Astronomy 401/Physics 903
Problem Set 6
Due in class, **Thursday April 4, 2019**

## 1 Galaxy cluster

Measurements of the radial recession velocities of five galaxies in a cluster give velocities of 9700, 8600, 8200, 8500, and 10,000 km $s^{-1}$ .

a) What is the distance to the cluster?

b) If every galaxy is roughly half a degree from the center of the cluster, estimate the mass of the cluster.

## 2 Absorption from a quasar outflow

A source moving with a radial velocity $v$ with respect to the Earth causes a photon with an emitted wavelength of $\lambda_{\mathrm{em}}$ to be observed at a wavelength $\lambda_{\mathrm{obs}}$, because of the Doppler effect. The ratio of these two wavelengths is given by

$$\frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \left(\frac{1+v/c}{1-v/c}\right)^{1/2}, \tag{1}$$

where $c$ is the speed of light. (Note that this is the full special relativistic form of the Doppler shift; for $v \ll c$, it reduces to the familiar approximation $\lambda_{\mathrm{obs}}/\lambda_{\mathrm{em}} = 1 + v/c$).

a) A quasar (QSO) is observed to have a redshift $z = 3.0$, where

$$z = \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{em}}}{\lambda_{\mathrm{em}}}. \tag{2}$$

If you interpret this redshift as being due to the Doppler effect (*not* a correct interpretation for cosmological redshifts), what is the apparent recessional velocity of the quasar, in terms of the speed of light?

b) The quasar has blown a hydrogen cloud out of its center with a velocity of $10^4$ km $s^{-1}$ (relative to the QSO) in the direction of Earth. This hydrogen cloud absorbs radiation from the QSO. What is the wavelength at which a spectrum of the QSO taken from the Earth reveals $\mathrm{Ly}\alpha$ absorption from the hydrogen cloud? The rest frame wavelength of $\mathrm{Ly}\alpha$ is 1216 Å.

c) Suppose that the hydrogen cloud is an ideal gas with a temperature of $10^4$ K, and that there are no bulk motions in the cloud—the only motions are random thermal motions. What is the typical width of a hydrogen absorption line in this cloud? Hint: The kinetic energy of an ideal gas is

$$E_{\mathrm{kin}} = \frac{3}{2}kT, \tag{3}$$

where $T$ is the temperature and $k = 1.38 \times 10^{-23}$ J $K^{-1}$ is Boltzmann's constant.

## 3 Uncertainties in distance measurements

a) Suppose that you mis-measure the apparent flux from a galaxy by 15%, and that you're using a standard candle to find the galaxy's distance. What percentage error would you make in the distance to the galaxy?

b) Now suppose that you are using the Tully-Fisher relation to find the distance to the galaxy. The Tully-Fisher relation has an intrinsic scatter of about 0.15 magnitudes—i.e. at a fixed rotation speed, there will be roughly a $\pm 0.15$ magnitude scatter in the luminosity, even for perfect measurements. Assuming no error in either the measurement of flux or velocity, what percentage distance uncertainty would this correspond to?

c) Given that the distance $r = (L/4\pi F)^{1/2}$, where $F$ is the flux and $L$ is the luminosity, what is the percentage distance uncertainty which results from the combination of the 15% uncertainty in the measured flux from part a) and the intrinsic 0.15 mag scatter in the Tully-Fisher relation in part b)?

d) Now consider the effect of adding an error in the inclination of the galaxy. Suppose you measure the galaxy's inclination to be $45° \pm 5°$. What percentage error in the galaxy's true rotation speed will you measure?

e) Assuming that the Tully-Fisher relationship is $L \propto v^4$, what is the percentage error in the luminosity which results from the uncertainty in the rotation speed derived in part d)?

## 4 Closed universe

a) Using the Friedmann equation for a matter-only universe,

$$\left[\frac{da(t)}{dt}\right]^2 - \frac{8\pi}{3}G\rho a^2(t) = -Kc^2, \tag{4}$$

find an expression for the maximum scale factor $a$ in a closed universe. Your answer will contain the current matter density $\rho_0$ and $K$.

b) Find the lifetime of a closed universe as a function of the density parameter $\Omega_0$. Express your answer as a multiple of the Hubble time $t_H$.

## 5 More uncertainties

**This problem is required only for students enrolled in Physics 903.**

In this problem you will explore another way of estimating uncertainties in derived parameters. Go to the course website and download the file `samplespectrum.dat`. This is a portion of a galaxy spectrum including the H$\alpha$ emission line. The columns of the file are rest-frame wavelength in Å, flux in erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$, and the 1-$\sigma$ uncertainty in the flux (the "error spectrum"), also in erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$. The distance to the galaxy is 82 Mpc.

a) The first step is to fit a Gaussian to the H$\alpha$ emission line and determine the star formation rate. To do this, follow essentially the same procedure you used for Problem 5 on Problem Set

4. The free parameters for your Gaussian will be the amplitude, center, width, and the level of the surrounding continuum. Estimating initial guesses for these parameters will help. Integrate the emission line flux of your best fit, use the distance to compute the H$\alpha$ luminosity $L(H\alpha)$, and convert the luminosity to the star formation rate using the relation

$$\text{SFR } (\mathrm{M}_\odot \text{ yr}^{-1}) = 7.9 \times 10^{-42} \; L(\mathrm{H}\alpha) \text{ (erg s}^{-1}). \tag{5}$$

What is the best-fit SFR?

b) Make a plot of the spectrum with error bars showing the uncertainty at each pixel, and overplot the continuum level and your best fit Gaussian.

c) The next step is to estimate the uncertainty in your measurement of the SFR. We'll assume we know the distance perfectly, and that the only source of error is given by the flux uncertainty of each pixel in the spectrum. We'll use a Monte Carlo procedure to estimate the uncertainties; this will involve using the error spectrum to create many fake realizations of the data, computing the SFR for each fake spectrum, and using the distribution of fake SFRs to calculate the uncertainty.

Use the error spectrum to make an array of random values the same length as the spectrum. Each element of this array should be drawn from a Gaussian distribution centered at zero with a standard deviation given by the value of the error spectrum at that pixel; e.g. the first element of the error spectrum is $4.486 \times 10^{-15}$, so the first random value will be drawn from a Gaussian distribution centered at zero and with a standard deviation of $4.486 \times 10^{-15}$. Add this array of random values to the original spectrum. Now you have a new spectrum in which each flux value has been perturbed by some amount corresponding to its uncertainty. Repeat step (a) on this new perturbed spectrum to determine the SFR.

Repeat the above process 1000 times, and save each measurement of the SFR. You should end up with an array of 1000 SFR values. What is the median value of your array of fake SFRs, and how does it compare to your fitted value from part (a)? The standard deviation of the array of fake SFRs is an estimate of the uncertainty in your measurement of the SFR. What is this uncertainty?

d) Make a histogram of your 1000 SFR values, and mark the fitted value from part (a), the median of the fake SFRs, and the $\pm 1\sigma$ uncertainties.