Test 2 - MATH 471

Name: _____

Directions: Show ALL work on the test paper. Please note that correct answers without supporting work will not receive full credit. To receive full credit for solutions, all problems should be completed using only the methods and techniques discussed in this class so far this semester. **Note: Your final answer to each question should be clearly labeled and/or circled.** You may not receive human help on this assessment. Any Python code you use in your solutions must be written by you, and you alone. Your Zoom camera should remain on throughout this assessment. NOTE: If you submit Python programs as solutions to the problems below, you need to provide a clearly-labeled individual Python *.py file for each respective problem.

1. (50 pts.) Write a computer program that uses the derivative approximation

   $$\Delta_2(h) = (4D_2(h) - D_2(2h))/3$$

   to approximate the first derivative at $x = 1$ for each of the following functions, using $h^{-1} = 4, 8, 16, 32$.

   (a) $g(x) = -\ln(\cos x)$
   (b) $f(x) = x^{x^x}$

2. (50 pts.) For each integral below, write a program to compute the trapezoid rule using the sequence of mesh sizes $h = 1/2(b-a), 1/4(b-a), 1/8(b-a), ..., 1/128(b-a)$ where $b - a$ is the length of the given interval.

   (a) $\int_0^1 (e^x - e^{-x})/2 \, dx$
   (b) $\int_0^1 \sqrt{1 - x^4} \, dx$

3. (50 pts.) For the function below, write a program that indicates the (minimum) number of iterations required to estimate the root on the interval $[1, 2]$, within an error tolerance of 10^{-5}. Make sure to print your final conclusions to the console, or write them as a comment in your code. Do this for the bisection method, the regula-falsi method, and Newton's method with $x_0 = 1$:

   $$f(x) = x^{x^x} - 3.$$