Astronomy 401/Physics 903
Problem Set 1
Due in class **Thursday January 31, 2019**

1 Lines of Sight and Olber's Paradox

a) Suppose that in Sherwood Forest, the average radius of a tree is $R = 1$ m and the average number of trees per unit area is $\Sigma = 0.005$ m^{-2}. If Robin Hood shoots an arrow in a random direction, how far, on average, will it travel before it strikes a tree? Hint: Consider the *mean free path*.

b) Suppose you are in an infinitely large, infinitely old universe in which stars are clumped into galaxies with a density $n_g = 1$ Mpc^{-3} and average radius $R_g = 2000$ pc. How far, on average, can you see in any direction before your line of sight hits a galaxy? Could we see this far in a universe that is 13.7 Gyr old?

2 Thermal and Magnetic Energy

a) Using the data in Table 1 of the notes for Lecture 2, estimate the average thermal energy density of neutral hydrogen gas in the disk of the Galaxy. Assume a temperature of 15 K, a typical value for hydrogen in the interstellar medium, and assume that the height of the gas disk is 160 pc.

b) Magnetic energy density is given by

$$u_m = \frac{B^2}{2\mu_0},$$

where B is the magnetic field strength and $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}. The strength of the Milky Way's magnetic field is discussed in Section 2.3.4 of the text. Using the value for the field strength given in the text, estimate the magnetic energy density in the spiral arms of the Milky Way. Note that the equation above requires the magnetic field to be given in tesla, and 1 G $= 10^{-4}$ T.

Compare your answer with the thermal energy density of the gas. Would you expect the magnetic field to play a significant role in the structure of the Galaxy? Why or why not?

3 Stellar Velocities

Determine the proper motion relative to the LSR for a star in a circular orbit about the Galactic center, at a distance $d = 5$ kpc from the Sun and at Galactic longitude $l = 45°$. Hint: the Galaxy's rotation curve is in the textbook, and in the lecture notes on the kinematics of the Milky Way.

4 Counting Stars

Even when distances to individual stars are not known, much can be learned simply by counting stars as a function of limiting flux. Suppose that, in our region of the Galaxy, the number density of stars with a particular luminosity L, $n(L)$, is independent of position.

a) Show that the number of such stars observed to have a flux greater than some flux F_0 obeys $N(F > F_0) \propto F_0^{-3/2}$. Hint: Consider the distance at which a star has some flux F_0, and then figure out how the number of stars with $F > F_0$ depends on this distance and hence on F_0.

b) Explain why the same behavior will occur even if the stars have a distribution of luminosities, as long as that distribution is the same everywhere. If you observed that the numbers do not grow with decreasing F_0 according to this relation, what could be the reason?

Parts c) and d) of this problem are required only for students enrolled in Physics 903. Please submit original code by email along with any plots required. A sample solution will be given in Python, and the use of Python is preferred. The use of programs such as Mathematica is not allowed. This problem, and the numerical problems on later problem sets, will count as extra credit for students in Astronomy 401.

Here you will repeat parts a) and b), numerically instead of analytically.

c) Write a program that creates a population of fake stars with random distances but uniform number density. First assume that all your stars have identical luminosity, and use the distance to each star to calculate its flux. You can use arbitrary values for distance and luminosity and ignore all numerical factors, since we are only interested in the scalings. Then figure out the number of stars above some flux F_0, i.e. $N(F > F_0)$ as in part a) above. Make a plot of $\log[N(F > F_0)]$ vs. $\log(F_0)$, and fit a line to your plot to show that your best fit slope is approximately -1.5.

d) Next, allow your stars to vary randomly in luminosity, assuming (incorrectly) that the distribution of luminosities is uniform between $L_{\rm min}$ and $L_{\rm max}$ (where $L_{\rm min}$ and $L_{\rm max}$ can again be chosen arbitrarily). Repeat the above, and show that your best fit slope is still approximately -1.5. Some hints for part d):

Hint #1: You will find that the stars at your maximum distance $d_{\rm max}$ behave strangely, since at this point in your distribution the fluxes will depend only on the varying luminosity. To eliminate these stars from your distribution, use only stars with fluxes $F > 2\langle L\rangle/d_{\rm max}^2$ (this excludes the stars at $d_{\rm max}$, since for a uniform distribution, $2\langle L\rangle = L_{\rm max} + L_{\rm min}$).

Hint #2: For a better measurement of the slope you may want to exclude the low N end of your $\log[N(F > F_0)]$ distribution when fitting your line, since it may not be well-sampled.