Astronomy 401/Physics 903
Problem Set 5
Due in class, **Thursday March 7, 2019**

1 The chemical evolution of galaxies

The gas phase metallicity Z of a galaxy is the fraction by mass of elements heavier than helium in the interstellar medium. It is also useful to define the yield y, a measurement of the mass of heavy elements produced by stars and returned to the ISM via stellar winds and supernovae (to be precise, y is the mass of metals produced and ejected by stars in units of the mass that remains locked in long-lived, low-mass stars and compact remnants).

a) Consider a galaxy with no inflows or outflows of gas (no gas leaves the galaxy, and no new gas flows into the galaxy). This galaxy has gas mass g and stellar mass s. Because no gas or stars enter or leave the galaxy, the total mass is constant, $M = g + s =$ constant, and

$$\frac{ds}{dt} = -\frac{dg}{dt} \tag{1}$$

and

$$dg = -ds. \tag{2}$$

It can also be shown that

$$-g\frac{dZ}{dg} = y. \tag{3}$$

This is a differential equation for the metallicity Z as a function of gas mass g. When the metallicity is zero, the gas mass is equal to the total mass of the system: $g(0) = M$. In other words, we are starting with a galaxy made entirely of metal-free gas, before it has formed any stars. Given this initial condition, solve the differential equation to show that the gas phase metallicity is given by

$$Z = y\ln\left(\frac{1}{f}\right), \tag{4}$$

where f is the gas fraction

$$f = \frac{g}{s+g} = \frac{g}{M}. \tag{5}$$

This is known as the "closed box" model of galactic chemical evolution, because it treats galaxies as closed boxes from which nothing enters or leaves. Note the expected inverse dependence of metallicity and gas fraction: metallicity increases as more and more of the gas is formed into stars and those stars return metals into the ISM.

b) Now suppose that we measure the metallicity and gas fraction of a large number of galaxies. We can define the effective yield

$$y_{\rm eff} = \frac{Z}{\ln(1/f)} \tag{6}$$

and compare this to the true yield to determine whether or not the closed box model is actually a good description of how galaxies evolve. If galaxies really are closed boxes, the effective yield will be equal to the true yield. But if galaxies are not closed boxes, they may expel gas and metals via supernova-driven winds, or accrete new, unenriched gas from outside. Both of these processes will decrease the metallicity of the galaxy, and the effective yield $y_{\rm eff}$ will be less than the true yield y.

Copy or download the data file found at
`http://www.cgca.uwm.edu/~dawn/astron401/yeff_vrot.dat.`
These are real galaxy data collected for local spiral and irregular galaxies. The first column in the file is the galaxy name, the second column is the effective yield $y_{\rm eff}$ computed from the metallicity and gas fraction as described above, and the third column is the galaxy's rotational velocity $v_{\rm rot}$ in km s^{-1}. Recall that more massive galaxies have higher rotational velocities.

Make a plot with $v_{\rm rot}$ on the x axis and $\log(y_{\rm eff})$ on the y axis. Draw a horizontal line at $\log(y_{\rm eff}) = -1.9$. This is the true yield y. Are high or low mass galaxies more likely to behave like closed boxes? What might be a possible physical explanation for your answer?

2 Timescales in Galaxies and Clusters

a) How long does it take the Sun to orbit the center of the Galaxy? Assume that we are moving at 220 km s^{-1}, and are 8 kpc from the Galactic center.

b) How many times would the Sun have completed an orbit during the age of the universe? Assume 13.7 Gyr for the age of the universe.

c) As an observational aside, how long would we have to wait to observe a star in Andromeda move through an angle of 1 arcsecond, i.e. to directly detect its rotation? Assume a rotation speed of 250 km s^{-1} and a distance to Andromeda of 750 kpc. You can neglect the inclination of the galaxy. Is an Earth-bound astronomer likely to be able to detect this rotation?

1 arcsecond would be easily measurable with ground-based telescopes, but the Hubble Space Telescope could measure a change of 0.1 arcsecond. Are we likely to be able to see a 0.1 arsecond change?

d) Now consider a cluster of galaxies. The Coma cluster has a velocity dispersion of roughly $\sigma = 1000$ km s^{-1}, a core radius of 0.3 Mpc, and a total radius of about 3 Mpc. How long would it take a galaxy to completely cross the Coma cluster, if it were moving at the typical velocity dispersion?

e) Given that the universe is 13.7 billion years old, do you think that the galaxies in Coma are gravitationally bound to it?

3 Colliding galaxies

Consider a disk galaxy with radius 15 kpc, thickness 1 kpc, and mass in stars $M_\star = 5 \times 10^{10}$ M$_\odot$. Estimate the average number density of stars in this galaxy (the number of stars per unit volume). Assume all of the stars have the same mass, and make a reasonable estimate of that mass.

Now consider two such galaxies, colliding. How far can a star in either of these galaxies travel before hitting another star? Are stars are likely to hit each other when galaxies collide?

4 Creation of the Magellanic Stream

The Large Magellanic Cloud has a mass of about 2×10^{10} $\mathrm{M}_\odot$, and orbits the Milky Way at a distance of 51 kpc. Its angular diameter is $460'$. The Small Magellanic Cloud is 60 kpc away and has one-tenth the mass of the LMC. The SMC has an angular diameter of $150'$, and its angular separation from the LMC is $21°$.

a) What is the distance between the LMC and the SMC?

b) Ignoring the influence of the Milky Way, estimate the present tidal radii of both the LMC and the SMC due to the other.

c) The Magellanic Clouds are in eccentric orbits around one another, and during the last few hundred million years they have been moving apart with an average speed of about 110 km s^{-1}. How recently did the LMC extend beyond its tidal radius? Assuming that the Magellanic Stream was formed by tidal stripping when the LMC and SMC were close to each other, when would you estimate that the Magellanic Stream was formed? (Note: Following this mutual tidal stripping, the gas was pulled from the LMC/SMC system by our Galaxy.)

5 Galaxy in a box

This problem is required only for students enrolled in Physics 903.

For this problem you will numerically evolve a galaxy in order to reproduce the chemical evolution result of Problem 1. Unlike a real galaxy, your galaxy will be a closed box: a cloud of gas which turns into stars over time, with no inflows or outflows of gas. Start your galaxy with an initial gas mass $M_{\mathrm{gas}} = 10^{10}$ $\mathrm{M}_\odot$, and assume that it has a star formation rate of 9 $\mathrm{M}_\odot$ yr^{-1}, which stays constant throughout. Set up a logarithmic time array (i.e., with time steps equally spaced on a logarithmic scale) running for 10^9 years, and compute the stellar mass, gas mass, and gas fraction $f = M_{\mathrm{gas}}/(M_{\mathrm{star}} + M_{\mathrm{gas}})$ at each time step.

Next, use Equation 3 of Problem 1 to compute the change in metallicity for each step, and then the total metallicity of the galaxy at each step. Assume that the yield $y = 0.02$. Make a plot of metallicity Z vs. gas fraction f, with a point for each time step. Overplot the analytic result from Equation 4 of Problem 1, and see how it compares to your simulated galaxy.

Please turn in your printed plot, and email your code to `erbd@uwm.edu` as usual.