Astronomy 401/Physics 903
Problem Set 3
Due in class, **Thursday February 21, 2019**

1 Stellar Populations of Spiral Galaxies

Because the light from galaxies is emitted by stars, the typical colors of galaxies can be compared to different types of stars. This provides information about the star formation history of the galaxy.

a) Table 3.2 of the text gives typical $B-V$ colors for spiral galaxies of different Hubble types. The $B-V$ color of an object is its apparent magnitude in the B (blue) band minus its apparent magnitude in the V (visual, actually green) band; a star with a smaller value of $B-V$ is bluer than a star with a larger value. Use the color information for main sequence stars given in the attached table ("Appendix G") to determine the approximate spectral type of the integrated light from each galaxy type.

b) What is the stellar mass-to-light ratio in units of $\mathrm{M}_\odot/\mathrm{L}_\odot$ of a star of each of these spectral types, as it would be measured in the B-band? To calculate this, you'll need to correct the total luminosities given in Appendix G into observed B-band luminosities using the bolometric corrections (BC) and $B-V$ colors also given in Appendix G. The *bolometric correction* is the difference between a star's bolometric (total) magnitude and its absolute visual magnitude M_V,

$$BC = M_{\mathrm{bol}} - M_V. \quad (1)$$

Some values of the stellar mass are missing in the table in Appendix G; interpolate between nearby values if you need to.

c) The stellar mass-to-light ratios you calculated in (b) differ from the dynamically determined mass-to-light ratios (M/L_B) of the galaxies given in Table 3.2 for two reasons. What are they?

d) Recall that the *main sequence* is the stage of stellar evolution during which stars spend the vast majority of their lives. The main sequence lifetime of a star is (very) approximately

$$\tau_{\mathrm{MS}} = 10^{10}\ \mathrm{yr} \times (M/L_{\mathrm{bol}}). \quad (2)$$

Stars older than τ_{MS} will have evolved off the main sequence. What is the approximate main sequence lifetime corresponding to the mean spectral types derived in part a)? What does this imply about the history of star formation in galaxies as a function of Hubble type?

2 Masses of Galaxies

a) Figure 1 below shows the rotation curve of the center of the galaxy M87, which is at a distance $d = 16$ Mpc. Just outside 0.1 arcsec from the center, the rotation curve is approximately Keplerian. Estimate the mass inside this region.

b) Figure 2 shows the rotation curve of the spiral galaxy NGC 3198. Use the rotation curve to estimate the mass within a radius of 30 kpc.

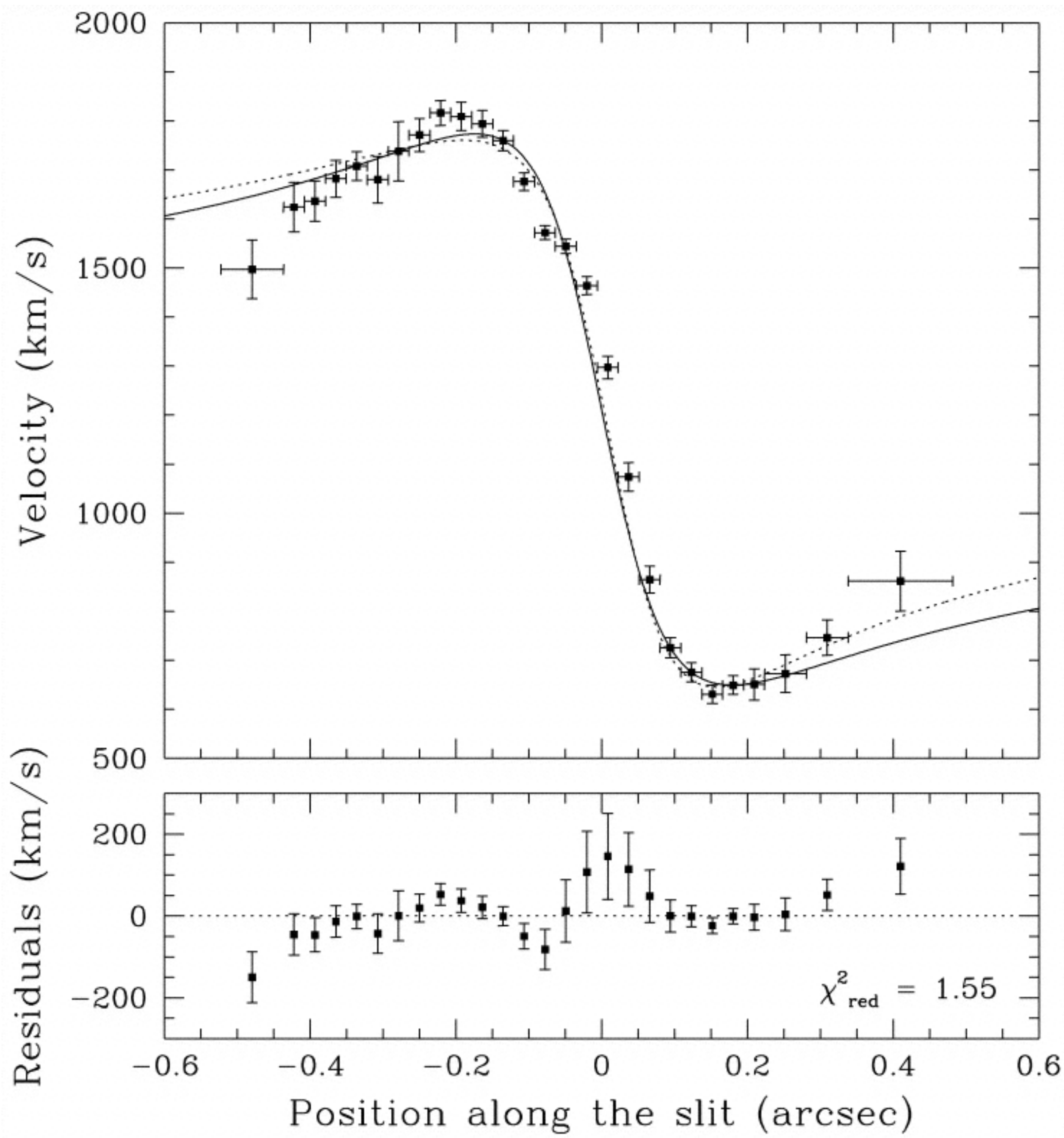

Figure 1: The rotation curve of the central portion of the galaxy M87. Part (b) shows the residuals to the best fit in part (a).

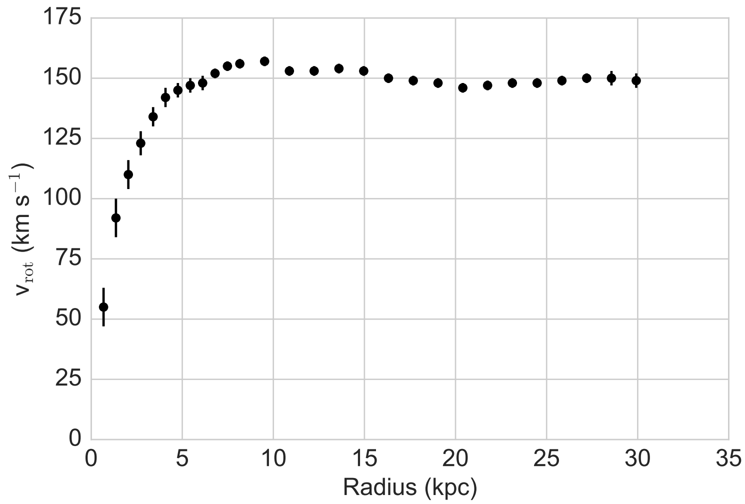

Figure 2: The rotation curve of the spiral galaxy NGC 3198.

c) Figure 3 shows the velocity dispersion σ measured at large radii in the halo of the Milky Way. Estimate the virial mass of the Milky Way within 80 kpc.

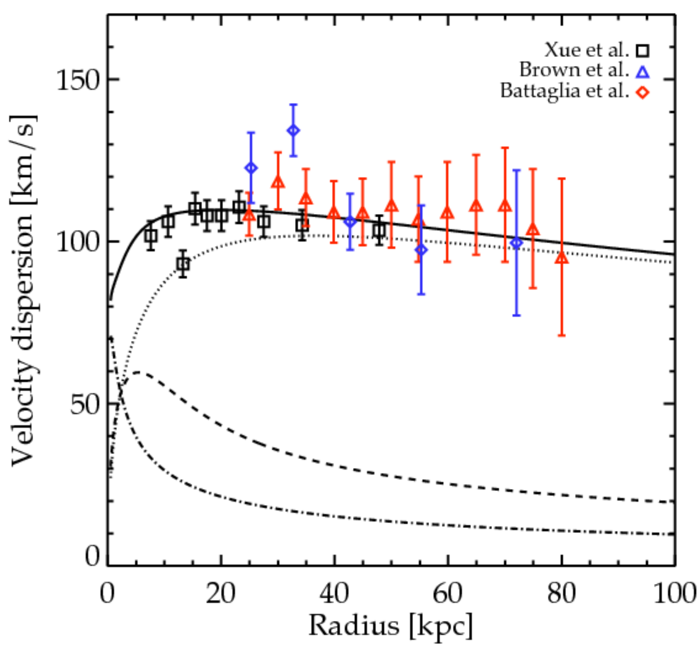

Figure 3: The velocity dispersion as a function of radius in the halo of the Milky Way.

3 Luminosity functions

The Schechter luminosity function in magnitudes is given by

$$\Phi(M) = (0.4 \ln 10)\, \Phi^* \, 10^{0.4(\alpha+1)(M^*-M)} \exp\left[-10^{0.4(M^*-M)}\right] \tag{3}$$

(see Equation 3.54 of the text; 3.40 in the first edition).

For the local field of galaxies near the Milky Way, $\alpha = -1.0$ and $M_B^* = -21$, while for galaxies in the Virgo Cluster $\alpha = -1.24 \pm 0.02$ and $M_B^* = -21 \pm 0.7$.

Make a plot of $\log \Phi(M)$, the logarithm of the Schechter luminosity function, for both the local field of galaxies near the Milky Way and the Virgo cluster over the range $-23 < M_B < -12$, using the values of α and M^* given above (you do not need to use the uncertainties in α and M_B^* for the Virgo Cluster). Shift your data so that $\log \Phi(-23) = 0$ for both groups of galaxies, and note that this renormalization means that you can ignore the numerical factors at the beginning of Equation 3 and that you do not need to know the value of Φ^*. You can do this either by explicitly taking the logarithm of $\Phi(M)$ and plotting it, or by simply using a logarithmic y axis for your plot.

What does your plot tell you about the relative numbers of bright and faint galaxies in the local field and the Virgo Cluster? Which parameter of the luminosity function tells you the answer to this question before you have made any plots?

4 Elliptical Galaxy

This problem is required only for students enrolled in Physics 903.

For this problem you will examine a hypothetical elliptical galaxy at a distance of 3 Mpc from the Milky Way. Suppose that you have measured the positions and taken spectra of 500 individual stars in this elliptical galaxy. Go to the course website and download the file `elliptical_stars.dat`; this file contains your measurements. The first column in the file is an ID number for the star; the second is the wavelength you have measured for the Hα absorption line in that star in Å; and the third and fourth are the Δx and Δy positions of the star on the sky, in arcminutes relative to the center of the galaxy. Pay careful attention to your units throughout this problem.

Your first step is to turn these data into the radial velocity of each star and its position in kpc. Assume that any relative velocity between the galaxy and the Milky Way has already been corrected for, so that for the purposes of your data we are at rest with respect to the center of mass of the galaxy. The rest wavelength of Hα is 6562.85 Å, and the wavelengths in your data file are Doppler shifted because of the radial velocities of the stars. Use this information to calculate the radial velocity r_v for each star in km s^{-1}.

Next use the distance to the galaxy to calculate the Δx and Δy positions of each star in kpc, relative to the center of the galaxy. Assume that the size of the galaxy r_e is the radius which contains 50% of the stars – what is this radius in kpc?

Make a plot of Δx vs. Δy in kpc, and somehow indicate the radial velocity of each star on your plot. You can do this by assigning a different color to different velocities, either in a smooth range or by binning the velocities into groups, or by plotting with different symbols or symbol sizes instead of colors. Does there appear to be any relationship between the position of a star and its radial velocity? Also add a circle to your plot at the radius r_e.

Plot a histogram of the radial velocities. What is the mean radial velocity of the stars in the galaxy? What is the radial velocity dispersion σ_r? Use the radius r_e and the appropriate velocity measurement to calculate the virial mass of the galaxy in $\mathrm{M}_{\odot}$.

APPENDIX G

Stellar Data

Main-Sequence Stars (Luminosity Class V)

Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	$M_{\rm bol}$	BC	M_V	$U-B$	$B-V$
O5	42000	499000	13.4	60	−9.51	−4.40	−5.1	−1.19	−0.33
O6	39500	324000	12.2	37	−9.04	−3.93	−5.1	−1.17	−0.33
O7	37500	216000	11.0	—	−8.60	−3.68	−4.9	−1.15	−0.32
O8	35800	147000	10.0	23	−8.18	−3.54	−4.6	−1.14	−0.32
B0	30000	32500	6.7	17.5	−6.54	−3.16	−3.4	−1.08	−0.30
B1	25400	9950	5.2	—	−5.26	−2.70	−2.6	−0.95	−0.26
B2	20900	2920	4.1	—	−3.92	−2.35	−1.6	−0.84	−0.24
B3	18800	1580	3.8	7.6	−3.26	−1.94	−1.3	−0.71	−0.20
B5	15200	480	3.2	5.9	−1.96	−1.46	−0.5	−0.58	−0.17
B6	13700	272	2.9	—	−1.35	−1.21	−0.1	−0.50	−0.15
B7	12500	160	2.7	—	−0.77	−1.02	+0.3	−0.43	−0.13
B8	11400	96.7	2.5	3.8	−0.22	−0.80	+0.6	−0.34	−0.11
B9	10500	60.7	2.3	—	+0.28	−0.51	+0.8	−0.20	−0.07
A0	9800	39.4	2.2	2.9	+0.75	−0.30	+1.1	−0.02	−0.02
A1	9400	30.3	2.1	—	+1.04	−0.23	+1.3	+0.02	+0.01
A2	9020	23.6	2.0	—	+1.31	−0.20	+1.5	+0.05	+0.05
A5	8190	12.3	1.8	2.0	+2.02	−0.15	+2.2	+0.10	+0.15
A8	7600	7.13	1.5	—	+2.61	−0.10	+2.7	+0.09	+0.25
F0	7300	5.21	1.4	1.6	+2.95	−0.09	+3.0	+0.03	+0.30
F2	7050	3.89	1.3	—	+3.27	−0.11	+3.4	+0.00	+0.35
F5	6650	2.56	1.2	1.4	+3.72	−0.14	+3.9	−0.02	+0.44
F8	6250	1.68	1.1	—	+4.18	−0.16	+4.3	+0.02	+0.52

Main-Sequence Stars (Luminosity Class V)

Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	M_{bol}	BC	M_V	$U-B$	$B-V$
G0	5940	1.25	1.06	1.05	+4.50	−0.18	+4.7	+0.06	+0.58
G2	5790	1.07	1.03	—	+4.66	−0.20	+4.9	+0.12	+0.63
Sun[a]	5777	1.00	1.00	1.00	+4.74	−0.08	+4.82	+0.195	+0.650
G8	5310	0.656	0.96	—	+5.20	−0.40	+5.6	+0.30	+0.74
K0	5150	0.552	0.93	0.79	+5.39	−0.31	+5.7	+0.45	+0.81
K1	4990	0.461	0.91	—	+5.58	−0.37	+6.0	+0.54	+0.86
K3	4690	0.318	0.86	—	+5.98	−0.50	+6.5	+0.80	+0.96
K4	4540	0.263	0.83	—	+6.19	−0.55	+6.7	—	+1.05
K5	4410	0.216	0.80	0.67	+6.40	−0.72	+7.1	+0.98	+1.15
K7	4150	0.145	0.74	—	+6.84	−1.01	+7.8	+1.21	+1.33
M0	3840	0.077	0.63	0.51	+7.52	−1.38	+8.9	+1.22	+1.40
M1	3660	0.050	0.56	—	+7.99	−1.62	+9.6	+1.21	+1.46
M2	3520	0.032	0.48	0.40	+8.47	−1.89	+10.4	+1.18	+1.49
M3	3400	0.020	0.41	—	+8.97	−2.15	+11.1	+1.16	+1.51
M4	3290	0.013	0.35	—	+9.49	−2.38	+11.9	+1.15	+1.54
M5	3170	0.0076	0.29	0.21	+10.1	−2.73	+12.8	+1.24	+1.64
M6	3030	0.0044	0.24	—	+10.6	−3.21	+13.8	+1.32	+1.73
M7	2860	0.0025	0.20	—	+11.3	−3.46	+14.7	+1.40	+1.80

[a] Values adopted in this text.