Astronomy 401/Physics 903
Problem Set 4
Due in class, **Thursday February 28, 2019**

1 The temperature of hydrogen clouds

The Boltzmann factor $e^{-(E_1-E_2)/kT}$ helps to determine the relative populations of energy levels in an atom. Using the Boltzmann factor, estimate the temperature required for a hydrogen atom's electron and proton to go from being anti-aligned to being aligned. Are the temperatures in H I clouds sufficient to produce this low-energy excited state?

2 Star formation efficiency and gas consumption timescales

In this problem you will study one of the most important relationships for star formation in galaxies. Go to `http://www.cgca.uwm.edu/~dawn/astron401/sf_gas_data.dat` and copy or download the data file there. This is a collection of real data for a wide range of galaxies, from normal spirals to very luminous starbursts. The columns in the file are the name of the galaxy, the gas mass density per unit area in $\mathrm{M}_\odot$ kpc^{-2}, the star formation rate (SFR) per unit area in $\mathrm{M}_\odot$ yr^{-1} kpc^{-2}, and the dynamical time $t_{\rm dyn}$, the approximate time it takes a star to complete one orbit around the galaxy, in years. We will call the gas surface density $\Sigma_{\rm gas}$ and the star formation rate surface density $\Sigma_{\rm SFR}$.

a) Make a plot with the gas surface density $\Sigma_{\rm gas}$ on the x axis and the SFR surface density $\Sigma_{\rm SFR}$ on the y axis.

b) Now take the logarithm of both quantities, and make a plot with $\log(\Sigma_{\rm gas})$ on the x axis and $\log(\Sigma_{\rm SFR})$ on the y axis. You should see a strong correlation. Find a straight line that is a good fit to the points. The equation of your line will have the form $\log(\Sigma_{\rm SFR}) = a\log(\Sigma_{\rm gas}) + b$. What are your values of a and b? (This relationship is known as the Schmidt law.)

Solve your equation for $\Sigma_{\rm SFR}$. You should find that the density of star formation increases with the gas surface density to the power of a. If $a = 1$, the relationship is linear, and star formation rate per unit area is directly proportional to the gas density. If $a > 1$, the star formation rate per unit area increases faster than the gas density, implying that galaxies with higher gas densities are more efficient at turning their gas into stars. What does your value of a imply about the efficiency of star formation?

c) Now let's look at this correlation in another way. Divide the gas density by the dynamical time $t_{\rm dyn}$, and make a plot with $\Sigma_{\rm gas}/t_{\rm dyn}$ on the x axis and $\Sigma_{\rm SFR}$ on the y axis. We are interested in $t_{\rm dyn}$ because if star formation is triggered by spiral arms or bars, we might expect a relationship between the star formation rate and the orbital timescale.

d) You will again find that this is better represented on a log-log plot. Take the logarithm of both quantities, and make a plot with $\log(\Sigma_{\rm gas}/t_{\rm dyn})$ on the x axis and $\log(\Sigma_{\rm SFR})$ on the y axis. You should again see a strong correlation. Find the appropriate intercept for a line of slope 1 that is a good fit to the points. Your line will have the form $\log\Sigma_{\rm SFR} = \log(\Sigma_{\rm gas}/t_{\rm dyn}) + C$.

Solve your equation for $\Sigma_{\rm SFR}$. You should find $\Sigma_{\rm SFR} = \epsilon(\Sigma_{\rm gas}/t_{\rm dyn})$, where ϵ represents the fraction of gas that is turned into stars per dynamical time. What is ϵ? How many dynamical times will it take to consume all the gas in the galaxy? What is the mean value of $t_{\rm dyn}$ in your sample, and what therefore is the mean timescale to consume all the gas in a galaxy?

3 The gas consumption time of the Milky Way

If the disk of the Milky Way contains $\sim 10^9$ $\rm M_\odot$ of neutral hydrogen within the solar circle ($R = 8$ kpc), estimate the time required for the Galactic disk to be depleted of interstellar gas via star formation. Use the Schmidt law you found in part b) of Problem 2 above. How might the period of star formation in the Galaxy be prolonged?

4 Galaxy spectra

In this problem you'll examine images and spectra of galaxies from the Sloan Digital Sky Survey (SDSS), one of the largest and most widely used surveys in astronomy. Over eight years of operation, SDSS obtained images of more than a quarter of the sky, covering more than 930,000 galaxies and more than 120,000 quasars, and several later, more focused versions of SDSS followed and continue today. Instead of measuring light in the familiar B and V bands, SDSS uses a different set of filters; from blue to red, the SDSS wavelength bands are u, g, r, i and z. To describe the color of a galaxy in the Sloan Survey, we will use $u - r$ instead of $B - V$.

Do the exercise found at
`http://cas.sdss.org/dr5/en/proj/advanced/galaxies/spectra.asp`,
answering all four questions (7-10). Number the galaxies 1-11. For the second question, use the image of the galaxy to classify it according to the Hubble sequence.

5 Galaxy spectra II

This problem is required only for students enrolled in Physics 903.

Go to the course website and download the file `spSpec-51688-0302-325.dat`. This is the SDSS spectrum of the first galaxy from Problem 1. The spectrum is in ASCII format, and the three columns are observed (i.e. not rest frame) wavelength in Å, flux density F_λ in units of 10^{-17} erg s^{-1} cm^{-2} Å^{-1}, and the 1-σ uncertainty on the flux density in the same units (the error spectrum). Note that these units mean than an integral of the spectrum over wavelength will have units of flux, erg s^{-1} cm^{-2}.

Your goal is to simultaneously fit the eight strongest emission lines in this spectrum, and then calculate the star formation rate and virial mass of the galaxy using your measurements of the $\rm H\alpha$ emission line.

The eight lines to be fit and their rest-frame wavelengths in Å are:

Hβ	4862.721
[O III]	4960.295
[O III]	5008.239
[N II]	6549.86
Hα	6564.614
[N II]	6585.27
[S II]	6718.29
[S II]	6732.68

All of these lines come from the H II regions surrounding the massive stars. You can assume that all of the lines have Gaussian profiles, and that they all have the same redshift and width; this will limit the number of parameters you need to determine.

a) Use only the portion of the spectrum between 4500 and 8000 Å. Fit and subtract the continuum. You can do this by fitting a linear model to the spectrum and then subtracting it. Your fit will be improved if you exclude the emission lines from the fit, but this isn't essential. Your continuum-subtracted spectrum should have an average value of about zero, except where there are features.

b) Construct the function to fit, and fit it to the spectrum. This should be a sum of eight Gaussians, with centers given by $\lambda_{\rm rest}(1+z)$, where $\lambda_{\rm rest}$ is the rest wavelength in the table above; this will allow you to fit for the redshift of the galaxy. (Recall that the redshift is $z = \lambda_{\rm obs}/\lambda_{\rm rest} - 1$.) Assume that all of the Gaussians have the same standard deviation σ, and that the continuum level is zero. This should result in ten free parameters: the redshift, the line width, and the amplitude of each Gaussian. You will need initial guesses for the values of these parameters, so that your program knows where to look for the Gaussians; you can get these from examining the spectrum or from estimating them from the plot below.

Use the error spectrum (this is the 1-σ uncertainty on each pixel) to determine uncertainties on your fitted parameters.

c) Report your best-fit parameters and their uncertainties, and make a plot of the spectrum and your best-fit model. The plot should look something like the figure below; it will be easier to see the lines if you make the plot in two panels, covering only the wavelength regions shown.

d) Integrate the Hα line to determine its flux. If the distance to the galaxy is 107 Mpc, what is the Hα luminosity in ergs s^{-1}?

The conversion between Hα luminosity and star formation rate is

$$\text{SFR } (\text{M}_\odot \text{yr}^{-1}) = 7.9 \times 10^{-42} L(\text{H}\alpha) \text{ (erg s}^{-1}). \tag{1}$$

What is the star formation rate of the galaxy?

You should have a best-fit line width (the standard deviation of your Gaussians) in Å. Line widths are usually expressed in km s^{-1}, and you can convert your line width from Å to km s^{-1}via the relation

$$\sigma_{\rm kms} = \sigma_{\rm Ang} \frac{c}{\lambda_{\rm obs}}, \tag{2}$$

where c is the speed of light in km s^{-1}and $\lambda_{\rm obs}$ is the observed wavelength of the line. What is your Hα line width in km s^{-1}?

The emission lines are broadened both by gravitational motions in the galaxy and by the spectrograph itself. We need to account for this instrumental resolution, and we do this by subtracting it in quadrature from the observed line width:

$$\sigma_{\rm true} = (\sigma_{\rm obs}^2 - \sigma_{\rm inst}^2)^{1/2}. \tag{3}$$

Assume that the instrumental resolution is 50 km s^{-1}. What is the true velocity dispersion of the galaxy?

As we discussed in class, the virial mass is given by $M = 5\sigma^2 r/G$. Using your true velocity dispersion from above and a size of $r = 0.8$ kpc, what is the mass of the galaxy in solar masses?

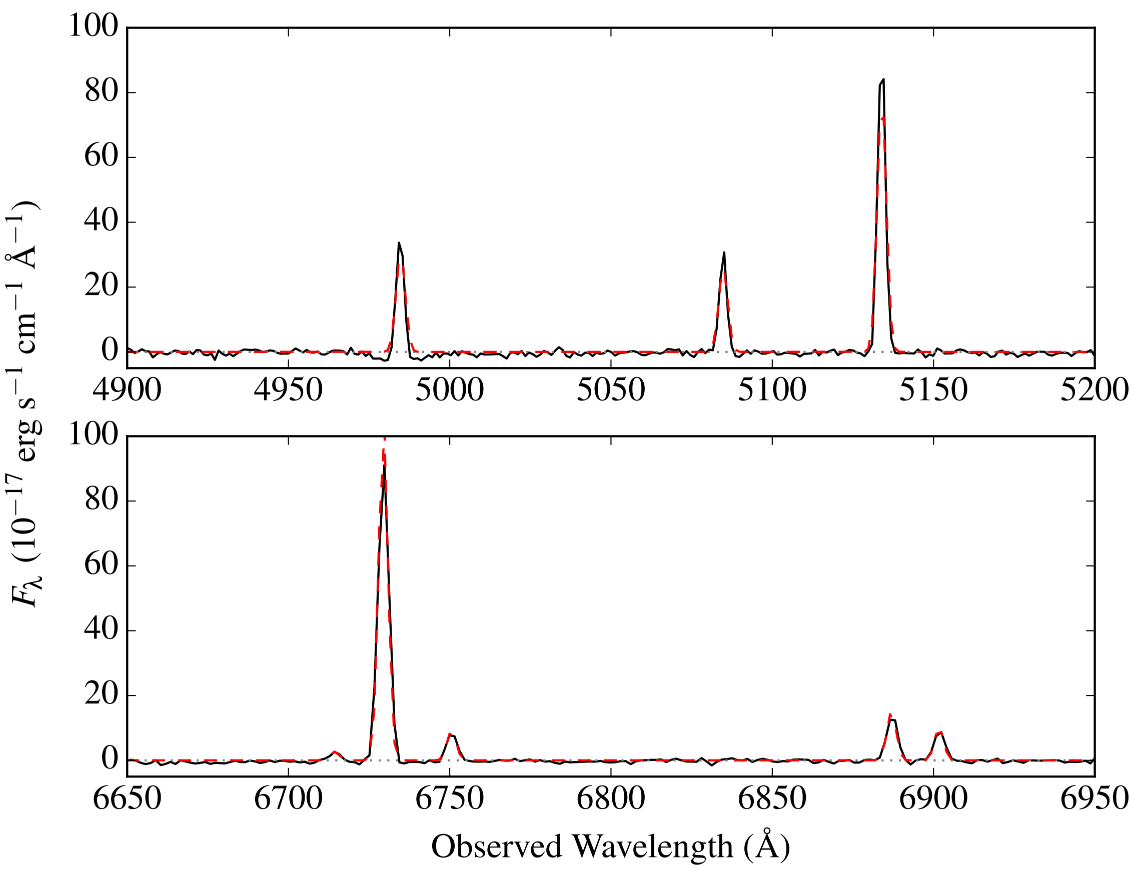

Figure 1: Continuum-subtracted SDSS spectrum of the galaxy, with a simultaneous fit to the eight strong emission lines shown by the dashed line.