**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

Given:

*μ* = 45 minutes

*σ*  = 8 minutes

The service manager plans to have the work begin 10 minutes after the car is dropped off, so the total time allotted for the service is 60−10=50 minutes.

So, x = 50 minutes

Now, we want to find the probability that the service time exceeds 50 minutes.

The z-score is calculated using the formula:

Z =

where:

x is the given value (50 minutes),

μ is the mean (45 minutes),

σ is the standard deviation (8 minutes).

P (Z) = = = 0.625

Now, we look up this z-score in the standard normal distribution table to find the corresponding probability.

Let's denote P (Z > 0.625) as the probability that the service time exceeds 50 minutes.

This is the probability that the service time exceeds 50 minutes.

We should keep in mind that the standard normal distribution table provides the probability for

Z, so we might need to look up -0.625 since the table typically provides probabilities for Z values to the left of the mean.

P (Z > 0.625) = 0.2676 = option B

This probability will give you the likelihood that the service manager cannot meet his commitment of having the car ready within 1 hour from drop-off.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

Given that

Number of employees (N) = 400

mean (*μ* )= 38

Standard deviation (*σ* ) = 6

A. More employees at the processing center are older than 44 than between 38 and 44.

To determine whether more employees are older than 44 than between 38 and 44, we can use the z-scores associated with these age ranges.

For the age of 44, the z-score is given by:

= = = = 1

For the age range 38 to 44, we are considering z-scores between 0 and 1.

The statement is True if the area under the normal curve for Z > 1 is greater than the area for

0 < Z ≤ 1.

P (Z > 1) = 1 = 1- 0.8413 = 0.1587 (area to the right side of z) (The z distribution table value for P (Z > 1))

P (0 < Z ≤ 1) = 0.8413 (area to the left side of z)

The area to the right of z=1 is Not greater, so the statement is false.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

To determine the number of employees under the age of 30, we need to find the z-score for

= = = - = - = - 1.34

P (Z = - 1.34) = 0.0901

Now, the corresponding probability associated with Z = − 1.34 = 0.0901

This will give us the proportion of employees under the age of 30.

The number of employees expected to be under 30 can be calculated using the formula:

Number of employees = Proportion × Total number of employees

Number of employees = 0.0901 × 400 = 36.4 ≈ 34

the calculated value is close to 36, the statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

Let's analyze the distributions and parameters of the random variables of 2 X1 and X1 + X2 whereX1 ~ N (μ, σ2) and X2 ~ N (μ, σ2) are independent and identically distributed (i.i.d.) normal random variables.

1. Distribution of 2X1:

* If X1 ~ N (μ, σ2), then 2 X1 follows normal distribution.
* The mean of 2 X1 is 2μ and the variance is 4σ2.
* So, 2X1 ~ N (2μ, 4σ2).

1. Distribution of X1 + X2.

* The sum of two independent normal random variables is also normally distributed.
* The mean of X1 + X2 is 2μ and the variance is 2σ2 (since they have the same variance).
* So, X1 + X2 ~ N (2μ, 2σ2).

In summary:

* **The Distribution of 2X1:** 2X1 ~ N (2μ, 4σ2).
* **The Distribution of X1 + X2:** X1 + X2 ~ N (2μ, 2σ2).

The key difference is the variance. The distribution of 2X1 has the variance that is 2 times that of X1 + X2.

This illustrates a general property when combining random variables. Multiplying a random variable by a constant increases the spread of the distribution (variance is scaled by the square of the constant), while adding independent random variables tends to increase the spread, but not as dramatically (variance is added linearly).

In summary, while both 2 X1 and X1 + X2 are normally distributed with mean 2μ, their variances differ due to the scaling effect in 2X1, making its distribution more spread out.

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:

To find two values, a and b, symmetric about the mean (μ) such that the probability of the random variable taking a value between them is 0.99, we can use the properties of the standard normal distribution and convert from the given normal distribution N (100, 202) to the standard normal distribution N (0, 12).

The standard normal distribution has a cumulative distribution function (CDF) denoted as Z, and it is often denoted by Φ(z). Using this, we can find the values a and b as follows:

P (a < X < b) = P () = P ()

Given that the probability is 0.99, we need to find the corresponding z-scores from the standard normal distribution. In the standard normal distribution table, the critical values of and are such that:

P (− < Z < ) = 0.99

You typically look up these values in a standard normal distribution table or use statistical software. For a 99% confidence interval, is approximately 2.576 and is approximately -2.576.

Now, we can set up the equations for a and b:

Solving for a and b:

A = −2.576 × 20 - 100 = 48.48 ≈ 48.5

b = 2.576 × 20 + 100 = 151.52 ≈ 151.52

The answer is option D. 48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

Given:

Profit1 follows a normal distribution N (5,) (mean = 5, variance = ).

Profit2 follows a normal distribution N (7, ) (mean = 7, variance = ).

The exchange rate is 1 USD = 45 INR.

Let's denote X as the total profit of the company, where

X = Profit1 + Profit2.

A. Rupee Range for 95% Probability:

* The total profit X follows a normal distribution N ( + , + ), where and are the means of Profit1 and Profit2, and and are their variances.
* For X, + 5 + 7 = 12 (mean of the sum) and = + = 9 + 12 = 25(variance of the sum).
* z is the z-score corresponding to the desired confidence level (for 95%, z≈1.65),
* The standard deviation = √25 = 5.
* = 5/100 = 0.05
* For two tailed test level of significance = = 0.025 Is similarly = 1 - 0.025 = 0.975 hence and are 2.5 and 97.5 percentile of normal distribution
* = , -2.81 = = (-2.86 × 5) – 12 = -14.8 + 12 = -2.8 million
* = , 1.96 = = (1.96 × 5) + 12 = 9.8 + 12 = 21.8 million

B. 5th Percentile of Profit in Rupees:

* The 5th percentile corresponds to the z-score z such that P (X ≤ x) = 0.05.
* = , -2.55 = = (-2.55 × 5) + 12 = - 12.75 + 12 = 0.75 millions

C. Probability of Making a Loss:

To compare the probability of making a loss for each division, we calculate the probability of profit being less than zero (negative) for each division based on their respective normal distributions.

For Profit1:

* Mean (μ1) = $5 million
* Standard Deviation (σ1) = √(32) = $5.657 million

Using these parameters, we calculate the Z-score for zero (since zero corresponds to the break-even point): = =−0.886

Then, we find the cumulative probability using the standard normal distribution table .

The probability of Profit1 being less than zero Is 0.1894.

For Profit2:

* Mean (μ2) = $7 million
* Standard Deviation (σ2) = √(42) = $6.4807 million

we calculate the Z-score for zero: = =−1.078

Then, we find the cumulative probability for Profit2 being less than zero is 0.1423.

Let's assume:

* Probability of making a loss for Profit1 = P(Profit1<0)
* Probability of making a loss for Profit2 = P(Profit2<0)

P(Profit1<0)>P(Profit2<0) = 01894 > 0.1423 ,

Therefore Profit1 has a higher probability of making a loss.