Astronomy 401/Physics 903
Problem Set 7
Due in class, **Thursday April 11, 2019**

1 Energy density and the scale factor

By inserting the equation of state $P = w\rho c^2$ into the fluid equation

$$\frac{d(a^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(a^3)}{dt}, \tag{1}$$

show that $a^{3(1+w)}\rho = \text{constant} = \rho_0$, where ρ_0 is the present value of ρ.

2 Density and pressure

a) Consider a model of the universe consisting of neutral hydrogen atoms for which the average (rms) speed of the atoms is 600 km s^{-1} (approximately the speed of the Local Group relative to the Hubble flow). Show that $\rho \gg P/c^2$ for the gas.

b) For an adiabatically expanding universe, for what value of a and z will $\rho = P/c^2$?

3 Matter and energy density in the Universe

Suppose that all of the baryonic (non-dark) matter in the universe were converted into energy in the form of blackbody radiation. Take the average density of matter to be $\rho = 4.17 \times 10^{-28}$ kg m^{-3}; this is the baryonic matter density found by the Wilkinson Microwave Anisotropy Probe (WMAP) studies of the cosmic microwave background radiation.

a) What would the temperature of the universe be in this situation?

b) At what wavelength would the blackbody spectrum peak? Would we be able to see the sky glow due to this radiation?

4 Temperature of the CMB

The relative populations of different excitation states of an atom (or molecule) are described by the Boltzmann equation,

$$\frac{n_1}{n_2} = \frac{g_1}{g_2}e^{-(\Delta E)/kT}, \tag{2}$$

where n_1 and n_2 are the number densities of atoms with excitation states in levels 1 and 2 respectively, g_1 and g_2 are the statistical weights of the levels, and $\Delta E = E_1 - E_2$ is the energy difference between the levels. The statistical weights account for the fact that some energy levels may be degenerate, with more than one state having the same energy; thus g_1 is the number of states having energy E_1 and g_2 is the number of states having energy E_2.

In 1941, microwave observations detected absorption lines due to cyanogen molecules (CN) in molecular clouds. A cyanogen molecule has three first excited rotational states, each of which is degenerate and has an energy that is 4.8×10^{-4} eV above the ground state. An analysis of the absorption lines shows that for every 100 molecules in the ground state, there are 27 others that are in one of the three first excited states.

Assuming that the molecular clouds are in thermal equilibrium with the CMB, use the Boltzmann equation to estimate the temperature of the CMB.

5 Gas clouds and the CMB

The carbon absorption lines that are formed when the light from a distant quasar, Q1331+70, passes through an intergalactic cloud have been measured by astronomer Antoinette Songaila and her colleagues. The relative strength of lines indicate that the temperature of the cloud is 7.4 ± 0.8 K, and the lines show a redshift of $z = 1.776$. How does the temperature of the cloud compare with the temperature of the CMB at that redshift? (If there are sources of heating for the cloud in addition to the CMB, then its temperature must be considered as an upper limit to the temperature of the CMB.)

6 Particle velocities and temperature

This problem is required only for students enrolled in Physics 903

The distribution of particle velocities v in a gas is given by the Maxwell-Boltzmann distribution,

$$n_v \, dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp\left(\frac{-mv^2}{2kT} \right) 4\pi v^2 \, dv \tag{3}$$

where m is the particle mass, n is the total number density, k is the Boltzmann constant, and T is the temperature. This equation gives the number density of particles with velocities between v and $v + dv$.

Go to the course website and download the file `velocities.dat`. This file gives the velocities of individual particles in a pure hydrogen gas in m s^{-1}. Use these data and the Maxwell-Boltzmann distribution to determine the temperature of the gas.