Astronomy 401/Physics 903
Lecture 7
Elliptical Galaxies

According to the Hubble sequence, elliptical galaxies are classified only by their degree of observed ellipticity. This turns out to be not particularly useful; unlike the properties used to classify spirals, the observed ellipticity shows very little correlation with other properties of the galaxies.

1 Types of elliptical galaxies

Elliptical galaxies have a much wider range in mass than spirals. A lot of different types:

- **cD galaxies**. These are the biggest and most massive galaxies in the universe. Rare, found near the centers of large galaxy clusters. Can be nearly 1 Mpc across, have masses between 10^{13} and 10^{14} $M_\odot$. Very high mass-to-light ratios, implying lots of dark matter.
- **Normal ellipticals**. Centrally condensed, high central surface brightness. Mass range 10^8 to 10^{13} $M_\odot$, sizes from < 1 kpc to almost 200 kpc. M/L ranges from 7 to > 100.
- **Dwarf elliptical (dE)**. Lower surface brightness than normal ellipticals, mass range 10^7 to 10^9 $M_\odot$, sizes $\sim$ 1–10 kpc.
- **Dwarf spheroidal (dSph)**. Very low luminosity, low surface brightness; detected only near the Milky Way. Masses $\sim 10^7$–10^8 $M_\odot$, diameters 0.1 to 0.5 kpc.
- **Blue compact dwarf galaxies (BCD)**. Small and unusually blue, indicating rapid star formation. Masses $\sim 10^9$ $M_\odot$, sizes < 3 kpc. Large gas masses and low M/L, consistent with their high star formation rates.

2 Velocity dispersion

Unlike spiral galaxies, elliptical galaxies generally aren't coherently rotating—the stars are on **random orbits**. To measure the velocities of a system like this, we use the **velocity dispersion**:

$$\sigma^2 \equiv \frac{1}{N}\sum_{i=1}^{N}(v_i - \langle v\rangle)^2 \tag{1}$$

where $v_i - \langle v\rangle$ is the difference between the velocity of the ith star and the mean velocity of the system of N stars. This should look familiar; this is the expression for the variance of a population. So the velocity dispersion σ is equivalent to the standard deviation of the velocity distribution.

Except for very nearby galaxies, it usually isn't possible to measure the velocities of individual stars, so instead this is measured from the widths of absorption or emission lines in the spectrum of a galaxy. The light from each star will be Doppler shifted due to the velocity of that star, so for a wider range of stellar velocities, the total width of a line produced by stars will be broader.

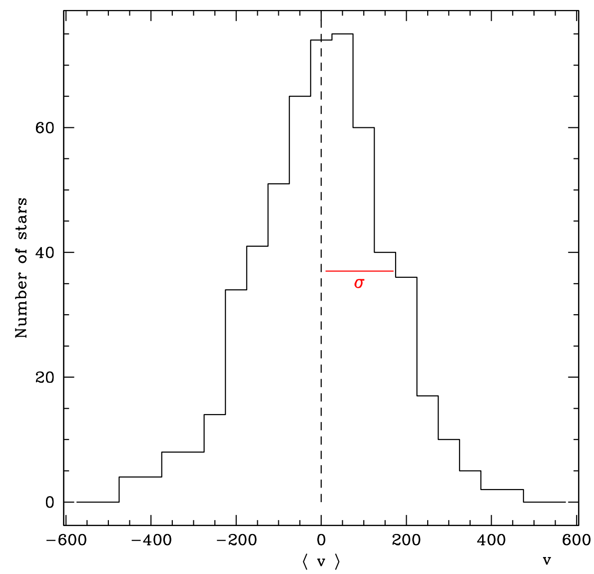

Figure 1: Dispersion of stellar velocities

3 Dynamics of elliptical galaxies

3.1 Virial mass

How do we estimate the mass of an elliptical galaxy?

We use the **virial theorem**, which can be written

$$\frac{1}{2}\left\langle \frac{d^2 I}{dt^2} \right\rangle - 2\langle K \rangle = \langle U \rangle. \tag{2}$$

I is the moment of inertia of the system, $\langle K \rangle$ is the average kinetic energy (due to stellar motions), and $\langle U \rangle$ is the gravitational potential energy due to the mass of the system. If a galaxy is in equilibrium, or "virialized,"

$$\left\langle \frac{d^2 I}{dt^2} \right\rangle = 0 \tag{3}$$

and

$$-2\,\langle K \rangle = \langle U \rangle. \tag{4}$$

For a galaxy of N identical stars of mass m and with no coherent motions (i.e. the stars are on random orbits), $\langle v \rangle = 0$, the kinetic energy is

$$\begin{aligned} \langle K \rangle &= \sum_{i=1}^{N} \frac{1}{2} m v_i^2 && (5) \\ &= \frac{N}{2} m \sigma^2 && (6) \\ &= \frac{M_{\rm tot}\sigma^2}{2} && (7) \end{aligned}$$

What is the potential energy?

Consider the potential energy of a spherical shell at radius r, surrounding a mass $M(<r)$. The mass of the shell is $dm = 4\pi r^2 dr \times \rho(r)$. The potential energy ($U$) of the shell is

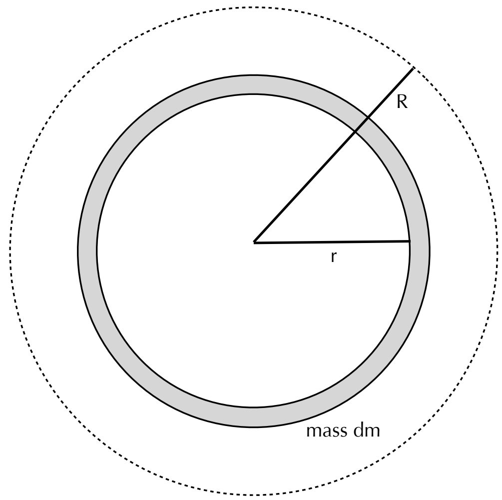

Figure 2: Spherical shell of mass dm.

$$dU(r) = -\frac{GM(<r)}{r} dm \tag{8}$$

$$= -\frac{GM(<r)}{r} 4\pi r^2 dr \times \rho(r) \tag{9}$$

We integrate over all shells out to some radius R:

$$U(R) = -4\pi G \int_0^R M(<r)\rho(r) r \, dr \tag{10}$$

For a uniform sphere of radius R ($\rho =$ constant),

$$M(<r) = \rho \frac{4\pi}{3} r^3 = M_{\rm tot} \frac{r^3}{R^3} \tag{11}$$

and the potential energy is

$$\langle U \rangle = -4\pi G \, M_{\rm tot} \, \rho \int_0^R r^4 \, dr \tag{12}$$

$$= -\frac{3}{5}\left(\frac{GM_{\rm tot}^2}{R}\right) \tag{13}$$

where we have substituted $\rho = M_{\rm tot}/(4\pi R^3/3)$ for the density.

Now return to the virial theorem:

$$-2\,\langle K \rangle = \langle U \rangle. \tag{14}$$

So we have

$$-2\left[\frac{M_{\rm tot}\sigma^2}{2}\right] = \frac{3}{5}\left(\frac{GM_{\rm tot}^2}{R}\right) \tag{15}$$

for a spherical, constant density galaxy (we can work out more accurate versions for any $\rho(r)$).

Rearranging:

$$M_{\rm tot} = \frac{5R}{G}\left[\frac{\sigma^2}{3}\right] \tag{16}$$

Bigger mass $\Rightarrow$ Larger velocity dispersion.

Now consider the *total* velocity dispersion. For a spherical system, the stellar velocities have three components:

$$\sigma^2 = \sigma_r^2 + \sigma_\theta^2 + \sigma_\phi^2 \tag{17}$$

σ_r^2 is the **radial velocity dispersion** and is the only one we can measure (the other two are perpendicular to our line of sight). In general,

$$\sigma_r^2 \neq \sigma_\theta^2 \neq \sigma_\phi^2. \tag{18}$$

However, if $\sigma_r^2 = \sigma_\theta^2 = \sigma_\phi^2$ (**isotropic** velocity dispersion),

$$\sigma^2 = 3\sigma_r^2 \tag{19}$$

and

$$M_{\rm tot} = \frac{5R\sigma_r^2}{G}. \tag{20}$$

The mass obtained this way is called the **virial mass**.

Problem: velocity dispersions aren't necessarily isotropic, which leads to significant uncertainty in measuring M.

4 Scaling relations for elliptical galaxies

There is a relationship for elliptical galaxies similar to the Tully-Fisher relation for spirals, relating the velocity dispersion to the luminosity rather than the rotational velocity. This is the **Faber-Jackson relation**:

$$L \propto \sigma^\alpha, \tag{21}$$

where as for the Tully-Fisher relation $\alpha \simeq 4$. This relationship has quite a bit of scatter, and it's found that we can reduce the scatter by including another parameter, the effective radius r_e (remember that this is the radius that encloses half the light of the galaxy); alternatively, we can use the surface brightness Σ in place of L. Observationally, we find

$$r_e \propto \sigma^{1.4} \langle\Sigma\rangle_e^{-0.85}, \tag{22}$$

where $\langle\Sigma\rangle_e$ is the average surface brightness within the effective radius r_e. This is one fit to a sample; as with all similar scaling relations, results differ somewhat for different data sets.

This is called the **fundamental plane** — elliptical galaxies fall on a plane in the three-dimensional space defined by luminosity (or surface brightness), velocity dispersion and effective radius. Both this and the Faber-Jackson relation can be roughly reproduced by considering relationships between mass and velocity (from the virial theorem), surface brightness and the mass-to-light ratio, as we showed with the Tully-Fisher relation.

What does the fundamental plane imply about the mass-to-light ratios of elliptical galaxies? From the virial theorem, we know that the mass within r_e is $M \propto \sigma^2 r_e$, and the relationship between luminosity and surface brightness is

$$L = 2\pi r_e^2 \langle\Sigma\rangle_e. \tag{23}$$

Combining these two expressions gives

$$r_e \propto \frac{L}{M} \frac{\sigma^2}{\langle \Sigma \rangle_e}. \tag{24}$$

So the virial theorem predicts $r_e \propto \sigma^2 \langle \Sigma \rangle_e^{-1}$ if M/L is constant.

In order for this to agree with the observed fundamental plane, Equation 25, we need

$$\frac{L}{M} \frac{\sigma^2}{\langle \Sigma \rangle_e} \propto \frac{\sigma^{1.4}}{\langle \Sigma \rangle_e^{0.85}} \tag{25}$$

or

$$\begin{aligned} \frac{M}{L} &\propto \frac{\sigma^{0.6}}{\langle \Sigma \rangle_e^{0.15}} & (26) \\ &\propto \frac{M^{0.3}}{r_e^{0.3}} \frac{r_e^{0.3}}{L^{0.15}}, & (27) \end{aligned}$$

where in the last step we have substituted $\sigma^2 \propto M/r_e$ and $\langle \Sigma \rangle_e \propto L\, r_e^{-2}$.

This then implies that the fundamental plane follows from the virial theorem provided that

$$\left(\frac{M}{L}\right) \propto M^{0.2} \tag{28}$$

or

$$\left(\frac{M}{L}\right) \propto L^{0.25}. \tag{29}$$

In other words, the fundamental plane implies that the mass-to-light ratio of elliptical galaxies increases slightly with mass. It is in fact observed that more massive galaxies have higher mass-to-light ratios.

Recent results using careful mass modeling of elliptical galaxies show that when mass is substituted for luminosity, the correlation becomes significantly tighter. The "mass plane" satisfies the expectation from the virial theorem $M \propto \sigma^2 r_e$, confirming that the deviation of the fundamental plane from the virial exponents is due to variations in M/L (see Figure 4).

5 What flattens elliptical galaxies?

Disk galaxies are relatively gas-rich, rotating systems. They are flattened by rotation: a self-gravitating, rotating ball of gas will bulge outward at the equator due to centrifugal forces (as does the Earth). Gas can also lose energy through radiation (stars also radiate, but this does not change the kinetic or potential energy of a system of stars). This energy loss in combination with angular momentum, which is much harder to lose, causes the gas to settle into a disk. So disk galaxies are disks because of rotation.

Are elliptical galaxies also flattened in this way? Or is the flatness due to an anisotropic velocity distribution? Clearly it's not rotation at least some of the time.

An example: The luminous E4 galaxy NGC 1600 (recall classification of ellipticals, ellipticity $\epsilon \equiv 1 - b/a$, where a is major axis and b is minor axis; classification is En, where $n = 10(1 - b/a)$. So an E4 galaxy has ellipticity 0.4) has $v_{\rm rot} = 1.9 \pm 2.3$ km s^{-1} and $v_{\rm rot}/\sigma < 0.013$—no significant rotation, and yet significantly flattened.

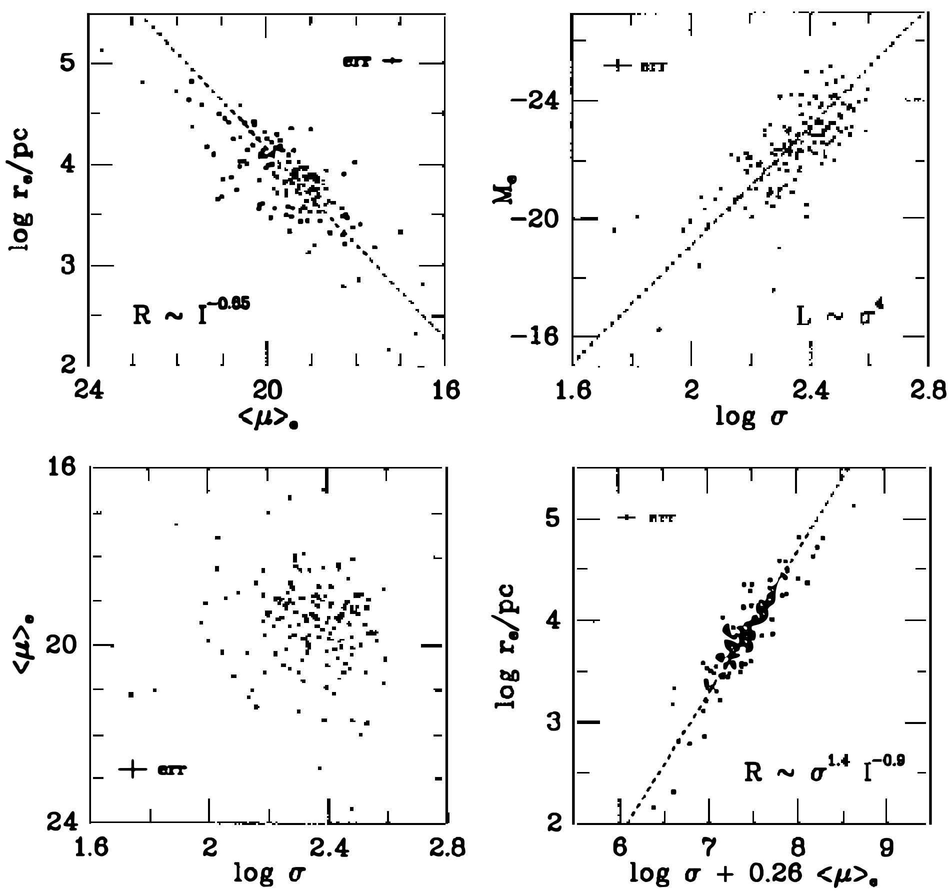

Figure 3: Projections of the fundamental plane. Upper left: the correlation between effective radius and surface brightness (μ indicates surface brightness in magnitudes per unit area, and I indicates surface brightness in units of luminosity per unit area). Upper right: the Faber-Jackson relation, the correlation between luminosity and velocity dispersion. Lower left: There is little correlation between mean surface brightness and velocity dispersion; this is the fundamental plane seen from above. Lower right: The fundamental plane viewed from the side. The tightest correlation is obtained by combining all three parameters. From Kormendy & Djorgovski, 1989, *Annual Reviews of Astronomy and Astrophysics*, 1989, 27, 235.

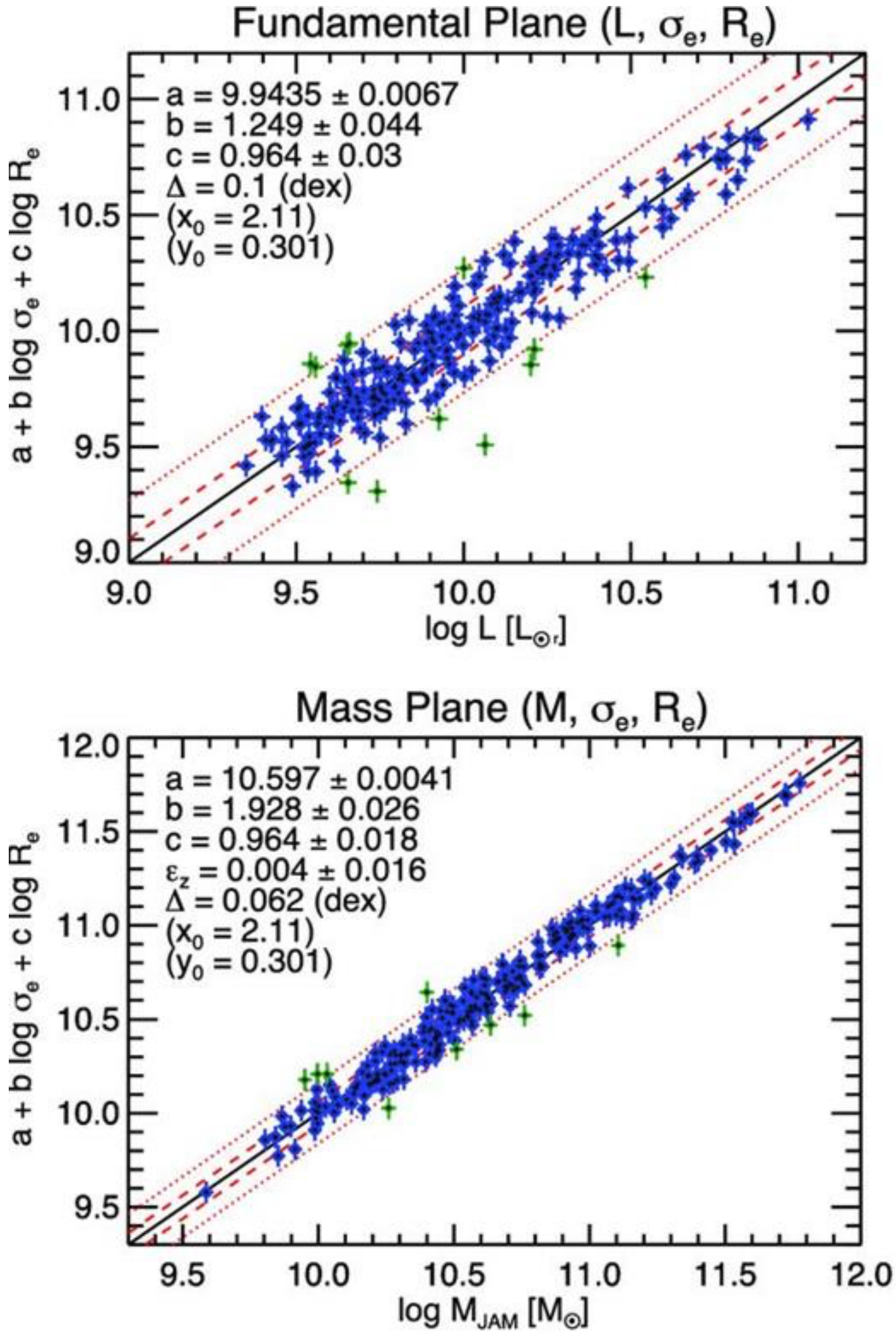

Figure 4: Top: A recent determination of the fundamental plane. Bottom: If mass is substituted for luminosity, the correlation becomes significantly tighter. The "mass plane" in the lower panel satisfies the expectation from the virial theorem $M \propto \sigma^2 r_e$, confirming that the deviation of the fundamental plane from the virial exponents is due to variations in M/L. From Cappellari et al, 2013, *Monthly Notices of the Royal Astronomical Society*, 432 1709.

We can formalize this a bit: stellar dynamics show that for a rotationally flattened system with ellipticity ϵ:

$$\frac{v_{\rm rot}}{\sigma} \approx \left(\frac{\epsilon}{1-\epsilon}\right)^{1/2} \tag{30}$$

So if an E4 galaxy is rotationally flattened, it should have $v_{\rm rot}/\sigma \approx 0.8$, much higher than the observed $v_{\rm rot}/\sigma < 0.013$. So NGC 1600 and similar luminous ellipticals are not flattened because of rotation; the flattening must have some other cause.

Ellipticals that aren't flattened by rotation may be flattened by **velocity anisotropy**. Example: start with a rotating, flattened galaxy. Take half the stars and rotate them in the opposite direction. Kinetic energy is the same, and the galaxy is flattened by the same amount, but there is no net rotation, and the circular velocity dispersion is now much bigger than the vertical one. Things like this are actually observed—"counter-rotating cores"—and probably indicate a prior merger or accretion of another galaxy. Ellipticals can have complicated orbital structure—different fractions of stars on radial and various circular orbits.

Low mass ellipticals are generally flatted by rotation, high mass by velocity anisotropy.

Galaxies are considered to be **rotationally supported** or **pressure supported**, depending on whether their stellar velocities are mostly ordered rotation or random motions.

6 Relaxation timescale

Are velocity distributions such as we have described for elliptical galaxies stable in time? If interactions between stars are frequent, we would expect these interactions to disturb the orbits of stars and result in the "thermalization" of the velocity distribution (recall that we discussed this in the context of the stars near the Galactic Center, where the stellar density is high and the density profile is close to that expected from a Maxwellian velocity distribution). We will estimate the timescale for such interactions between stars and the changes in direction they cause.

We consider the *relaxation timescale* by pair collisions for a system of N stars of mass M, with total mass M, size R and mean stellar density $n = 3N/(4\pi R^3)$. We define the relaxation time t as the characteristic time in which a star changes the direction of its velocity by $\sim 90°$ due to interactions with other stars. The relaxation time is

$$t_{\rm relax} \approx \frac{R}{6v}\frac{N}{\ln(N/2)} \tag{31}$$

or

$$t_{\rm relax} \approx \frac{t_{\rm cross}}{6}\frac{N}{\ln(N/2)}, \tag{32}$$

where $t_{\rm cross} = R/v$ is the *crossing time*, the time it takes a star to cross the stellar system (see section 3.2.4 of the text for a derivation of the relaxation time).

We'll now consider a typical large elliptical galaxy, with $t_{\rm cross} \sim 10^8$ yr and $N \sim 10^{12}$ (so $\ln N \sim 30$). We find that the relaxation time is $t_{\rm relax} \sim 10^{18}$ yr, much longer than the age of the Universe. This means that interactions between stars play no role at all in the evolution of stellar orbits in elliptical galaxies, and the dynamics of the orbits are determined only by the large-scale gravitational field.

The stars behave like a collisionless gas: they are stabilized by dynamical pressure, and the galaxies are elliptical because the stellar orbits are anisotropic in velocity space — the *velocity anisotropy* discussed above.

Non-baryonic dark matter dark matter is also collisionless, since both weak and gravitational interactions between particles (assumed to be WIMPs, weakly interacting massive particles) are negligible in any galactic context. Gas, on the other hand, is not; this is why gas-rich galaxies lose energy and settle into disks, while dark matter halos remain roughly spherical (although some flattening is observed).