Algorithmic Study of Kernel Contraction in $\mathcal{EL}$

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Our Goal

- Investigate contracting knowledge bases represented in Description Logic $\mathcal{EL}$.
- Provide a graph approach to the contraction operation.
- Design heuristics for choosing the "best" set of beliefs to be removed, if there are multiple options.

Outline

1. Belief Contraction
2. Description Logic $\mathcal{EL}$
3. Kernels
4. Contraction using Graphs
5. Incision functions:
   - Localization
   - Specificity

Belief Contraction

- Birds can fly
- Penguins are birds
- Penguins can fly

- Mumble is a penguin
- Mumble can fly

Belief Contraction

- Birds can fly
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Description Logic $\mathcal{EL}$

TBox

- Bird ⊑ CanFly
- Penguin ⊑ Bird
- Penguin ⊑ CanFly

ABox

- Penguin(Mumble)
- CanFly(Mumble)

Polynomial-time subsumption algorithm: (Tbox ⊢ (A ⊑ B)) ?

A ⊑ B : General Concept Inclusion (GCI)

Description Logic $\mathcal{EL}$

TBox

- Bird $\sqsubseteq$ CanFly
- Penguin $\sqsubseteq$ Bird
- Penguin $\sqsubseteq$ CanFly

ABox

- Penguin(Mumble)
- CanFly(Mumble)

Polynomial-time subsumption algorithm: (Tbox $\vdash$ (A $\sqsubseteq$ B)) ?

A $\sqsubseteq$ B : General Concept Inclusion (GCI)

Kernels

TBox

- Bird ⊑ CanFly
- Penguin ⊑ Bird
- Penguin ⊑ CanFly

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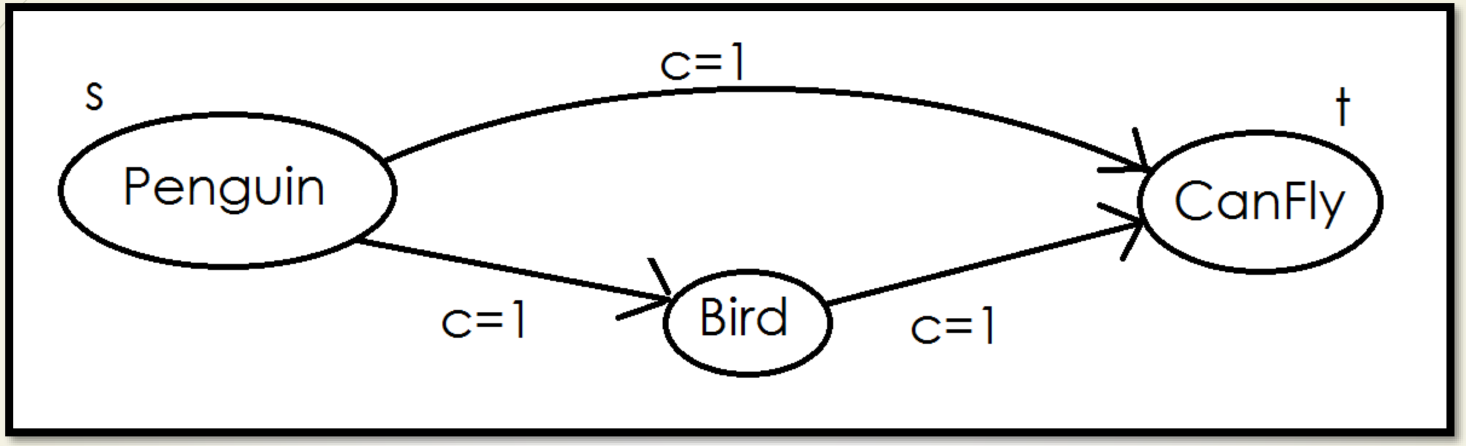
- Smallest subsets of a Tbox that imply a certain belief

Contraction Using Graphs

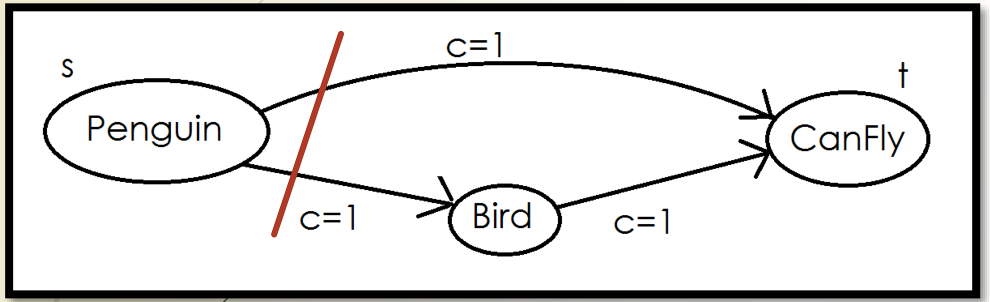
- Reduce the problem to Network-Flow:
  - Transform the TBox into a Graph.
  - Define source, sink, and capacities.
  - Run Ford-Fulkerson algorithm, and compute maximum flow.
  - Compute minimum cut.
  - Remove the edges forming minimum cut.
  - Transform the Graph into TBox.

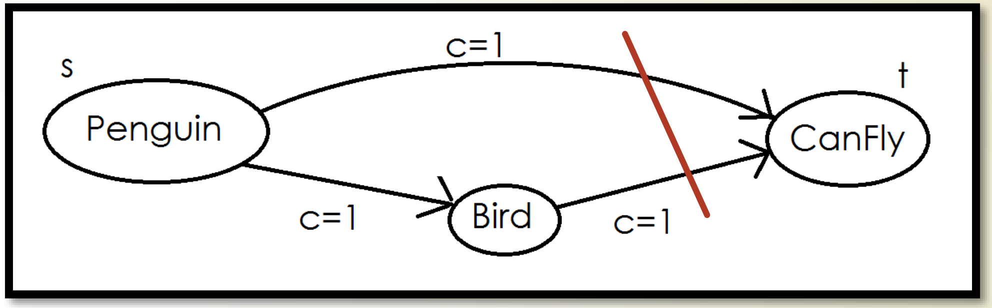
Contraction Using Graphs



- Complexity is $O(E^2)$ – E is the number of edges
- Applicable only under some settings

Contraction Using Graphs





- Complexity is $O(E^2)$ – E is the number of edges
- Applicable only under some settings

Incision Functions

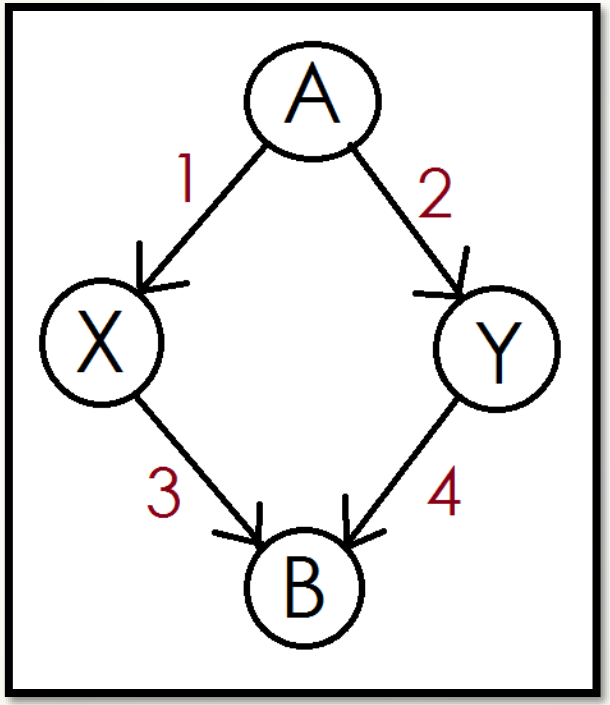
Already have a set of kernels computed – possibly using minimum hitting set algorithm?

Incision Functions: Localization

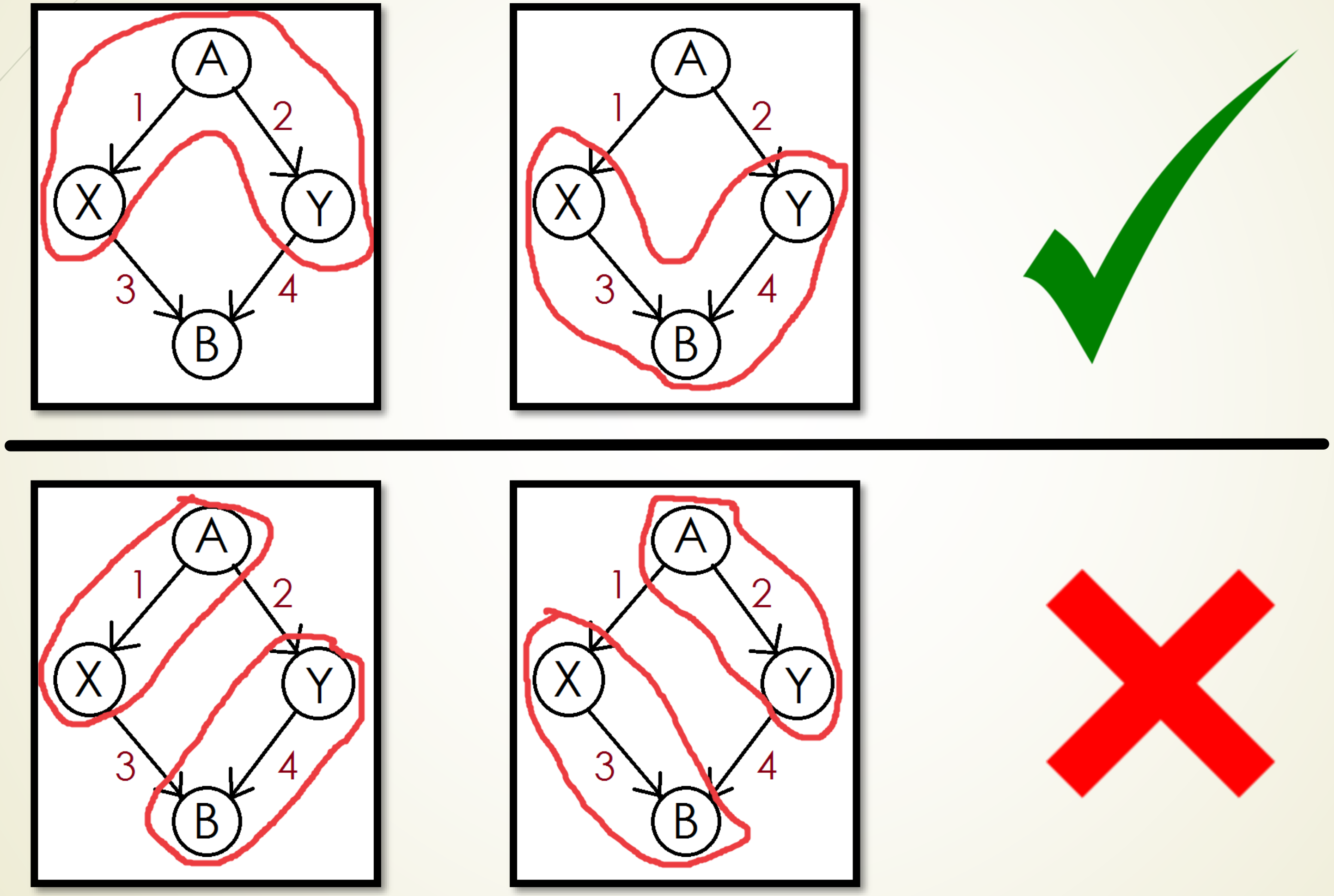
- Intuition:
  - Ensure change is only applied to smallest portion of the knowledge base by affecting least number of concepts: prefer removing GCIs that share concepts or roles to ones that do not.

- Method:
  - Compute the number of strongly-connected components in each potential GiveUpSet.
  - Choose the GiveUpSet with the minimum such number.

Incision Functions: Localization



Incision Functions: Localization



Incision Functions: Specificity

- Intuition:
  - Ensure the effect of contraction is minimized by removing beliefs related to most specific concepts rather than general ones.

- Method:
  - Assign a weight to each GCI based on it's position in the subsumption hierarchy.
  - Assign a weight to each potential GiveUpSet to be the sum of the GCIs' weights.
  - Choose the GiveUpSet with minimum weight.

Incision Functions: Specificity

TBox

- Vertebrate ⊑ Animal weight=2
- Mammal ⊑ Vertebrate weight=1
- Lion ⊑ Mammal weight=0

- Lion ⊑ Animal

Summary

- The contributions of this study are:
  - An graph algorithm for kernel contraction.
  - A revised version of Localization algorithm.
  - A Specificity-based incision function.

Questions?