Astronomy 401/Physics 903
Problem Set 8
Due in class, **Thursday April 18, 2019**

1 Redshift surveys and K-corrections

This problem explores some complications associated with observing galaxies in the distant universe.

Because redshift increases the wavelength of light propagating from distant galaxies,

$$\lambda_{\rm obs} = (1+z)\lambda_{\rm rest}, \tag{1}$$

when we measure the light from a distant galaxy we are actually measuring light emitted at bluer wavelengths. This is important, because galaxies don't emit the same amount of light at all wavelengths.

The following is a list of the wavelength centers and ranges of the five filters used in the Sloan Digital Sky Survey (SDSS):

Filter	$\lambda_{\rm center}$ (Å)	FWHM (Å)
u	3585	556
g	4858	1297
r	6290	1358
i	7706	1547
z	9222	1530

a) Suppose we select galaxies using their magnitudes in the r filter. At what approximate redshift would a galaxy be if our r-band observations are actually detecting light which the galaxy emitted in the center of the g filter? At what redshift would the galaxy be if we are detecting light which the galaxy emitted in the center of the u filter?

b) Astronomers routinely detect galaxies at redshifts of $z \sim 2$. At this redshift, what wavelength of light would we need to detect in order to measure the i-band light output of the galaxy? What wavelength of light would we need to detect in order to measure the $\mathrm{H}\alpha$ emission line from one of these galaxies? Would these measurements require observations in the optical, infrared or submillimeter part of the spectrum?

c) Briefly discuss how this effect might change how we should interpret observations of galaxies at different redshifts (e.g. star formation rates, relative brightnesses, morphologies).

d) Assuming no change in the frequency of different types of galaxies with redshift, would we be more likely to detect a higher fraction of spiral galaxies at low or at high redshifts?

There is an additional effect we need to consider when interpreting galaxy magnitudes at high redshift. Because of the wavelength stretching, we observe a smaller wavelength *range* of a high redshift galaxy's spectrum. This makes galaxies appear fainter. Correcting a galaxy's magnitude for both the wavelength shift and the decreased wavelength range is known as applying a K-correction. In other words, this corrects for the fact that we're observing a bluer part of the spectrum than if the galaxy were not redshifted, and for the fact that we're observing a smaller part of the spectrum. The K-correction is

$$m_{\text{true}} = m_{\text{obs}} - K. \tag{2}$$

K is usually but not always positive, which means that the "true" magnitude which would have been measured in some filter without the effect of redshift would have been brighter than the magnitude we actually measured.

e) Will the size of the K-correction (i.e. K) vary with redshift? Why or why not?

f) Will the K-correction be different for different types of galaxies (e.g. ellipticals vs. spirals)? Why or why not? If so, which way will it vary?

2 The CMB on TV

Channel 6 on your television consists of radio waves with wavelengths between 3.41 m and 3.66 m (or used to, in the days of analog television). Consider a 25,000 watt television station located 70 km from your home. Use the equation for the energy density of blackbody radiation to estimate the ratio of the number of channel 6 photons to the number of CMB photons that your television antenna picks up in this wavelength band. (Hint: For the television broadcast, recall that the energy density of an electromagnetic wave is related to the time-averaged Poynting vector by $u = \langle S \rangle / c$.)

3 When did the universe begin to accelerate?

Use the acceleration equation to show that the acceleration of the universe changed sign (from negative to positive) when the scale factor was

$$a_{\text{accel}} = \left(\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}} \right)^{1/3}. \tag{3}$$

Evaluate the value of a_{accel} and z_{accel} at this time with the Planck 2015 values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$.

4 Einstein's static universe

Consider the static universe originally proposed by Einstein, in which the attractive force of the matter density ρ_m is exactly balanced by the repulsive force of the cosmological constant. Suppose that some of the matter is converted into radiation (by stars, for example). Will the universe remain static, or start to expand or contract? Explain your answer with reference to the appropriate equations.

5 More K-corrections

This problem is required only for students enrolled in Physics 903.

For this problem you will determine the K-corrections for real galaxies, using a red galaxy and a blue galaxy from the Sloan Digital Sky Survey. This will allow you to see the effects of redshift on measurements of galaxies directly. You will integrate the spectra of the galaxies over the transmission curve of a filter, which will allow you to determine the magnitude of the galaxy in the i-band. We'll do this for the galaxies at a redshift of zero, and again at a redshift of $z = 0.85$.

a) Go to the course website and download the files `spSpec-51820-0400-608.dat` and `spSpec-51612-0280-549.dat`; these are spectra from the Sloan Digital Sky Survey of a red galaxy and a blue galaxy respectively. You will also need the transmission curve for the i filter, `i_filter.dat`. The spectra have rest-frame wavelength in Å in the first column, and flux in units of 10^{-17} erg s^{-1} cm^{-2} Å^{-1} in the second column. The first column of the filter transmission curve gives wavelength in Å, and the second column is the transmission of the filter, given as a fraction between 0 and 1.

First plot the spectra and the transmission curve to see what they look like. Your plot should look like Figure 1 below. Note that you will need to scale the transmission curve by some arbitrary factor to make both the spectra and the transmission curve visible on the same plot.

b) The next step is to figure out the i-band magnitude of each galaxy by integrating the spectra over the transmission curves. As a first step, convert your spectra from f_λ units (erg s^{-1} cm^{-2} Å^{-1}) to f_ν units (erg s^{-1} cm^{-2} Hz^{-1}) using the relationship

$$f_\nu = f_\lambda \times \frac{\lambda^2}{c}. \tag{4}$$

In other words, multiply each flux value by the square of its corresponding wavelength, and divide by c. Remember that your wavelengths are in Å, and **be very careful to make sure your units come out correctly!** You will need your spectra in frequency units in order to convert to magnitudes later.

The flux in the filter is given by the product of the spectrum and the filter transmission curve integrated over wavelength, normalized by the integral of the transmission curve over wavelength:

$$f_{\nu,\mathrm{filt}} = \frac{\int f_\nu T_\lambda \lambda \, d\lambda}{\int T_\lambda \lambda \, d\lambda}, \tag{5}$$

where f_ν is the galaxy spectrum in frequency units and T_λ is the filter transmission as a function of wavelength. The additional factor of wavelength λ in both integrals accounts for the fact that CCDs count photons, not energy; since $E = h\nu = hc/\lambda$, multiplying by λ is the same as dividing by photon energy (the factor of hc cancels since we do this in both the numerator and denominator).

You will first need to interpolate the filter transmission curve so that it has the same wavelength array as each of the spectra. You should end up with two new wavelength arrays and two new transmission arrays for the filter curves; the wavelength arrays should be identical to the wavelength arrays of your red and blue spectra, and your new transmission arrays should have a value

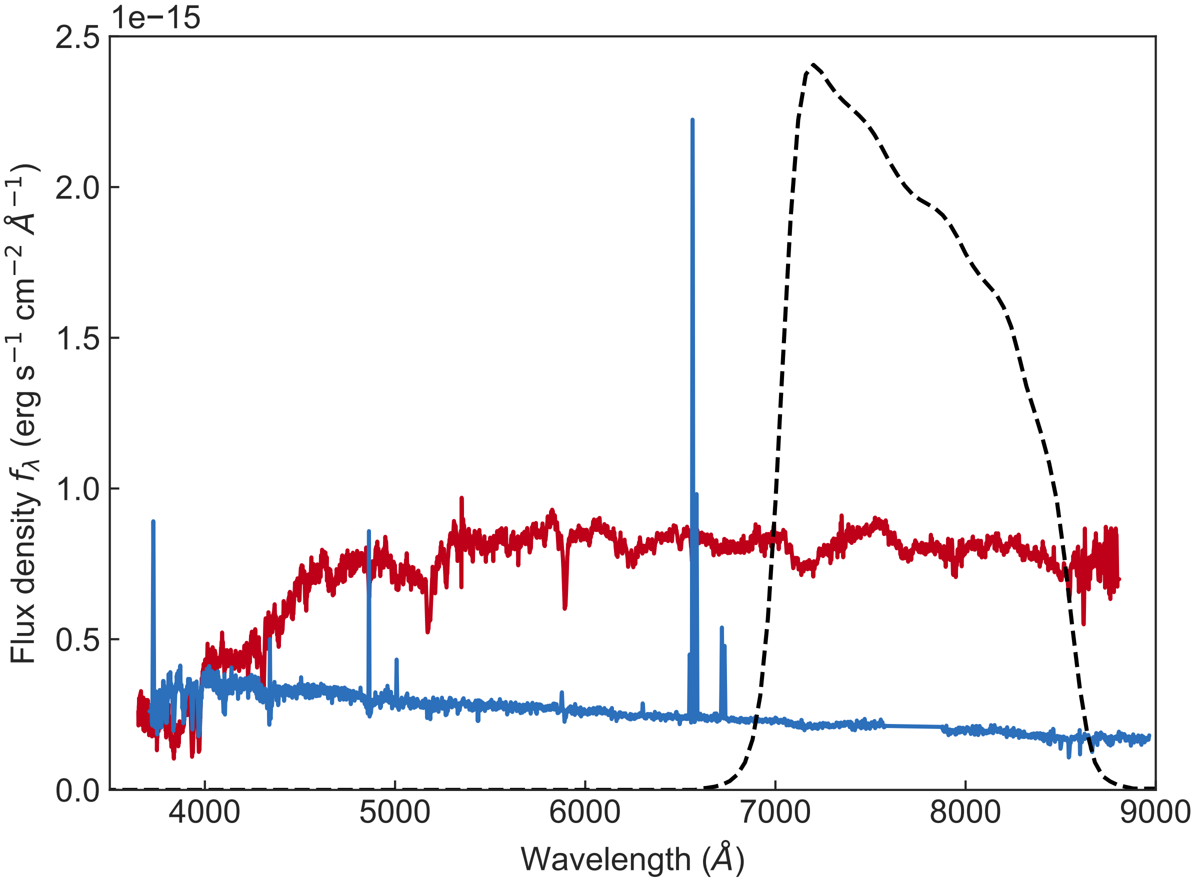

Figure 1: Spectra of the red and blue galaxies, with the scaled transmission curve of the i-band filter shown with the dashed line. Note that the galaxies have similar fluxes at around 4000 Å, but the red galaxy is brighter at longer wavelengths.

for each wavelength. Once you've done this, do the integrals numerically and compute $f_{\nu,\text{filt}}$ for each spectrum.

Finally, we compute the magnitude from the flux in the filter. We will use AB magnitudes, which are related to the flux in the filter by the following equation:

$$m = -2.5 \log f_{\nu,\text{filt}} - 48.6. \tag{6}$$

What are the i-band magnitudes of the two galaxies?

c) Now we'll see what happens when the galaxies are redshifted. Redshift both galaxies to $z = 0.85$ by multiplying both wavelength arrays by $(1 + z)$, and repeat part b): convert your redshifted spectra to f_ν units, interpolate the filter transmission curve onto the wavelengths of your redshifted spectra, and integrate to find $f_{\nu,\text{filt}}$ for your redshifted spectra. What are the i-band magnitudes of the two galaxies when they are at $z = 0.85$?

Make a plot similar to Figure 1, showing the redshifted spectra and the filter transmission curve. This should allow you to see what parts of the spectra are being sampled by the filter when the galaxies are redshifted to $z = 0.85$. Briefly describe what you see here: what parts of the galaxies' spectra are you now measuring with the filter, and how will this change your measurements?

d) The K-correction is given by

$$K = m_{\mathrm{obs}} - m_{\mathrm{true}}, \tag{7}$$

where m_{obs} is the magnitude of the redshifted galaxy you found in part c) and m_{true} is the magnitude in the rest-frame you found in part b).

What are the K-corrections for the two galaxies? Do these results agree with your expectations from your answers to Problem 1?