Astronomy 401/Physics 903
Lecture 8
The Luminosity Function

1 The luminosity function of galaxies

When we discuss populations of galaxies, it is important to be able to describe the relative numbers of galaxies with different luminosities. We do this by specifying the number density of galaxies as a function of luminosity. This is done with the **luminosity function**, where $\Phi(L)\,dL$ represents the number density of galaxies with luminosity between L and $L + dL$.

The global galaxy distribution can be roughly approximated by the *Schechter luminosity function*:

$$\Phi(L) = \left(\frac{\Phi^*}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} \exp(-L/L^*). \tag{1}$$

For bright galaxies, the number density drops exponentially, while the distribution of faint galaxies has a slope α. L^* is a characteristic luminosity that separates the two regimes, and Φ^* has units of number density and gives the normalization of the distribution. The luminosity function $\Phi(L)$ itself has units of number density divided by luminosity, as can be see from Equation 1; $(L/L^*)^\alpha$ and $\exp(-L/L^*)$ are dimensionless and determine the shape of the function. So $\Phi(L)$ is the number density of galaxies per luminosity interval, and $\Phi(L)\,dL$ is the number density of galaxies in the interval dL.

A schematic plot of the Schechter luminosity function is shown in Figure 1.

We can integrate over the Schechter luminosity function to calculate the *total* number density of galaxies

$$n_{\text{tot}} = \int_0^\infty \Phi(L)\,dL = \Phi^*\,\Gamma(\alpha+1), \tag{2}$$

where the Gamma function $\Gamma(x)$ is defined by

$$\Gamma(x) = \int_0^\infty y^{(x-1)} e^{-y}\,dy. \tag{3}$$

For positive integers, $\Gamma(n+1) = n!$. For $\alpha \le -1$ the total number density of galaxies diverges, but this is not a problem because the luminosity function does not actually extend to arbitrarily small luminosities (i.e., galaxies have a minimum luminosity).

We can also calculate the total *luminosity density* of galaxies:

$$l_{\text{tot}} = \int_0^\infty L\,\Phi(L)\,dL = L^*\Phi^*\,\Gamma(\alpha+2). \tag{4}$$

The total luminosity density is finite for $\alpha \ge -2$. For $\alpha \sim -1$, a typical value, most of the luminosity is from galaxies with $L \sim L^*$, Φ^* is a good estimate of the number density of L^* galaxies, and $l_{\text{tot}} \sim \Phi^* L^*$ is a good approximation for the total luminosity density.

The luminosity function is often expressed in magnitudes instead of luminosity, and in this case the function is much more complicated. An interval dL in luminosity corresponds to an interval dM in absolute magnitude, and $dL/L = -0.4 \ln 10\, dM$. Since $\Phi(L)\,dL = \Phi(M)\,dM$, the luminosity function in magnitudes is then

$$\Phi(M) = (0.4 \ln 10)\,\Phi^*\,10^{0.4(\alpha+1)(M^*-M)} \exp\left[-10^{0.4(M^*-M)}\right] \tag{5}$$

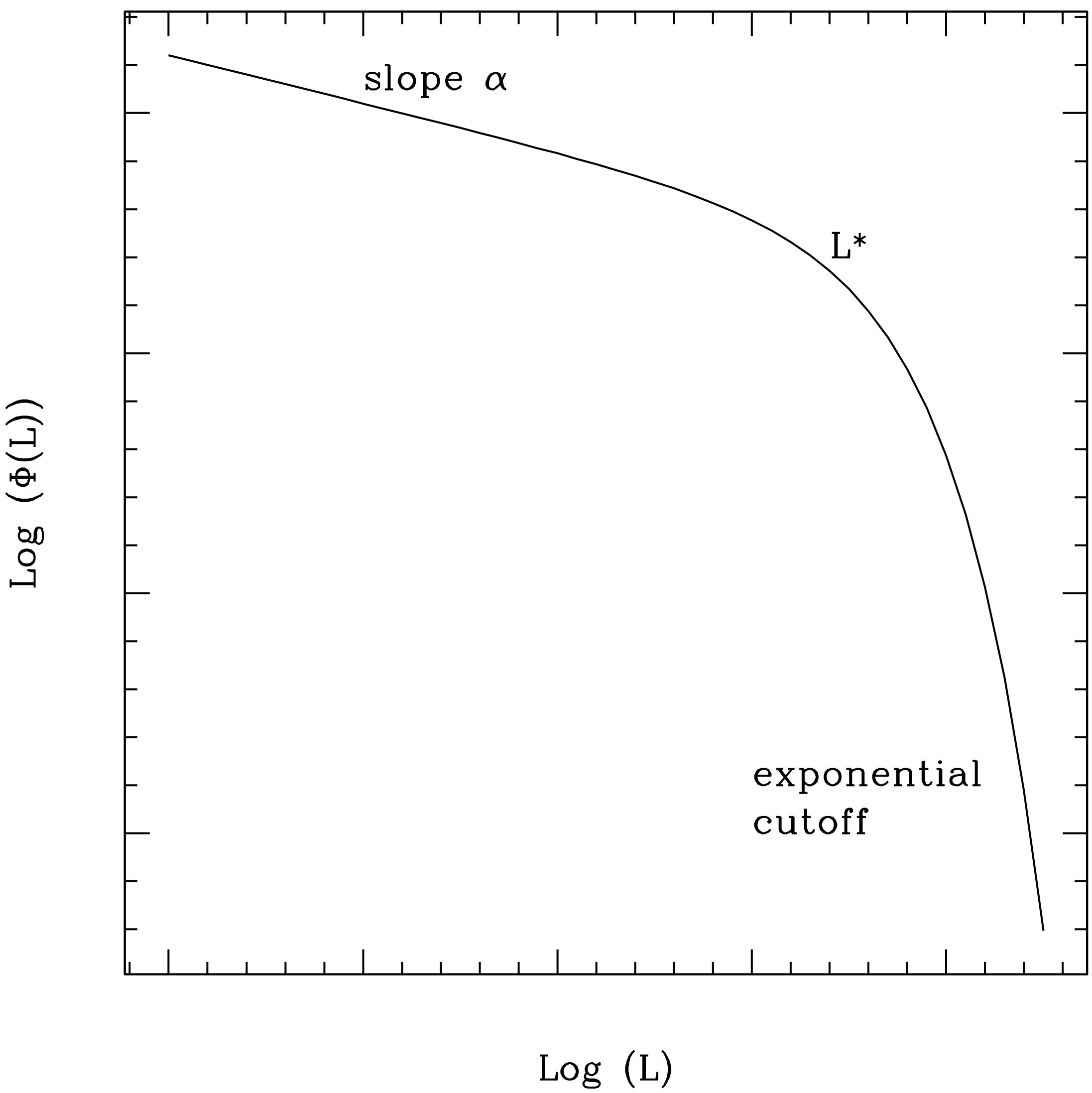

Figure 1: Schematic plot of the Schechter luminosity function showing the slope α (a typical value is $\alpha \sim -1$) for faint galaxies, the characteristic luminosity L^*, and the exponential cutoff for bright galaxies. Note that this is a log-log plot.

(see Equations 3.53 and 3.54 of the text).

The galaxy luminosity function is not universal: different types of galaxies have different luminosity functions, and the luminosity function may also evolve over time. An example luminosity function from the literature is shown in Figure 2. When plotted in magnitudes, luminosity functions are typically shown with bright galaxies (smaller magnitudes) on the left and faint galaxies (larger magnitudes) on the right.

This example is for galaxies at $z \sim 2$ (blue points)[1] and $z \sim 3$ (red points), and shows the rest-frame UV luminosity function; $M_{\rm AB}$ (1700 Å) is the absolute magnitude at a wavelength of 1700 Å (AB magnitudes have a zeropoint defined to make it easy to convert between AB magnitudes and flux; useful, but not important at the moment). Contours showing the best-fit values of α and M^* are shown in the inset plot. Note that the best-fit value of α is $\alpha = -1.73$; in the local universe, typical values are $\alpha \simeq -1$. This steep faint-end slope indicates that faint galaxies were apparently more numerous relative to bright galaxies in the distant universe than they are today.

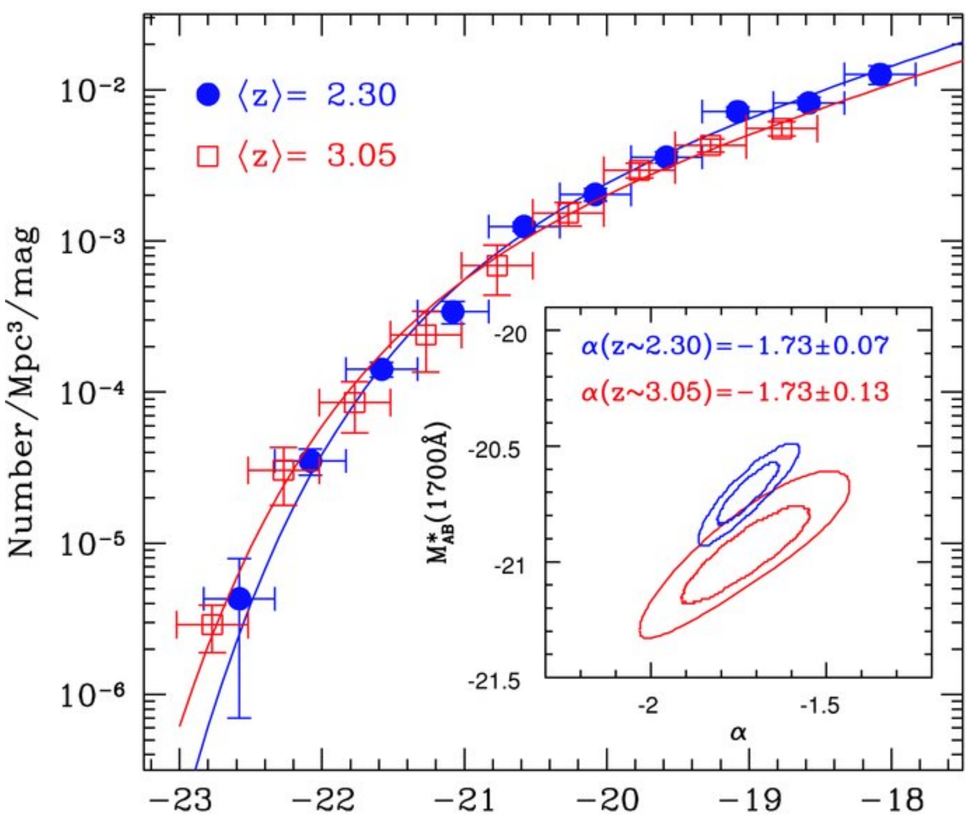

Figure 2: The rest-frame ultraviolet luminosity functions of galaxies at $z \sim 2$ and $z \sim 3$. From Reddy & Steidel 2009 (*The Astrophysical Journal*, 692, 778).

[1] At $z \sim 2$, the universe was roughly $\sim 25\%$ of its current age. More on that later in the course.