Final Exam

#2) Solving for M:

$$f(x) = \frac{x}{2} - \sin x$$
$$f'(x) = \frac{1}{2} + \cos x$$
$$f''(x) = \sin x$$

$$M = \frac{\max_{x \in I} \{f''(x)\}}{2 \min_{x \in I} \{f'(x)\}}$$

$$= \frac{\max_{x \in I} \{|\sin x|\}}{2 \min_{x \in I} \{|\frac{1}{2} - \cos x|\}} = \frac{\sin(\pi/2)}{|\frac{1}{2} + \cos(3)|} = \frac{1}{|-.49|} \doteq 1.0204$$

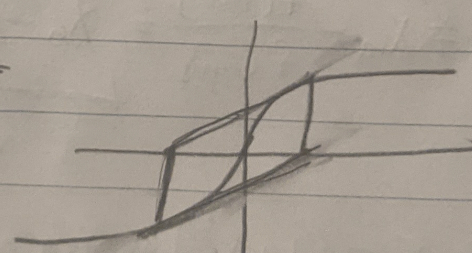
$$= 1.020423789$$

$$|x - x_0| < \frac{1}{M}$$

X_0 has to be within M of X_0 to converge

3) $f(x) = \arctan(x)$ $\quad f'(x) = \frac{1}{1+x^2}$ $\quad f''(x) = \frac{-2x}{(1+x^2)^2}$

$$M = \frac{\max_{x \in \mathbb{R}} |f''(x)|}{\min_{x \in \mathbb{R}} |f'(x)|} = \frac{.6495190528}{\quad}$$



$$f'''(x) = \frac{2(-3x^2+1)}{(x^2+1)^3} = 0$$

$$x = \pm\sqrt{1/3}$$

Max a $x = -\sqrt{1/3}$

$$\frac{+2\sqrt{1/3}}{(1+1/3)^2} = .6495190528$$

min of $f'(x)$ is tricky since it is never zero but infinitely gets smaller if ~~we have a $x_0=\beta$ where it osolates between $-\beta,\beta$~~

then ~~the following is true,~~

$$\cancel{\alpha - x_0 = \alpha - x_1 = \alpha - x_2 = \dots}$$

$$\cancel{(\alpha - x_0)^2 M = (\alpha - x_1)^2 M = \dots = |\alpha - x_n|}$$

$$\cancel{(\alpha - x_0)^2 = (\alpha - x_1)^2}$$

$$x_{n+1} = \cancel{x_n - \frac{f(x_n)}{f'(x_n)} = (x_{n+1})(-1)} \quad \text{where } \cancel{x_n \text{ is } -x_{n+1}}$$

$$\cancel{x_{n+1} = -x_{n+1} - \frac{f(-x_{n+1})}{f'(-x_{n+1})}}$$

$$\cancel{x_n - \frac{f(x_n)}{f'(x_n)} = x_{n+1} = -x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}}$$

We are looking for the x value where

$$x_0 = x_2 = x_4 = \dots =$$

$$x_1 = x_3 = x_5 = \dots = -$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \underbrace{x_n - (1+x_n^2)\arctan(x_n)}$$

We are looking for an x that satisfies

$$-x = x - (1+x^2)\arctan x$$

$$0 = 2x - (1+x^2)\arctan x$$

$x = \beta = 1.39174520027$

but now can we get that?

Well we have this equation that if it holds true the x intercepts are the x values where newtons method equals it's self indefinitly, we get $x=0$ which makes sense since that is the root, but we also get this $x = 1.3917452007$ number where if we plug into our newtons method it is indefinet