

Electronic Homework 4: Decision/Game Theory

VE492: Introduction to AI

Due: 11:59pm, June 13, 2022

1 Decision Theory

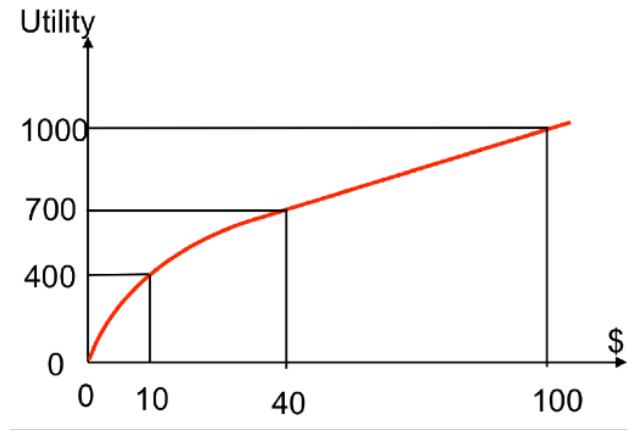
1.1 Rationality of Utilities

1. (2 points) Consider a lottery $L = [0.25, o_1; 0.3, o_2; 0.35, o_3; 0.1, o_4]$, where the utility values of each of the outcomes are $U(o_1) = 4$, $U(o_2) = 3$, $U(o_3) = 2$, $U(o_4) = 5$. What is the expected utility of this lottery, $EU(L)$?
2. (2 points) Consider a lottery $L_1 = [0.25, o_1; 0.75, L_2]$, where $U(o_1) = 6$, and $L_2 = [0.5, o_2; 0.5, o_3]$ is a lottery, and $U(o_2) = 4$, $U(o_3) = 0$. What is the expected utility of the first lottery, $EU(L_1)$?
3. (2 points) Assume $A \succ B$, $B \succ L$, where $L = [0.5, C; 0.5, D]$, and $D \succ A$. Assuming rational preferences, which of the following statements are guaranteed to be true?
A. $A \succ L$ D. $B \succ C$
B. $A \succ C$ E. $B \succ D$
C. $A \succ D$

Sample answer: ADE

1.2 Certainty Equivalent Values

4. (2 points) Consider the following utility function U shown below:



Recall the certainty equivalent monetary value in dollars (\$) of a lottery L is the value X such that $U(X) = EU(L)$. Under the above utility function, what is the certainty equivalent monetary value of the lottery $[0.6, \$0; 0.4, \$100]$?

1.3 Preferences and Utilities

Our Pacman board now has food pellets of 3 different sizes—pellet P_1 of radius 1, P_2 of radius 2, and P_3 of radius 3. In different moods, Pacman has different preferences among these pellets. In each of the following questions, you are given Pacman’s preference for the different pellets. We consider the following cases for the utility functions, which are functions of the pellet radius r and are defined over non-negative values of r .

- | | |
|--------------------|---------------------------|
| A. $U(r) = 0$ | F. $(U(r)) = r^2$ |
| B. $U(r) = 3$ | G. $U(r) = -r^2$ |
| C. $U(r) = r$ | H. $U(r) = \sqrt{r}$ |
| D. $U(r) = 2r + 4$ | I. $U(r) = -\sqrt{r}$ |
| E. $U(r) = -r$ | J. Irrational preferences |

The last case J corresponds to the case where Pacman’s stated preferences are not compatible with expected utility.

5. (2 points) Which of the previous cases are consistent with $P_1 \sim P_2 \sim P_3$?

Sample answer: ABC

6. (2 points) Which of the previous cases are consistent with $P_1 \prec P_2 \prec P_3$?

7. (2 points) Which of the previous cases are consistent with $P_1 \succ P_2 \succ P_3$?

8. (2 points) Which of the previous cases are consistent with $P_1 \prec P_2 \prec P_3$ and $P_2 \prec [0.5, P_1; 0.5, P_2]$?

9. (2 points) Which of the previous cases are consistent with $P_1 \succ P_2 \succ P_3$ and $P_2 \succ [0.5, P_1; 0.5, P_2]$?
10. (2 points) Which of the previous cases are consistent with $P_1 \prec P_2 \prec P_3$ and $[0.5, P_2; 0.5, P_3] \prec [0.5, P_1; 0.5, P_3]$?
11. (2 points) Which of the previous utility functions model a risk-seeking behavior? That is, for which utility function(s) would Pacman prefer entering a lottery for a random food pellet, with expected size s over receiving a pellet of size s ?

2 Game Theory

2.1 Basics

Consider the matrix described by the following matrix:

| | L | C | R |
|---|------|--------|------|
| T | 2, 4 | 5, 1 | 2, 3 |
| M | 1, 5 | 2, 4 | 4, 4 |
| B | 5, 1 | 10, 10 | 1, 2 |

12. (a) (2 points) What is the maximin value for the row player? Recall this is the value that a player can guarantee whatever the other player does. What is her optimal strategy according to the maximin criterion?

Sample answer: 10 T

- (b) (2 points) What is the maximin value for the column player? What is her optimal strategy according to the maximin criterion?
- (c) (2 points) Does this strategy profile lead to a Nash Equilibrium? Answer *yes* or *no*.

13. (a) (2 points) Give the Pareto optimal solution of the previous game.

Sample answer: (B,L)

- (b) (2 points) Is it a Nash equilibrium? Answer *yes* or *no*.

2.2 Nash Equilibrium

14. Consider the following normal-form games:

| | | (a) | |
|--|--|-----|-------------|
| | | L | R |
| | | T | 1, 4 2, 3 |
| | | B | 0, 2 3, 5 |

| | | (b) | |
|--|--|-----|-------------|
| | | L | R |
| | | T | 1, 4 2, 5 |
| | | B | 7, 1 2, 2 |

| | | (c) | |
|--|--|-----|-------------|
| | | L | R |
| | | T | 2, 1 5, 1 |
| | | B | 5, 1 3, 1 |

- (a) (6 points) Compute all (mixed and pure) Nash equilibria for game (a).

Sample answer: (T,L),(T,R),((1/3,2/3),(3/4,1/4))

Especially, if no mixed equilibria, write as: (T,L),(T,R)

If one player is indifferent for any mixed strategy, write as: ((any),(p,1 - p))

- (b) (6 points) Compute all (mixed and pure) Nash equilibria for game (b).

- (c) (6 points) Compute all (mixed and pure) Nash equilibria for game (c).