Astronomy 401/Physics 903
Lecture 11
Galaxy Spectra I

The colors of a galaxy (the integrated light) can tell us about its stellar population, but we can learn even more information from its spectrum. Galaxies contain gas and stars; unsurprisingly, the spectrum of a galaxy looks like some mixture of the spectra of stars and clouds of gas, and it varies depending on the type of the galaxy.

## 1 The production of spectral lines and the Bohr atom

Before the reasons for it were understood, it was observed that hydrogen gas emitted lines with wavelengths given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \tag{1}$$

where $m$ and $n$ are integers with $m < n$ and $R_H$ is the experimentally determined Rydberg constant for hydrogen.

This was explained by Niels Bohr, who proposed that the angular momentum of the orbiting electron was quantized: it could only have certain values given by multiples of $\hbar$: $L = nh/2\pi = n\hbar$.

The kinetic and electrical potential energies of the atom can be derived classically. We will use the reduced mass $\mu$,

$$\mu = \frac{m_e m_p}{m_e + m_p} = 0.999 m_e \approx m_e \tag{2}$$

and the total mass $M = m_p + m_e \approx m_p$. The hydrogen atom can then be modeled as a proton of mass $M$ that is at rest and an electron of mass $\mu$ that follows a circular orbit of radius $r$ about the proton.

To find the kinetic energy, we start with the electric force between the proton and electron given by Coulomb's law,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}. \tag{3}$$

This electric force produces the centripetal acceleration of the electron, $v^2/r$, so, from Newton's second law,

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = -\mu\frac{v^2}{r} \tag{4}$$

where the negative sign comes from the fact that the acceleration is directed inward. With $q_1 = e$ and $q_2 = -e$, we have

$$-\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = -\mu\frac{v^2}{r}. \tag{5}$$

This expression can be solved for the kinetic energy, $\frac{1}{2}\mu v^2$:

$$K = \frac{1}{2}\mu v^2 = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r}. \tag{6}$$

The electrical potential energy can be found through a derivation analogous to that of gravitational potential energy. The result is

$$U = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -2K. \tag{7}$$

The total energy of the atom is then

$$E = K + U = K - 2K = -K = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r}. \tag{8}$$

We then introduce the quantization of angular momentum

$$L = \mu v r = n\hbar \tag{9}$$

and use it to rewrite the kinetic energy:

$$K = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r} = \frac{1}{2}\mu v^2 = \frac{1}{2}\frac{(\mu v r)^2}{\mu r^2} = \frac{1}{2}\frac{(n\hbar)^2}{\mu r^2}. \tag{10}$$

We can solve this for $r$ to see that only certain values are allowed:

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}n^2 = a_0 n^2, \tag{11}$$

where $a_0 = 5.29 \times 10^{-11}$ m $= 0.0529$ nm is the Bohr radius. Only certain orbits are allowed, with $r = a_0$, $4a_0$, $9a_0$, etc.

We then insert this expression for the radius into our expression for the total energy of the atom to find the allowed energies:

$$E_n = -\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = -13.6 \text{ eV } \frac{1}{n^2}. \tag{12}$$

The integer $n$ is the principal quantum number. When the electron is in the ground state, $n = 1$ and it takes at least 13.6 eV to ionize the atom. When it's in the first excited state, $n = 2$ and the atom's energy is higher, $E_2 = -13.6/4\text{eV} = -3.40$ eV.

Electrons can move between energy levels. If an electron goes from a higher state to a lower state, it emits a single photon with energy $E = E_{\text{high}} - E_{\text{low}}$, or

$$E = \frac{hc}{\lambda} = E_{n,\text{high}} - E_{n,\text{low}}. \tag{13}$$

Using the expression for $E_n$, we find

$$\frac{1}{\lambda} = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c}\left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right), \tag{14}$$

which is the observed relationship we started with, with the combination of constants equal to the Rydberg constant. Spectral lines of hydrogen appear in different series: the Balmer series were the first observed, with $n_{\text{low}} = 2$. The Lyman series has $n_{\text{low}} = 1$ and the Paschen series has $n_{\text{low}} = 3$. The most commonly observed lines are the Balmer lines in the optical: H$\alpha$ is the strongest at 6563 Å, then H$\beta$ at 4861 Å, H$\gamma$, etc.

Electrons can also move from a lower to a higher energy level by absorbing a photon with energy equal to the energy difference between two levels.