Astronomy 401/Physics 903
Lecture 6
Spiral Galaxies

Goal: understand the 3-d structure of galaxies. How do we get to this from what we can observe?

1 Surface brightness profiles

Surface brightness profiles of spiral galaxy disks are generally well-described with an exponential decay:

$$\Sigma(r) = \Sigma_0 e^{-r/h}, \tag{1}$$

where Σ_0 is the central surface brightness and h is the scale length. When expressed in magnitudes rather than luminosity units this is linear. From the definition of magnitudes, $m_1 - m_2 = -2.5\log(F_1/F_2)$, the surface brightness in magnitudes is

$$\begin{aligned} \mu(r) &= C - 2.5\log\Sigma(r) & (2)\\ &= C - 2.5\log\Sigma_0 - 2.5\log e^{-r/h} & (3)\\ &= [C - 2.5\log\Sigma_0] + [2.5\log e]\times r/h & (4) \end{aligned}$$

where C is some constant that defines the zeropoint of our magnitude scale. We define $\mu_0 = C - 2.5\log\Sigma_0$; this is the central surface brightness in mag arcsec^{-2}. Then

$$\mu(r) = \mu_0 + 1.086\,\frac{r}{h}. \tag{5}$$

So we can make a linear plot of surface brightness as a function of radius; the central surface brightness is the intercept, and the slope measures the scale length.

Bulges of spiral galaxies (and elliptical galaxies) are often described by a different surface brightness profile:

$$\begin{aligned} \Sigma(r) &= \Sigma_e 10^{\{-3.33[(r/r_e)^{1/4}-1]\}} & (6)\\ &= \Sigma_e \exp\left(-7.67[(r/r_e)^{1/4} - 1]\right). & (7) \end{aligned}$$

The length scale r_e is the **effective radius** or **half-light radius**, and the constant 3.33 is chosen so that half of the total luminosity of the galaxy is emitted inside r_e: $L(r_e) = 1/2L_{\rm tot}$, and Σ_e (or μ_e in mag arcsec^{-2}) is the surface brightness at r_e. This is often called the $r^{1/4}$ law, or the **de Vaucouleurs profile**.

Written in magnitudes per square arcsec, this is

$$\mu(r) = \mu_e + 8.33\left[\left(\frac{r}{r_e}\right)^{1/4} - 1\right]. \tag{8}$$

Note that this implies that the central surface brightness (the surface brightness at $r = 0$) is more than 8 mag arcsec^{-2} brighter than the surface brightness at r_e; this is a factor of ≈ 2000, so this is very centrally concentrated.

A similar but more generalized profile is also often used, in which the $1/4$ exponent is replaced with the more general $1/n$. This is called a **Sérsic profile**:

$$\Sigma(r) = \Sigma_e \exp(-b_n[(r/r_e)^{1/n} - 1]) \tag{9}$$

and again the constant b_n is chosen so that half of the total light is emitted inside r_e. A good approximation is $b_n \approx 1.999n - 0.327$.

Expressed in magnitudes, this is

$$\mu(r) = \mu_e + 2.5 \log(e)\, b_n \left[\left(\frac{r}{r_e} \right)^{1/n} - 1 \right]. \tag{10}$$

The exponential disk profile is a special case with $n = 1$, when μ_0 and h are written in terms of μ_e and r_e respectively. Most galaxies are fit with profiles with n between 1/2 and 10, and the best-fit value of n correlates with galaxy size and luminosity, such that bigger and brighter galaxies tend to be fit with larger n.

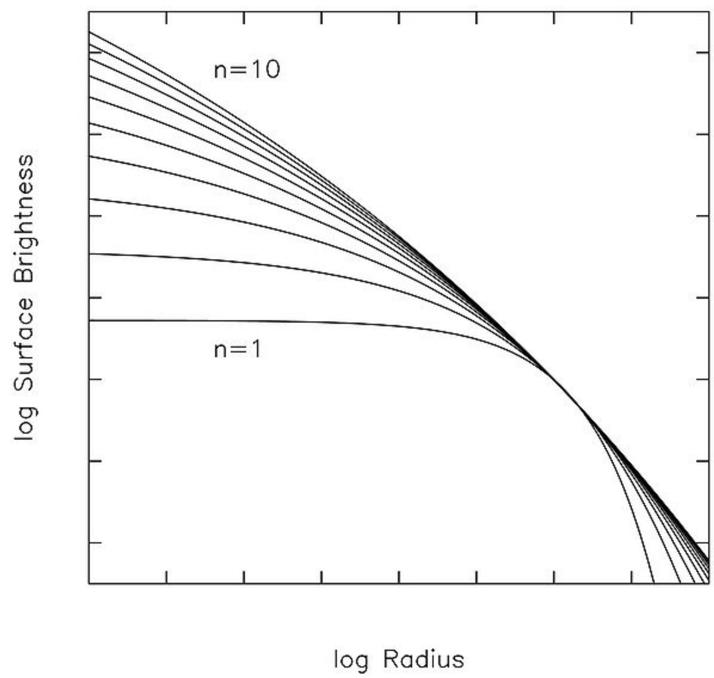

Figure 1: Surface brightness profiles with different Sersic indices.

2 Kinematics of spiral galaxies

Measuring the surface brightness profiles of galaxies tells us about the distribution of luminous matter, but not about the total distribution of mass. In order to measure this, we need to study the kinematics of galaxies.

We saw that the rotation curve of the Milky Way flattens at large radii, with a constant rotational velocity of ~ 220 km s^{-1} . In the 1970s and 80s Vera Rubin showed that spiral galaxies generally have flat rotation curves (surprise!), and that therefore there is a lot of mass beyond the luminous matter we can see.

What do flat rotation curves tell us about the mass distribution? Consider a particle of mass m, in a circular orbit at radius r around a spherically symmetric mass distribution M. The gravitational force on the particle is

$$F_{\rm grav} = \frac{GMm}{r^2} \tag{11}$$

and the centripetal force is

$$F_{\mathrm{c}} = \frac{mv_c^2}{r} \tag{12}$$

where v_c is the (constant with radius!) circular velocity of the particle. These forces are equal:

$$v_c^2 = \frac{GM}{r}. \tag{13}$$

We can use this to determine the dark matter density profile in the outer parts of the galaxy.

Solve for M and differentiate:

$$\frac{dM}{dr} = \frac{v_c^2}{G}. \tag{14}$$

We also know

$$\frac{dM}{dr} = 4\pi r^2 \rho(r). \tag{15}$$

Setting these equal,

$$\frac{v_c^2}{G} = 4\pi r^2 \rho(r) \tag{16}$$

and

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}. \tag{17}$$

So we've shown that the density of dark matter goes as r^{-2} at large radii! This is called an **isothermal sphere**—constant velocities inside sphere.

It's often suggested that the density profile of dark matter halos may be universal, i.e. the same over a wide mass range. This is still a matter of debate, but a commonly used form is the **NFW profile** (Navarro, Frenk and White, 1996):

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_0}{(r/a)(1+r/a)^2}. \tag{18}$$

This behaves like an r^{-2} profile over much of the halo, but is shallower ($\sim 1/r$) near the center and steeper ($\sim 1/r^3$) near the edge.

3 The Tully-Fisher relation

Vera Rubin's work also showed that the maximum rotational velocity of spiral galaxies depends on the galaxy type and luminosity: spirals of earlier type have larger $v_{\max}$, and more luminous galaxies have larger $v_{\max}$.

The correlation between the luminosity and maximum rotational velocity of spiral galaxies is known as the **Tully-Fisher relation** (1977). In luminosity terms, this is $L \propto v_{\max}^{\alpha}$, with $\alpha \simeq 4$; the details depend somewhat on the type of galaxy and the band. As usual in astronomy, we plot this logarithmically to make it a straight line. For example:

$$M_B = -10.2 \log v_{\max} + 2.71, \tag{19}$$

where v is in km s^{-1}. This particular form of the Tully-Fisher relation is a fit to observations of Sb galaxies in the B band — the numerical values depend on both the type of galaxy and the band in which we observe it. Recall that the absolute magnitude $M = M_{\odot} - 2.5\log(L/L_{\odot})$, so $L \propto v^4$ gives $\log L \propto 4 \log v$, and $M = -10 \log v +$ const.

There is less scatter in the Tully-Fisher relation at IR wavelengths: it is less affected by dust, and the light comes from stars that are a better tracer of the overall luminous mass distribution (the B-band light is mostly from young stars in regions of recent star formation).

Note also: this can be a distance indicator! If we can measure the maximum rotational velocity of a spiral galaxy and have reason to believe it should fall on the Tully-Fisher relation, we can estimate its absolute magnitude and thus its distance.

Can we understand where this relationship comes from?

We saw above that for flat rotation curves,

$$v_c^2 = \frac{GM}{r}. \tag{20}$$

We write the mass M in terms of the mass-to-light ratio[1], $M = (M/L) \times L$, and $L = \Sigma \pi r^2$, where Σ is the average surface brightness (luminosity per unit area) of the galaxy, so $r^2 = L/(\Sigma\pi)$. Then square both sides and substitute:

$$v_c^4 = \frac{G^2 M^2}{r^2} = G^2 (M/L)^2 L^2 \frac{\Sigma\pi}{L} = \left[G^2 \pi (M/L)^2 \Sigma \right] L, \tag{24}$$

or $L \propto v_c^4$. Notice that we've assumed that all galaxies have the same M/L and the same average surface brightness, neither of which is true—so the fact that this is actually observed is somewhat surprising. Another way to think about this: the radius in our original expression for velocity is the radius enclosing all the mass, while the radius we've used for the luminosity (surface brightness) is the radius enclosing all the light—these aren't the same. So the fact that this correlation is observed suggests that the total mass-to-light ratios don't actually vary all that much.

This also explains some of the variation with Hubble type and observed band, since M/L and surface brightness depend on these things.

4 Spiral structure

The spiral structure of spiral galaxies varies.

The classic spiral galaxy is called a **grand design spiral**, with two symmetric and well-defined arms. Others have more than two arms, or arms that appear fragmented. Galaxies without well-defined arms that can be traced over a large angular distance are called **flocculent spirals**.

[1] A simple way to get information about the types of stars responsible for the generation of light. Consider the thin disk of the Milky Way: $M \simeq 6.5 \times 10^{10}$ M$_\odot$ (stars and gas), and (blue) luminosity $L_B = 1.8 \times 10^{10}$ L$_\odot$. Divide these (always in solar units!) to get the mass-to-light ratio $M/L_B \approx 3$ M$_\odot$/L$_\odot$ (the units M$_\odot$/L$_\odot$ are usually implied).

Recall that a star's luminosity depends strongly on its mass:

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot} \right)^\alpha \tag{21}$$

where $\alpha \simeq 4$ for $M \gtrsim 0.5$ M$_\odot$and $\alpha \simeq 2.3$ for $M \lesssim 0.5$ M$_\odot$. Solve for M in terms of M/L and substitute our observed M/L to find an average mass

$$\langle M \rangle = (M/L)^{1/(1-\alpha)}\, \mathrm{M}_\odot \tag{22}$$
$$\simeq 0.7\, \mathrm{M}_\odot \tag{23}$$

for $\alpha = 4$ and $M/L = 3$. This indicates that the disk is dominated by stars slightly less massive than the sun. This makes sense, since low mass stars are the most common and M dwarf stars are the most common type of star observed near the Sun.

Grand design: $\sim 10\%$
Multiple arm: $\sim 60\%$
Flocculent: $\sim 30\%$

Optical images of spiral galaxies are dominated by the arms, especially in blue light, because massive, hot (O and B) stars are found in the spiral arms. These stars live for only ~ 10 Myr, so their presence indicates active star formation. The bulk of disk of the galaxy is dominated by older, redder stars. Spiral arms also have gas and dust.

4.1 Trailing and leading spiral arms

There are two possibilities for how the spiral arms can be oriented with respect to the rotation of the galaxy. The tips of **trailing arms** point in the opposite direction from the direction of rotation, and **leading arms** are the opposite. Intuitively, it looks as if the arms should be trailing, and this is the case in most galaxies in which it can be measured. A few galaxies have a combination of leading and trailing arms, however; this is probably a result of an encounter with another galaxy.

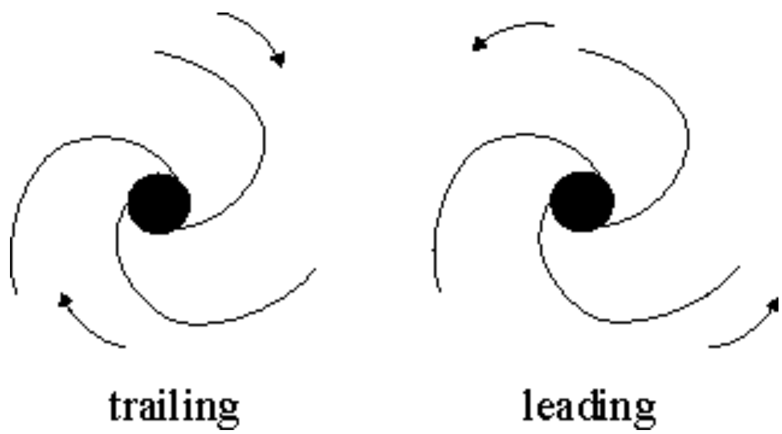

Figure 2: Trailing and leading spiral arms

4.2 The winding problem

An obvious suggestion is that spiral structure is due to galactic rotation. For galaxies with a flat rotation curve (constant rotational velocity V), the *angular velocity* $\Omega = V/R$ decreases with increasing radius, so stars farther from the center will take longer to complete an orbit. This will naturally generate spiral arms, as shown in Figure 3. The problem is that after a few orbits, the arms will be too tightly wound to be observed (recall that the Sun has orbited the center of the Milky Way ~ 20 times). This is called the **winding problem**. This also shows that the spiral arms can't be fixed components of the same stars and gas.

4.3 The origin of spiral structure: density waves

Leading theory for the origin of spiral structure is the **Lin-Shu density wave theory**. This says that spiral structure is due to long-lived **quasistatic density waves**. These are regions in the galactic disk where the mass density is $\sim$10 to 20% higher than average. Stars and gas clouds move through these regions of enhanced density as they orbit around the center of the galaxy. This is like cars moving through a traffic

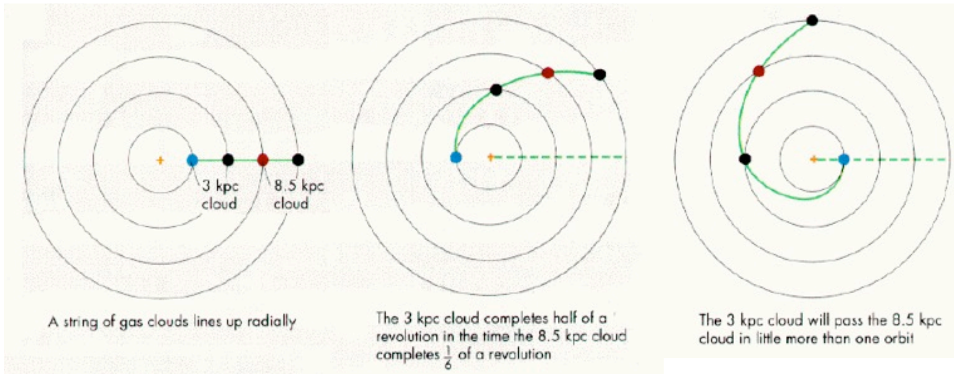

Figure 3: The winding problem for spiral arms

jam—the density increases in the traffic jam, and the cars slowly move through it, but the traffic jam itself doesn't move (much).

Define the **global pattern speed** Ω_{gp}: this is the angular speed of the spiral pattern. Viewed in a noninertial reference frame rotating with Ω_{gp}, the spiral pattern is stationary.

The stars aren't necessarily stationary, however. Stars near the center of the galaxy can have orbital speeds shorter than the density wave pattern ($\Omega > \Omega_{gp}$), so they will overtake a spiral arm, move through it, and continue on until they reach the next arm. Stars far from the center of the galaxy will be moving more slowly than the density wave pattern ($\Omega < \Omega_{gp}$), so they will be overtaken by the spiral arm. At some distance from the center the stars and the density wave will have the same angular speed. This is called the **corotation radius** (R_c). In this noninertial reference frame, stars with $R < R_c$ will appear to pass through the arms moving in one direction, and stars with $R > R_c$ will appear to move through in the opposite direction.

This theory explains observations:

- Star formation is concentrated in spiral arms: as gas in the galaxy passes through the density wave it's compressed, and this increase in density makes it more likely to collapse and form stars.
- Distribution of star formation: collapse of gas into stars takes some time, so star formation will be observed somewhat downstream from the leading edge of the spiral arm.
- O and B stars concentrated in spiral arms, red stars distributed throughout disk: The most massive O and B stars don't live very long, so they don't have time to pass entirely through the spiral arm and become generally distributed throughout the galaxy. Longer-lived, redder stars do.

Where does the density wave come from, and how is it maintained?

Very generally speaking....

- Stellar orbits about the center of the galaxy aren't perfectly circular (we saw before that the Sun has a peculiar velocity (the solar motion) with respect to the local standard of rest).
- We can describe these orbits as motion about an equilibrium position that is moving in a perfectly circular orbit.
- This is simple harmonic motion—oscillations superimposed on a perfectly circular orbit.

- The resulting motion of the star is a non-closing rosette pattern, when viewed in an inertial frame.
- In a noninertial frame (rotating with pattern speed Ω_{gp}), the orbits are closed (they overlap, the star returns to the same place it was before) and elliptical. These orbits can have major axes aligned, producing a bar structure, or each one can be rotated relative to those next to it. This produces a spiral patterned density enhancement which is stationary in the noninertial frame (observed in a non-rotating frame, the spiral pattern will appear to rotate with an angular speed Ω_{gp}). The pattern depends on the number of oscillations per orbit.
- The orbits appear to be simplified ovals only when viewed in the reference frame rotating with Ω_{gp}; in a non-rotating inertial reference frame the spiral pattern will appear to move with an angular speed Ω_{gp}. The "traffic jam" of stars being packed together where their oval orbits approach each other leads to the density waves.

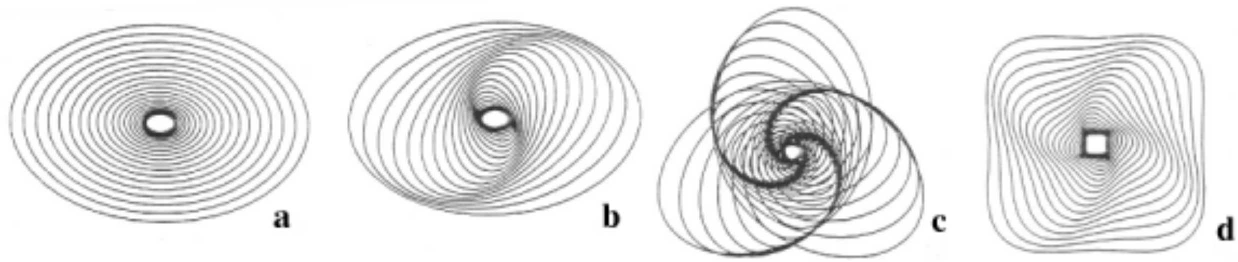

Figure 4: By aligning a series of concentric elliptical (2/1) orbits, a bar can be produced (a). If each ellipse is given an azimuthal offset proportional to $r^{1/2}$, the effect is a two armed spiral of orbits (b). A set of (3/2) orbits produces a three armed spiral (c) and (4/1) produces a four armed pattern (d).

4.4 The origin of spiral structure: stochastic, self-propagating star formation

The density wave model describes the production of grand design spirals, but many of the galaxies we observe are flocculent spirals (recall that this means that the galaxies have fragmentary spiral arms that cannot be traced over a large angular distance). This appearance may be produced by a superposition of several stable density perturbations combined with patchiness in the interstellar medium.

There is also another theory for spiral structure in flocculent spirals known as **stochastic, self-propagating star formation (SSPSF)**. In this model, a region of the galaxy undergoes star formation, and the shock waves from core-collapse supernovae then triggers new star formation in a nearby region. Star formation thus propagates across the galaxy like a forest fire. The spiral pattern arises when differential rotation of the galaxy draws the regions with new star formation into trailing arms. This model reproduces flocculent spiral structure in computer simulations, but does not explain the observations of OB stars transitioning to red stars across the arms of grand design spirals. It may be that both theories apply to different degrees in different types of galaxies.