Astronomy 401/Physics 903
Lecture 29
Observational Cosmology II: Cosmic Microwave Background Anisotropies

One of the most important methods of determining cosmological parameters is through measurements of the temperature fluctuations in the cosmic microwave background radiation. As we have discussed, the CMB has a nearly perfect blackbody spectrum with temperature $T = 2.7$ K. However, there are small variations in temperature at different places on the sky, with magnitude

$$\frac{\delta T}{T} \sim 10^{-5}. \tag{1}$$

These small variations tell us that the universe wasn't perfectly homogeneous at the time of last scattering ($z \sim 1100$). This is expected, since the universe is not smooth today; as soon as the CMB was discovered, it was expected that there would be anisotropies in temperature reflecting the anisotropies in the matter distribution. Based on the structures present in the universe today, it was expected that the amplitude of the CMB fluctuations would be $\delta T/T \sim 10^{-3}$. This expectation was based on our model of the growth of structure via gravitational collapse: in order to have density fluctuations (galaxies and clusters of galaxies) as large as we see today, fluctuations of order $\delta T/T \sim 10^{-3}$ are required at $z \sim 1100$. Such fluctuations were not seen, however, and when the temperature anisotropies finally were detected (by the COBE satellite in 1992), they were two orders of magnitude smaller than expected. As we will see, this is another strong piece of evidence for the existence of dark matter.

1 Origin of the temperature fluctuations

Fluctuations in the temperature of the CMB reflect the conditions in the universe at the time of last scattering, at $z \sim 1100$. Because the universe was ionized and the photons and electrons were coupled until this time, the variations in photon temperature tells us about variations in the matter distribution at the time: variations in temperature correspond to variations in the wavelengths of the photons as they respond to variations in the matter density.

If, at the time of decoupling, a photon was in a slightly denser region of space, it had to spend some of its energy against the gravitational attraction of the denser region in order to move away from it, thus becoming slightly colder than the average temperature of the photons. And vice versa, photons that were located in a slightly less dense portion of space lost less energy when leaving it than other photons, thus appearing slightly hotter than average.

The CMB anisotropies are divided into two categories: *primary anisotropies* originate at the time of recombination, and *secondary anisotropies* are imprinted on the CMB at a later time, as the photons propagate through the universe. Both types of anisotropies contain useful astrophysical information, but we will focus on the primary anisotropies today.

There are several mechanisms responsible for the primary anisotropies in the CMB. First and perhaps most importantly, we will discuss *acoustic oscillations*.

At the time of recombination, most of the mass density in the universe consisted of nonbaryonic, pressureless dark matter (as it does now as well). Mixed with the dark matter and sharing the same pattern of inhomogeneities was a relativistic gas of baryons and radiation: the *photon-baryon fluid*. This fluid is well-described by the equation of state for relativistic particles,

$$P = \frac{1}{3}\rho c^2. \tag{2}$$

From the equation of state, we can calculate the speed of sound in the photon-baryon fluid,

$$c_s = \sqrt{\frac{dP}{d\rho}} = \frac{c}{\sqrt{3}}. \tag{3}$$

The gravitational potential of the inhomogeneities in the mass density attracts the photon-baryon fluid, which becomes compressed in the denser regions and more rarefied in the underdense regions. However, the fluid has a pressure which opposes the compression: this pressure causes an expansion that stops only when the density has become lower than the equilibrium density and the gas in the originally overdense region has become underdense. This results in periodic expansion and contraction of the fluid: standing sound waves in the photon-baryon fluid, at all wavelengths. These are the **acoustic oscillations** which become imprinted on the CMB.

These oscillations happen on all spatial scales, but the only ones that can react to their environment are those on scales which are in causal contact: scales equal to or smaller than the particle horizon

$$d_h(t) = a(t) \int_0^t \frac{c\, dt'}{a(t')}. \tag{4}$$

As we saw previously, during the matter-dominated era (when recombination occurs) $a(t) \propto t^{2/3}$ and the horizon distance is $d_h(t) = 3ct$.

We are interested in regions which can develop acoustic oscillations, and the size of a region that can develop such an oscillation is determined by the time available for a sound wave to cross the region. This is called the sonic horizon or acoustic horizon, and because the sound speed is $c_s = c/\sqrt{3}$, the sonic horizon is related to the particle horizon by

$$d_s = d_h/\sqrt{3} = \sqrt{3}ct. \tag{5}$$

Using the time of decoupling $t_{\rm dec} \approx 3.8 \times 10^5$ yr (see Lecture 22),

$$d_s(t) = \sqrt{3}ct_{\rm dec} = 6.2 \times 10^{21} \text{ m} = 0.2 \text{ Mpc}. \tag{6}$$

This is the sound-crossing horizon at the time of recombination, and it is the largest scale at which an acoustic oscillation can be present in the photon-baryon fluid. This physical scale is a standard ruler: the angle subtended on the sky by the sound horizon at recombination can be predicted for different curvatures of spacetime, and measurement of that angle then directly reveals the geometry of the universe, as shown in Figure 1.

By measuring the angular size of the largest fluctuations, we can use the relationship between the physical and angular sizes (the angular diameter distance) to constrain the geometry. The dependence of the angular diameter distance on the cosmological parameters is shown in Figure 2, and Figure 3 shows the angular size corresponding to a proper size of 0.2 Mpc for universe models with different curvature. For a flat universe with our current best-fit cosmological parameters, 0.2 Mpc corresponds to one degree on the sky. For a closed universe with positive curvature, the angular size is larger, while an open universe with negative curvature produces a smaller angular size.

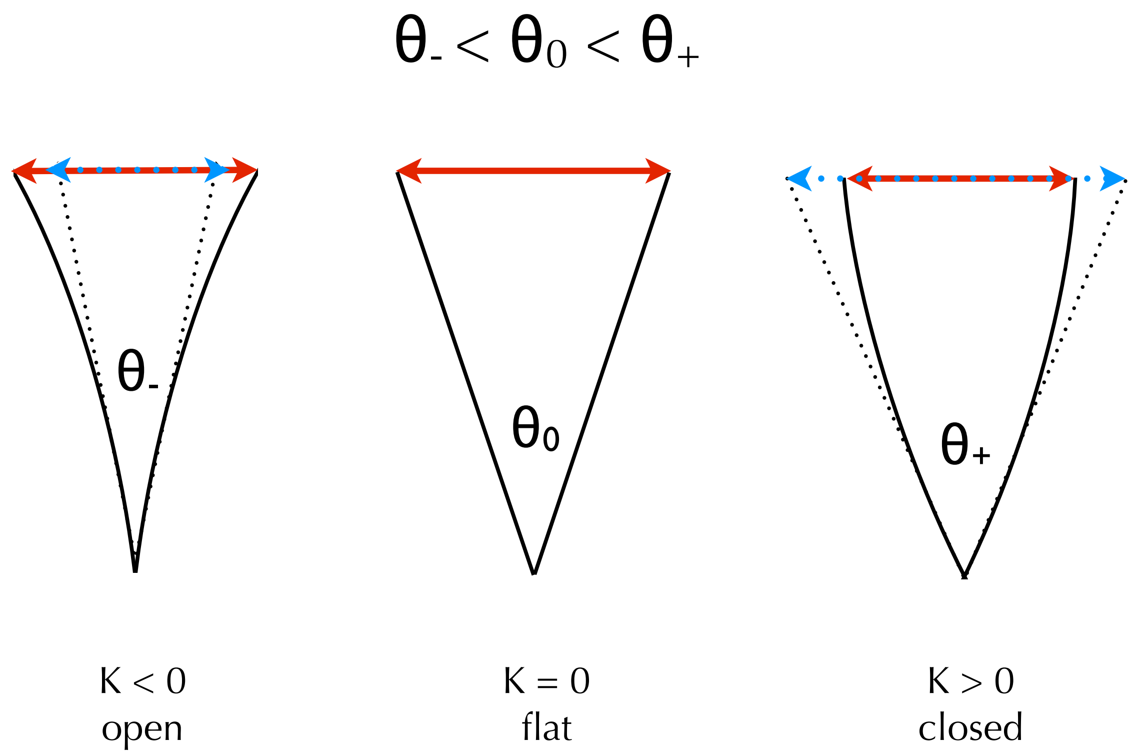

Figure 1: The angular size of the sound horizon at recombination depends on the curvature of the universe. The solid red arrow in each diagram shows the sound horizon at the time of recombination. The path light rays will take from either side of the sound horizon will bend depending on the curvature of the universe. In a flat universe, the angles of the triangle sum to 180 degrees, and the angle subtended is given by Euclidean geometry. In an open universe, the angles of the triangle sum to less than 180 degrees, and the angle the sound horizon subtends is smaller than the $K = 0$ case. If $K > 0$, the angles add to more than 180 degrees, and the angle is larger than in the flat case.

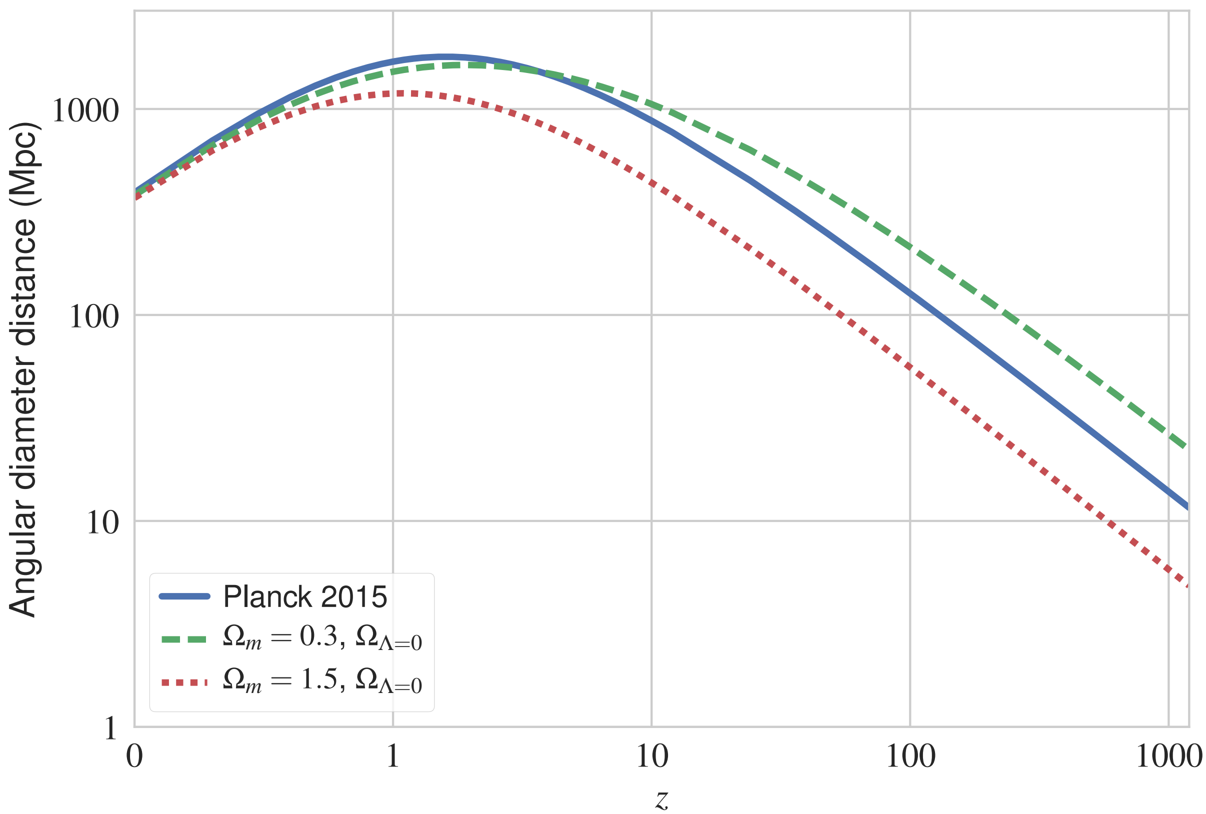

Figure 2: The angular diameter distance as a function of redshift, for three different cosmological models.

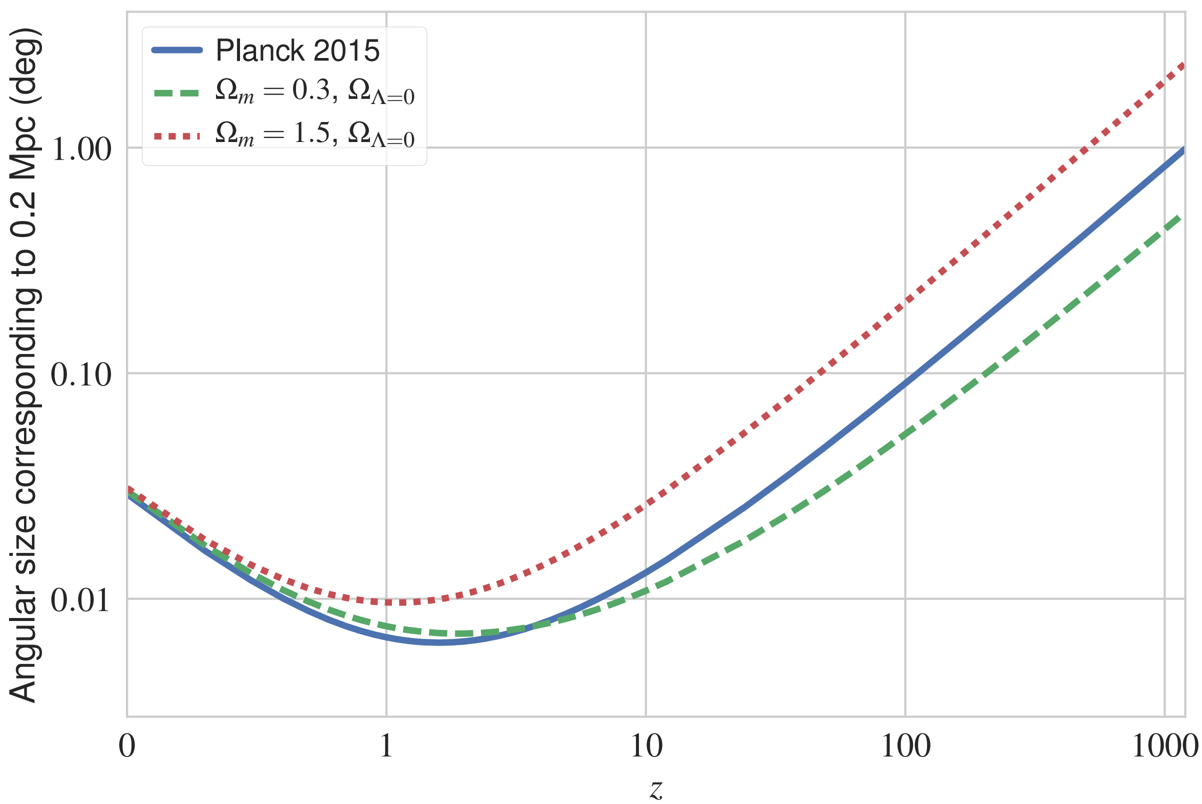

Figure 3: The sound horizon at recombination was 0.2 Mpc, and the corresponding angular scale on the sky depends on the cosmological model.

2 The power spectrum

How do we actually see the acoustic oscillations in the CMB?

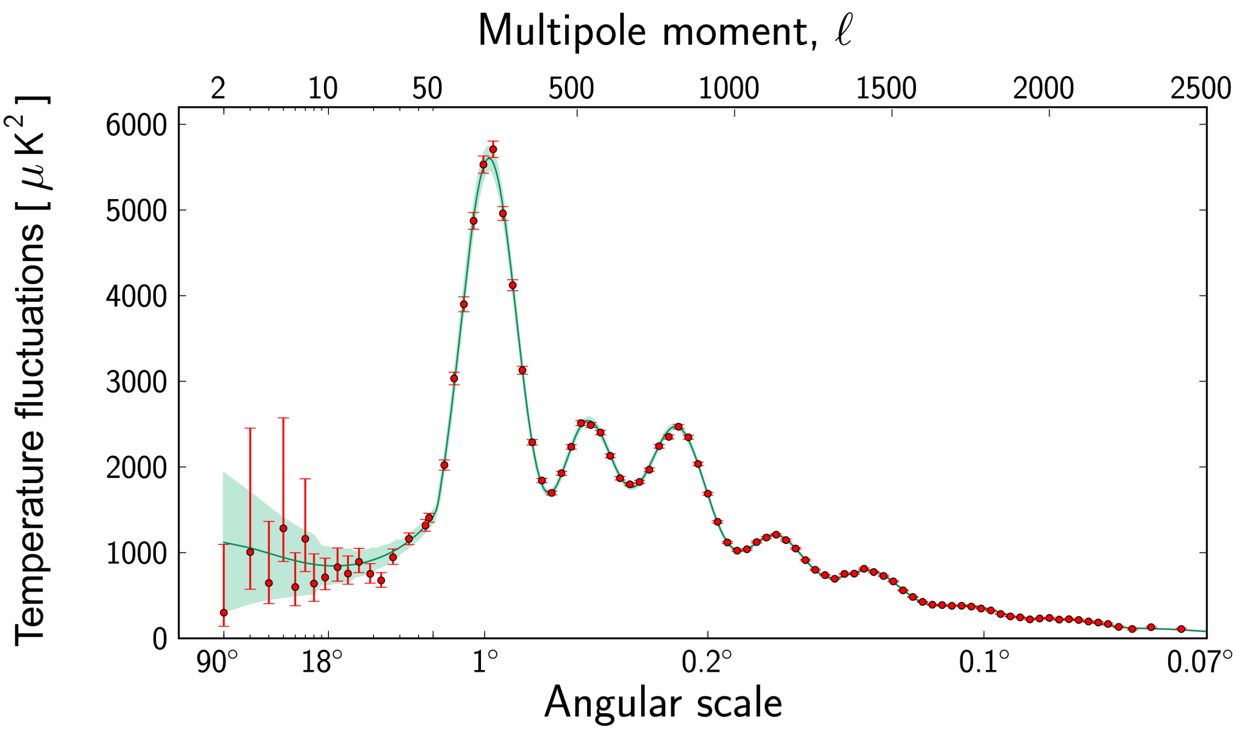

Figure 4: The CMB power spectrum shows the amplitude of temperature fluctuations as a function of angular size. Data points are from the Planck mission, and the green line shows the best-fit model.

The CMB anisotropies are presented in a plot such as Figure 4. This is the **power spectrum** of the cosmic microwave background: it is a measurement of how strong the fluctuations are at different angular scales. The mathematics of how this is calculated are briefly discussed in the text; the *multipole moment* ℓ on the top axis corresponds to the angular scale, plotted on the bottom axis, roughly as $\theta \sim \pi/\ell = 180°/\ell$.

The physical scale of the strongest peak in the power spectrum is the sound horizon at recombination, and as described above, the angular scale at which we observe this peak is directly related to the curvature of the universe. For an open universe with $\Omega < 1$, the angle subtended by the sound horizon would be smaller, and the peak of the power spectrum shifts to the right, to a higher multipole ℓ corresponding to a smaller angular scale. If $\Omega > 1$, the first peak would be observed at a larger angular scale. The position of the first peak has been extremely well-constrained by the WMAP and Planck observations of the CMB anisotropies; the Planck 2015 results give

$$\Omega = 1.000 \pm 0.009. \tag{7}$$

So the universe is extremely close to flat, and as we have already discussed, there are reasons to expect it to be exactly flat, since $\Omega = 1$ is an unstable condition. If the universe is flat it will always be flat, but if it deviates even slightly from $\Omega = 1$ the deviation will grow rapidly until Ω is very different from 1. So a value that is extremely close to 1 but not exactly 1 is unlikely.

We'll now consider fluctuations on other angular scales. The origin of fluctuations with $\theta > \theta_s$ is different from those with $\theta < \theta_s$, so we will consider them separately.

3 Sub-horizon scale fluctuations

We'll start by reviewing the production of acoustic oscillations and discussing them in more detail.

- Fluctuations on scales smaller than the sonic horizon $\theta \leq \theta_s$ come from regions small enough that sound waves have had time to cross them by the time of decoupling. This means that the fluctuations depend on the behavior of photons and baryons.

- Immediately before decoupling, photons, electrons and protons make a single photon-baryon fluid. Its energy density is about a third of the dark matter, so it moves primarily under the gravitational influence of the dark matter rather than under its own self-gravity.

- If the photon-baryon fluid is in a potential well, it will fall to the center. If the size of the well is larger than the sonic horizon, the fluid, traveling at the sound speed $c_s < c$, will not have time to fall to the center by the time of last scattering. This is why the motions of photons and baryons don't matter on scales $\theta > \theta_s$ and why the fluctuations on these large scales are determined only by the dark matter distribution, as we will see later.

- On scales smaller than the horizon, $\theta < \theta_s$, oscillations develop. The falling photon-baryon fluid is compressed by gravity, and its pressure rises until it's sufficient to make the fluid expand outward. Expansion continues until pressure drops enough for gravity to cause it to fall inward again. This results in standing waves in the fluid called acoustic oscillations.

- The temperature is higher than average in regions of compression, and lower than average in regions of rarefaction. We see the imprint of these oscillations in the temperature fluctuations of the CMB.

- The oscillations can be described as a simple harmonic oscillator in which the frequency of oscillations of size L is given by
$$\omega = \frac{c_s}{L}. \tag{8}$$
This shows that larger density fluctuations will oscillate more slowly, and that the frequency of oscillation depends on the sound speed, which depends on the baryon density and the equation of state.

- There is an additional important effect: the dark matter which makes up most of the mass density has no pressure, and does not interact with the photon fluid. This means that the dark matter does not participate in the oscillations. It can form clumps unopposed by pressure, and the gravitational force of these clumps acts as an additional forcing term on the harmonic oscillator. The effect is to shift the equilibrium position of the oscillations to favor compression over rarefaction.

- This is the behavior of a *forced harmonic oscillator*. The displacement of a forced harmonic oscillator for different frequencies is shown in Figure 5. The solid line is the unforced solution, with no added gravitational force: these are oscillations about the origin. The dashed curves are the forced solutions for two different frequencies. In both cases the zeropoint of the oscillations shifts in the direction of the force, and the effect is more dramatic for lower frequencies.

- The lower panel of Figure 5 shows the square of the oscillator position as a function of time. All three oscillators show a series of peaks at $\omega t = [(2n-1)/2]\pi$ corresponding to the maxima and minima of the oscillations in the upper panel. The odd modes are compression, and the even modes are rarefaction. In the case of the unforced oscillator, the heights of the peaks are identical, but in the forced case the heights of the odd peaks are greater than the heights of the even peaks, and the

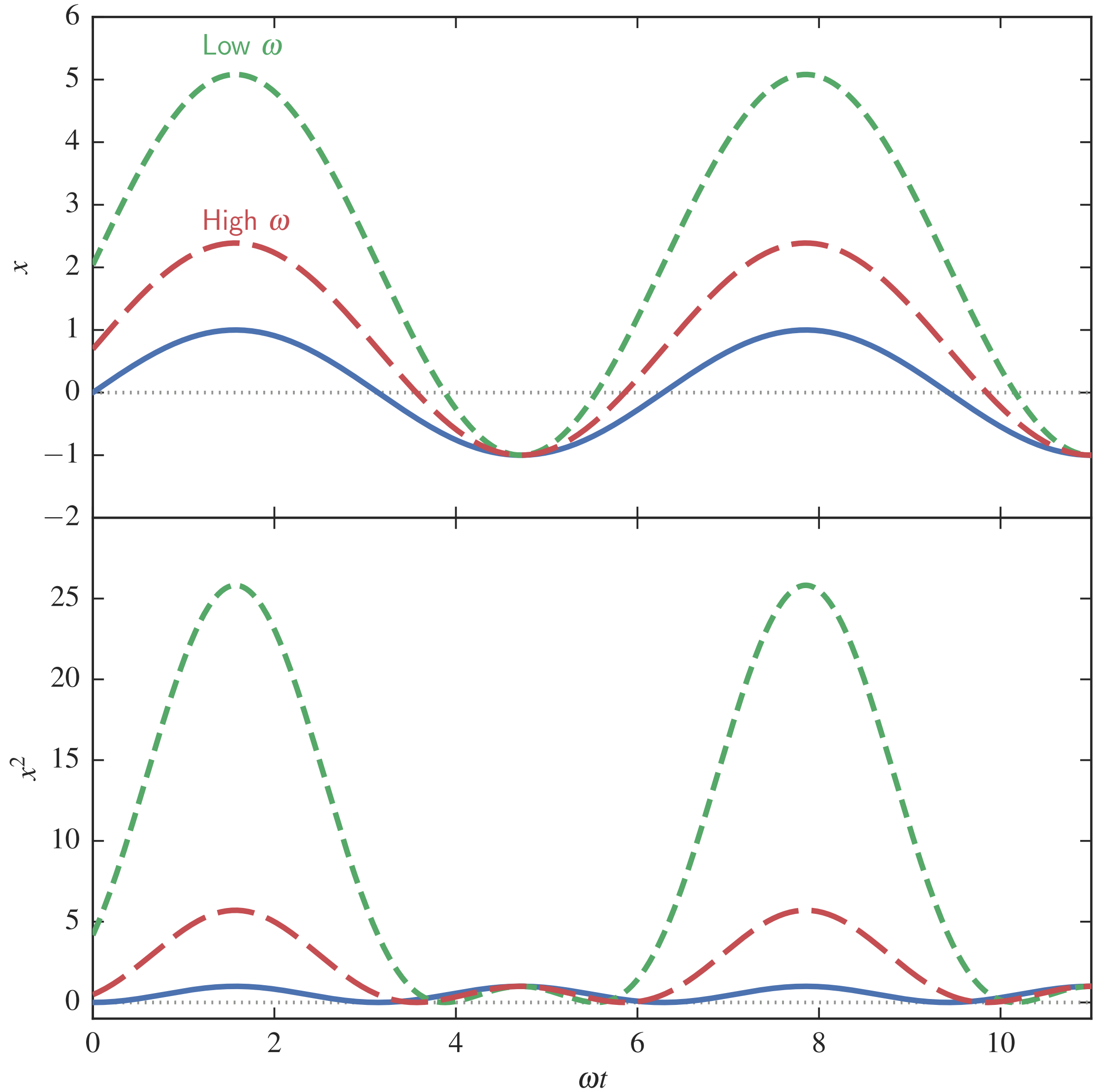

Figure 5: Displacement of a harmonic oscillator with varying frequencies ω. The solid blue line shows an unforced oscillator, with oscillations centered at zero. The dashed lines show the addition of a forcing term, which shifts the oscillations in the direction of the force. The forcing effect is stronger for lower frequencies. The lower panel shows the square of the displacement. Even and odd peaks are of equal strength for the unforced oscillator, but the forcing term enhances the strength of the odd peaks relative to the even peaks, with the effect being stronger for lower frequencies.

effect is most dramatic for low frequencies. This means that the compressions in the direction of the gravitational field are of greater magnitude than the rarefactions.

- This tells us that in the early universe, because of the gravitational effect of dark matter, collapsing fluctuations are favored over expanding ones. Because compressions are of greater magnitude than rarefactions, we expect the peaks of the angular power spectrum to be enhanced for odd harmonics (compressions) and diminished for even harmonics (rarefactions).

- The frequency depends on the baryon content of the universe, since baryons are heavy and reduce the sound speed. This means that both the position and relative heights of the additional peaks are very sensitive to the baryon density Ω_b and to the relative amounts of dark and baryonic matter. Careful modeling of the CMB power spectrum is complex, but matching of these models to observations gives us precise measurements of the total density Ω_0, the baryon density Ω_b, and the total matter density Ω_m.

- Now we consider fluctuations on different scales at the time of recombination. The smaller a fluctuation is, the sooner it is included by the sound horizon and can begin oscillating. The first peak of the power spectrum is due to the compression of a large region that reached maximum compression at the time of decoupling. The first trough is produced by a smaller region that started oscillating earlier and is able to oscillate more quickly, so it arrives at $\delta T = 0$ at the time of decoupling. The second peak is due to the oscillation of a still smaller region that has passed through its maximum compression and reached maximum rarefaction at the time of decoupling. Note that the magnitude of δT for the first peak is larger than that for the second peak, because of the biasing effect of dark matter discussed above.

- So, the first peak in the CMB power spectrum corresponds to maximum compression at the time of last scattering, and the size is equal to the sonic horizon at the time of last scattering. Higher l peaks will be from higher harmonics.

- The suppression of the second peak increases as Ω_b increases, since a greater baryon density slows the oscillations. This means that the relative heights of the first two peaks gives Ω_b. The third peak corresponds to a second maximum compression. This is sensitive to the density of dark matter, and the comparable heights of the second and third peaks tell us that most of the matter is dark.

- The effects of dark matter also explain why the temperature fluctuations in the CMB are of order $\delta T/T \sim 10^{-5}$, instead of $\delta T/T \sim 10^{-3}$ as originally expected. Because dark matter does not have pressure, it could begin to collapse at earlier times, when the collapse of baryons was still resisted by the pressure of the photons to which they were coupled; this effectively gives structure formation a head start. This means that, at recombination, the dark matter fluctuations were larger than those of the photon-baryon fluid, and a prediction of density fluctuations at $z \sim 1100$ based on the observed structure today tells us more about the amplitude of dark matter fluctuations at recombination than about baryons. So the amplitude of the fluctuations is strong evidence for the presence of dark matter.

- On very small angular scales (at high $\ell \gtrsim 1000$), the peaks in the power spectrum decrease. This is damping due to the distance a photon travels between scatterings. On small scales, photons can diffuse from hotter regions to colder ones, washing out the temperature fluctuations. The scale on which this occurs depends on the mean free path between scatterings

$$\lambda_{\rm mfp} = \frac{1}{n_e \sigma_T}. \tag{9}$$

Because n_e is proportional to the baryon density when the universe is ionized, models with smaller baryon density have larger mean free paths, the photons can diffuse farther, and the damping sets in at larger scales. So the damping at high values of l also tells us about the density of baryons. This is called **Silk damping**.

4 Large scale fluctuations

The large-scale fluctuations with angular size $\theta > \theta_s$ arise from the gravitational effect of density fluctuations in the distribution of nonbaryonic dark matter. Inhomogeneities in the matter distribution lead to inhomogeneities in the gravitational potential. As photons climb out of the potential well of regions with higher gravitational potential, they lose energy and are redshifted; this is the *gravitational redshift*. The result is that photons from regions of higher density have lower temperature. This is called the **Sachs-Wolfe effect**, and it is the dominant source of CMB fluctuations on large angular scales of above ~ 10 degrees.

5 Joint constraints on cosmological parameters

We saw previously that observations of distant Type 1a supernovae require a model of the universe with current acceleration and previous deceleration, i.e. a positive cosmological constant. The cosmic microwave background radiation is sensitive to nearly all of the cosmological parameters, with the strongest constraint coming from the position of the first peak indicating that the universe is flat. The supernovae and the CMB thus provide *orthogonal* constraints in the Ω_m–Ω_Λ plane, as shown in Figure 6. The SNe tell us that we need Λ, and the CMB tells us that the universe is flat. Both are consistent with measurements of the matter density from studies of galaxy clusters, which tell us that $\Omega_m \simeq 0.3$ (the number of massive galaxy clusters is sensitive to the matter density). This plot summarizes our model of the universe, which is often called the Concordance Model. It is one of a general category of models called **ΛCDM**: the primary components of the universe are the cosmological constant Λ and cold dark matter (CDM), where *cold* refers to the velocity of the dark matter particles and indicates that they move slowly relative to the speed of light.

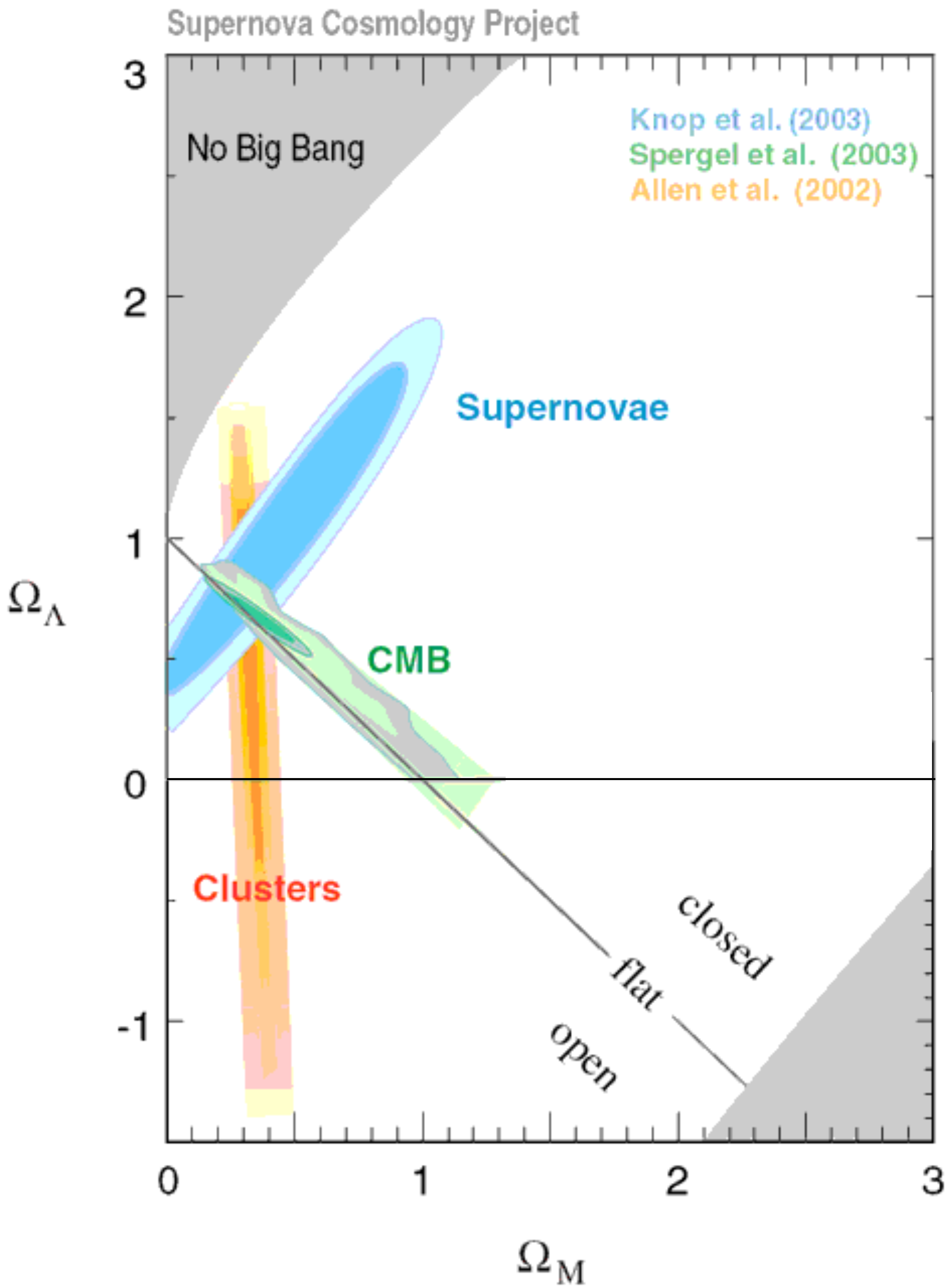

Figure 6: Confidence intervals on Ω_m and Ω_Λ from Type Ia SNe, the CMB and galaxy clusters.