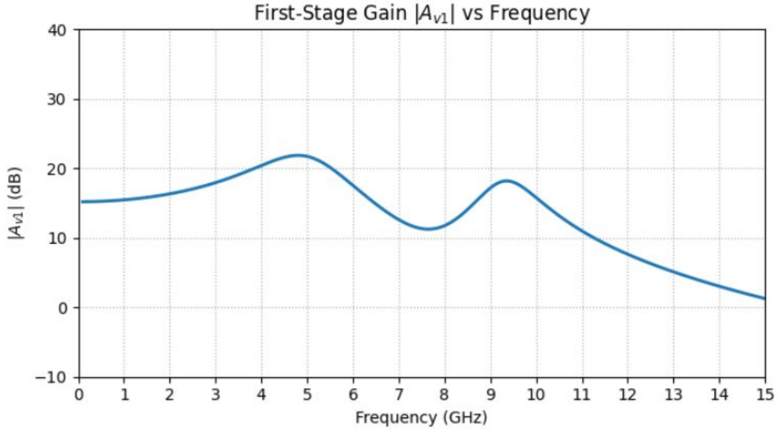


X



=

With Q1=8.2,Q1=2



When C11 and C12 are equal
K = 0.55

First Stage Gain

- The plot shows the simulated first-stage voltage gain magnitude

$$|A_{v1}(f)| = |G_{m1}(f)| \times |Z_L(f)|$$

  over the 0–15 GHz range.

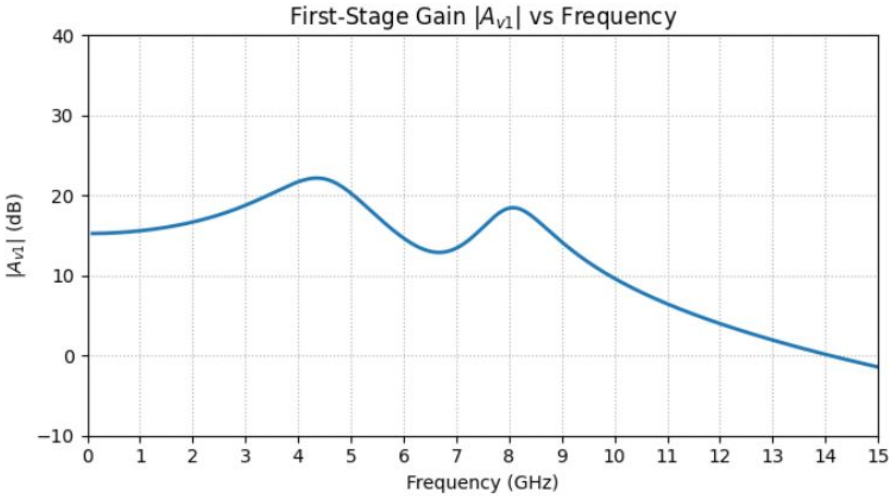
- Two gain peaks appear at:
  - $f_1 \approx 4.2$ GHz — lower-mode resonance
  - $f_2 \approx 8.2$ GHz — upper-mode resonance

  These originate from the coupled LC tanks at the drain.

- The valley between ∼6–7 GHz results from mode splitting and mutual coupling (parameter k).

Interpretation:

- **Peak 1 (∼4.2 GHz):** Dominated by tank 1 resonance and large $|Z_L|$. Limited by Q_1 and finite $|G_{m1}|$.
- **Valley (∼6–7 GHz):** Occurs due to coupling-induced impedance splitting between even and odd tank modes.
- **Peak 2 (∼8.2 GHz):** Driven by tank 2 resonance. Smaller because $|G_{m1}|$ rolls off at high frequency.



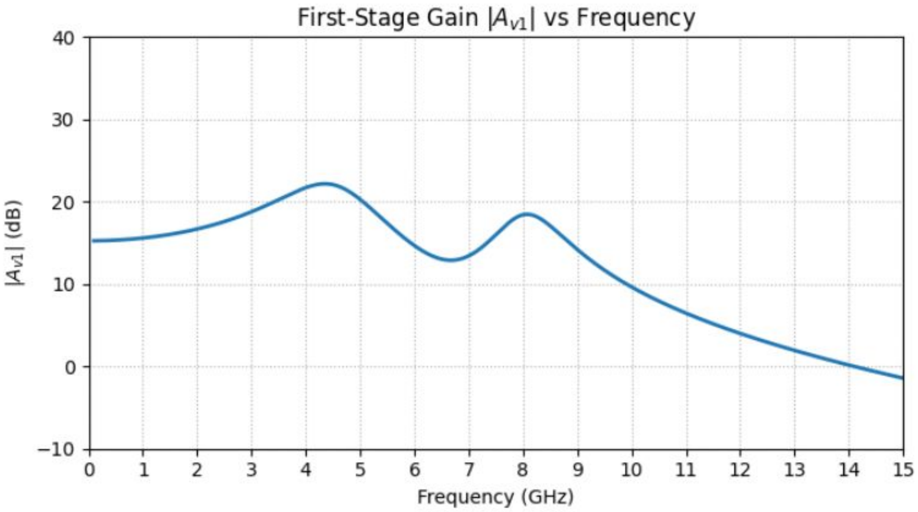
Parameter Setup:

- $L_{11} = 1.9\,\text{nH}, \quad L_{12} = 1.6\,\text{nH}, \quad C_{11} = C_{12} = 0.4\,\text{pF}, \quad k = 0.55$
- $Q_1 = 8.2, \ Q_2 = 2.0$
- $g_{m1} = 0.22\,\text{S}, \ g_{m2} = 0.10\,\text{S}$
- $L_s = 0.25\,\text{nH}, \ L_g = 0.85\,\text{nH}, \ R_s = 50\ \Omega$

First Stage Gain

What Changes What:

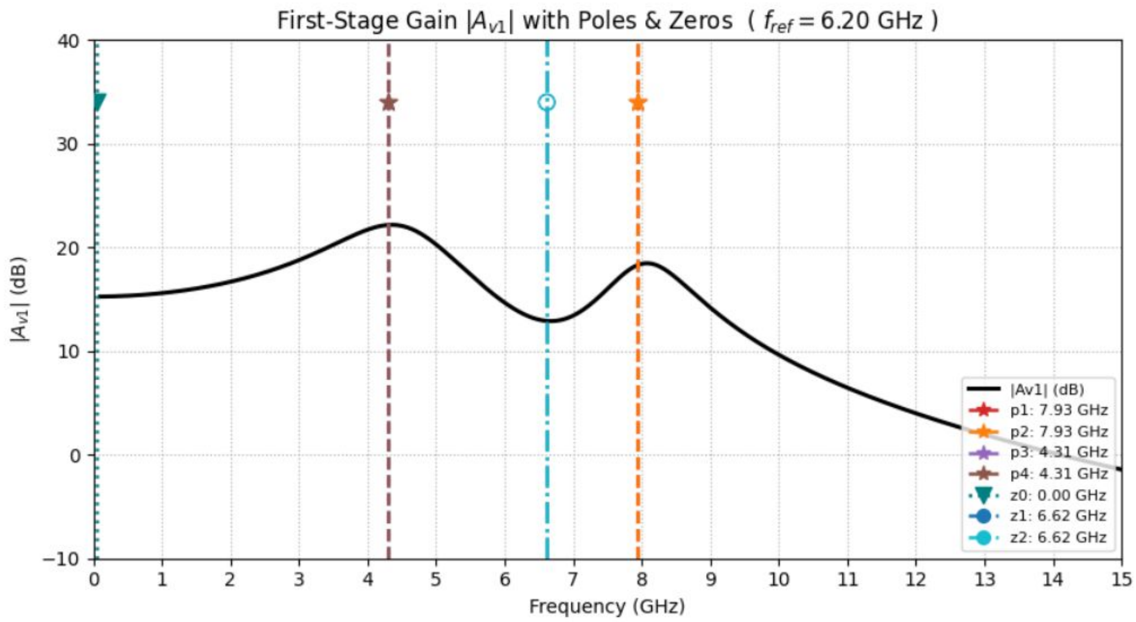
- Increase $Q_1 \rightarrow$ raises f_1 peak amplitude (sharper resonance).
- Decrease $Q_1 \rightarrow$ lowers f_1 peak; flattens the valley.
- Increase $Q_2 \rightarrow$ raises f_2 peak amplitude.
- Increase $k \rightarrow$ widens peak separation (larger mode split).
- Increase C_{12} or $L_{12} \rightarrow$ shifts f_2 lower, affects symmetry.
- Increase g_{m1} or reduce $L_s \rightarrow$ raises overall gain level.



Conclusion:

- Confirms the broadband dual-resonance behavior of the coupled tanks.
- Gain remains $> 11\,\text{dB}$ across 3.7–9 GHz, consistent with the paper's trend.
- Tuning Q_1/Q_2, k, and C_{12} allows fine control of bandwidth and peak symmetry.

First stage gain with Poles and Zeros




```
f_ref = 6.200 GHz

Poles (s-plane):
  p1: -5.237844e+09  + j 4.982125e+10  rad/s   -->  |Im|/(2π) = 7.929298 GHz
  p2: -5.237844e+09  - j 4.982125e+10  rad/s   -->  |Im|/(2π) = 7.929298 GHz
  p3: -4.431713e+09  + j 2.705643e+10  rad/s   -->  |Im|/(2π) = 4.306165 GHz
  p4: -4.431713e+09  - j 2.705643e+10  rad/s   -->  |Im|/(2π) = 4.306165 GHz

Zeros (s-plane):
  z0:  0.000000e+00 + j0.000000e+00  rad/s   -->  0.000000 GHz
  z1: -8.021923e+09  + j 4.156637e+10  rad/s   -->  |Im|/(2π) = 6.615494 GHz
  z2: -8.021923e+09  - j 4.156637e+10  rad/s   -->  |Im|/(2π) = 6.615494 GHz
```

Implementation of equation 5 & 6

Input Impedance Match Analysis

Derivation of equation(5,6)

$$s = j\omega.$$

KCL at G; Gate current:

$$I_{in} = j\omega C_{gs1}(V_g - V_s) + j\omega C_{gd1}(V_g - V_d), \qquad Z_{in} = \frac{V_g}{I_{in}}.$$

KCL at D, solve Vd: sum of current leaving D = 0

$$\frac{V_d}{Z_D} + sC_{gd1}(V_d - V_g) + g_{ds1}(V_d - V_s) + g_{m1}(V_g - V_s) = 0.$$

Group by node voltage and and define; $A \equiv \frac{1}{Z_D} + sC_{gd1} + g_{ds1}.$

$$V_d = \frac{(sC_{gd1} - g_{m1})V_g + (g_{ds1} + g_{m1})V_s}{A}.$$

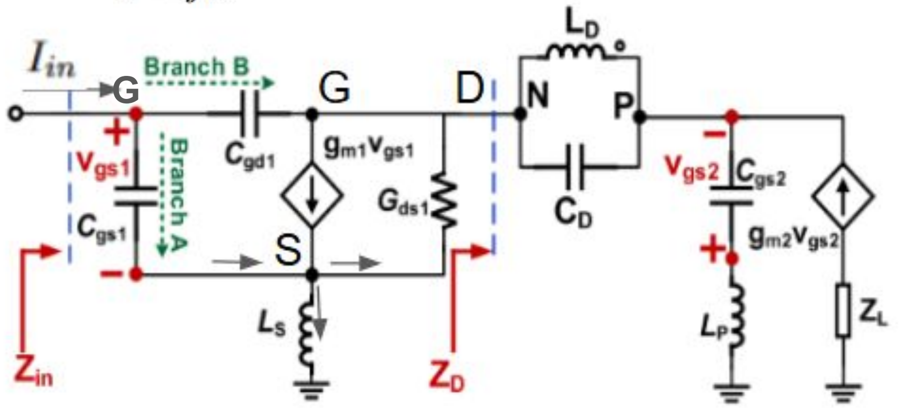


Fig. 3. Small-signal equivalent circuit of the input stage for Z_{in} derivation.

KCL at S, solve Vs/Vg: current leaving S

$$\frac{V_s}{sL_S} + sC_{gs1}(V_s - V_g) + g_{ds1}(V_s - V_d) - g_{m1}(V_g - V_s) = 0.$$

Define

$$B \equiv \frac{1}{sL_S} + sC_{gs1} + g_{ds1} + g_{m1},$$

so that

$$B\,V_s = (sC_{gs1} + g_{m1})V_g + g_{ds1}V_d.$$

Substitute Vd and solve for Vs/Vg; $\frac{V_s}{V_g} = \frac{A(sC_{gs1} + g_{m1}) + g_{ds1}(sC_{gd1} - g_{m1})}{AB - g_{ds1}(g_{ds1} + g_{m1})}$

Branch currents leaving the nodes

$$i_{gs1} = sC_{gs1}(V_g - V_s), \qquad i_{gd1} = sC_{gd1}(V_g - V_d),$$
$$i_{ds1} = g_{ds1}(V_d - V_s), \qquad i_{gm1,\text{from } D} = g_{m1}(V_g - V_s),$$
$$i_{Z_D} = \frac{V_d}{Z_D}, \qquad i_{L_S} = \frac{V_s}{sL_S}.$$

Derivation of equation(5,6) continue . . .

For compactness, denote $r = V_s/V_g$ and $r = N_r/D_r$ with

$$N_r = A(sC_{gs1} + g_{m1}) + g_{ds1}(sC_{gd1} - g_{m1}), \qquad D_r = AB - g_{ds1}(g_{ds1} + g_{m1})$$

From KCL@ D, express it as Vd/Vg

$$\frac{V_d}{V_g} = \frac{(sC_{gd1} - g_{m1}) + (g_{ds1} + g_{m1})\, r}{A}$$

Input current and Zin ,

$$I_{in} = j\omega C_{gs1}(V_g - V_s) + j\omega C_{gd1}(V_g - V_d), \qquad Z_{in} = \frac{V_g}{I_{in}}.$$

Then divided by Vg, use r = Vs/Vg and Vd/Vg

$$\frac{I_{in}}{V_g} = sC_{gs1}(1 - r) + sC_{gd1}\left[1 - \frac{(sC_{gd1} - g_{m1}) + (g_{ds1} + g_{m1})\, r}{A}\right]$$

Write each part over its natural denominator and combine:

$$\frac{I_{in}}{V_g} = sC_{gs1}\frac{D_r - N_r}{D_r} + sC_{gd1}\frac{A - (sC_{gd1} - g_{m1}) - (g_{ds1} + g_{m1})\, r}{A}$$

With $r = N_r/D_r$ and common denominator $A\,D_r$:

$$\frac{I_{in}}{V_g} = \frac{sC_{gs1}A(D_r - N_r) + sC_{gd1}\left[AD_r - A(sC_{gd1} - g_{m1})D_r - (g_{ds1} + g_{m1})N_r\right]}{A\,D_r}$$

Therefore Zin,

$$Z_{in} = \frac{V_g}{I_{in}} = \frac{A\,D_r}{sC_{gs1}A(D_r - N_r) + sC_{gd1}\left[AD_r - A(sC_{gd1} - g_{m1})D_r - (g_{ds1} + g_{m1})N_r\right]}$$

Substitute back, $s = j\omega$

$$N_r = A(sC_{gs1} + g_{m1}) + g_{ds1}(sC_{gd1} - g_{m1}), \qquad D_r = AB - g_{ds1}(g_{ds1} + g_{m1})$$

To simplifie introduce T and CT

$$T = sL_S\,(g_{m1} + sC_{gs1} + g_{ds1}), \qquad C_T = C_{gs1} + \frac{g_{ds1}}{s},$$

expand $A = \frac{1}{Z_D} + sC_{gd1} + g_{ds1}$ and $B = \frac{1}{sL_S} + sC_{gs1} + g_{ds1} + g_{m1}$, and simplify to obtain

$$Z_{in} = \frac{\big(1 + sC_{gd1}Z_D\big)(1 + T) + Z_D g_{ds1}\big(1 - \omega^2 L_S C_{gs1}\big)}{jw\Big\{(Z_D + sL_S)\left[g_{ds1}C_T + C_{gd1}(g_{m1} + sC_{gs1})\right] + C_T\Big\}}$$

where:

- $C_T = C_{gs1} + C_{gd1}$ (note: the document states $C_T = C_{gs1} + g_{ds1}$, but this appears to be a typographical error, as the units do not match; the correct term is likely $C_{gs1} + C_{gd1}$ for dimensional consistency),

Derivation of equation(5,6) continue . . .

Drain looking-In Impedance ZD

- From node D (M1 drain), the right network consists of: Series peaking inductor LP into the common-gate input of M2, whose shunt input is gm2 // jwCgs2, giving

$$Z_{\mathrm{CG}+L_P} = jwL_P + \frac{1}{g_{m2} + jwC_{gs2}} = \frac{1 + jwL_P g_{m2} - \omega^2 L_P C_{gs2}}{g_{m2} + jwC_{gs2}} = \frac{1 - \omega^2 L_P C_{gs2}}{g_{m2} + jwC_{gs2}}$$

Note : - The small term ***jwLPgm2*** is typically negligible and omitted in the paper

- Parallel tank LD // CD

$$Z_{LD\|CD} = \frac{jwL_D}{1 - \omega^2 L_D C_D}$$

- Summing Everything together Gives :

$$Z_D = \frac{1 - \omega^2 L_P C_{gs2}}{g_{m2} + jwC_{gs2}} + \frac{jwL_D}{1 - \omega^2 L_D C_D}$$

Paper assumptions used for these plots

- **Equations:** Eq. (5) for Z_{in} and simplified Eq. (6) for Z_D:

$$C_T = C_{gs1}, \qquad T = j\omega L_S\big(g_{m1} + j\omega C_{gs1} + G_{ds1}\big),$$

$$Z_D(\omega) \approx \frac{1}{g_{m2}} + \frac{j\omega L_D}{1 - \omega^2 L_D C_D}.$$

- **Neglected in Eq. (6):** C_{gs2} and L_P, under the paper's conditions $g_{m2} \gg \omega C_{gs2}$ and $\omega^2 L_P C_{gs2} \ll 1$.
- **No explicit losses:** ideal L/C (infinite Q); no shunt/series damping (R_p, R_{pad}), no gate resistance R_g, no pad/bond/parasitics.
- **Reference impedance:** $Z_0 = 50\ \Omega$.
- **Device constants :** $C_{gs1} = 0.24$ pF, $C_{gd1} = 90$ fF, $G_{ds1} = 33$ mS, $g_{m1} = 0.22$ S.
- **Port modeling:** single-ended input with series L_G included explicitly.

Common device/constants (used in both Fig. 4a & 4b)

Calculated (Eq. 5–6), CT = Cgs1, no explicit loss (no Rp, no series R)

Parameter	Paper (units)	Used (units)	Offset
CT= Cgs1	0.24 pF	0.24 pF	0%
Cgd1	90 fF	90 fF	0%
gm1	0.22 S	0.22 S	0%
Gds1	33 mS	33 mS	0%
Z0 (for S11)	50 Ω	50 Ω	0%
Cgs2 (Eq. 6)	not used (assumed ≈0)	0	N/A
LP (Eq. 6)	not used (assumed ≈0)	0	N/A

Case: LD = 1 nH (green “Cal.”)			
Parameter	**Paper (units)**	**Used (units)**	**Offset**
LG	0.85 nH	0.85 nH	0%
LS	0.25 nH	0.25 nH	0%
LD	1.00 nH	1.00 nH	0%
CD	2.5 pF	2.5 pF	0%
gm2	0.10 S	0.10 S	0%

Eq.(6) simplified per paper: no Cgs2,LP

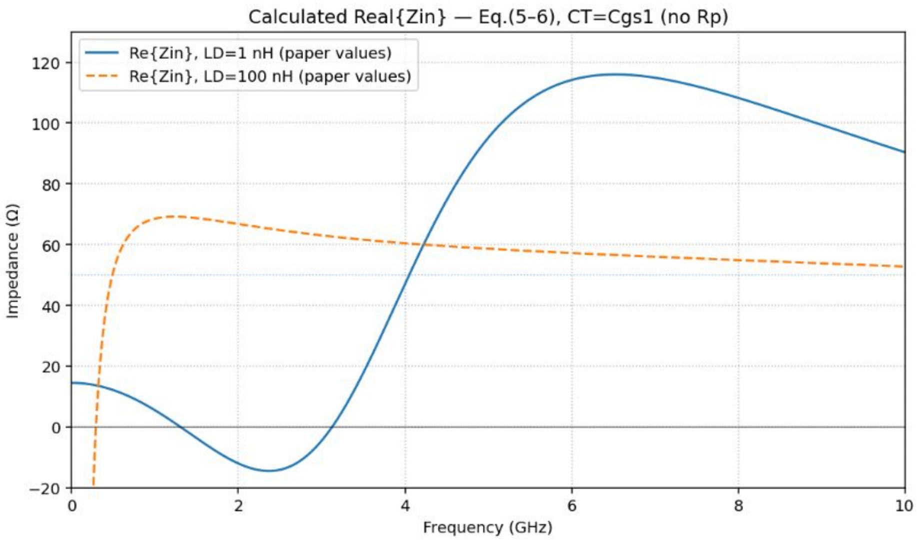
Common device/constants (used in both Fig. 4a & 4b)

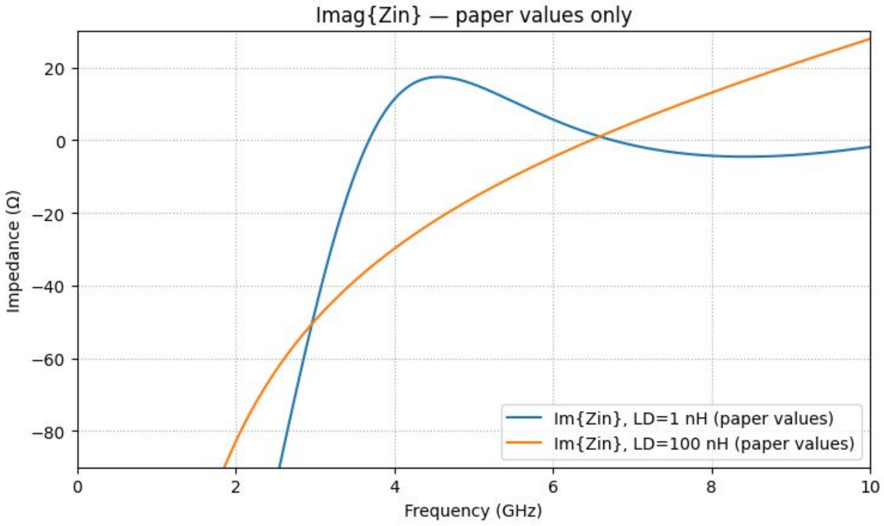
Case: LD = 100 nH (blue “Cal.”)			
Parameter	**Paper (unit)**	**Used (unit)**	**Offset**
LG	0.90 nH	0.90 nH	0%
LS	0.19 nH	0.19 nH	0%
LD	100 nH	100 nH	0%
CD	2.9 pF	2.9 pF	0%
gm2	not published	0.05 S (Assumed)	N/A

Notes

- For the 100 nH case the paper doesn’t publish gm2; they state LG,LS,CD were set for optimum S11 and gm2 was adjusted. I used 0.05 S as the nominal value when plotting.
- Eq. (6) was used in its simplified form (neglecting Cgs2 and LP), per the paper’s stated assumptions.

Plot with only Eqn (5) & (6) and papers assumption





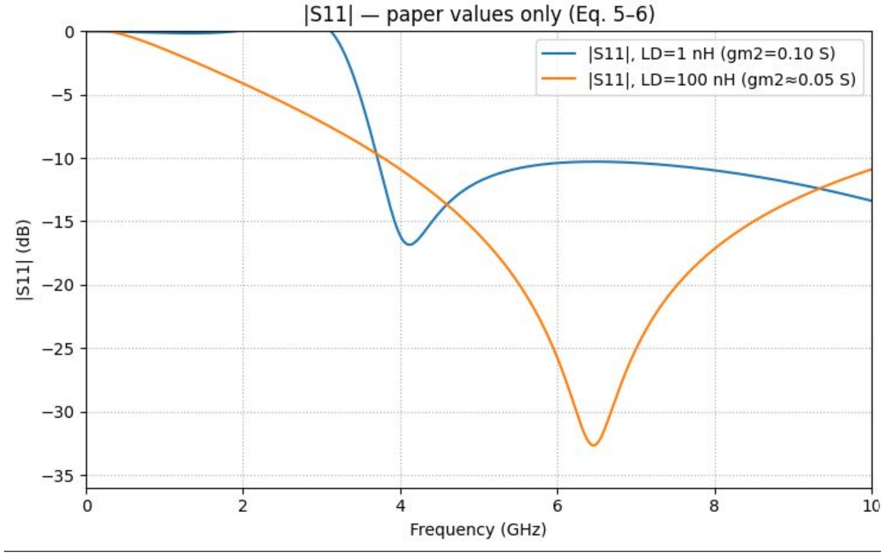
For LD=1 nH
Im = 0, at ~ 4GHZ, Re ~45 Ohm

For LD=1 nH
Im = 0, at ~ 4GHZ

DIp location: find the zero of Im{Zin}
Dip depth : Evaluate R = Re{Zin} at that frequency used

$$|S_{11}|_{\text{dip}} = \left|\frac{R-50}{R+50}\right|, \quad \text{Depth (dB)} = 20\log_{10}\left|\frac{R-50}{R+50}\right|$$

|S11|



For LD =1 nH
R ~ 45 Ohm
20 log10^(40-50)/(40+50) =~-17 dB
|s11| dip is ~ -17 dB

Change LS,LG, and gm2 to see what's happening

Calculated (Eq. 5–6), CT = Cgs1, used Rp.

Parameter	Paper value (unit)	Used with Rp (unit)	Offset
Cgs1	0.24 pF	0.24 pF	0%
Cgd1	90 fF	90 fF	0%
Gds1	33 mS	33 mS	0%
gm1	0.22 S	0.22 S	0%

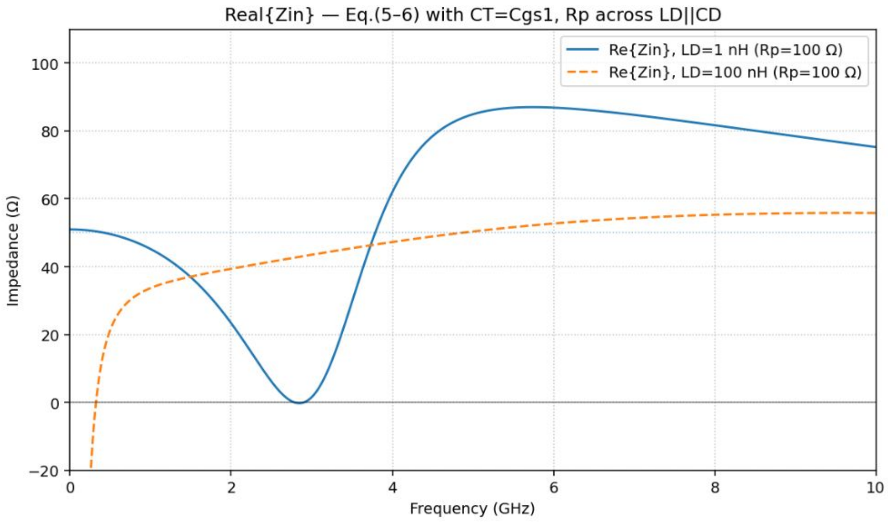
Case: LD = 100 nH (blue “Cal.”)			
Parameter	Paper value (unit)	Used (Rp) (unit)	Offset
LG	0.85 nH	0.85 nH	0%
LS	0.25 nH	0.25 nH	0%
LD	1.00 nH	1.00 nH	0%
CD	2.5 pF	2.5 pF	0%
gm2	0.10 S	0.10 S	0%

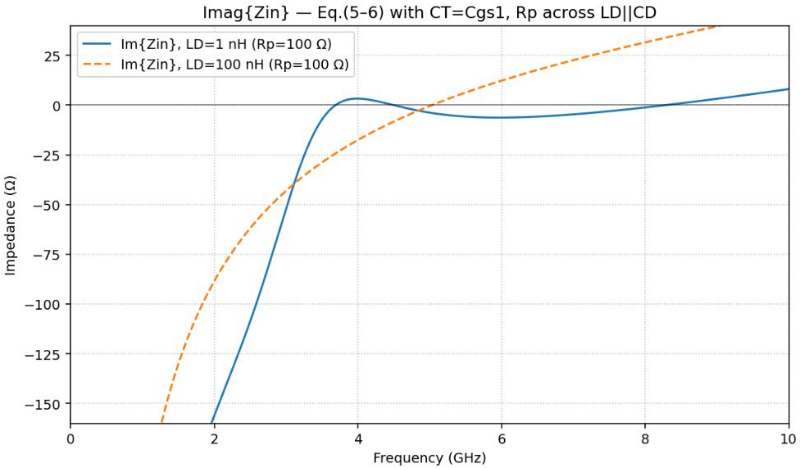
Change some variables to see what's happening

Case: LD = 1 nH ("Cal.")			
Paramet er	Paper value (unit)	Used (Rp) (unit)	Offset
LG	0.85 nH	0.84nH	-1.2 %
LS	0.25 nH	0.34nH	+36 %
LD	1.00 nH	1.00nH	0%
CD	2.5 pF	2.6 pF	+4%
gm2	0.1 S	0.1 S	0%

As The paper stated that "Actually, as long as Re{Zin} is positive, the circuit is stable." so, to lift up Re{Zin} above 0, at LD = 1nH some parameters need to be swept which are LS,LG, and gm2

Impedance vs Frequency plot with parallel Rp



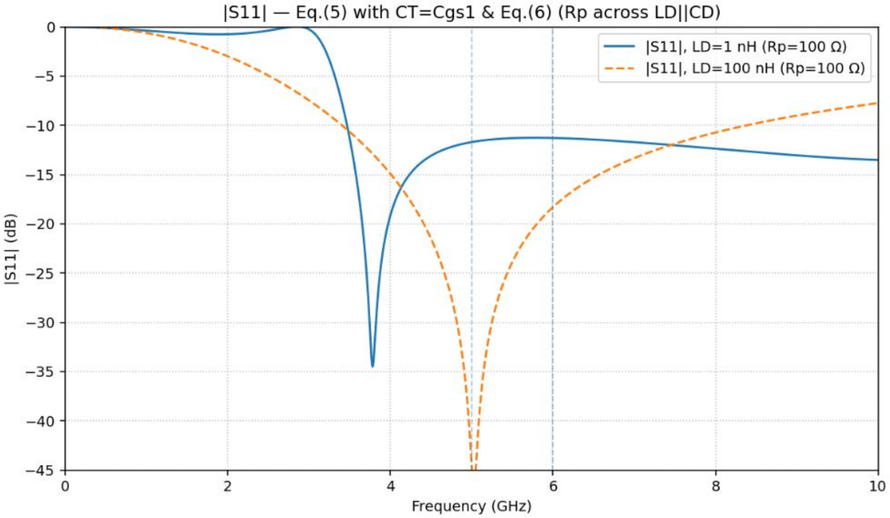


- At Im=0, if R > 50 Ω (dip too shallow)→ ↑gm2 or ↓LS
- At Im=0 , if R < 50 Ω (over-deep/narrow) → ↓gm2 or ↑LS
- If Im above 0 (near dip) → ↑gm2;if below 0 → ↓gm2
- Rp and gm2, moves both Re and Im part up and down

- Im curve sits below 0 → decrease gm2 a bit.
- Im curve sits above 0 around the dip → increase gm2 a bit

Large Ls and Lower gm2 helps to ensure Re{Zin} > 0

|S11|



Shift dip left/right:

- ↑CD,↑LG move left (←)
- ↓CD,↓LG move to the right (→)

- Use LG and CD (plus Cgs1 if needed) to place the dip.
- Decrease Lg or CD dip too low in frequency
- Increase Lg or CD dip too high in frequency
- Use LS, gm1, gm2 to set depth (via the real plateau).
- Use Rp / finite Q to control Q and stability (broaden/shalllow), with minimal frequency pull.

Keep Parameters consistency throughout first stage design

A

First stage design constants from equation(3) - (6) used these base device parameters.

- gm1 = 0.22, # S
- gm2 = 0.10, # S
- Cgs1 = 0.24e-12, # F
- Cgd1 = 90e-15, # F
- Gds1 = 33e-3, # S

B

On top of those constants from A, only equation (3),(5), and (6) depends on LS,LG

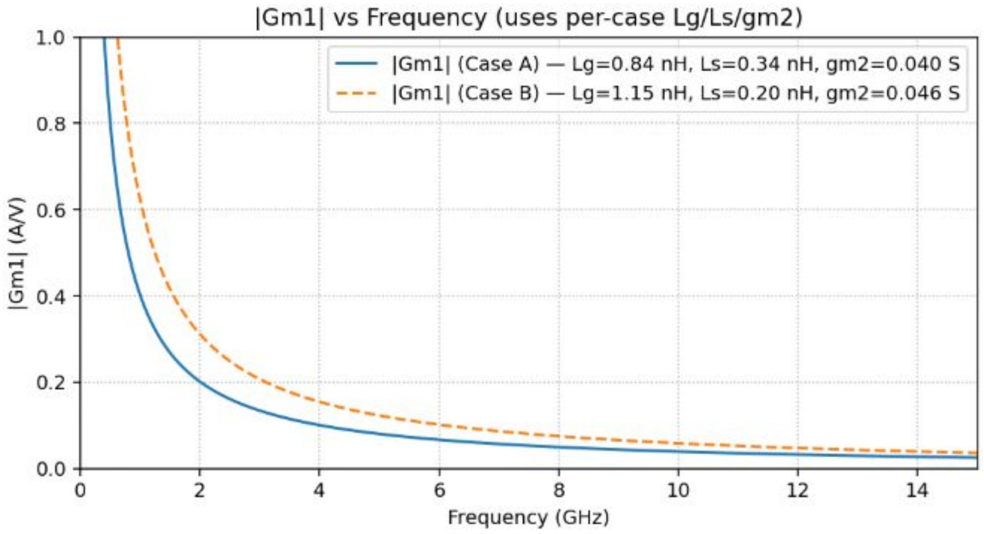
- Lg = 0.85e-9 # H
- Ls = 0.25e-9 # H

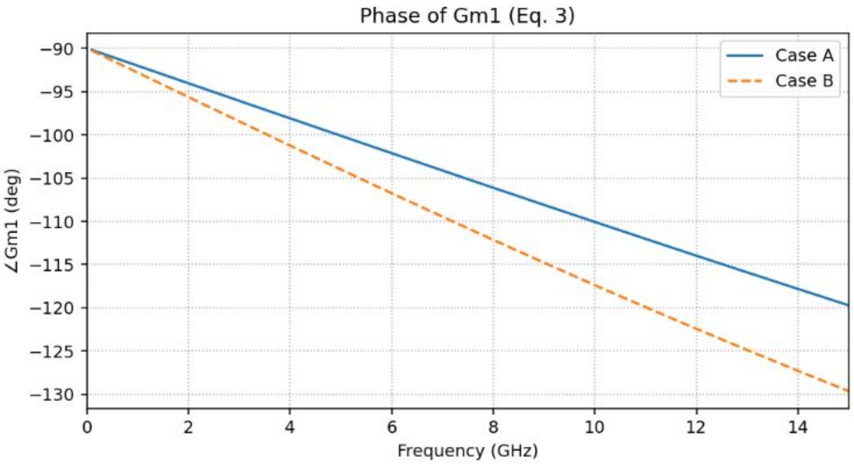
If Ls and Lg value sweeps, only Gm1, Re{Zin},Im{Zin}, and |S11| plots behavior should change because these plots are depend on the value used for Lg and Ls in addition to Part A parameters.

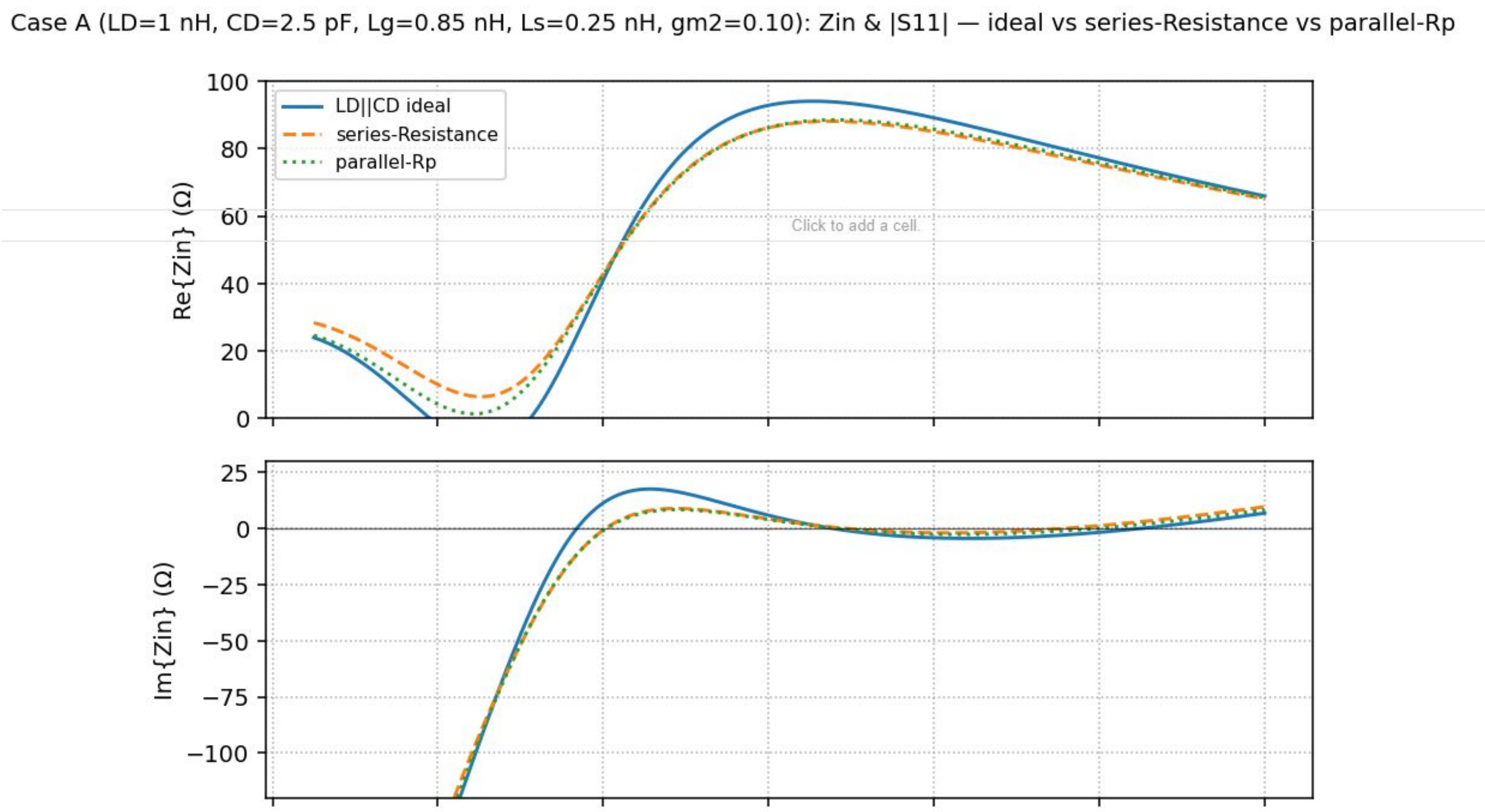
C

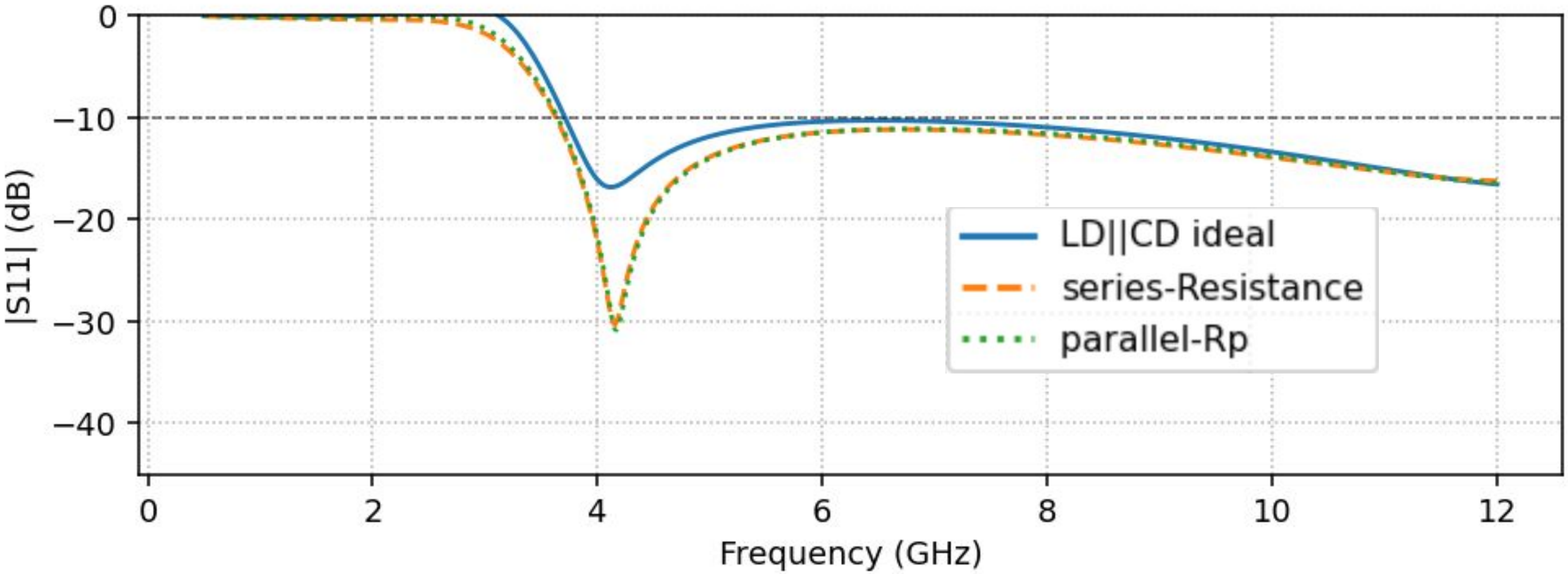
Coupled-tank load (transformer)

- L11 = 1.9e-9
- L12 = 1.6e-9
- C11 = 0.5e-12
- C12 = 0.5e-12
- k = 0.55
- Q1 = 8.2
- Q2 = 2.0


|Gm1| vs Frequency (uses per-case Lg/Ls/gm2)
|Gm1| (Case A) — Lg=0.84 nH, Ls=0.34 nH, gm2=0.040 S
|Gm1| (Case B) — Lg=1.15 nH, Ls=0.20 nH, gm2=0.046 S
|Gm1| (A/V)
Frequency (GHz)


Phase of Gm1 (Eq. 3)
Case A
Case B
∠Gm1 (deg)
Frequency (GHz)


Case A (LD=1 nH, CD=2.5 pF, Lg=0.85 nH, Ls=0.25 nH, gm2=0.10): Zin & |S11| — ideal vs series-Resistance vs parallel-Rp
LD||CD ideal
series-Resistance
parallel-Rp
Re{Zin} (Ω)
Im{Zin} (Ω)
Click to add a cell.
100
80
60
40
20
0
25
0
−25
−50
−75
−100



|S11| <= -10 dB

Lg=0.84nH, Ls=o.25nH —> exact with papers table II values

how the curves move what to tweak, and why

Why the real part $\Re\{Z_{in}\}$ sits on a plateau

Starting from Eq. (5) with the paper's intuition limits (set $C_{gd1} \to 0$, $G_{ds1} \to 0$ just to expose the dominant term),

$$Z_{in} = \frac{1+T}{j\omega C_{gs1}}, \qquad T = j\omega L_S\big(g_{m1} + j\omega C_{gs1} + G_{ds1}\big),$$

which gives

$$Z_{in} = \underbrace{\frac{1}{j\omega C_{gs1}}}_{\text{imag}} + \underbrace{\frac{g_{m1} L_S}{C_{gs1}}}_{\text{real, flat plateau}} + \underbrace{j\omega L_S}_{\text{imag}}.$$

Hence the main *lifter*

$$\Re\{Z_{in}\} \approx \frac{g_{m1}\, L_S}{C_{gs1}}.$$

What moves what (plateau height).

- $\uparrow L_S \Rightarrow \uparrow$ plateau (higher $\Re\{Z_{in}\}$).
- $\uparrow g_{m1} \Rightarrow \uparrow$ plateau; $\uparrow C_{gs1} \Rightarrow \downarrow$ plateau.
- Finite G_{ds1} and any real part seen through the drain path (via C_{gd1}) also add positive real part, helping to keep $\Re\{Z_{in}\} > 0$.

Drain path seen through C_{gd1}

Using Eq. (6) with the paper's stated limits $g_{m2} \gg \omega C_{gs2}$ and $\omega^2 L_P C_{gs2} \ll 1$,

$$Z_D \approx \frac{1}{g_{m2}} + \frac{j\omega L_D}{1 - \omega^2 L_D C_D}.$$

This Z_D couples to the input through C_{gd1}, contributing:

- an *effective capacitance* $C_D^{\mathrm{eff}}(\omega)$ that grows with frequency (from the L_D–C_D tank),
- a *real* term (notably $1/g_{m2}$, and any tank loss such as finite Q or a parallel R_p), which further lifts $\Re\{Z_{in}\}$ and broadens/shallows the match.

What sets the Dip frequency and how low it goes

From S_Parameters

$$S_{11} = \frac{Z_{in} - 50}{Z_{in} + 50}, \qquad Z_{in} = R + jX \qquad |S_{11}| = \frac{\sqrt{(R-50)^2 + X^2}}{\sqrt{(R+50)^2 + X^2}}, \qquad R = \Re\{Z_{in}\},\ X = \Im\{Z_{in}\}$$

Dip frequency in |S11|: where the Im plot crosses 0, or where the reactance is zero $X = \mathrm{Im}\{Z_{in}\} = 0$

$$|S_{11}|_{\mathrm{dip}} = \left|\frac{R-50}{R+50}\right|, \quad \text{Depth (dB)} = 20\log_{10}\left|\frac{R-50}{R+50}\right|$$

At Zero-crossing X = 0, the formula reduced to

SO, the closer R = Re{Zin} is to 50 Ω at that same frequency, the deeper the dip

- If $R = 50\ \Omega$ → ideal match → $|S_{11}| = 0$ (−∞ dB).
- If $R = 60\ \Omega$ → depth ≈ $20\log_{10}(10/110) \approx -20.8$ dB.
- If $R = 40\ \Omega$ → depth ≈ $20\log_{10}(10/90) \approx -19.1$ dB.

$$Z_D(s) = \frac{1}{g_{m2}} + Z_{\text{tank}}(s) \quad Z_{\text{tank}}(s) = \frac{sL_D}{1 + \frac{sL_D}{R_p} + s^2 L_D C_D}$$

Tank Poles

Parallel Rp||LD||CD that sits inside ZD:

For a parallel RLC the characteristic equation is

$$s^2 + \frac{1}{R_p C_D} s + \frac{1}{L_D C_D} = 0$$

$$s_p = -\frac{1}{2R_p C_D} \pm j\sqrt{\frac{1}{L_D C_D} - \frac{1}{(2R_p C_D)^2}}.$$

From this:

$\omega_0 = \frac{1}{\sqrt{L_D C_D}}$,

$Q \approx R_p \sqrt{\frac{C_D}{L_D}}$ (for light damping).

$$s_{p1,2} = -2.403846 \times 10^8 \pm j\,1.961014 \times 10^{10} \text{ rad/s}$$

Tank Zeros

Its the frequency where Im{Zin}=0

$$f_{\text{dip}} \approx \frac{1}{2\pi\sqrt{L_G C_{\text{eff}}}}, \qquad C_{\text{eff}} \approx C_{gs1} + C_D^{\text{eff}}(\omega) \text{ (seen through } C_{gd1})$$

Knobs : ↑LGL or ↑CD→ dip left or ↓CD → dip right

$$s_{z1,2} = \frac{-L_D\left(g_{m2} + \frac{1}{R_p}\right) \pm \sqrt{\left[L_D\left(g_{m2} + \frac{1}{R_p}\right)\right]^2 - 4L_D C_D}}{2L_D C_D}.$$

$$s_{z1,2} = -7.932692 \times 10^9 \pm j\,1.793566 \times 10^{10} \text{ rad/s}$$

The poles and zeros above is just for LD=1nH. For LD=100nH can be managed to get using the formulas above. Main thing to note is that if we change variables of the Tank, we don't get the same poles and zeros. Therefore, use the formula.

Why |S11| <= -10 dB Return Loss Matters

Return loss (RL) is → $20 \cdot \log_{10}|\Gamma|$

For RL = −10 dB ⇒ $|\Gamma| = 10^{(-10/20)} \approx 0.316$

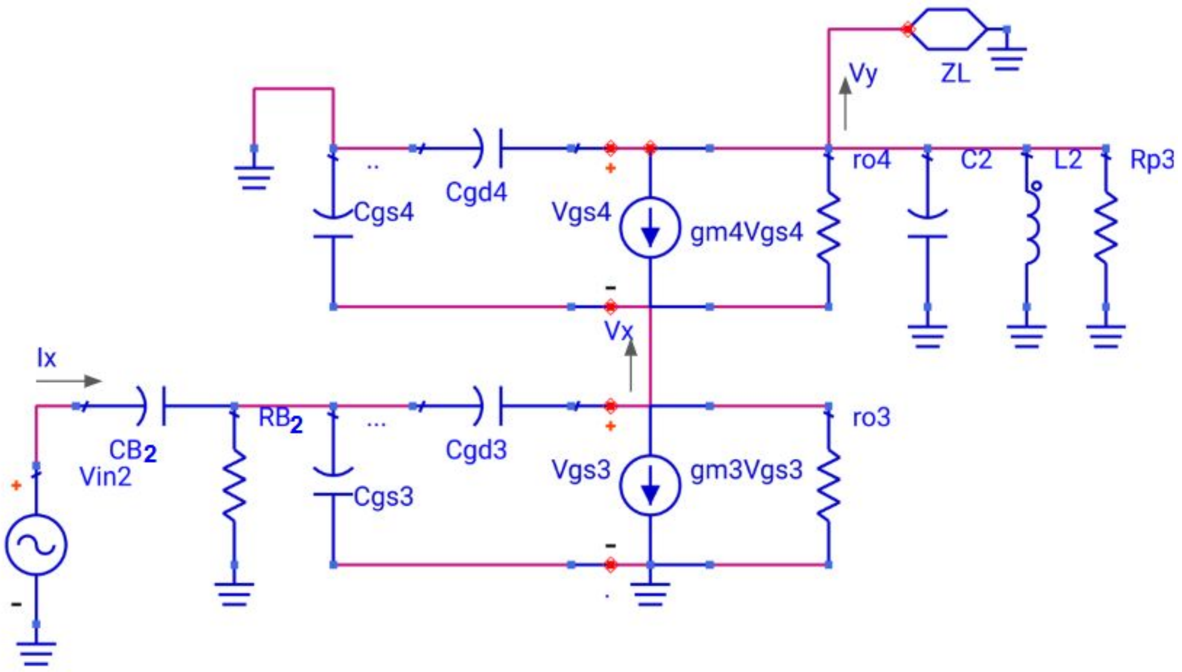
VSWR = $(1+|\Gamma|) / (1-|\Gamma|)$ ⇒ VSWR ≈ 1.92.

Power reflection coefficient = $|\Gamma|^2 \approx 0.10$ ⇒ about 10% reflected, 90% accepted.

Mismatch loss = $-10 \cdot \log_{10}(1-|\Gamma|^2) \approx 0.46$ dB.

Derivation of 2nd & 3rd stage gain

Small signal Model for 2nd and 3rd stage Gain analysis



- ➔ 2nd & 3rd stage small signal are exactly similar. Just only number different such as instead of L2,C2,gm3Vgs3 &4, Rp3, use L4,L4,gm5Vgs5 & 6, Rp4. for the rest refer the papers schematics Fig.2 (a)

Gain Derivation (M3-M4, M5-M6) with CB2,CB3 and RB1,RB2

Model & Definitions

Used the midband Razavi small-signal model:

- M3 and M5 (CS): controlled source gm3vgs3,gm5vgs5 from drain to source and ro3,ro5 from drain to source, source is AC ground, so *vgs3=vg3, vgs5=vg5*.
- M4 and M6 (CG cascode): gate AC-GND; controlled source gm4vgs4 from drain to source with *vgs4=-Vx, Vgs6= -Vx* (where Vx is M3/M5 drain/M4/M6 source), and ro4/ro6 between M4/M6 drain(Vy) and source (Vx).
- Shunt tanks at the source node Vy: L2||C2||Rp3 and L3||C3||Rp4. (Rp1 &Rp2 used in first stage transformer tank)
- The input gate high-pass: formed by Cb2/CB3 and RB1/RB2.

$$H_{\mathrm{HP2}}(s) \triangleq \frac{v_{g3}}{v_{\mathrm{in},2}} = \frac{s\,R_B C_{B2}}{1+s\,R_B C_{B2}}, \qquad H_{\mathrm{HP3}}(s) \triangleq \frac{v_{g5}}{v_{\mathrm{in},3}} = \frac{s\,R_B C_{B3}}{1+s\,R_B C_{B3}}, \qquad s=j\omega.$$

- Load admittance at Vy:

$$Y_L(s) = \frac{1}{R_{p3}} + sC_2 + \frac{1}{sL_2}, \qquad Y_{L3}(s) = \frac{1}{R_{p4}} + sC_3 + \frac{1}{sL_3}, \quad Z_L(s) = \frac{1}{Y_L(s)}.$$

Gain Derivation (M3-M4) with CB2 and RB

KCL at Vy (output node):

$$\left(Y_L(s) + \tfrac{1}{r_{o4}}\right) v_y + \left(g_{m4} - \tfrac{1}{r_{o4}}\right) v_x = 0.$$

Solve for v_y:

$$v_y = -\frac{g_{m4} - \frac{1}{r_{o4}}}{Y_L(s) + \frac{1}{r_{o4}}}\, v_x. \qquad (1)$$

At Vx (Cascode Source):

$$\left(\tfrac{1}{r_{o3}} + \tfrac{1}{r_{o4}} - g_{m4}\right) v_x - \tfrac{1}{r_{o4}}\, v_y = g_{m3}\, v_{g3} \qquad (2)$$

Substitute (1) into (2) and solve for v_x/v_{g3}:

$$\left[\frac{1}{r_{o3}} + \frac{1}{r_{o4}} - g_{m4} + \frac{\frac{1}{r_{o4}}\left(g_{m4} - \frac{1}{r_{o4}}\right)}{Y_L(s) + \frac{1}{r_{o4}}}\right] v_x = g_{m3}\, v_{g3},$$

$$\Rightarrow \quad \frac{v_x}{v_{g3}} = \frac{g_{m3}}{\frac{1}{r_{o3}} + \frac{1}{r_{o4}} - g_{m4} + \frac{\frac{1}{r_{o4}}\left(g_{m4} - \frac{1}{r_{o4}}\right)}{Y_L(s) + \frac{1}{r_{o4}}}}$$

Then v_y/v_{g3} follows from (1)

$$\frac{v_y}{v_{g3}} = -\frac{g_{m4} - \frac{1}{r_{o4}}}{Y_L(s) + \frac{1}{r_{o4}}}\, \frac{v_x}{v_{g3}} \qquad (3)$$

Gate-referenced stage gain:

Combine equation (2) and (3)

$$A_{v2,\text{gate}}(s) \equiv \frac{v_y}{v_{g3}} = -\,g_{m3} \frac{g_{m4} - \frac{1}{r_{o4}}}{\left(\frac{1}{r_{o3}} + \frac{1}{r_{o4}} - g_{m4}\right)\left(Y_L(s) + \frac{1}{r_{o4}}\right) + \frac{1}{r_{o4}}\left(g_{m4} - \frac{1}{r_{o4}}\right)}$$

A useful identity to simplify the denominator is

$$\left(\tfrac{1}{r_{o3}} + \tfrac{1}{r_{o4}} - g_{m4}\right)\tfrac{1}{r_{o4}} + \tfrac{1}{r_{o4}}\left(g_{m4} - \tfrac{1}{r_{o4}}\right) = \frac{1}{r_{o3} r_{o4}},$$

so the full denominator equals

$$\left(\tfrac{1}{r_{o3}} + \tfrac{1}{r_{o4}} - g_{m4}\right) Y_L(s) + \frac{1}{r_{o3} r_{o4}}.$$

Including the Input high-pass

Since $v_{g3} = H_{\text{HP2}}(s)\, v_{\text{in},2}$, the input-referenced 2nd-stage gain is

$$A_{v2}(s) \equiv \frac{v_y}{v_{\text{in},2}} = H_{\text{HP2}}(s)\; A_{v2,\text{gate}}(s) = \frac{sR_B C_{B2}}{1 + sR_B C_{B2}}\; A_{v2,\text{gate}}(s).$$

Final Gain Expression for 2nd and 3rd stage

The derivation is similar for both stage gain expression:

$$A_{v2}(s) = \underbrace{\frac{s\,R_B C_{B2}}{1 + s\,R_B C_{B2}}}_{\text{input HPF}} \left[-g_{m3} \frac{g_{m4} - \dfrac{1}{r_{o4}}}{\left(\dfrac{1}{r_{o3}} + \dfrac{1}{r_{o4}} - g_{m4}\right)\left(\underbrace{\dfrac{1}{R_{p2}} + sC_2 + \dfrac{1}{sL_2}}_{Y_{L2}(s)} + \dfrac{1}{r_{o4}}\right) + \dfrac{1}{r_{o4}}\left(g_{m4} - \dfrac{1}{r_{o4}}\right)} \right]$$

Second Stage Gain

$$A_{v3}(s) = \underbrace{\frac{s\,R_B C_{B3}}{1 + s\,R_B C_{B3}}}_{\text{input HPF}} \left[-g_{m5} \frac{g_{m6} - \dfrac{1}{r_{o6}}}{\left(\dfrac{1}{r_{o5}} + \dfrac{1}{r_{o6}} - g_{m6}\right)\left(\underbrace{\dfrac{1}{R_{p4}} + sC_3 + \dfrac{1}{sL_3}}_{Y_{L3}(s)} + \dfrac{1}{r_{o6}}\right) + \dfrac{1}{r_{o6}}\left(g_{m6} - \dfrac{1}{r_{o6}}\right)} \right]$$

Third stage Gain

Final Gain Expression for 2nd and 3rd stage

The derivation is similar for both stage gain expression:

$$Z_{L2}(s) = \frac{1}{\frac{1}{R_{p3}} + sC_2 + \frac{1}{sL_2}}, \qquad Z_{L3}(s) = \frac{1}{\frac{1}{R_{p4}} + sC_3 + \frac{1}{sL_3}}$$

$$H_{\mathrm{HP2}}(s) = \frac{sR_BC_{B2}}{1 + sR_BC_{B2}}, \qquad H_{\mathrm{HP3}}(s) = \frac{sR_BC_{B3}}{1 + sR_BC_{B3}}$$

Voltage divider in the s-domain

$$\frac{v_{g3}}{v_{\mathrm{in},2}} = \frac{Z_R}{Z_R + Z_C} = \frac{R_B}{R_B + \frac{1}{sC_{B2}}} = \frac{sR_BC_{B2}}{1 + sR_BC_{B2}}$$

$$\boxed{r_{o,\mathrm{cas2}} = \frac{1}{g_{ds3}} + \frac{1}{g_{ds4}} + \frac{g_{m4}}{g_{ds3}g_{ds4}}} \qquad \boxed{r_{o,\mathrm{cas3}} = \frac{1}{g_{ds5}} + \frac{1}{g_{ds6}} + \frac{g_{m6}}{g_{ds5}g_{ds6}}}$$

$$\boxed{A_{v2}(s) = H_{\mathrm{HP2}}(s)\left[-\,g_{m3}\left(Z_{L2}(s) \parallel r_{o,\mathrm{cas2}}\right)\right]}$$

$$\boxed{A_{v3}(s) = H_{\mathrm{HP3}}(s)\left[-\,g_{m5}\left(Z_{L3}(s) \parallel r_{o,\mathrm{cas3}}\right)\right]}$$

Compacted Second and Third stage Gain expression

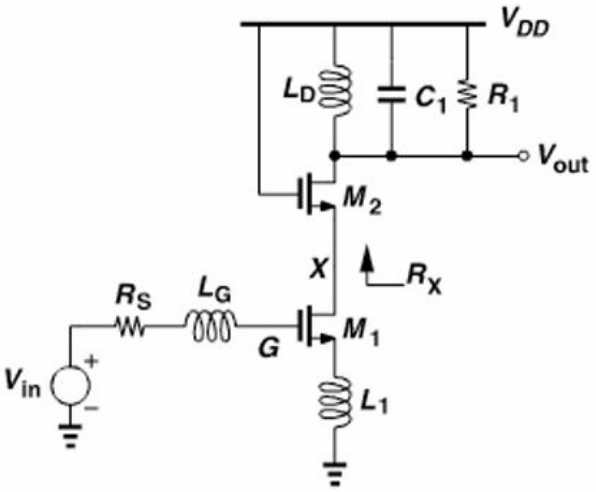
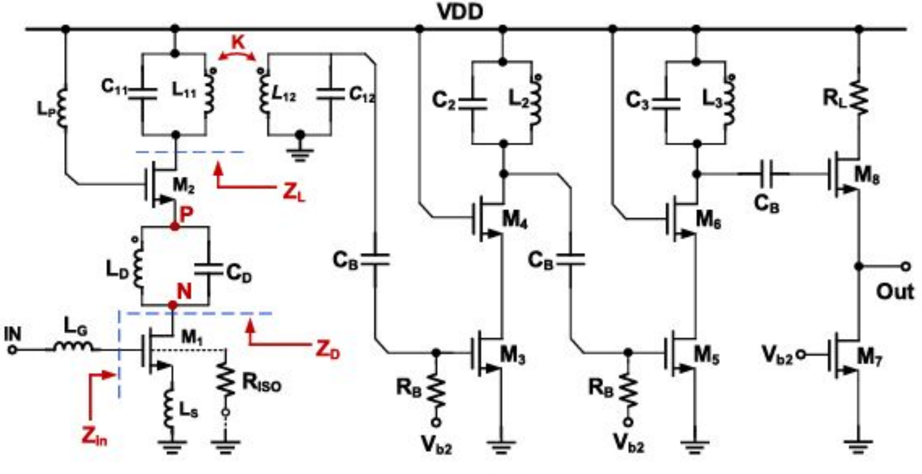


Figure 5.37 *Inductively-degenerated cascode CS LNA.*



(a)

Razavi page 290

A Cryogenic Broadband Sub-1-dB NF CMOS Low Noise Amplifier for Quantum Applications

Razavi's Cascode CS stage with Inductive Degeneration

$$R_X = \frac{R_1 + r_{O2}}{1 + g_m r_{O2}}.$$ Eqn(5.99)

$$\frac{V_X}{V_G} = \frac{R_S}{L_1\omega_0} \cdot \frac{R_1 + r_{O2}}{(1 + g_{m2} r_{O2})(R_S + L_G\omega_0)}. \quad (5.100)$$

- Razavi’s Rx→ “Resistance seen looking up from node x into M2 + tank”
- Ours ro,cas2 → “Resistance seen looking up from the drain of M3 into M4 + ro’s (and similarly for stage 3)”
- Eqn 5.99 of Razavi’s is playing same role as the effective load $(Z_L(j\omega_0) \parallel r_{o,\text{cas}})$
- Eqn 5.100 Razavi’s gain $A_v = \frac{V_X}{V_G} \cdot \frac{V_{\text{out}}}{V_X}$

  equivalent to my notations $A_v = H_{\text{HP}}(s) \cdot [-g_m(Z_L \parallel r_{o,\text{cas}})]$

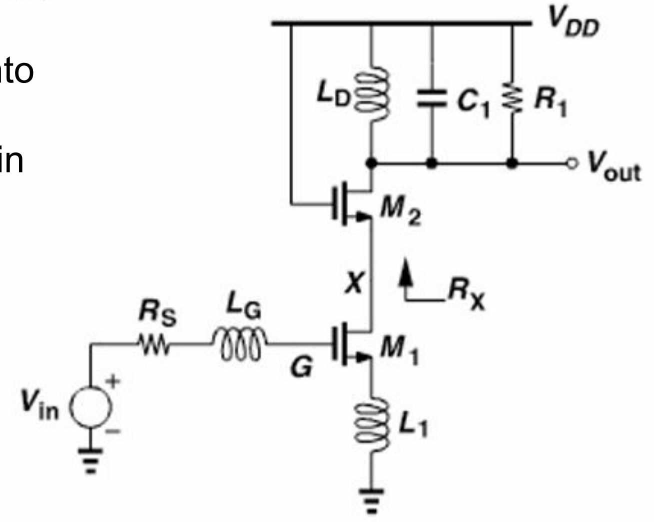


Figure 5.37 *Inductively-degenerated cascode CS LNA.*

Equivalent relation at resonant

$$R_X \longleftrightarrow (Z_{L2} \parallel r_{o,\text{cas2}}), \quad g_{m2} \longleftrightarrow g_{m4}, \quad R_1 \longleftrightarrow R_{p3},$$

Final Gain expression from Razavi

$$A_v \triangleq \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\omega_T R_1}{2\,\omega_0 R_S} = \frac{R_1}{2L_1\omega_0}$$

The first term

$$A_v = \frac{\omega_T R_1}{2\,\omega_0 R_S} \qquad (5.97)$$

Transconductance x load resistance

$$A_v = \frac{R_1}{2L_1\omega_0} \qquad (5.98)$$

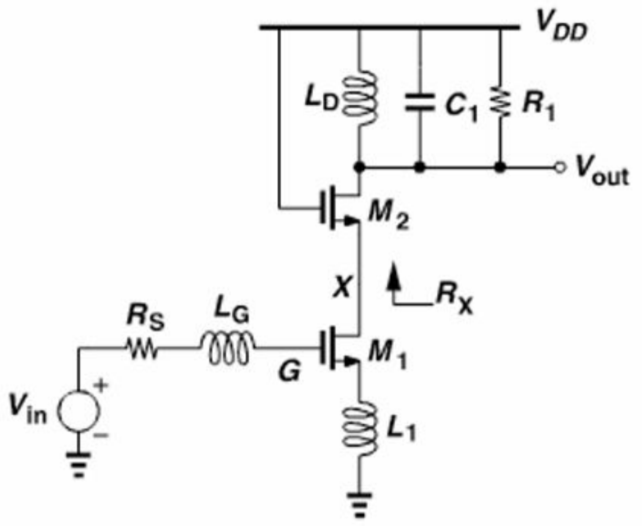


Figure 5.37 *Inductively-degenerated cascode CS LNA.*

Note

My gain expressions are longer because I keep the full s-domain transfer (tank frequency response, RC high-pass, finite cascode ro, and tank Rp).Razavi's final formula for Fig. 5.37 is the special case at resonance under matching assumptions, which reduces our result to Av≈−gmRp and then further to ωTR1/(2ω0RS)

To show how my long expression collapses to Razavi's one

- If I impose the same assumptions Razavi uses, my formula shrinks:

1. Evaluate at resonance:

   $s = j\omega_{02}$, so $Z_{L2}(j\omega_{02}) \approx R_{p3}$

2. If the cascode is strong:

   $r_{o,\text{cas2}} \gg R_{p3}$, then $Z_{L2} \parallel r_{o,\text{cas2}} \approx R_{p3}$

3. If the high-pass ≪ω02

   $|H_{\text{HP2}}(j\omega_{02})| \approx 1$

   Then the final expression becomes to

$$A_{v2,\text{max}} \approx -g_{m3} R_{p3}$$

He then rewrites g_m and R_1 in terms of $\omega_T,\ \omega_0,\ R_S,\ L_1$ to get that very compact $\frac{\omega_T R_1}{2\omega_0 R_S}$

Razavi's CS with Inductive Load gain Formulas

Fig. 5.11, Eq. 5.3.1

$$Z_T = \frac{L_1 s + R_S}{L_1 C_1 s^2 + R_S C_1 s + 1}, \tag{5.30}$$

and the tank voltage by $(I_X - g_m V_X) Z_T$. Adding the voltage drop across C_F to the tank voltage, we have

$$V_X = \frac{I_X}{C_F s} + (I_X - g_m V_X) Z_T. \tag{5.31}$$

Substitution of Z_T from (5.30) gives

$$Z_{in}(s) = \frac{V_X}{I_X} = \frac{L_1(C_1 + C_F)s^2 + R_S(C_1 + C_F)s + 1}{[L_1 C_1 s^2 + (R_S C_1 + g_m L_1)s + 1 + g_m R_S] C_F s}. \tag{5.32}$$

This is the same parallel-RLC load expression used in my stages:

$$Z_{L2}(s) = \left(\frac{1}{R_{p3}} + sC_2 + \frac{1}{sL_2}\right)^{-1}, \quad Z_{L3}(s) = \left(\frac{1}{R_{p4}} + sC_3 + \frac{1}{sL_3}\right)^{-1}$$

$$\frac{Y(s)}{Q(s)} = \frac{1}{1 + H(s)}. \tag{12.15}$$

Proper choice of $H(s)$ can yield a *high-pass* response; for example, if $H(s)$ is an ideal integrator [Fig. 12.13(b)],

$$\frac{Y(s)}{Q(s)} = \frac{s}{1 + s}. \tag{12.16}$$

"The gate of M3/M5 is AC-coupled through $C_{B2}(C_{B3})$ and biased by $R_{B2}(R_{B3})$, forming a standard first-order RC high-pass:

$$H_{\mathrm{HP}}(s) = \frac{sR_BC_B}{1 + sR_BC_B}.$$

This has the same single-pole high-pass form as Razavi's $\frac{Y(s)}{Q(s)} = \frac{s}{1+s}$ in *Design of CMOS Phase-Locked Loops* (Eq. 12.16), but here it is implemented with an explicit R_B–C_B coupling network."

$$H_{\mathrm{HP}}(s) = \frac{sR_BC_B}{1 + sR_BC_B}.$$

define a normalized variable

$$\tilde{s} = sR_BC_B,$$

$$H_{\mathrm{HP}}(s) = \frac{\tilde{s}}{1 + \tilde{s}},$$

Zeros and poles for 2nd and 3rd stage

The derivation is similar for both stage gain expression:

Zeros

1. From the tank: as $s \to 0$, $Y_L(s) \to \infty$ due to $\frac{1}{sL_2}, \frac{1}{sL_3}$ so $A_{v2,v3,\text{gate}}(s), \to 0$ linearly in s.

   $\Rightarrow\ s = 0$ is a zero of $A_{v2,v3,\text{gate}}(s)$

2. From the input high-pass: $H_{\text{HP}}(s)$ contributes a zero at $s = 0$.

   $\Rightarrow\ s = 0$ is a second (coincident) zero of $A_{v2,v3}(s)$.

Thus, the *input-referenced* stage has a double zero at the origin:

zeros: $z_1 = 0,\ z_2 = 0.$

Zeros and poles for 2nd and 3rd stage

The derivation is similar for both stage gain expression:

Poles for Av2 :Tank pair are the roots of;

$$(g_{ds3} + g_{ds4} - g_{m4})\, C_2\, s^2 + \left[(g_{ds3} + g_{ds4} - g_{m4}) \frac{1}{R_{p2}} + g_{ds3} g_{ds4} \right] s + (g_{ds3} + g_{ds4} - g_{m4}) \frac{1}{L_2} = 0,$$

With,

$$s_{p1,2} = \frac{-\left[(g_{ds3} + g_{ds4} - g_{m4}) \frac{1}{R_{p2}} + g_{ds3} g_{ds4} \right] \pm \sqrt{\left((g_{ds3} + g_{ds4} - g_{m4}) \frac{1}{R_{p2}} + g_{ds3} g_{ds4} \right)^2 - 4 (g_{ds3} + g_{ds4} - g_{m4})^2 \frac{C_2}{L_2}}}{2 (g_{ds3} + g_{ds4} - g_{m4}) C_2}.$$

HPF adds one real pole at $p_0 = -1/(R_B C_{B2})$

Zeros and poles for 2nd and 3rd stage

The derivation is similar for both stage gain expression:

Poles for Av3 :Tank pair are the roots of;

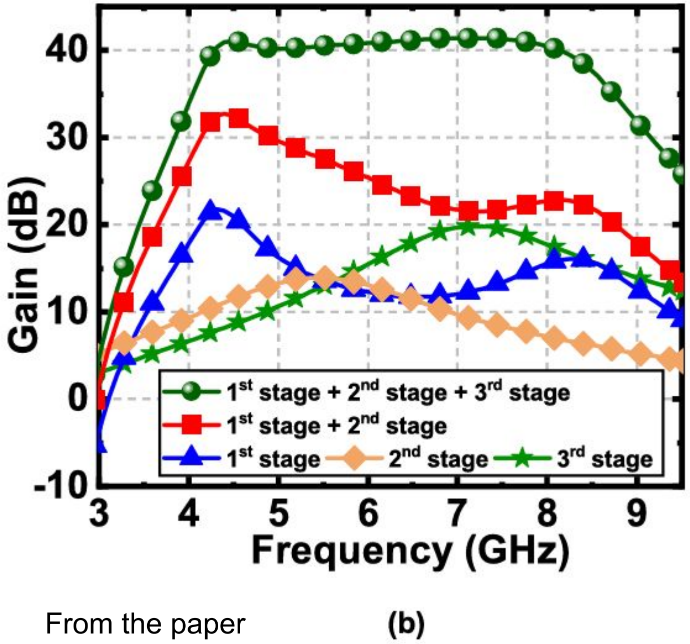
$$\left(g_{ds5}+g_{ds6}-g_{m6}\right)C_3\,s^2+\left[\left(g_{ds5}+g_{ds6}-g_{m6}\right)\frac{1}{R_{p4}}+g_{ds5}g_{ds6}\right]s+\left(g_{ds5}+g_{ds6}-g_{m6}\right)\frac{1}{L_3}=0,$$

With,

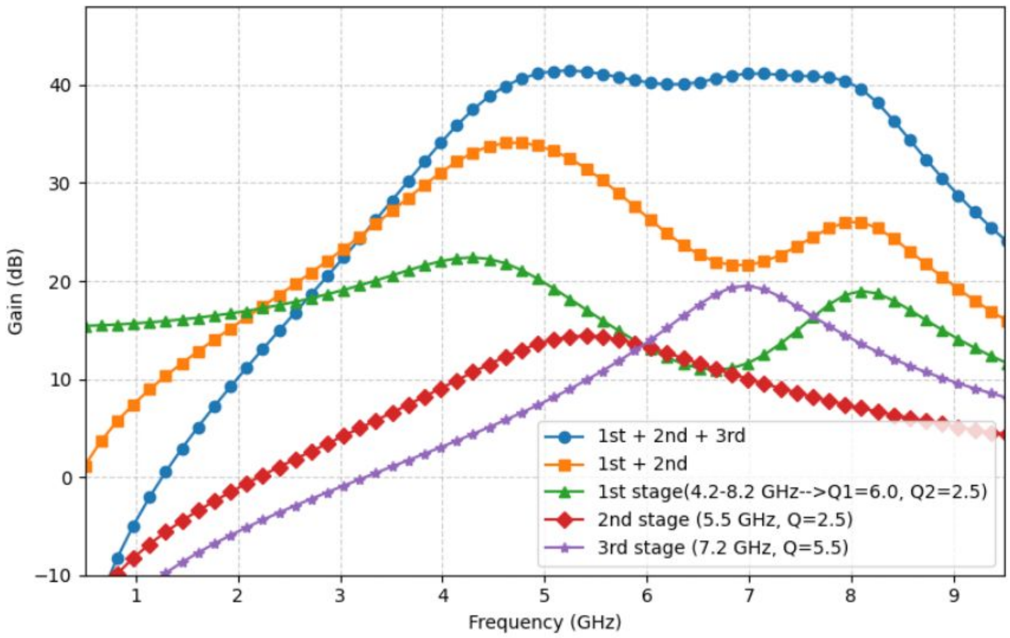
$$s_{q1,2}=\frac{-\left[\left(g_{ds5}+g_{ds6}-g_{m6}\right)\frac{1}{R_{p4}}+g_{ds5}g_{ds6}\right]\pm\sqrt{\left(\left(g_{ds5}+g_{ds6}-g_{m6}\right)\frac{1}{R_{p4}}+g_{ds5}g_{ds6}\right)^2-4\left(g_{ds5}+g_{ds6}-g_{m6}\right)^2\frac{C_3}{L_3}}}{2\left(g_{ds5}+g_{ds6}-g_{m6}\right)C_3}$$

HPF adds one real pole at $p_0=-1/(R_BC_{B3})$

Gain Plots



From the paper



Reproducing work

$$Q = \frac{R_p}{\omega_0 L} = \omega_0 C R_p$$

$$R_p = Q\,\omega_0 L, \qquad C = \frac{1}{\omega_0^2 L}$$

$$R_{p1} \approx 370\ \Omega, \quad R_{p2} \approx 198\ \Omega, \quad R_{p3} \approx 86\ \Omega, \quad R_{p4} \approx 249\ \Omega.$$

$$\omega_0 = 2\pi f_0,$$
$$C = \frac{1}{\omega_0^2 L},$$
$$R_p = \frac{Q_p}{\omega_0 C} = Q_p \omega_0 L.$$

Stage-2 target:

$f_{0,2} = 5.50\,\mathrm{GHz},\ Q_{p2} \approx 5.0,\ L_2 = 1.00\,\mathrm{nH}.$

$$\omega_{0,2} = 2\pi \cdot 5.50 \times 10^9/\mathrm{s} \approx 3.46 \times 10^{10}/\mathrm{s},$$
$$C_2 = \frac{1}{\omega_{0,2}^2 L_2} \approx \frac{1}{(3.4558\times 10^{10})^2 \cdot 1.0\times 10^{-9}} \approx 0.837\,\mathrm{pF},$$
$$R_{p2} = Q_{p2}\,\omega_{0,2} L_2 \approx 5.0 \cdot (3.4558\times 10^{10}) \cdot 1.0\times 10^{-9} \approx 173\,\Omega.$$

Stage-3 target: $f_{0,3} = 7.20\,\mathrm{GHz},\ Q_{p3} = 8.2,\ L_3 = 1.00\,\mathrm{nH}.$

$$\omega_{0,3} = 2\pi \cdot 7.20 \times 10^9/\mathrm{s} \approx 4.52 \times 10^{10}/\mathrm{s},$$
$$C_3 = \frac{1}{\omega_{0,3}^2 L_3} \approx \frac{1}{(4.5239\times 10^{10})^2 \cdot 1.0\times 10^{-9}} \approx 0.489\,\mathrm{pF},$$
$$R_{p3} = Q_{p3}\,\omega_{0,3} L_3 \approx 8.2 \cdot (4.5239\times 10^{10}) \cdot 1.0\times 10^{-9} \approx 371\,\Omega.$$

Pick gm from a peak-gain target

$$A_{v\mathrm{pk}} \approx g_m R_p \quad \Longrightarrow \quad g_m \approx \frac{A_{v\mathrm{pk}}}{R_p} \quad (\text{with } A_{v\mathrm{pk}} \text{ in linear, not dB)}$$

Stage-2: target peak $\approx 15.0\,\mathrm{dB}$,

$$A_{v\mathrm{pk}} = 10^{15/20} \approx 5.623 \Rightarrow g_{m3} \approx \frac{5.623}{173} \approx 0.0330\,\mathrm{S}.$$

Stage-3: choose g_{m5} strong but not overpowering,

$$g_{m5} = 0.0700\,\mathrm{S} \quad \Rightarrow \quad A_{v\mathrm{pk}} \approx g_{m5} R_{p3} \approx 0.07 \times 371 \approx 26 \ (\approx 28.3\,\mathrm{dB}).$$

Gate caps from an fT guess

With a simple RF MOS estimate,

$$C_{\mathrm{sum}} \approx \frac{g_m}{2\pi f_T}, \qquad C_{gs} \approx 0.8\,C_{\mathrm{sum}},\quad C_{gd} \approx 0.2\,C_{\mathrm{sum}}$$

Using $f_T = 100\,\mathrm{GHz}$:

Stage-2 devices (from $g_{m3} \approx 0.0330\,\mathrm{S}$, $g_{m4} = 0.0900\,\mathrm{S}$):

$$C_{\mathrm{sum},3} = \frac{0.033}{2\pi \cdot 10^{11}} \approx 52.0\,\mathrm{fF}, \qquad C_{gs3} \approx 41.4\,\mathrm{fF}, \quad C_{gd3} \approx 10.4\,\mathrm{fF},$$

$$C_{\mathrm{sum},4} = \frac{0.090}{2\pi \cdot 10^{11}} \approx 143\,\mathrm{fF}, \qquad C_{gs4} \approx 115\,\mathrm{fF}, \quad C_{gd4} \approx 28.6\,\mathrm{fF}.$$

Stage-3 devices (from $g_{m5} = 0.0700\,\mathrm{S}$, $g_{m6} = 0.110\,\mathrm{S}$):

$$C_{\mathrm{sum},5} = \frac{0.070}{2\pi \cdot 10^{11}} \approx 112\,\mathrm{fF}, \qquad C_{gs5} \approx 89.2\,\mathrm{fF}, \quad C_{gd5} \approx 22.3\,\mathrm{fF},$$

$$C_{\mathrm{sum},6} = \frac{0.110}{2\pi \cdot 10^{11}} \approx 175\,\mathrm{fF}, \qquad C_{gs6} \approx 140\,\mathrm{fF}, \quad C_{gd6} \approx 35.0\,\mathrm{fF}.$$

Input high-pass corners (series C_B into R_B)

Ignoring the small gate caps for the corner estimate,

$$f_{\mathrm{HP}} \approx \frac{1}{2\pi R_B C_B}.$$

Stage-2: $R_{B2} = 50.0\,\mathrm{k\Omega}$, $C_{B2} = 4.00\,\mathrm{pF} \Rightarrow f_{\mathrm{HP}} \approx \dfrac{1}{2\pi \cdot 50\times10^{3} \cdot 4\times10^{-12}} \approx 0.800\,\mathrm{MHz}.$

Stage-3: $R_{B3} = 50.0\,\mathrm{k\Omega}$, $C_{B3} = 1.00\,\mathrm{pF} \Rightarrow f_{\mathrm{HP}} \approx \dfrac{1}{2\pi \cdot 50\times10^{3} \cdot 1\times10^{-12}} \approx 3.18\,\mathrm{MHz}.$

Implementation of
Noise Analysis of M1

Derivation of Noise Figure