Astronomy 401/Physics 903
Problem Set 2
Due in class, **Thursday February 14, 2019**

1 Stars, Supermassive Black Holes and Tidal Forces

In this problem we will calculate whether or not stars swallowed up by the supermassive black hole at the center of the Milky Way are ripped apart before they pass the event horizon of the black hole. The tidal force is the *differential* force across a body due to an external gravitational force; the gravitational force will act more strongly on the side of the body closer to the massive external object. If the tidal force is stronger than the force holding the object together, the object will be ripped apart.

Consider a star of mass $M_\star$ and radius $r_\star$ a distance r from a black hole of mass $M_{\rm BH}$. If the differential tidal force across the star is

$$\Delta F = \frac{G M_{\rm BH} M_\star}{r^3} r_\star \tag{1}$$

and the gravitational force holding the star together is

$$F_{\rm grav} = \frac{G M_\star^2}{r_\star^2}, \tag{2}$$

how close can the star come to the black hole before it is ripped apart? Consider a star with the mass and radius of the Sun, and a black hole of mass $M_{\rm BH} = 3.7 \times 10^6$ $\rm M_\odot$. What about a star with mass 60 $\rm M_\odot$ and radius 13 $\rm R_\odot$? Do these stars cross the Schwarzchild radius of the black hole before they are ripped apart?

If a star is swallowed whole by a black hole it doesn't produce a major outburst of radiation before it crosses the event horizon, but if it's ripped apart first its gas forms a hot and bright accretion disk around the black hole. Would we expect to see an accretion disk around the black hole at the center of the Milky Way?

2 Exponential Disks

The radial surface brightness distribution of disks of spiral galaxies is well-described by a decaying exponential with scale length h:

$$\Sigma(r) = \Sigma_0 e^{-r/h} \tag{3}$$

a) Show that the total luminosity of a disk with central surface brightness Σ_0 and disk scale length h is given by $2\pi\Sigma_0 h^2$.

b) What is the radius which contains half the light, in terms of the scale length h? (This is often known as the effective radius r_e or the half-light radius.) Hint: You will end up with an expression which is most easily solved numerically, using Mathematica, Wolfram Alpha, or any other program you prefer.

c) What is the radius which contains 95% of the light, in terms of the scale length h?

3 Making a Mass Model for the Milky Way

In this problem you will build up an approximate mass-model for the Milky Way galaxy, which will give you roughly the correct rotation curve for the galaxy.

The Milky Way disk has an exponential scale length h which is thought to be about 3.5 kpc. For simplicity, you may assume that the mass surface density Σ_m (with units $\mathrm{M}_\odot$ pc^{-2}) of the disk is exponential, so that $\Sigma_m(r) = \Sigma_{m0} e^{-r/h}$. Also assume that you can calculate the circular velocity due to the disk as if it were a *spherically* distributed mass; this isn't strictly the case for a disk, but it's a close enough approximation for this exercise. You will be neglecting the effect of the bulge for simplicity. All radii should be given in kiloparsecs, masses in $\mathrm{M}_\odot$, velocities in km s^{-1}, densities in $\mathrm{M}_\odot$ kpc^{-3}, and surface densities in $\mathrm{M}_\odot$ kpc^{-2}.

a) Find an expression for the rotational velocity as a function of radius for the disk alone, in terms of h and Σ_{m0}. To do this, calculate the mass enclosed within radius r, and then find the velocity corresponding to this mass. Your results from the previous problem may be helpful here.

b) What value of Σ_{m0} will give you a maximum rotational velocity of $\approx$ 200 km s^{-1} for the disk? You can find this by plotting your expression from part (a) for some value of Σ_{m0}, looking at the resulting maximum velocity, and then figuring out how to scale your value of Σ_{m0} to get $v_{\max} \approx 200$ km s^{-1}.

c) The surface density of the Milky Way disk is measured to be around 75 $\mathrm{M}_\odot$ pc^{-2} (7.5×10^7 $\mathrm{M}_\odot$ kpc^{-2}) at the solar circle (around $r = 8$ kpc). How does this compare with the surface density at the solar radius you would expect from your result from (b)?

d) The rotation curve of the Milky Way becomes asymptotically flat at a velocity of $v_{\rm rot} \approx 220$ km s^{-1}. At very large radii, the disk of the Milky Way contributes much less mass to the rotation curve than does the dark matter halo. Thus the dark matter halo alone must be capable of producing a flat rotation curve at large radii. Assuming that the density profile of the dark matter halo is a "modified isothermal sphere,"

$$\rho_{\rm halo} = \rho_0 \frac{a^2}{r^2 + a^2}, \tag{4}$$

where a is the "core radius," what must ρ_0 be to produce a flat rotation curve at large radii with the correct value of $v_{\rm rot}(r \sim \infty) \approx 220$ km s^{-1}? Solve this by finding an expression for $M(< r)$, the mass enclosed within radius r. You may assume that in the region of interest $r \gg a$, and your answer will be in terms of the core radius a. If $a = 7$ kpc, what is the value of ρ_0?

e) The *total* rotation curve of the galaxy is the rotational velocity due to the combined masses of the disk and halo. To find the total rotational velocity, add the masses of the two components (your expressions for $M(< r)$), and then find the circular velocity corresponding to this combined mass. Using the numerical values from the previous sections, plot this total rotation curve. Also plot the rotation curves of the disk and halo individually, and label each component. Plot your rotation curves out to a radius of about 25 kpc. Note that we are now interested in the shape of the total rotation curve at all radii, so it is no longer appropriate to assume $r \gg a$.

f) For your model, what is the total mass of the Milky Way within 4 disk scale lengths (14 kpc)?

4 Using the Tully-Fisher Relation

You have pointed your telescope at an Sb spiral galaxy and measured a maximum rotational velocity of 324 km s^{-1} and an apparent magnitude of $B = 12.22$ mag, after correcting for the effects of dust.

a) Estimate the galaxy's absolute magnitude in the B band from the Tully-Fisher relation.

b) Determine the distance to the galaxy using its distance modulus.

c) Spiral galaxies of types Sa-Sc show a correlation between radius and luminosity, such that radius increases with increasing luminosity, independent of Hubble type. We define the radius R_{25} to be the radius of the galaxy at which the surface brightness is 25 B-mag arcsec^{-2}. The relation between radius and luminosity is then given by

$$\log R_{25} = -0.249 M_B - 4.00, \tag{5}$$

with R_{25} in kpc. What is R_{25} for the galaxy you have observed?

d) Find the mass of the galaxy interior to R_{25}.

e) What is the luminosity of the galaxy in the B band? Hint: Consult Appendix A.4.4 of the textbook. The absolute magnitude of the Sun in the B band is $M_{\odot \mathrm{B}} = 5.45$, with corresponding luminosity $L_{\odot \mathrm{B}} = 1.99 \times 10^{26}$ W.

f) What is the galaxy's mass-to-light ratio in the B band, interior to R_{25}?

5 Exponential Disks II

This problem is required only for students enrolled in Physics 903.

Go to the course website and download the file `exponentialdisk.py`. This short program produces the plot below, which is a 2D exponential profile with central surface brightness `a` and scale length `h` given on lines 15 and 16 of the code (we also set a background level `b` on line 17). Notice that noise has been added to the profile, and the amount of noise can be changed by changing the value of the `rms` parameter on line 19. Add to this code to do the following steps:

a) Fit a two-dimensional exponential profile to this image, recovering values for the central surface brightness, scale length, and background. How do your recovered values compare with the input values?

b) Add three circles to the plot in Figure 1. They should have radii equal to the best fit scale length, r_e, and r_{95}, where r_e and r_{95} are written in terms of the scale length as you found in parts b) and c) of Problem 2.

c) Now change the value of the background to 5 and the value of the `rms` to 1. Repeat steps a) and b) above.

d) Change the value of the background to 30 and the value of the `rms` to 10. Repeat steps a) and b) above.

Please turn in printed versions of your plots if possible, and written answers explaining how your best fit values change. Also turn in your finished code by email to `erbd@uwm.edu`.

This problem gives you a sense of what an exponential disk profile looks like with varying image quality, but note that this is not a realistic simulation of fitting a galaxy in a real astronomical image, since the form we are fitting is known in advance here. A real galaxy would have an unknown surface brightness profile and would probably not be face-on, meaning that it would not be symmetric in x and y. All of these factors make recovering the properties of a real galaxy more complicated.

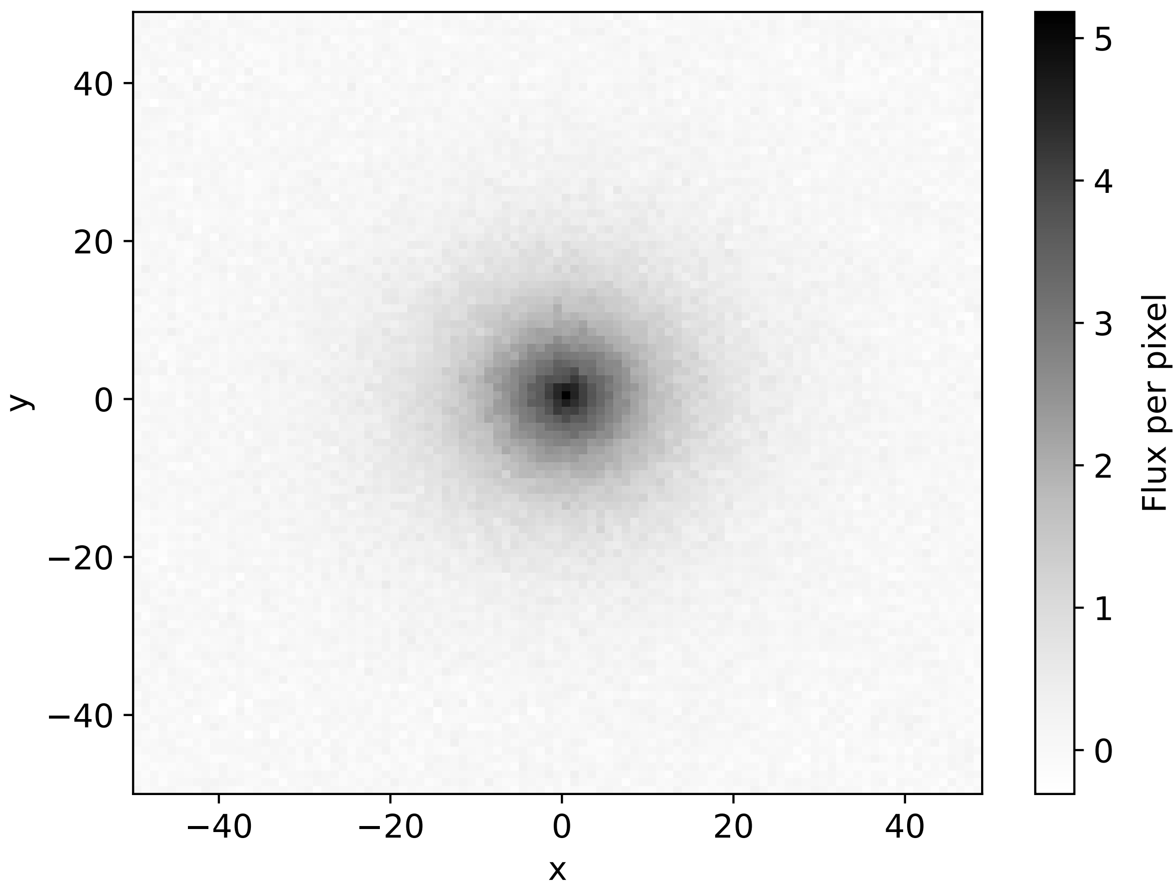

Figure 1: A simulated exponential disk observed face-on.