Astronomy 401/Physics 903
Lecture 3
Kinematics of the Milky Way

1 Coordinate systems

1.1 Galactic coordinates

We usually assign coordinates to astronomical objects using an Earth-based coordinate system (projection of latitude and longitude onto the sky), which is useful for knowing when and from where it's possible to observe an object—but if we want to know where an object is located in the Galaxy, this doesn't help at all. For that we define another coordinate system based on the Galaxy.

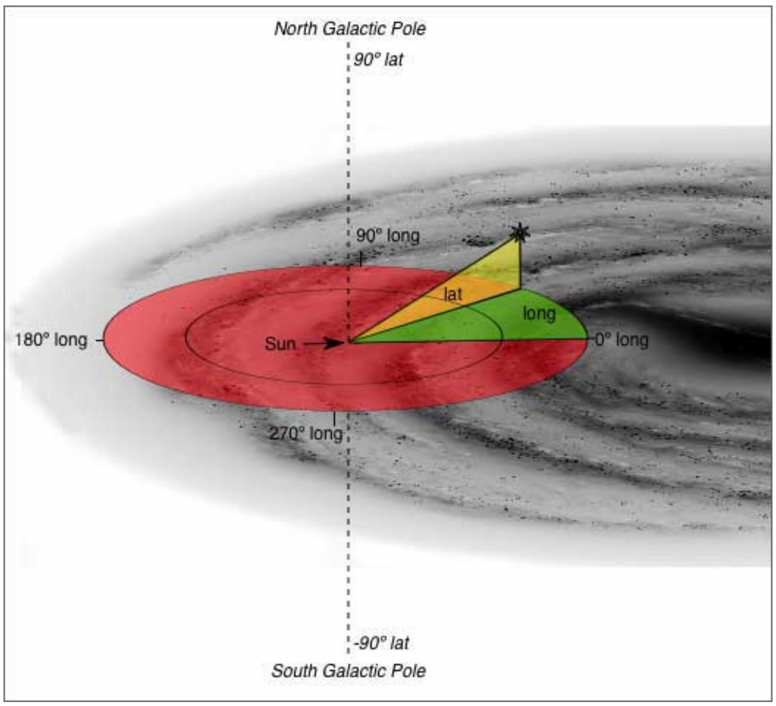

Figure 1: The Galactic coordinate system

The Sun is at the origin. Galactic latitude (b) and longitude (ℓ) are measured in degrees: longitude in the Galactic plane, and latitude perpendicular to the plane.

1.2 Cylindrical coordinates for Galactic motions

This is useful for describing the locations of things in the galaxy, but not very useful for studying kinematics; it's centered on the Sun, which is moving around the center of the Galaxy. When we're studying Galactic motions, it makes more sense to define coordinates with the center of the Galaxy at the origin.

Radial coordinate R increases outward, angular coordinate θ increases in the direction of rotation of the Galaxy, and vertical coordinate z increases out of the plane to the north.

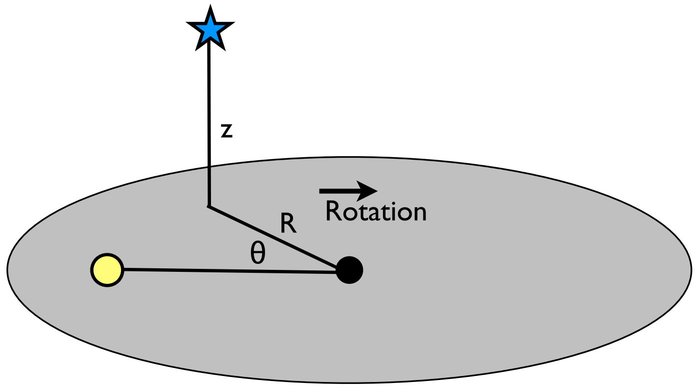

Figure 2: The cylindrical coordinate system used for Galactic motions

We also define corresponding velocity components:

$$U \equiv \frac{dR}{dt}, \quad V \equiv R\frac{d\theta}{dt}, \quad W \equiv \frac{dz}{dt}. \tag{1}$$

2 The local standard of rest and peculiar velocities

All measurements of velocities in the Galaxy are made from the Sun (actually the Earth, but the Earth-Sun velocity is small and we correct for it), so we first need to understand and correct for the Sun's motion. To do this we define the **local standard of rest (LSR)**. This is defined to be a point centered on the Sun and moving in a perfectly circular orbit around the Galactic center. The Sun isn't actually in such an orbit—it's moving slowly inward and upward out of the plane—and the Sun's velocity relative to the LSR is called the **solar motion**. A star's velocity relative to the LSR is called its **peculiar velocity**.

The Sun is orbiting the Galactic center at about 220 km s^{-1} ; this is the internationally agreed upon velocity of the LSR: $V_0(R_0) = 220$ km s^{-1} . By definition, $U_{\rm LSR} \equiv 0$, $V_{\rm LSR} \equiv V_0$, and $W_{\rm LSR} \equiv 0$.

Let's estimate the mass of the Milky Way inside the Sun's orbit. We'll use Kepler's third law

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3, \tag{2}$$

where a is semi-major axis, which is $R_0 = 8$ kpc for the Galaxy, and P is the orbital period:

$$P_{\rm LSR} = \frac{2\pi R_0}{V_0} = 230 \text{ Myr}. \tag{3}$$

So it takes about 230 Myr for the Sun to complete one orbit around the Galaxy. How old is the Sun, and how many times has it made this orbit? About 4.6 Gyr, so about 20 times. We will also assume our test particle (which could be the Sun) is much less massive than the mass enclosed, $m_1 \ll m_2 = M(< R_0)$, and that the mass is spherically distributed. We then have

$$M(< R_0) = \frac{4\pi^2 R_0^3}{G P_{\rm LSR}^2} = 8.8 \times 10^{10}\ \mathrm{M}_\odot. \tag{4}$$

3 Stellar velocities

In order to completely measure the velocity of a star, we need to make two measurements:

- the **radial velocity** v_r , the component of velocity along the line of sight—measure this by the red or blue Doppler shift in the spectrum of the star ($\Delta\lambda/\lambda = v_r/c$)
- the **transverse** or **tangential velocity** v_t, across the sky. We measure this by measuring the **proper motion** μ of the star—how fast it moves across the sky over time, in arcseconds per year—and convert that to a velocity if we know the distance to the star d:

$$\mu = \frac{v_t}{d}. \tag{5}$$

The star's total velocity is then $v^2 = v_r^2 + v_t^2$.

4 Differential Galactic rotation and Oort's constants

Now we'll look at rotational motion in other parts of the Galaxy. The basic principles of Galactic rotation were determined by Jan Oort in 1927. Different types of rotation:

- Rigid body rotation: rotation as a single solid body, like a wheel, with a constant angular speed and $V \propto R$, so stars on larger orbits would move faster.
- Keplerian rotation: all mass concentrated at the center (M = constant), $V \propto R^{-1/2}$ and stars farther from the center move more slowly.
- Constant orbital speed: V = constant, $M \propto R$. This is a reasonable approximation for most of the Galaxy. Note that this means that stars at different radii move with different angular velocities ($\Omega = V/R$); this is called **differential rotation**.

Now we analyze Galactic motions.

We consider a star (or other object) at a distance d from the Sun and a Galactic longitude ℓ—see Figure 1. The star has circular velocity $V(R)$ about the Galactic center. We measure the star's radial and transverse velocities. These are:

$$v_r = V\cos\alpha - V_0\sin\ell \tag{6}$$
$$v_t = V\sin\alpha - V_0\cos\ell, \tag{7}$$

where V_0 is the circular orbital velocity of the LSR (the Sun, in our diagram).

We then define angular velocity

$$\Omega(R) \equiv \frac{V(R)}{R} \tag{8}$$

(recall $V(R) \equiv R\ d\theta/dt$, so this is just the angular velocity $d\theta/dt$). Referring to Figure 1 and considering the right triangle defined by the Sun, the Galactic Center, and point T, we see that

$$R\cos\alpha = R_0\sin\ell \tag{9}$$
$$R\sin\alpha = R_0\cos\ell - d. \tag{10}$$

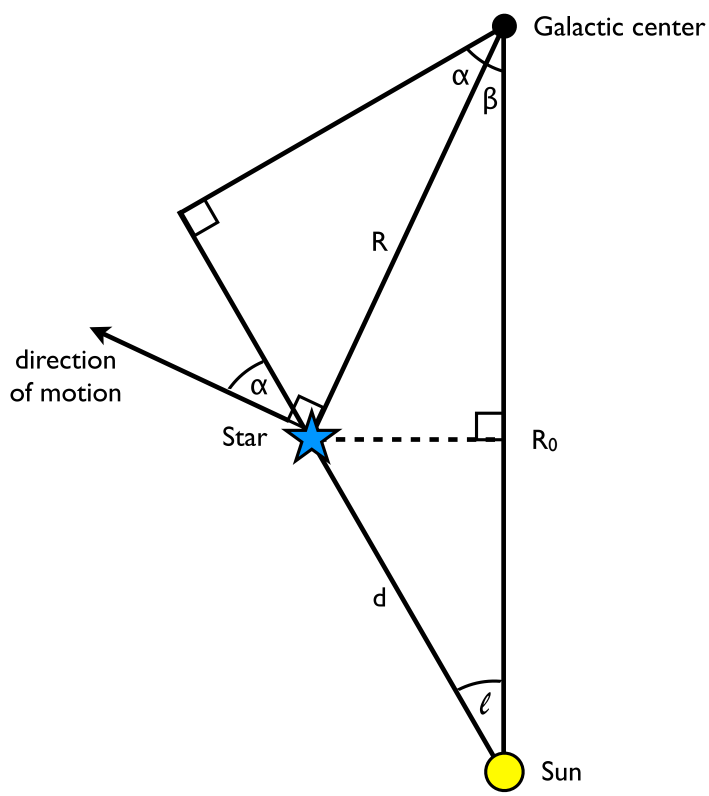

Figure 3: Geometry for analyzing differential rotation in the Galactic plane.

Substituting this into the expressions for radial and transverse velocity, we find

$$\begin{aligned} v_r &= (\Omega - \Omega_0) R_0 \sin \ell && (11) \\ v_t &= (\Omega - \Omega_0) R_0 \cos \ell - \Omega d. && (12) \end{aligned}$$

These are the **Oort equations**. If we can measure d as well as v_r and v_t, these equations allow us to determine the star's angular velocity $\Omega(R)$. The difficulty is that d is hard to measure reliably unless the star is close enough that we can measure the parallax. It's also hard to measure stars close to the Galactic center because of extinction. We can therefore simplify this further for stars near the sun ($d \ll R_0$, which also implies $R \approx R_0$).

Assume $\Omega(R)$ is a smoothly varying (decreasing) function of R, and then we can linearly approximate it near R_0 (this is a Taylor expansion about R_0, where we use only the first order term):

$$\Omega(R) = \Omega_0(R_0) + \left.\frac{d\Omega}{dR}\right|_{R_0} (R - R_0) + ... \tag{13}$$

So to first order the difference between Ω and Ω_0 is

$$\Omega - \Omega_0 \simeq \left.\frac{d\Omega}{dR}\right|_{R_0} (R - R_0) \tag{14}$$

and the value of Ω is approximately

$$\Omega \simeq \Omega_0 \tag{15}$$

since $R \simeq R_0$.

Now we substitute $\Omega = V/R$ into the Oort equations and rearrange:

$$\begin{aligned} v_r &\simeq \left[\left.\frac{dV}{dR}\right|_{R_0} - \frac{V_0}{R_0} \right] (R - R_0) \sin \ell && (16) \\ v_t &\simeq \left[\left.\frac{dV}{dR}\right|_{R_0} - \frac{V_0}{R_0} \right] (R - R_0) \cos \ell - \Omega d. && (17) \end{aligned}$$

We also see from Figure 1 that

$$\begin{aligned} R_0 &= d \cos \ell + R \cos \beta && (18) \\ &\simeq d \cos \ell + R, && (19) \end{aligned}$$

where the latter approximation comes from the small angle approximation $\cos \beta \approx 1$, since $d \ll R$ implies that $\beta \ll 1$ radian. We then define the **Oort constants**:

$$\begin{aligned} A &\equiv -\frac{1}{2} \left[\left.\frac{dV}{dR}\right|_{R_0} - \frac{V_0}{R_0} \right] && (20) \\ B &\equiv -\frac{1}{2} \left[\left.\frac{dV}{dR}\right|_{R_0} + \frac{V_0}{R_0} \right]. && (21) \end{aligned}$$

Using the appropriate trigonometric identities, we find that the radial and tangential velocities are

$$v_r \simeq Ad \sin 2\ell \tag{22}$$
$$v_t \simeq Ad \cos 2\ell + Bd. \tag{23}$$

What does this tell us? These equations describe the radial and tangential velocities of stars **near the sun** as a function of Galactic longitude. A is a measurement of shear, the degree to which the disk of the Galaxy doesn't rotate like a rigid body; for rigid body rotation, angular velocity doesn't depend on R and A would be zero. B is a measurement of vorticity of the disk material, its tendency to circulate around a given point. Note that the Oort constants are an approximation which describe the motions of stars near the Sun (within ~ 1 kpc); for stars that are farther away, we need to use the full velocity expressions (Equations 10 and 11).

The angular velocity at the Sun is given by

$$\Omega_0 = A - B \tag{24}$$

and the local gradient of differential rotation—the slope of the rotation curve—is given by

$$\left.\frac{dV}{dR}\right|_{R_0} = -(A + B). \tag{25}$$

Best values are $A = 14.8 \pm 0.8$ km s^{-1} kpc^{-1}, $B = -12.4 \pm 0.6$ km s^{-1} kpc^{-1}.

5 Measuring Galactic rotation

5.1 21-cm H emission

It's hard to see stars to large distance (especially toward the Galactic center) because of dust extinction, so a better way to map the structure of the Milky Way is by looking at gas. We use radio telescopes to look at 21-cm emission from neutral hydrogen. This emission line is produced by the spin flip of the electron in an atom of neutral hydrogen: the spins of the electron and the proton can be aligned in the same direction or in opposite directions. A hydrogen atom that has the spins of the electron and proton aligned in the same direction (parallel) has slightly more energy than one where the spins of the electron and proton are in opposite directions (anti-parallel).

The lowest orbital energy state of atomic hydrogen has hyperfine splitting arising from the spins of the proton and electron changing from a parallel to antiparallel configuration. When this transition occurs, the atom emits a photon with wavelength 21.10611405413 cm, or frequency 1420.40575177 MHz. This transition is highly forbidden with an extremely small probability of 2.9×10^{-15} s^{-1}, which means that the time for a single isolated atom of neutral hydrogen to undergo this transition is around 10 million (10^7) years and so is unlikely to be seen in a laboratory on Earth. However, because the total number of atoms of neutral hydrogen in the interstellar medium is very large, this emission line is easily observed by radio telescopes.

We measure Galactic rotation by using Doppler shifts in the wavelength of the 21 cm line to measure the radial velocities of gas clouds along a line of sight.

The usefulness of this technique depends on where the gas clouds are located. For clouds with $\ell < 90$ and $\ell > 270$ ($\cos \ell > 0$, or looking "inwards" toward the Galactic center), the radial velocity will reach a

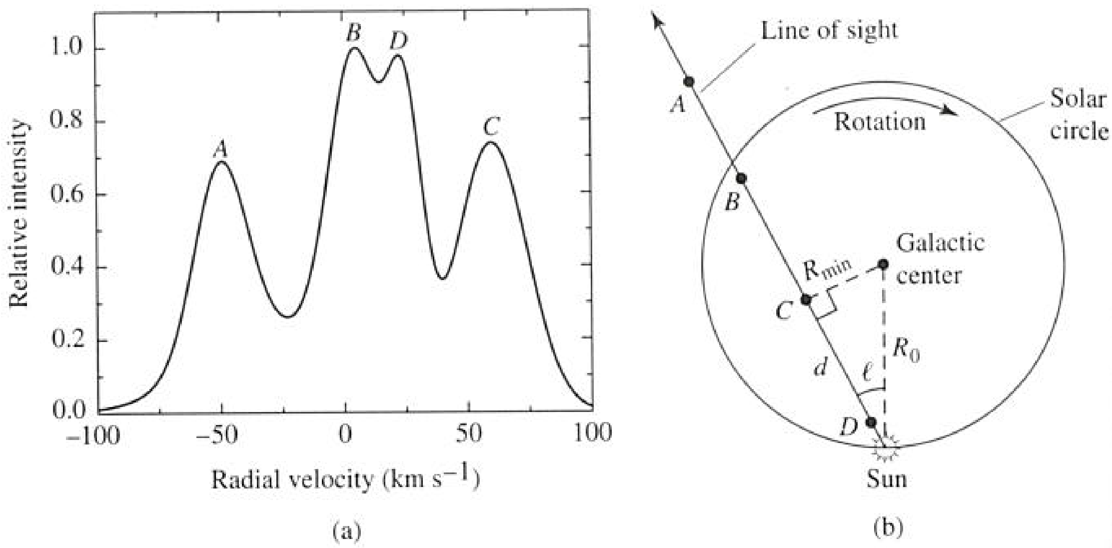

Figure 4: An example of a 21 cm line profile (left), and the locations of gas clouds A, B, C and D along the line of sight (right).

maximum at the point along the line of sight closest to the Galactic center. The distance to the cloud is then

$$d = R_0 \cos \ell \tag{26}$$

and the cloud is at a distance $R_{\rm min}$ from the Galactic center, where

$$R_{\rm min} = R_0 \sin \ell. \tag{27}$$

The line of sight is tangent to the orbit at this point, so the velocity is purely radial and (if the cloud is actually at this point and on a circular orbit) we are measuring the actual rotational velocity of the cloud at this point. This is called the tangent point, and this is the **tangent point method** of determining Galactic rotation.

We measure lots of lines of sight, and assume that the cloud with the highest velocity on each line of sight is at the tangent point (cloud C, in the example in Figure 2). For clouds with lower velocities, there is some ambiguity since we don't know the distance to the cloud; we can try to estimate it from the angular size, but not all clouds are the same size. Also harder for $90 < l < 270$, since the line of sight is never tangent to the orbit and there is no unique orbit with a maximum radial velocity. We really do need to know the distance to the cloud then, via some more reliable distance indicator.

5.2 The rotation curve and dark matter

A plot of rotational velocity vs radius is called a rotation curve. Measurements of velocities of objects at $R > R_0$ (Cepheid variables for which we can measure distances; more on that later) show that the rotational speed $V(R)$ doesn't significantly decrease with distance beyond R_0. This was a big surprise: if most of the mass were concentrated in the center, as appears to be the case from the distribution of stars and gas, stars would follow Keplerian orbits and velocity would decrease with increasing radius as $V \propto R^{-1/2}$. Instead the rotation curve is flat to the edge of the measurements, $V(R) \sim$ constant. This implies a mass distribution $M(R) \propto R$, and that there is significant mass beyond R_0—the dark matter halo.

Other spiral galaxies also have rotation curves like this. We'll talk about what this implies about the distribution of mass in the outer parts of galaxies when we talk about spiral galaxies later.

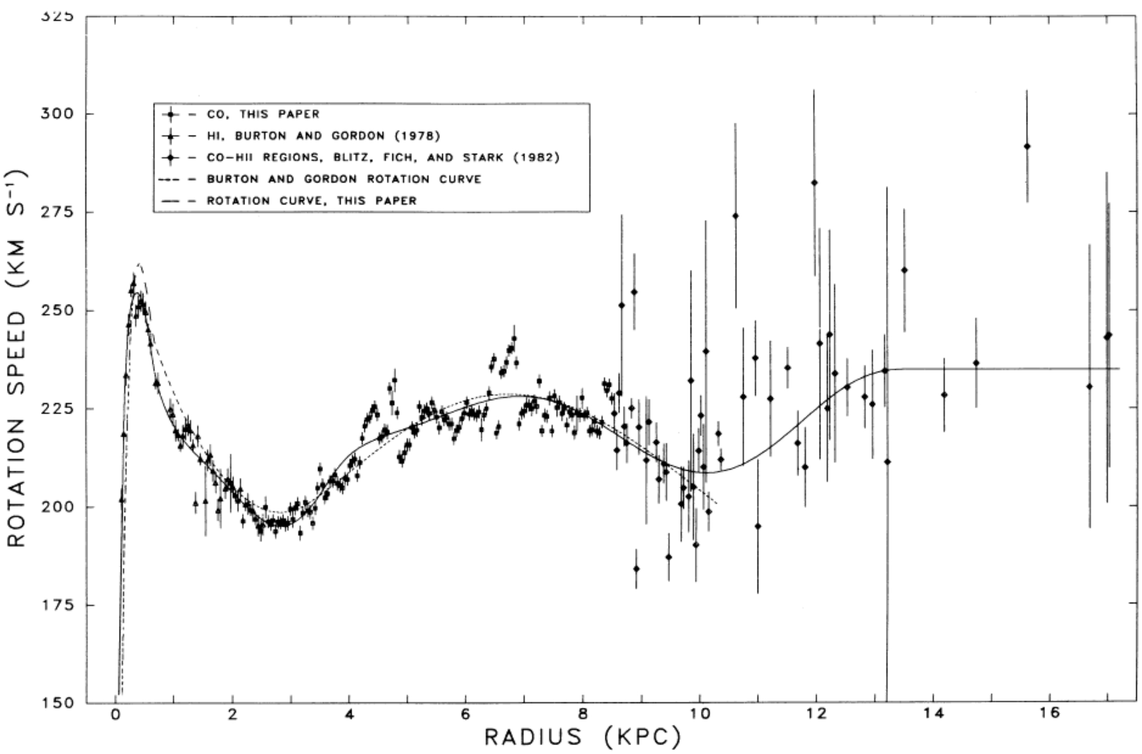

Figure 5: Rotation curve of the Milky Way. Note that the measurements are much more uncertain beyond the Solar radius, since independent distance measurements are required to measure the velocity in this case.

Note also that near the center of the galaxy, the rotation speed rises rapidly with radius. This is consistent with rigid body rotation, and implies that that the mass is roughly spherically distributed and the density is nearly constant.