Astronomy 401/Physics 903
Lecture 1
Overview of the Universe

1 Class overview

Prerequisites and requirements, topics covered—see syllabus.

2 Units and scales

Galaxies and cosmology deal with sizes and masses that are both very large and very small.

Distance units: AU, distance from Earth to Sun, 1.5×10^{11} m. Not so useful beyond the solar system, so we use parsec (pc), the distance at which 1 AU subtends an angle of 1 arcsec: 1 pc = 3.1×10^{16} m = 3.26 light years. We are 1.3 pc from Proxima Centauri (nearest star) and 8000 pc = 8 kpc from center of Galaxy. For intergalactic distances, we use megaparsec (Mpc): 1 Mpc = 3.1×10^{22} m. We are 0.7 Mpc from M31 (Andromeda galaxy) and 15 Mpc from the Virgo Cluster (nearest big cluster of galaxies). The size of the observable Universe is $\sim$ 10 Gpc (gigaparsec, 10^9 pc), very roughly.

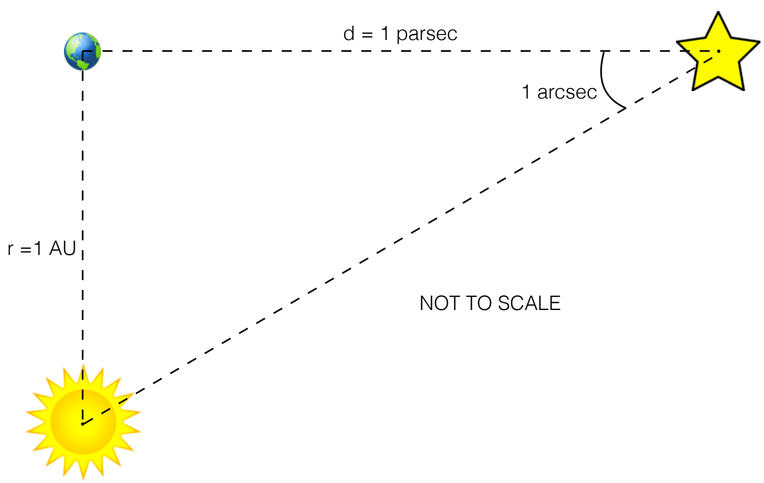

Figure 1: A parsec is the distance at which 1 AU subtends an angle of 1 arcsec. We use the small angle approximation $\tan\theta = r/d \simeq \theta$ where θ is in radians. 1 arcsec = 1/3600 deg = 4.8×10^{-6} radians, so $d \simeq r/\theta = 1$ AU /$4.8 \times 10^{-6} = 206{,}625$ AU = 1 pc.

Mass: standard unit of mass is solar mass, $M_\odot$: 1 $M_\odot$= 2.0×10^{30} kg. Mass of Milky Way $\approx 10^{12}$ $M_\odot$. Sun also provides standard unit of power or luminosity: 1 $L_\odot$= 3.8×10^{26} W. Total luminosity of Milky Way, $L_{\rm gal} = 3.6 \times 10^{10}$ $L_\odot$. (Approximately how many stars are in the galaxy?)

Time: 1 year = 3.2×10^7 s. In a cosmological context we use Gyr, 1 Gyr = 10^9 yr = 3.2×10^{16} s. The Universe is 13.7 Gyr old.

Angle and solid angle: we are often interested in the angle (1d) or solid angle (2d) that an object subtends on the sky. Relevant angles are typically given in arcseconds (galaxy sizes); see definition of parsec above.

A small object nearby may subtend the same solid angle as a larger object farther away. For example, although the Moon is much smaller than the Sun, it is also much closer to Earth. From any point on Earth, both objects have approximately the same solid angle and apparent size. This is evident during a solar eclipse.

Solid angle is the two-dimensional angle in three-dimensional space that an object subtends at a point. It is a measure of how large the object appears to an observer looking from that point. Unit is *steradian* (dimensionless, symbol sr). The full sky is 4π sr – this is the solid angle of a sphere measured from any point in its interior. Solid angles can also be measured in square degrees: 1 sr = $(180/\pi)^2$ square degrees.

Note: Most astronomers use CGS (centimeter-gram-second) units, instead of the MKS (meter-kilogram-second) units you are used to from physics classes. We will mostly use MKS units in this class, but some CGS quantities may appear. Useful conversion factors:

Energy: 1 J $= 10^7$ erg
Power: 1 W $= 10^7$ erg s^{-1}
Force: 1 N $= 10^5$ dyne
Mass density: 1 kg m^{-3} $= 10^{-3}$ g cm^{-3}
Number density: 1 m^{-3} $= 10^{-6}$ cm^{-3}

3 Olbers' Paradox: Why is the sky dark at night?

We'll begin with a simple question that turns out to involve most of the fundamental observations we make about the universe.

Olber's Paradox: named after Heinrich Olbers, who wrote a paper on the subject in 1826, but first proposed by Thomas Digges in 1576. Reference: Barbara Ryden, *Introduction to Cosmology*, section 2.1.

Let's suppose the universe is infinite and static, as Isaac Newton believed; a universe that isn't infinite will collapse inward due to its own self-gravity. Now let's compute how bright we expect the night sky to be in an infinite universe.

Let $n_\star$ be the average number density of stars in the universe – this is $n_\star \sim 10^9$ Mpc^{-3}, averaged over large scales.

Let $R_\star$ be the typical radius of a star. We'll assume the Sun is typical: $R_\star \sim R_\odot = 7.0 \times 10^8$ m $=$ 2.3×10^{-14} Mpc.

Suppose we're looking out through the universe in some direction. Draw a cylinder of radius $R_\star$ around line of sight – if a star's center lies within that cylinder, the star will block our view of more distant objects. See Figure 2.

Length of cylinder is λ, volume is $V = \lambda\pi R_\star^2$, and average number of stars that have centers inside the cylinder is

$$N = n_\star V = n_\star \lambda \pi R_\star^2. \tag{1}$$

It only requires one star to block the view, so the typical distance we can see before a star blocks the line of sight is the distance λ for which $N = 1$. This is

$$\lambda = \frac{1}{n_\star \pi R_\star^2}. \tag{2}$$

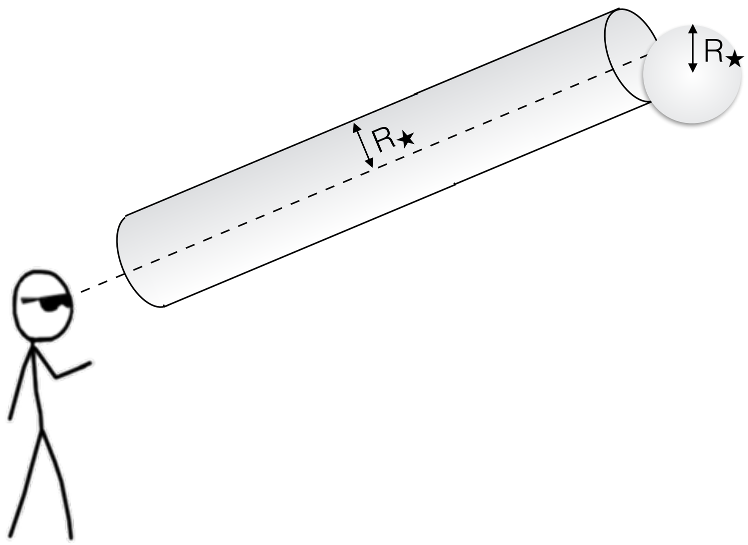

Figure 2: A line of sight through the universe eventually encounters an opaque star.

Using the above numbers $n_\star \sim 10^9$ Mpc^{-3} and $R_\star \sim R_\odot = 7.0 \times 10^8$ m $= 2.3 \times 10^{-14}$ Mpc, the distance we can see until our line of sight intercepts a star is

$$\lambda = \frac{1}{(10^9 \text{ Mpc}^{-3})(10^{-27} \text{ Mpc}^2)} \sim 10^{18} \text{ Mpc}. \tag{3}$$

This is a very large distance, but it is finite.

This means that in an infinitely large universe (or a universe that extends for at least 10^{18} Mpc in all directions), the sky will be completely covered with stars.

How bright is the sky in this universe? A star of radius $R_\star$ is at a distance $r \gg R_\star$, and its angular area in steradians is

$$\Omega = \frac{\pi R_\star^2}{4\pi r^2} = \frac{R_\star^2}{4r^2}. \tag{4}$$

If the star's luminosity is $L_\star$, its flux measured at a distance r is

$$f = \frac{L_\star}{4\pi r^2}. \tag{5}$$

The surface brightness (luminosity per unit area) of the star, in watts per square meter per steradian, will be

$$\Sigma_\star = \frac{f}{\Omega} = \frac{L_\star}{4\pi R_\star^2}. \tag{6}$$

Note that this does not depend on the distance to the star (we will discuss this again later this semester, when we talk about galaxies).

This means that the surface brightness of a sky covered with stars will be equal to the surface brightness of an individual star: in an infinite universe, the night sky should be as bright as the disk of the Sun.

Let's put more numbers in. The surface brightness of the Sun is $\Sigma_\odot \approx 5 \times 10^{-3}$ watts m^{-2} arcsec^{-2}; the surface brightness of the night sky is $\Sigma \sim 5 \times 10^{-17}$ watts m^{-2} arcsec^{-2}. Our estimate of the surface brightness of the night sky is off by a factor of 10^{14}, or 100 trillion.

Why is this wrong? Discuss.

- Interstellar matter that absorbs starlight? No, because the matter would be heated by starlight until it has the same temperature as the surface of a star, and then it would emit as much light as it absorbs and glow as brightly as the stars.
- Assumed that number density and mean luminosity of stars are constant throughout the universe; distant stars might be less numerous or less luminous than nearby stars.
- Assumed that universe is infinitely large. If universe extends to a maximum distance $r_{\rm max} \ll \lambda$, then only a fraction $F \sim r_{\rm max}/\lambda$ of the sky will be covered with stars. Note that this result will also be found if the universe is infinitely large, but has no stars beyond a distance $r_{\rm max}$.
- Assumed that the universe is infinitely old. When we see stars farther away, we're also seeing stars farther back in time. If universe has finite age $t_0 \ll \lambda/c$, then we can't yet see stars at a distance greater than $r \sim ct_0$, and only a fraction $F \sim ct_0/\lambda$ of the night sky will be covered with stars. This results also applies if the universe is infinitely old, but stars have only existed for time t_0.
- Assumed that the surface brightness of a star is independent of distance; this might not be true. We will see later than in an expanding universe, the surface brightness of distant objects is smaller than what it would be in a static universe.
- Primary resolution: universe has a finite age, and the light from stars beyond some distance—called the horizon distance—hasn't had time to reach us yet. First person to suggest this was Edgar Allen Poe in 1848: "Were the succession of stars endless, then the background of the sky would present us an [sic] uniform density ...since there could be absolutely no point, in all that background, at which would not exist a star. The only mode, therefore, in which, under such a state of affairs, we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all."

4 Basic observations

The cosmological principle:

- The Universe is homogenous. There are no preferred locations: the universe looks the same anywhere.
- The Universe is isotropic. There is no preferred direction: the universe looks the same in all directions.

This is only true on large scales; obviously not true on the scale of a person or a planet or even a galaxy or cluster of galaxies. On scales of ~ 100 Mpc, Universe is homogenous and isotropic; this is roughly the scale of superclusters of galaxies and the voids between them.

Also called the Copernican principle, after Copernicus, who determined that the Earth is not the center of the Universe. There is no center.

- Galaxies show a redshift proportional to their distance.

Consider light at a particular wavelength observed from a distant galaxy. $\lambda_{\rm obs}$ is the *observed* wavelength of some absorption or emission feature in the galaxy's spectrum. $\lambda_{\rm em}$ is the wavelength at which that feature is *emitted* by the galaxy. In general, $\lambda_{\rm obs} \neq \lambda_{\rm em}$: the galaxy has a **redshift** z given by

$$z \equiv \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}}. \tag{7}$$

Most galaxies have redshifts—this is because the Universe is expanding.

In 1929 Edwin Hubble plotted galaxy redshifts against their distance (redshifts are easy to measure, but distances are hard), and showed that the redshift of a galaxy was linearly proportional to its distance. This is now known as Hubble's law:

$$z = \frac{H_0}{c} r, \tag{8}$$

where H_0 is a constant now called the **Hubble constant**.

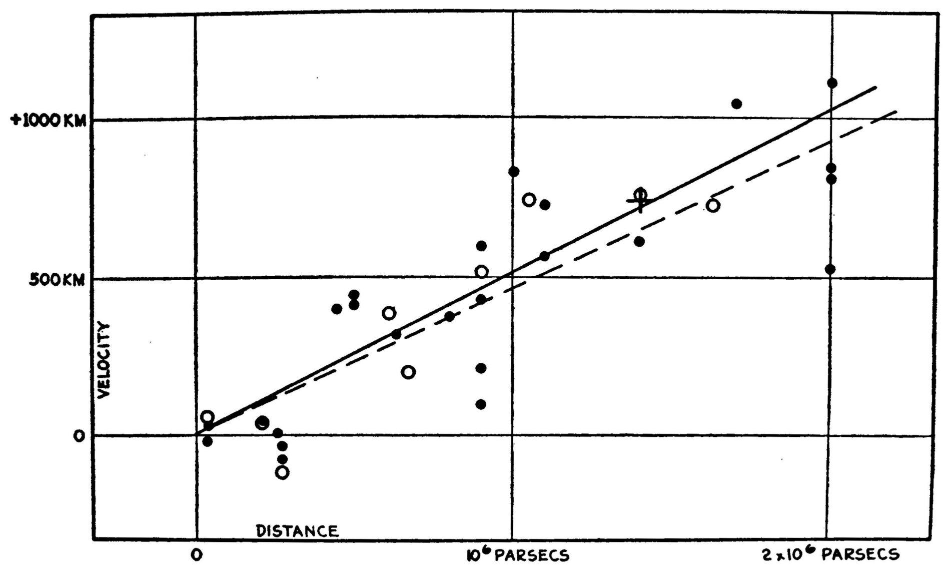

Figure 3: Edwin Hubble's original plot of the relation between redshift (vertical axis) and distance (horizontal axis). Note that the vertical axis actually plots cz rather than z and that the units are accidentally written as km rather than km/s. (from Hubble 1929, Proc. Nat. Acad. Sci., 15, 168, via Ryden)

If we interpret redshifts as Doppler shifts (not strictly true, but we'll talk about that later), $z = v/c$ and Hubble's law takes the form

$$v = H_0 r, \tag{9}$$

where v is the radial velocity of the galaxy. Therefore we can get the Hubble constant from dividing velocity by distance, and it has the units km s^{-1} Mpc^{-1} (note that the actual units are inverse time). Hubble's original estimate was $H_0 = 500$ km s^{-1} Mpc^{-1}, but he was severely underestimating the distances to galaxies. The Hubble constant has been measured with precision only in the last ~ 20 years; current value is $H_0 = 67.8 \pm 0.8$ km s^{-1} Mpc^{-1} (2015, from measurements of the cosmic microwave background from the Planck mission).

We can also use the Hubble constant to define a time. If galaxies are moving apart from each other, they must have been together at some point in the past. Consider two galaxies separated by a distance r and moving at a constant velocity v with respect to each other. The time elapsed since the galaxies were in contact is

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1}, \tag{10}$$

independent of r. The time H_0^{-1} is called the **Hubble time**, and is an approximate timescale for the age of the Universe (it is only equal to the age of the Universe if galaxy velocities were the same at all times in the past). $H_0^{-1} = 13.8$ Gyr.

Let's use this to return to Olbers' paradox. We saw above that in an infinite universe, we could see a distance of $\lambda \sim 10^{18}$ Mpc before our line of sight hits a star. In a young universe where light can travel a maximum distance $d \sim c/H_0 \sim 4000$ Mpc, the probability that we'll see a star along a randomly chosen line of sight is $P \sim d/\lambda \sim 4 \times 10^{-15}$: tiny! So instead of seeing a sky completely covered with stars, we see very few, and the average surface brightness is $\Sigma \sim P\Sigma\odot \sim 2 \times 10^{-17}$ watts m^{-2} arcsec^{-2}. This rough estimate comes surprisingly close to the observed surface brightness of the night sky.

For the night sky to be completely covered with stars, the universe would have to be over 100 trillion times older than it is, and you'd have to keep the stars shining during all that time.