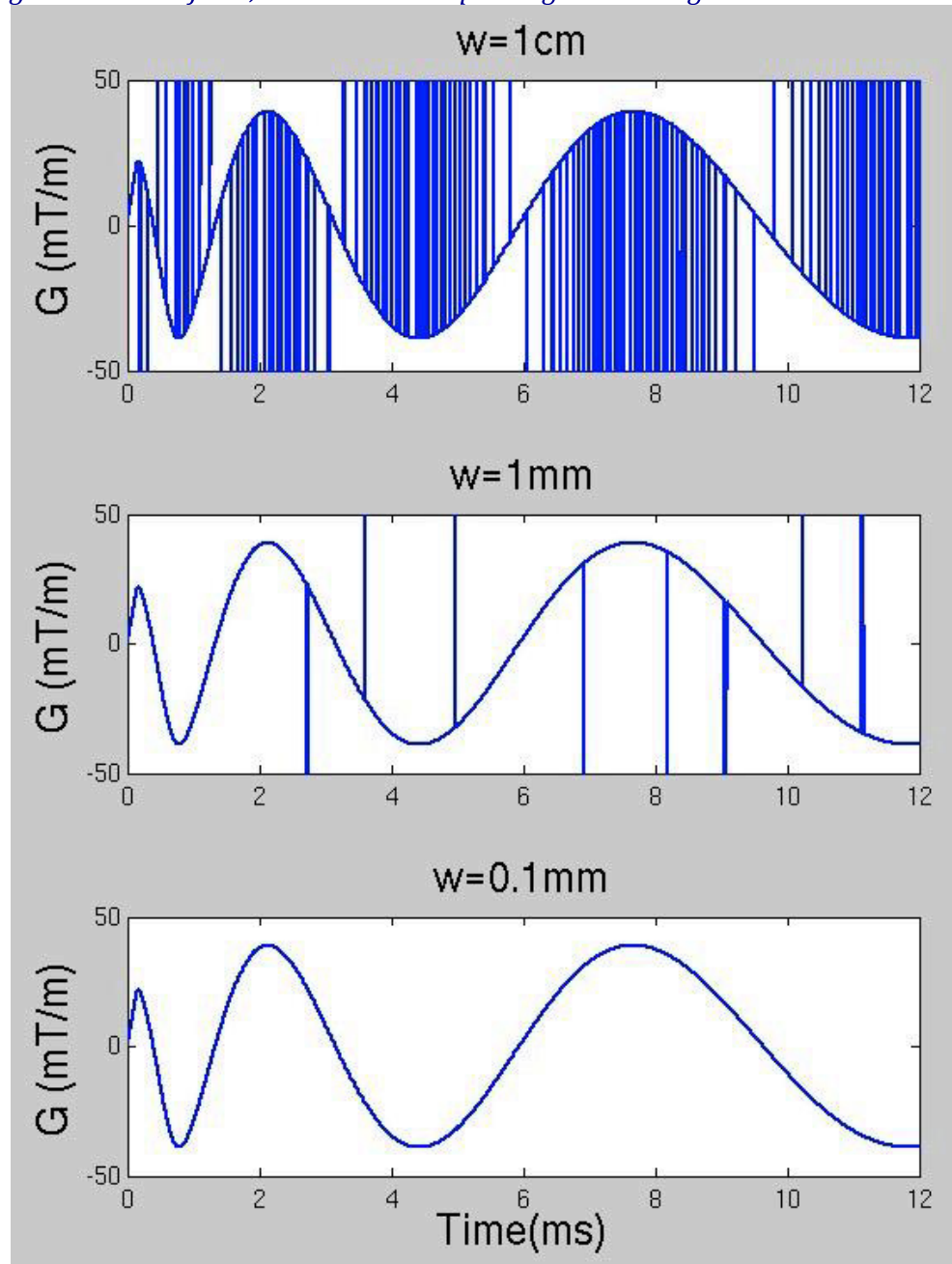


gradient waveform, or intravoxel dephasing leads to signal loss.



2. **MR Fingerprinting:** Here we will look at a very simple form of MR fingerprinting that uses a gradient-spoiled sequence with $TR/TE=10,2$ ms and a randomized RF flip angle. For simplicity define $\langle a, b \rangle$ as the inner product $\text{sum}(a \cdot \text{conj}(b))$ in Matlab. Load the file **mrf.mat** and note the contents of the file: **dict** is a “dictionary” whose columns are the evolution of signal for the given flip angle train **fa** (magnitude in degrees, and phase is the phase of the RF pulse), and the corresponding T_1 and T_2 in **T1list** and **T2list** in ms.

- a. Build an EPG simulation that produces the signal evolution as a function of T_1 and T_2 for the array of N flip angles, **fa**. Note the fact that the flip angle includes a phase for the RF! Call this evolution $f(t, T_1, T_2)$. *Note that you could use this to create the dictionary **dict** above, but we did it for you!*

The Matlab code is here:

```
%
%      function s = mrfevol(T1,T2,fa)
%
%      Calculate MR "fingerprint" for T1 and T2.
%      Assume a gradient-echo sequence with TR=10ms and
%      TE=2ms. T1 and T2 are in ms. fa is in degrees.
%
%
function s = mrfevol(T1,T2,fa)

TR = 10;
TE = 2;

N = length(fa); % # TRs
F = [0;0;1];    % Equilibrium Magnetization.
                % [F+; F-; Z], all longitudinal in Z0 state.

S1 = zeros(N,1);

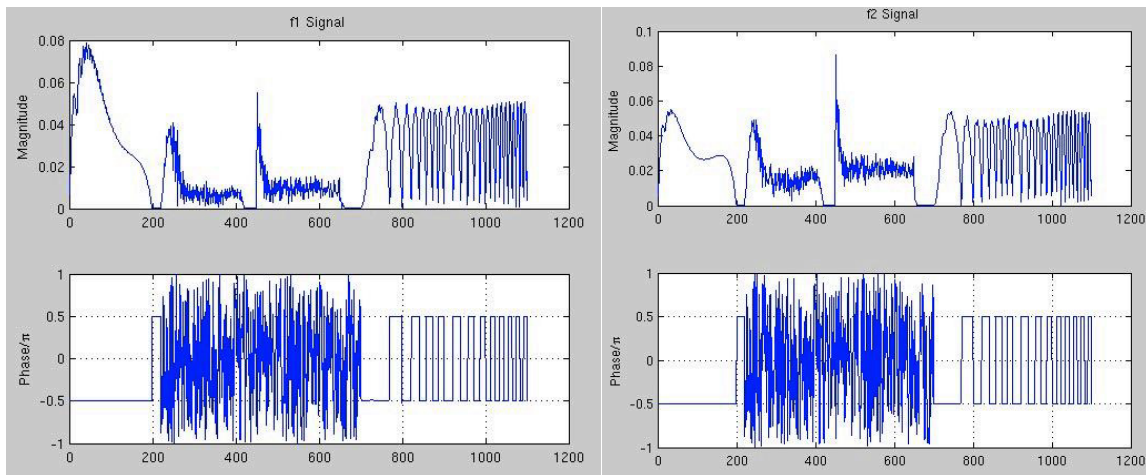
fa = pi/180*fa;

for k=1:N
    F = epq_rf(F,abs(fa(k)),angle(fa(k))); % RF
    S1(k) = F(1,1); % Record TE=0 signal.
    F = epq_grelax(F,T1,T2,TR,1,0,1,0); % Relaxation, spoiling
end;

s = S1 * exp(-TE/T2);
```

- b. Find the magnitude and phase of this evolution for $T_1=1000$, $T_2=200$ and for $T_1=300$, $T_2=80$, call these (complex) vectors f_1 and f_2 . Normalize f_1 by $\sqrt{\langle f_1, f_1 \rangle}$ and f_2 by $\sqrt{\langle f_2, f_2 \rangle}$ and plot the magnitude and phase of each, as a function of TR number. *You should check these against the signal in **dict** generated for similar T_1/T_2 !*

*Plots are shown below, and obviously match **dict** since this uses the same code! Note that they are not all that different.*



- c. Using the normalized values from (b), calculate $\langle f1, f2 \rangle$.

$$\langle f1, f2 \rangle = 0.95 \quad (\text{The "fingerprints" are not very orthogonal!})$$

- d. Now calculate $\langle f1, \text{dict} \rangle$ for all columns of dict, and find the column for which the magnitude of the inner product is maximized. In the arrays **T1list**, **T2list**, what does this correspond to? (ie what is your estimate of T_1 and T_2 ?) Repeat for $\langle f2, \text{dict} \rangle$.

$$\text{For } f1: (T1, T2) = (992, 201)$$

$$\text{For } f2: (T1, T2) = (317, 79)$$

- e. Add noise $\text{sg} * \text{randn}(\text{size}(\text{fa}))$ to the original (non-normalized) **f1** and **f2** and plot the noisy signal (magnitude and phase), then do part (d) for values $\text{sg} = 0.005, 0.01, 0.02$ and 0.05 .

(Answers may vary due to random noise!)

S_g	0.005	0.01	0.02	0.05
$T1, T2$ for $f1$	992, 201	992, 201	992, 201	827, 187
$T1, T2$ for $f2$	317, 87	270, 82	270, 82	367, 101

3. **Fat/Water Separation and B_0 mapping:** Begin here by loading the file **me.mat**, which contains k-space data that is (k_x, k_y, TE) with a TE spacing of 0.5ms. There are 16 echoes for a duration of ~8ms, which is convenient. You can reconstruct the images with $\text{dat} = \text{fftshift}(\text{fft}(\text{fftshift}(\text{dat}, n), [], n), n)$; where n is the dimension that you are Fourier transforming over (ie 1, and 2, and maybe 3 in some cases). For the problem, you will not use all 16 echoes (though you could reconstruct images over x, y, f for fun!).

- a. Fourier transform in x and y to form images. Display the magnitude images for echo times 1, 2, 3 and 4.