More on kernel contraction in $\mathcal{EL}$

by

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Abstract

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Dedication

Acknowledgements

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Introduction

Learning is one of the most intuitive functions of the human mind. It involves translating human's perception into knowledge, and adapting the state of mind according to that knowledge. Seeing the sunrise in the morning, one would intuitively change his mind and believe that it is not dark anymore; that would logically imply that one gives up the current belief that it is dawn time and replace it with a new belief that it is sunrise time. This is an example of belief revision. Revision is a kind of change in which the new belief ("it is sunrise") is conflicting with the current state of mind (believing that "it is dawn").

Learning can be thought of as the manipulation of beliefs according to perception. When humans perceive any change in their world, they change their knowledge accordingly. So changing the knowledge (or beliefs[1]) of an agent[2] can be seen as the reality of learning. One example of belief change is **Revision**, which involves changing the knowledge base in order to add a conflicting belief. This can actually be broken down into two processes: changing the knowledge base to account for the conflict, and adding the new belief.

The first process involves removing the beliefs that caused that conflict. This process is called **Belief Contraction**. The second process involves adding the new belief and expanding the knowledge base accordingly. This process is called **Belief Expansion**. Contraction is the type of change we are mainly concerned with in this study. In the following chapter, we will discuss in some level of detail these types of change with more focus on contraction.

The knowledge of an agent can be represented as a set of beliefs. An agent is assumed to believe in *A* if *A* is a member of its *belief set*. In this study, the language used to represent belief sets is *Description Logic*. Description Logic is a family of formalisms that are used to represent knowledge. They use concepts to represent classes of individuals and roles to represent relationships between them. They have different expressive powers and different

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[1]Throughout this study we will be using the terms "knowledge" and "beliefs" interchangeably, although they are not exactly the same. Knowledge is usually assumed to be a special kind of beliefs. However, for convenience, when we use the word "knowledge" we will be referring to "belief."

[2]The word "agent" here means humans, computers, or any thing that has knowledge base and perception.

reasoning mechanism with different complexities. They vary according to the set of logical operators they use. Here we use $\mathcal{EL}$, which is a member of the Description Logic family.

We start the study by discussing the basic types of belief change, but before that, we build a ground for them by defining the framework that was introduced by Gardenfors. We define epistemic states and attitudes that will help in understanding the mechanics of belief change. We then explain some postulates introduced in the AGM framework; those postulates are considered rationality rules for belief change operations. Then we discuss what description logic is, what logical operators are used and what it is composed of. In that discussion, we focus on the most relevant variation, which is $\mathcal{EL}$. And we explain the most important reasoning algorithm for $\mathcal{EL}$.

Chapter 2

Background

2.1 Belief change

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Chapter 3

Kernel contraction

Belief contraction is the process of removing beliefs from belief sets. It can either be done by selecting one of the largest subsets of the belief set that do not logically imply the abandoned belief, or by breaking all the minimal sets that logically imply that belief. The latter approach is the one we are going to focus on in this study. We call such minimal sets *kernels*. Kernels will be discussed in more details in the coming section. But we need to go over some definitions first.

Consider the following set:

$$\{a, a \rightarrow b, b, c\}$$

To contract the set by b, it is not enough to remove b from the set. If we do so, we get:

$$\{a, a \rightarrow b, c\}$$

which still implies b. We need to prevent the resulting set of beliefs from implying b. The minimal subset that implies b is

$$\{a, a \rightarrow b\}$$

We call it a b-kernel. Since the kernel is a minimal subset that implies b, removing only one element of that kernel is enough to make it not imply b. The only other b-kernel of the original set is $\{b\}$; we can only break this kernel by removing b. Breaking the other kernel can be done either by removing a, or by removing $a \rightarrow b$. So the two possible resulting belief sets after contraction are:

$$\{a, c\} \text{ and } \{a \rightarrow b, c\}$$

This is how kernel contraction works. Every b-kernel is one way to infer b. To contract b every kernel should be broken by removing at least one belief.

3.1 What are kernels?

Kernel contraction was introduced by Hansson[7] as a variant of an older approach called "safe contraction"[2]. In both approaches, contracting a knowledge base $\mathbf{K}$ by α is done by discarding beliefs that contribute to make $\mathbf{K}$ imply α. Beliefs that contribute to make $\mathbf{K}$ imply α are members of some α-kernels of $\mathbf{K}$, and all members of every α-kernel contribute to make $\mathbf{K}$ imply α. In the previous example, there were two b-kernels: $\{b\}$ and $\{a, a \rightarrow b\}$. We need to remove at least one element from each kernel to contract the original set by b. We refer to the set of α-kernels of $\mathbf{K}$ by $\mathbf{K} \perp \alpha$.

Definition 3.1. (Kernel Set) According to [6], for a belief set $\mathbf{K}$ and a belief α, $\mathbf{K} \perp \alpha$ is a set such that $X \in \mathbf{K} \perp \alpha$ if and only if:

1. $X \subseteq \mathbf{K}$,

2. $X \vdash \alpha$, and

3. if $Y \subset X$ then $Y \nvdash \alpha$.

$\mathbf{K} \perp \alpha$ is a kernel set that contains all α-kernels of $\mathbf{K}$.

Kernels are the smallest subsets of the knowledge base that imply a specific belief. Therefore, in order to give up that belief, every one of those kernels needs to be incised. If those kernels are not "minimal", there might contain some beliefs that do not contribute to make the kernel imply that specific belief. For example, if $\{\alpha, \beta\}$ is an α-kernel, β is a belief that does not contribute to make the kernel imply α. In that case, removing β only would not help in giving up α.

So, because of the minimality of the kernels, every belief in the kernel is significant to implying the belief that we want to give up (α). And that's why removing one belief from a kernel is sufficient to make that kernel not imply α. So, we use an incision function that cuts every kernel in the kernel set. When the incision function is given an α-kernel set, it selects at least one element from each kernel to be removed.

The incision function is defined in [6] as follows:

Definition 3.2. (Incision function) An incision function σ is a function such that for all beliefs α:

1. $\sigma(A \perp \alpha) \subseteq \bigcup(A \perp \alpha)$

2. If $\phi \neq X \in A \perp \alpha$, then $X \cap \sigma(A \perp \alpha) \neq \phi$

Contraction is done by removing the beliefs that are selected by the incision function from the original knowledge base. Contraction $\approx_\sigma$ using the incision function σ can be defined as:[6]

Definition 3.3. (Contraction)

$$A \approx_\sigma \alpha = A \smallsetminus \sigma(A \perp \alpha)$$

3.2 Computing kernels

Kernel contraction is about contracting knowledge bases using kernels. Kernels are the minimal subsets of the knowledge base that imply a certain consequence. The incision function can perform the contraction by cutting through all kernels of a specific belief. So, the first step in contraction is computing all kernels that imply the belief that needs to be removed. For that purpose, we use the pinpointing algorithm introduced in [5].

To show how the algorithm works, we use the example introduced in [5]. Given an $\mathcal{EL}$ TBox T consisting of the following four GCI axioms:

$$T = \{ax_1 : A \sqsubseteq \exists r.A, \quad ax_2 : A \sqsubseteq Y, \quad ax_3 : \exists r.Y \sqsubseteq B, \quad ax_4 : Y \sqsubseteq B\},$$

we can see that $A \sqsubseteq B$ holds according to T, i.e. $A \sqsubseteq_T B$. Let:

$$\alpha = A \sqsubseteq B.$$

According to the definition of kernel sets:

$$T \perp \alpha = \{\{ax_2, ax_4\}, \{ax_1, ax_2, ax_3\}\}$$

The algorithm introduced in [5] computes all kernels using a modified version of the $\mathcal{EL}$ subsumption algorithm. It works by finding a monotone boolean formula called *"pinpointing formula"*. The propositions of the pinpointing formula are GCIs of the TBox, and a propositional *valuation* represents the kernels. In that sense, a valuation is a set of propositional variables that satisfy the formula, and these variables are the GCIs that constitute a kernel. So, by finding all valuations of the pinpointing formula we can get all kernels.

In worst case, this approach takes exponential time (w.r.t the size of the TBox) to find all kernels. This is the case when there are exponentially many kernels. However, [5] also gives a polynomial-time algorithm that computes only one kernel. Because it is not part of the scope of this study, we are not going to discuss and analyze details of the pinpointing algorithm. All that matters is the worst-case time complexity, as it will affect the complexity of the kernel contraction algorithm that uses it.

3.3 Previous work

We use the pinpointing algorithm to get the set of all kernels. We then need to remove one element from each kernel to perform contraction. This is the role of the incision function; it picks an element from each kernel so that it cuts through all kernels. In this section, we look at some of the incision function implementations discussed in [9]. The goal of this study is to continue the work done in [9], and to discuss some of what has already been done.

Given an $\mathcal{EL}$ TBox T and a belief α, $(T \perp \alpha)$ is the set of α-kernels. The main contraction algorithm look like the following:

Algorithm 1 Contraction algorithm

1: **procedure** CONTRACT(T, A)
2:     kernelset = PINPOINT(T, A)
3:     giveUpSet = CUT(kernelset)
4:     T = T / giveUpSet
5: **end procedure**

"kernelset" refers to $(T \perp \alpha)$, and the *giveUpSet* is the set of beliefs selected by the incision function (CUT) to be removed. The algorithm is straight forward. It works by generating the set of kernels using the pinpoint formula as discussed in the previous section. Then it calls the incision function that picks up beliefs from every kernel, ensuring that it cuts all kernels. The beliefs are then removed from the knowledge base, i.e. from the TBox.

This general approach can be used with different incision functions. The call to the CUT function can be replaced with a call to another implementation of the incision function. The first implementation is the most straight-forward one, where it removes a random belief from every kernel. It looks like the following:

Algorithm 2 Random removal

1: **function** RANDOMREMOVE(kernelset)
2:     giveUpSet = {}
3:     **for** $kernel \in kernelset$ **do**
4:         choose a random belief α from *kernel*
5:         giveUpSet = giveUpSet $\cup \alpha$
6:     **end for**
7:     **return** giveUpSet
8: **end function**

The time complexity of the RandomRemove function is polynomial: $O(m)$, where m is the number of kernels (size of the kernelset, and assuming that the random selection takes constant time. However, this is clearly not a good algorithm. In this example:

$$kernelset = \{\{a, c\}, \{b, c\}\},$$

one of the possible outcomes of the RandomRemove algorithm is:

$$giveUpSet = \{c, b\}$$

which unnecessarily removes b. This could happen when the algorithm selects c from the first kernel and b from the second kernel. This solution:

$$giveUpSet = \{c\}$$

seems more concise and removes less beliefs. And it could be done easily if the algorithm is smart enough to check if the second kernel has already been incised or not.

3.4 Minimal change

According to information economy principal (minimality), in every change of the epistemic state loss of information should be minimum[10]. This means that a system should choose an epistemic change outcome that minimizes loss of information. To satisfy the requirement of the minimum change we need an algorithm that removes the least number of GCIs while hitting all the kernels; but the kernels are nothing but sets of beliefs (GCIs). Luckily, this is exactly the *minimal hitting set problem.*

3.4.1 Hitting set approach

The minimal hitting set problem can be defined as follows:

Definition 3.4. Given a set $S = \{s_1, s_2, ..., s_n\}$ of n non-empty sets, a minimal hitting set d is a set such that:

$$\forall_{s_i \in S}[s_i \cap d \neq \emptyset] \wedge \nexists_{d' \subset d}[\forall_{s_i \in S}(s_i \cap d' \neq \emptyset)][1]$$

In the context of kernel contraction in $\mathcal{EL}$, we can define the minimal hitting set contraction as:

Definition 3.5. (Minimal hitting set contraction) Given a kernelset $K = \{k_1, k_2, ..., k_n\}$ of n kernels, a minimal hitting set $giveUpSet$ is a set such that:

$$\forall_{k_i \in K}[k_i \cap giveUpSet \neq \emptyset] \wedge \nexists_{giveUpSet' \subset giveUpSet}[\forall_{k_i \in K}(k_i \cap giveUpSet' \neq \emptyset)]$$

And since kernels are subsets of the $TBox$, and our goal to find a $giveUpSet$ that hits all kernels, we can use approaches to the minimal hitting set problem to solve it. There are good approximation algorithms, such as the one introduced in [1], for the minimal hitting set problem, that are feasible in terms of running time.

We can now use them to implement contraction for $TBox$, by implementing the minimal incision function that adopts one of them, to achieve minimum change (which we can call then, a minimal incision function). We are not going to implement such function in this study, but for now, we will assume that there is a function:

```
min-hit-CUT(kernelSet)
```

that takes a set of kernels, and returns a minimal hitting set of sentences to give up. We can use this function, as if it is implemented, and may implement it in some future work.

3.4.2 Reduction to Network-flow problem

Now, we introduce another technique for kernel contraction in $\mathcal{EL}$ that is based on graphs. One of the motivating reasons for this approach is that a lot of work has been done in the area of Graph Theory, which makes it easy to use the already existing algorithms

to solve some graph problems. So if we can reduce a problem to a graph problem, that has a polynomial-time algorithm, we can solve our problem in polynomial time. Another motivating reason is that graphs are easy to imagine and work with.

Our goal is to reduce the kernel contraction problem to a graph problem, and, using some efficient graph algorithms, perform kernel contraction. We start by describing the algorithm, and then explain each of its main steps.

3.4.3   Main algorithm

We assume that every GCI of the form $C \sqsubseteq D \sqcap E$ can be decomposed into two GCIs: $C \sqsubseteq D$ and $C \sqsubseteq E$, where C, D and E are arbitrary concept expressions. Given an $\mathcal{EL}$ *TBox* T, and a GCI A (where A is of the form $C \sqsubseteq D$, such that C and D are arbitrary concept expressions), we contract T by A using Algorithm 3:

Algorithm 3 Contraction using graph approach

1: **function** GRAPHCONTRACT(T, A)
2:       complete(T)
3:       graph = transform(T)
4:       paths = getPaths(graph, C, D)
5:       cutPaths(graph, paths)
6:       T = de-transform(graph)
7: **end function**

The complete function in step 2 uses the subsumption algorithm to compute all subsumptions of T. All the subsumptions computed are added to T. The algorithm proceeds in four steps:

1. Normalize the TBox.

2. Translate the normalized TBox into a graph.

3. Complete the graph using completion rules.

4. Read off the subsumption relationship from the normalized graph.[3]

The algorithm is explained in full details in [3]. We now need to explain the transformation of the *TBox* T into a graph.

3.4.4   Transforming the *TBox* into a graph

Assuming the *TBox* T contains GCIs of the form $C \sqsubseteq D$, we construct a graph $graph = (V, E)$, where V is a set of nodes and E is a set of directed edges. Starting with an empty V and E, for every GCI $C \sqsubseteq D$, add C and D to V, and add (C, D) to E. This way every $v \in V$ represents a concepts (or a conjunction of concepts), and every pair $(C, D) \in E$ represents the subsumption relation $C \sqsubseteq D$.

Algorithm 4 Transforming a TBox to a graph

1: **function** TRANSFORM(T)
2:     result = new Graph(V, E)
3:     **for** every $C \sqsubseteq D$ in T **do**
4:         $V = V \cup \{C, D\}$
5:         $E = E \cup \{(C, D)\}$
6:     **end for**
7:     **return** result
8: **end function**

3.4.5 Transformation analysis

It is worth mentioning that even though subsumption can be -using the algorithm in [4]-
decided in polynomial time with respect to the size of the $TBox$, computing all subsump-
tions can be exponential in the number of concepts. For example, with $T = \{Human \sqsubseteq Mammal, Mammal \sqsubseteq Animal\}$, the new T after adding all subsumptions would be $\{Human \sqsubseteq Human, Human \sqsubseteq Mammal, Human \sqsubseteq Animal, Mammal \sqsubseteq Mammal, Mammal \sqsubseteq Animal, Animal \sqsubseteq Animal\}$. The new $TBox$ contains a lot of unneeded GCIs. This can
be avoided by adding a small step to the subsumption algorithm in [4], such that after com-
pleting the subsumption graph using the completion rules, we can remove all subsumptions
of the form $X \sqsubseteq X$ (e.g. $Human \sqsubseteq Human$); also we need to remove all subsumptions
$C \sqsubseteq D$ if $C \sqsubseteq X$ and $X \sqsubseteq D$ (e.g. exclude $Human \sqsubseteq Animal$ if $Human \sqsubseteq Mammal$, and
$Mammal \sqsubseteq Animal$ are in the TBox) are already there.

Now we have a graph *graph* that represents the $TBox$ T. The next step is to compute the
paths (using getPaths() function)from C to D (where C and D are graph nodes) using depth-
first search. Obviously, computing the paths using depth-first search can take polynomial
time. Assuming the TBox is acyclic as we mentioned before, hence the graph too, this can
be done as in Algorithm 5.

This can also be done if the graph contains cycles. It would require using a more
complicated algorithm that keeps track of the number of edges and the nodes visited during
execution. Finding paths in cyclic graphs in explained in details in [8].

3.4.6 Graph kernel contraction

The function cutPath is the contraction function. Every path from C to D is actually a
subsumption path that entails $C \sqsubseteq D$. Removing an edge from such path would break the
subsumption between C and D, and C would no longer be subsumed by D – unless there
is another subsumption line. So, in order to contract $C \sqsubseteq D$ it is enough to remove one
edge from each of the paths from C to D; as each path represents a *kernel* of $C \sqsubseteq D$ and
breaking them is sufficient to remove it.

Algorithm 5 Computing all paths between two nodes

1: **function** GETPATHS(graph, C, D)
2:      result = {}
3:      Stack path = new empty Stack
4:      computeAllPaths(graph, C, D, path, result)
5:      **return** result
6: **end function**

1: **function** COMPUTEALLPATHS(graph, C, D, path, paths)
2:      graph = (V, E)
3:      **for** every $(C, X) \in E$ **do**
4:          **if** X = D **then**
5:              Stack temp = new empty Stack
6:              temp.pushAll(path) // adds all edges without changing path
7:              temp.push((C, X))
8:              $paths = paths \cup \{temp\}$ // adding the path to the list of paths
9:          **else**
10:              path.push((C, X))
11:              computeAllPaths(graph, C, X, path, paths)
12:              path.pop()
13:          **end if**
14:      **end for**
15: **end function**

So, a simple and very easy implementation (though inefficient) for the contraction function is to go over the set of paths, and remove a random edge from every path. Suppose that the graph in Figure 3.1 is interpreted as C is the most specific concept that subsumes A ($C \sqsubseteq A$), and as it goes down the graph the concepts get more general. Here there are four C-D paths: (C, X, Z, A, E, D), (C, X, Z, A, F, D), (C, Y, Z, A, E, D), and (C, Y, Z, A, F, D). A function that removes random edges from each path might remove (C, X), (Y, Z), (E, D), and (F, D), which will actually remove the subsumption between C and D; however, it would be better (in terms of minimal change) to remove only (Z, A), which will also guarantee that subsumption between C and D is removed.

As we mentioned earlier, sometimes we prefer to remove GCIs that involve most specific concepts during contraction. To contract such GCIs, we can just remove, from each $C - D$ path, the edges emanating from C. In Figure 3.1, it would mean removing two edges: (C, X) and (C, Y). But removing such edges does not guarantee the minimum change. So we might end up having to choose which strategy is more preferred: minimal change or change with most specific concepts. The minimal hitting set approach that was mentioned earlier might not always satisfy specificity. So the user might have to choose which one to apply first, and which one to use as a tie breaker.

Removing the edges that involve nodes representing most specific concepts is straightforward; remove the edges that emanate from the source node (C in our example). But it
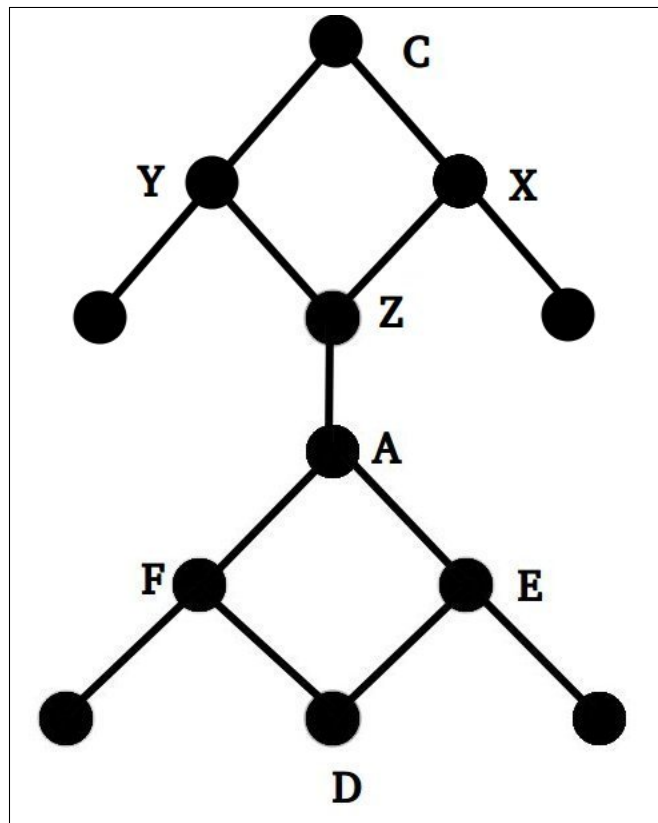
Figure 3.1: Four different paths between C and D, that share one edge.

is not clear if one chooses to remove least number of edges, instead, how this can be done. For this, we introduce, in the coming section, an approach that uses the Minimum Cut algorithm to determine the minimum number edges that need to be removed and identify them.

3.4.7 Reduction to network flow problem

As explained in [8], the network flow problem is the problem of solving a flow in a network -represented as a graph- by finding the minimum cut of the network (or the maximum flow). The input to the Ford-Fulkerson algorithm, that solves this problem, would be:

- A graph $G = (V, E)$.

- A source $s \in V$.

- A sink $t \in V$.

- Capacity function $C : E \rightarrow \mathbb{N}$ representing the maximum capacity of each edge.

To contract $C \sqsubseteq D$ given the $TBox$ graph G, we choose C to be the source, D to be the sink, and we assume that the capacities of all edges are the same, equal to 1. The capacity is not important here, as it is normally used to determine the bottleneck edges of the network to optimize the flow, while in our case, they are only needed to determine which edges are being used; so, an edge is used in the flow if the flow passing through it is 1, and it can never be greater than 1. The algorithm will find the maximum flow from C to D, which is equal to the capacity of the minimum cut (the sum of capacities of the cut edges); we can then extract the edges that form that minimum cut and remove them.

The approach minimum cut of the graph is equivalent to the approach of the minimal hitting set of the kernels. The minimal hitting set is the set containing the minimum number of elements that hit all sets, while the minimum cut of the graph is the minimum number of edges (since they all have the same capacity) that cover all paths from C to D (where edges represent GCIs of the TBox and paths represent kernels). So using the minimum cut approach should guarantee the minimum change for kernel contraction, as the minimal hitting set approach does.

Assuming we have a function "Ford-Fulkerson(graph, s, t)" that computes the maximum flow in the network "graph" from source node "s" to a sink "t" with edge-capacities "1", and returns the set of edges of the minimum cut, we can modify the contraction algorithm to adopt the minimum cut approach as in Algorithm 6.

For special cases such as contracting $C \sqsubseteq D1 \sqcap D2$, it is sufficient to contract $C \sqsubseteq D1$ first, and then contract $C \sqsubseteq D2$. But since the graph is already normalized (using complete() function), rules of the form $C \sqsubseteq D1 \sqcap D2$ are already broken down into two: $C \sqsubseteq D1$, and $C \sqsubseteq D2$. Therefore, the conjunction $\sqcap$ would only appear on the left hand

Algorithm 6 Another version of contraction algorithm

1: **function** GRAPHCONTRACTUSINGMINCUT(T, A)
2:     complete(T)
3:     graph = transform(T)
4:     $A = C \sqsubseteq D$
5:     min-cut = Ford-Fulkerson(graph, C, D)
6:     remove min-cut edges from graph
7:     T = de-transform(graph)
8: **end function**

side of a GCI in a normalized TBox (e.g. $A1 \sqcap A2 \sqsubseteq B$). In that case, for contracting $A1 \sqcap A2 \sqsubseteq B$, there will be a node $A1 \sqcap A2$, which we will use as a source node; the sink would be the node representing B.

As in [8], the minimum cut algorithm runs in polynomial time. So using it in the contraction algorithm will not have a significant effect on the complexity of the main algorithm (will not elevate the complexity from being polynomial to being exponential).

In some applications, the decision about which strategy to follow for choosing the edges to remove might vary depending on the situation. So the user can be asked in such case about which strategy to follow – specificity or minimality.

The last step of the algorithm is to transform the graph back to $\mathcal{EL}$. This can be done as follows: starting with an empty $TBox\ T'$, for every edge (X, Y), add $X \sqsubseteq Y$ to T'. The resulting $TBox$ would be the result of contracting T by A. This is discussed in Algorithm 7.

Algorithm 7 Transforming a graph back to a TBox

1: **function** DE-TRANSFORM(graph)
2:     result = {}
3:     graph = (V, E)
4:     **for** every $(X, Y) \in E$ **do**
5:         $result = result \cup \{X \sqsubseteq Y\}$
6:     **end for**
7:     **return** result
8: **end function**

Since the running time of every step of the contraction algorithm starting from "transform(T)" until the last step is polynomial in the size of the TBox, the complexity of the algorithm will depend on the complexity of the first step (the complete() function). If building the subsumption hierarchy by generating all subsumptions of an $\mathcal{EL}\ TBox$ can be done in polynomial time, then the contraction algorithm will in turn take polynomial time. But if generating all subsumptions takes exponential time, then the algorithm will take exponential running time as well.

Chapter 4

Heuristics to contraction

Considering algorithms that guarantee contraction with minimum change is not always enough for optimal contraction. Optimality in semantics is always important to consider. This means that if we have two possible *giveUpSets* of the same size, one could be better than the other. So far, we have only implemented algorithms that compute the smallest *giveUpSets* without looking at what is being removed.

4.1 Localization

4.2 Specificity

Sometimes we need some domain-specific and language-specific knowledge to decide which beliefs to give up – if we have two equally minimum *giveUpSets*. The subsumption hierarchy enforced by subsumption relation in $\mathcal{EL}$ somehow categorizes the beliefs (or GCIs) into different levels of generality and specificity. Consider the kernel in Figure 4.1, which implicitly entails

$$Lion \sqsubseteq Animal$$

And suppose we would like to contract the $TBox$ by $Lion \sqsubseteq Animal$. We have three options: we can remove $Lion \sqsubseteq Mammal$, $Mammal \sqsubseteq Vertebrate$, or $Vertebrate \sqsubseteq Animal$. Removing any of the three GCIs would guarantee minimum change. However removing $Lion \sqsubseteq Mammal$ sounds much more reasonable than removing $Vertebrate \sqsubseteq Animal$; $Lion \sqsubseteq Mammal$ involves concepts that are more specific than the ones involved

$$Lion \sqsubseteq Mammal$$
$$Mammal \sqsubseteq Vertebrate$$
$$Vertebrate \sqsubseteq Animal$$

Figure 4.1: Simple Animal kernel

in $Vertebrate \sqsubseteq Animal$. Removing $Vertebrate \sqsubseteq Animal$ may affect more concepts in the subsumption hierarchy (in worst case) than those affected by removing $Lion \sqsubseteq Mammal$.

So, it is preferable to remove GCIs that involve specific concepts rather than removing GCIs that involve more general ones. To account for specificity, we can assign a label to each of the concepts representing its level (or weight) – where levels increase with generality. And during contraction, we consider contracting GCIs that involve concepts at lower level before considering GCIs that involve concepts at higher level.

The approach we follow in this study for adopting the preference of removing sentences that contain more specific concepts is by assigning weights to every GCI that reflects its generality, then we select the kernels with minimal overall weight. Every GCI gets a numeric weight depending on the level of generality of the concepts involved in it (depending on their position in the subsumption graph). A GCI $A \sqsubseteq B$ gets weight '0' if A has no children, i.e. if there is no rule of the form $X \sqsubseteq A$, where X is an arbitrary concept; and the GCI involving its parent will have weight '1' in that case. So the weight somehow represents the level in the subsumption graph, starting from level '0' at the trees' roots. For simplicity, we assume that the $TBox$ is acyclic – so we avoid the problems that will be caused by loops.

Similar to the hierarchy of the subsumption graph of an $\mathcal{EL}$ TBox, we give a definition to *parent* and *child* concepts as follows:

Definition 4.1. For a GCI $A \sqsubseteq B$:

- A is a *child* concept, and

- B is a *parent* concept.

So by looking again at Figure 4.1, we can now say that:

- Mammal is a *parent* of Lion, and Lion is a *child* of Mammal

- Vertebrate is a *parent* of Mammal, and Mammal is a *child* of Vertebrate

- Animal is a *parent* of Vertebrate, and Vertebrate is a *child* of Animal

- Animal is the most *general* concept, i.e. Animal subsumes all other concepts

- Lion is the most *specific* concept (least *general*), i.e. it is subsumed by all other concepts

- $Lion \sqsubseteq Mammal$ has weight = '0'

- $Mammal \sqsubseteq Vertebrate$ has weight = '1'

- $Vertebrate \sqsubseteq Animal$ has weight = '2'

The algorithm we introduce now removes kernels that contain GCIs involving most specific concepts by first computing the weights of every kernel based on the weights of the GCIs inside it. Following Definition 4.1, we start the algorithm by building the *children* labels for all the concepts in the *TBox* that we will use to assign weights to the GCIs. Given a *TBox T*, the algorithm proceeds as in Algorithm 8.

Algorithm 8 Building the children graph

1: **function** GRAPHBUILD(T)
2:      $children(X) = \emptyset$, for every concept X in T
3:      **for** every $A \sqsubseteq B \in T$ **do**
4:          $children(B) = children(B) \cup \{A\}$
5:      **end for**
6: **end function**

Now we have a graph composed of concepts and their *children* relation; the nodes are concepts and the edges are the child-parent relationship between them. The roots are now concepts with no parents, and the leaves are concepts with no children. The GCIs involving leaves are assigned weight '0' and it increases by '1' every step towards the roots. So, the roots are most general concepts, and the leaves are most specific. If a GCI contains concepts $\alpha \sqsubseteq \beta$, where α has no parents and β has no children, then it gets weight '0' according to Algorithm 9.

Now we have every GCI in T assigned a weight relative to its level of generality. So, in every kernel we have a preference level of what to give up. Based on the weights, we will choose the sentences with less weight over the ones with more weight to remove. This preference can be used as a tie-breaker after we apply the minimal hitting set algorithm and end up with more than one minimal set. If we arrive at two minimal hitting sets for the kernels, we can compute the overall weight of each of the hitting sets and remove the sentences in the one with lower weight. This is shown in Algorithm 10. It uses the min-hit-CUT algorithm that gets the minimum hitting sets given a kernel set. There could be more than one minimum hitting set.

Now the CUT method in Algorithm 1 can be re implemented using the algorithms introduced here to account for specificity heuristics. Given a TBox *T* and a GCI *A*, the contraction algorithm can look like Algorithm 11.

The contraction algorithms in this version guarantees the minimality and specificity of the removed beliefs. It guarantees the minimality by reducing the selection of beliefs from the kernelset to the minimal hitting set problem. The better the performance and the optimality of the hitting set algorithms, the more minimal and efficient the contraction algorithm is. If the minimal set algorithm produces more than one solution of the same size, the specificity heuristic is used as a tie-breaker. It selects the solution with more specific beliefs to be removed.

Algorithm 9 Assigning Weights

1: **function** ASSIGNWEIGHTS(T)
2:     **for** every $A \sqsubseteq B \in T$ **do**
3:         $weight(A \sqsubseteq B) = GetWeight(A \sqsubseteq B)$
4:     **end for**
5: **end function**

1: **function** GETWEIGHT($X \sqsubseteq Y$)
2:     **if** $weight(X \sqsubseteq Y) \neq NULL$ **then**
3:         **return** $weight(X \sqsubseteq Y)$
4:     **end if**
5:     **if** $children(X) = \emptyset$ **then**
6:         **return** 0
7:     **else**
8:         **return** $1 + GetMaxWeight(X)$
9:     **end if**
10: **end function**

1: **function** GETMAXWEIGHT(X)
2:     max $= -1$
3:     **for** every Z in children(X) **do**
4:         w = GetWeight($Z \sqsubseteq X$)
5:         **if** w > max **then**
6:             max = w
7:         **end if**
8:     **end for**
9:     **return** max
10: **end function**

Algorithm 10 Removing specific hitting set

1: **function** GETMOSTSPECIFICHIT(kernelset)
2:     min-hit-sets = min-hit-CUT(kernelSet)
3:     $min = \infty$
4:     hit = NULL
5:     w(s) = 0, for every $s \in min-hit-sets$
6:     **for** every s in min-hit-sets **do**
7:         **for** every $A \sqsubseteq B$ in s **do**
8:             $w(s) = w(s) + weight(A \sqsubseteq B)$
9:         **end for**
10:         **if** w(s) < min **then**
11:             min = w(s)
12:             hit = s
13:         **end if**
14:     **end for**
15:     **return** hit
16: **end function**

Algorithm 11 Contraction algorithm – modified

1: **procedure** CONTRACT(T, A)
2:     kernelset = PINPOINT(T, A)
3:     graphBuild(T)
4:     assignWeights(T)
5:     giveUpSet = GETMOSTSPECIFICHIT(kernelset)
6:     T = T / giveUpSet
7: **end procedure**

The time complexity of Algorithm 9 is polynomial. The worst-case time complexity is $O(nl)$, where n is the size of the TBox T and l is the depth of the subsumption graph; it works as depth-first search because of the recursive call in *getWeight* function. The *graphBuild* function takes $O(n)$ steps to build the parent-child relationship graph, where n is the size of the TBox T. Finally, the *getMostSpecificHit* function takes $O(nm)$ steps in the worst case, where n is the size of the TBox T, and m is the number of kernel sets. So the contraction algorithm takes polynomial time if the minimum hitting set algorithm runs in polynomial time. But, if the minimum hitting set algorithm is not polynomial-time, then the contraction algorithm will not be. Thus, it is safe to say that the time complexity of the contraction algorithm is lower-bounded by the time complexity of the minimum hitting set algorithm's time complexity.

Chapter 5

Conclusion

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Appendix A

Code