Final Exam - MATH 471

Name: ____________________

Directions: Show ALL work on the test paper. Please note that correct answers without supporting work will not receive full credit. To receive full credit for solutions, all problems should be completed using only the methods and techniques discussed in this class so far this semester. **Note: Your final answer to each question should be clearly labeled and/or circled.** You may not receive human help on this assessment. Any Python code you use in your solutions must be written by you, and you alone. Your Zoom camera should remain on throughout this assessment. NOTE: If you submit Python programs as solutions to the problems below, you need to provide a clearly-labeled individual Python *.py file for each respective problem.

1. (50 pts.) Based on the material in Chapter 3, develop Python code which writes each of the following numbers in the form $a = b \times 2^k$, where $b \in (1/4, 1]$ and k is even. Then, use Newton's method to find the square root of b and hence of a.

   (a) a = 1/10

   (b) a = 1/1000

   (c) a = 1000

   (d) a = 0

   Be sure to use the initial value as generated by linear interpolation on $(1/4, 1]$, and doing only as many iterations as are necessary for 10^{-16} accuracy. Compare your values to the intrinsic square root function in the math library for Python.

2. (50 pts.) Consider the function $f(x) = x/2 - \sin(x)$. Code Newton's Method in Python to show that f has a root on the interval [1,3]. Also, write Python code which uses the Newton error estimate formula to help you to determine how close x_0 has to be to the root to guarantee convergence.

3. (50 pts.) For the function $f(x) = \arctan(x)$, write a program that estimates the (only) real root of f by computing 10 iterations of the bisection method (on the interval [-1,1]), as well as 10 iterations of Newton's method (with an initial value of $x_0 = 1$).

   Additionally, for Newton's method, formulate an equation that must be satisfied by the value $x = \beta$, in order to have the Newton iteration cycle back and forth between β and $-\beta$. Run Newton's Method again for $f(x)$ with 20 iterations and $x_0 = \beta$.