NORTHWESTERN UNIVERSITY

# EECS 336 – Homework 1


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## 1 QUESTION 1

### 1.1 QUESTION 1 PART A

From the question of part a, we need to order following 12 functions by growth:

$$(\frac{3}{2})^{2n}, n^3, (lg^2 n)^2, lg(n!), \left(2^{2^n}\right)^2, n^{\frac{1}{lgn}}$$
$$(ln(lnn))^2, lg^* n, \left(n2^n\right)^2, n^{lglgn}, (lnn)^2, 1$$

ranking can be solved by the following rules:
1.exponential functions grow faster than polynomial functions than logarithmic functions
2.the base of logarithm does not matter
3.$n^{lglgn} = (lgn)^{lgn}$
4.$lg(n!) = \Theta(nlgn)$
5.$n^{\frac{1}{lgn}} = 2$

So the final order will be:
$\left(2^{2^n}\right)^2 > \left(n2^n\right)^2 > (\frac{3}{2})^{2n} > n^{lglgn} > n^3 > lg(n!) > (lg^2 n)^2 > (lnn)^2 > (ln(lnn))^2 > lg^* n > n^{\frac{1}{lgn}} and 1$

### 1.2 QUESTION 1 PART B

As said on the piazza,we can find one upper bound function and one lower bound function for all 12 functions and then modify those two functions to become strict upper bound function and strict lower bound function. Then make a combination.

So the final result I get is:

$$f(n) = \left((1 + cosx)2^{2^n}\right)^2$$

# 2 Question 2

## 2.1 Question 2 Part a

$$\begin{aligned}
f(n) &= lg(lg^* n) + 2^{lgn} * lgn^{lgn} \\
&= \Theta(1) + n^{lg(lgn)+1} \\
&= \Theta(n^{lg(lgn)+1})
\end{aligned}$$

## 2.2 Question 2 Part b

$$\begin{aligned}
f(n) &= 2^{lg^* n} * log(lgn^{lgn}) + 4^{lgn^3} \\
&= \Theta(1) + lgn * lg(lgn) + 2^{6lgn} \\
&= \Theta(1) + lgn * lg(lgn) + \Theta(n^6) \\
&= \Theta(n^6)
\end{aligned}$$

## 2.3 Question 2 Part c

$$\begin{aligned}
f(n) &= (\sqrt{2})^{lgn} + e^n + log(\sqrt{lgn}) \\
&= \sqrt{n} + \Theta(e^n) + \Theta(lg(lgn)) \\
&= \Theta(e^n)
\end{aligned}$$

## 2.4 Question 2 Part d

$$\begin{aligned}
f(n) &= n^2 * 4^{lgn^3} + (\sqrt{2})^{lgn} \\
&= \Theta(n^8) + (\sqrt{2})^{lgn} \\
&= \Theta(n^8)
\end{aligned}$$

### 2.5 Question 2 Part e

$$
\begin{aligned}
f(n) &= n! * (lgn)! * (\sqrt{l}gn) + 2^{lg^* n} \\
&= n! * (lgn)! * (\sqrt{l}gn) + \Theta(1) \\
&= n^{n+\frac{1}{2}} * e^{-n} * lgn^{lgn} * (lgn) \\
&= \Theta(lgn * n^{lg(lgn)+n+\frac{1}{2}} * e^{-n})
\end{aligned}
$$

### 2.6 Question 2 Part f

$$
\begin{aligned}
f(n) &= = 2^{(n+1)!} + (\sqrt{2})^{lgn} \\
&= 2^{(n+1)!} + \Theta(n) \\
&= \Theta(2^{n^{n+\frac{1}{2}} * e^{-n}})
\end{aligned}
$$

## 3 Question 3

### 3.1 Question 3 Part a

if the condition is $f(n) \notin \overset{\infty}{\Omega}(g(n))$, so there is only finite integer which satisfies $f(n) >= cg(n)$ ,
if $n_0$ is the maximal value of all $n$ , which means for the $n >= n_0 + 1, cg(n) >= f(n) >= 0$.
so it will satisfies either of the following condition of $f(n) \in \overset{\infty}{\Omega}(g(n))$ or $f(n) \in O(g(n))$.

### 3.2 Question 3 Part b

Advantage:it can analyze a large more range of complexity, especially when Omega is hard to find. Disadvantage: The range is too big,which will influence the accuracy and the range is not fixed.

### 3.3 Question 3 Part c

if we have$f(n) = O^{'}(g(n))$, if and only if $|f(n)| = O(g(n))$,
from $f(n) = \Theta(g(n))$,
we can get $|f(n)| = O(g(n))$ and $f(n) = \Omega(g(n))$,
then from $|f(n)| = O(g(n))$ and $f(n) = \Omega(g(n))$,
we can't get $f(n) = \Theta(g(n))$
So the direction change from two-way to one direction

### 3.4 Question 3 Part d

$\tilde{\Omega} = f(n)$
$f(n) \geq cg(n) lg^k(n)$ for all $n \geq n_0$
$\tilde{\Theta} = f(n)$
$cg(n) lg^k(n) \leq f(n) \leq cg(n) lg^k(n)$ for all $n \geq n_0$

# 4 Question 4

### 4.1 Question 4 Part a

$f(n) = n-1, c = 2$
then: $n-k \leq 2, k \geq n-2$
and: $n-(k-1) > 2, k < n-3$
so: $k = \Theta(n)$

### 4.2 Question 4 Part c

$f(n) = \frac{n}{2}, c = 3$
then: $(\frac{n}{2})^k \leq 3, k \geq lg(\frac{n}{3})$
and: $(\frac{n}{2})^{k-1} > 3, k < lg(\frac{n}{3}) - 1$
so: $k = \Theta lg(n)$

### 4.3 Question 4 Part e

$f(n) = \sqrt{n}, c = 4$
then: $(n)^{\frac{1}{2^k}} \leq 4, k \geq lg(lgn) - 1$
and: $(n)^{\frac{1}{2^{(k-1)}}} > 4, k < lg(lgn)$
so: $k = \Theta lg(lgn)$

### 4.4 Question 4 Part g

$f(n) = n^{\frac{1}{3}}, c = 4$
then $(n)^{\frac{1}{3^k}} \leq 4, k \geq \frac{lg(lgn)-1}{lg3}$
and: $(n)^{\frac{1}{3^{k-1}}} > 4, k < \frac{lg3-1+lg(lgn)}{lg3}$
so: $k = \Theta lg(lgn)$

## 4.5 Question 4 Part h

$f(n) = \frac{n}{lgn}, c = 4$
then: $\frac{n}{\sqrt{n}} \leq \frac{n}{lgn} \leq \frac{n}{2}$
then for the left: $f(n) = \frac{n}{\sqrt{n}} = \sqrt{n}, c = 4$
and for the left: $(n)^{\frac{1}{2^{k_1}}} \leq 4, k_1 \geq lg(lgn) - 1$
and for the left: $(n)^{\frac{1}{2^{(k_1-1)}}} > 4, k_1 < lg(lgn)$
so for the left: $k_1 = \Theta lg(lgn)$
so for the left: $k_1 = \Omega(lg(lgn))$
then for the right: $f(n) = \frac{n}{2}, c = 4$
then for the right: $(\frac{n}{2})^{k_2} \leq 3, k_2 \geq lg(\frac{n}{3})$
and for the right: $(\frac{n}{2})^{k_2-1} > 3, k < lg(\frac{n}{3}) - 1$
so for the right: $k_2 = \Theta lg(n)$
so for the right:$k_2 = O(lgn)$
so: then lower bound is :$lg(lgn)$
the upper bound is :$lgn$