

Constants and conversions:

$$1 M_{\odot} = 2.0 \times 10^{30} \text{ kg}$$

$$1 L_{\odot} = 3.8 \times 10^{26} \text{ W}$$

$$R_{\odot} = 7.0 \times 10^8 \text{ m}$$

$$c = 3 \times 10^8 \text{ m s}^{-1}$$

$$G = 6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} = 4.3 \times 10^{-6} \text{ kpc (km s}^{-1})^2 M_{\odot}^{-1}$$

$$h = 6.6 \times 10^{-34} \text{ J s}, \hbar = h/2\pi$$

$$m_p \approx m_n \approx m_H = 1.7 \times 10^{-27} \text{ kg}$$

$$m_e \approx m_p/1800 = 9.1 \times 10^{-31} \text{ kg}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ pc} = 3.1 \times 10^{16} \text{ m}$$

$$1 \text{ yr} = 3.2 \times 10^7 \text{ s}$$

$$1 \text{ radian} = 206265 \text{ arcsec}$$

Redshift

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} \quad (1)$$

Doppler shift

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c} \quad (2)$$

$$z = v/c \text{ for small } z$$

Hubble law

$$v = H_0 d \quad (3)$$

$$H_0 = 70.4_{-1.4}^{+1.3} \text{ km s}^{-1} \text{ Mpc}^{-1}, \text{ Hubble time } t_H = 1/H_0 = 13.8 \text{ Gyr}$$

$$\text{Magnitudes and fluxes: } m_2 - m_1 = 2.5 \log(F_1/F_2)$$

$$\text{Distance modulus: } m - M = 5 \log(d/10 \text{ pc})$$

Kepler's third law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3, \quad (4)$$

Schwarzschild radius:

$$R_{\text{Sch}} = \frac{2GM_{\text{BH}}}{c^2} \quad (5)$$

Exponential surface brightness profile (scale height h , central surface brightness Σ_0): $\Sigma(r) = \Sigma_0 e^{-r/h}$

or

$$\mu(r) = \mu_0 + 1.086 \frac{r}{h}. \quad (6)$$

with surface brightness in magnitudes.

de Vaucouleurs or $r^{1/4}$ profile:

$$\mu(r) = \mu_0 + 8.33 \left(\frac{r}{r_e} \right)^{1/4} \quad (7)$$

where r_e is the effective radius, the radius which encloses half of the total light.

Sérsic profile:

$$\mu(r) = \mu_0 + 8.33 \left[\left(\frac{r}{r_e} \right)^{1/n} - 1 \right]. \quad (8)$$

Circular velocity

$$v_c^2 = \frac{GM}{r} \quad (9)$$

Isothermal density profile of dark matter halos

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}. \quad (10)$$

Tully-Fisher relation, $L \propto v^4$; in magnitudes and km s^{-1} :

$$M_B = -10.2 \log v_{\max} + 2.71 \quad (11)$$

(this is in the B -band, and for Sb galaxies).

Virial theorem

$$-2 \langle KE \rangle = \langle PE \rangle. \quad (12)$$

and virial mass

$$M = \frac{5R\sigma^2}{G} \quad (13)$$

Faber-Jackson relation $L \propto \sigma^4$, fundamental plane $L \propto \sigma^{2.65} r_e^{0.65}$

Galaxy interactions

Tidal radii

$$l_1 = a \left[0.500 - 0.227 \log \left(\frac{M_2}{M_1} \right) \right] \quad (14)$$

$$l_2 = a \left[0.500 + 0.227 \log \left(\frac{M_2}{M_1} \right) \right], \quad (15)$$

where a is the distance between M_1 and M_2 .

Dynamical friction

$$f_d = C \frac{G^2 M^2 \rho}{v_M^2} \quad (16)$$

AGN luminosity and accretion rate (where η is the efficiency):

$$\dot{M} = \frac{L}{\eta c^2} = 0.018 \text{ M}_\odot \text{ yr}^{-1} \left(\frac{L}{10^{37} \text{ W}} \right) \left(\frac{\eta}{0.1} \right)^{-1} \quad (17)$$

Eddington limit

$$\dot{M}_E = \frac{L_E}{\eta c^2} = 2 \text{ M}_\odot \text{ yr}^{-1} \left(\frac{M_{\text{bh}}}{10^8 \text{ M}_\odot} \right) \left(\frac{\eta}{0.1} \right)^{-1} \quad (18)$$