Heuristic Route Search in Public Transportation Networks

BY

TIMOTHY MERRIFIELD
B.S. in Computer Science, Ohio University, Athens, Ohio, 2004

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Computer Science
in the Graduate College of the
University of Illinois at Chicago, 2010

Chicago, Illinois

This thesis is dedicated to my beautiful wife Amber and my parents Tim and Linda for their relentless encouragement and support.

ACKNOWLEDGMENT

This thesis is a joint effort with Dr. Jakob Eriksson of the BITS Networked Systems Laboratory. Without his guidance, effort and infinite patience none of this work would have been possible. I would also like to thank James Biagioni for guiding me through the TransitGenie/Graphserver code base and helping me so much along the way.

TM

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Public transportation is a particularly attractive method of travel in urban areas. Travelers choose public transportation for a variety of reasons including: reduced cost, environmental concerns and convenience. However, despite the popularity of public transportation, only relatively recently have researchers begun to develop algorithms to efficiently compute shortest paths in this class of networks.

In recent years a tremendous amount of work has been done to speed up shortest path computations on static road networks. However, not all of that work is applicable to public transportation networks. This is due to the time dependency in a public transportation network. The shortest path between a source and a destination in a public transportation network can change depending upon the time at which the query is executed. We are also interested in supporting real-time updates to the schedule (based on GPS data) which adds additional complexity.

This thesis presents a precomputation-based heuristic that speeds up shortest path queries in public transportation networks while preserving route optimality. Our heuristic is also schedule independent and thus it is general enough to be used along with real-time data.

1.1 A Practical Motivation

Given an origin and a destination, TransitGenie (11) returns real-time, multi-modal public transportation routes to the user. The modes currently supported are buses, trains and walking. The searches are performed over the graph of the Chicagoland area which contains over 1.5 million vertices and over 4 million edges. The routing engine is derived from Graphserver (13) and uses a simple Dijkstra search that terminates when the search expands the destination vertex.

With this search strategy and a fairly large graph it is not surprising that many queries will perform poorly. Figure 1 demonstrates a search from the Chicago downtown area to a near-suburban area. Dijkstra's algorithm visits over 200,000 vertices during the search and takes 2.5 seconds to execute.

Figure 2 shows a CDF demonstrating this issue using 5,000 actual queries from the Transit-Genie web logs (times shown are solely based on the search algorithm and exclude any HTTP Request/Response overhead). The CDF shows that over 15% of the queries executed on the system take over a second and 5% take over 2.5 seconds. To improve the performance and scalability of the TransitGenie routing engine we began looking for algorithmic solutions to our slow searches.

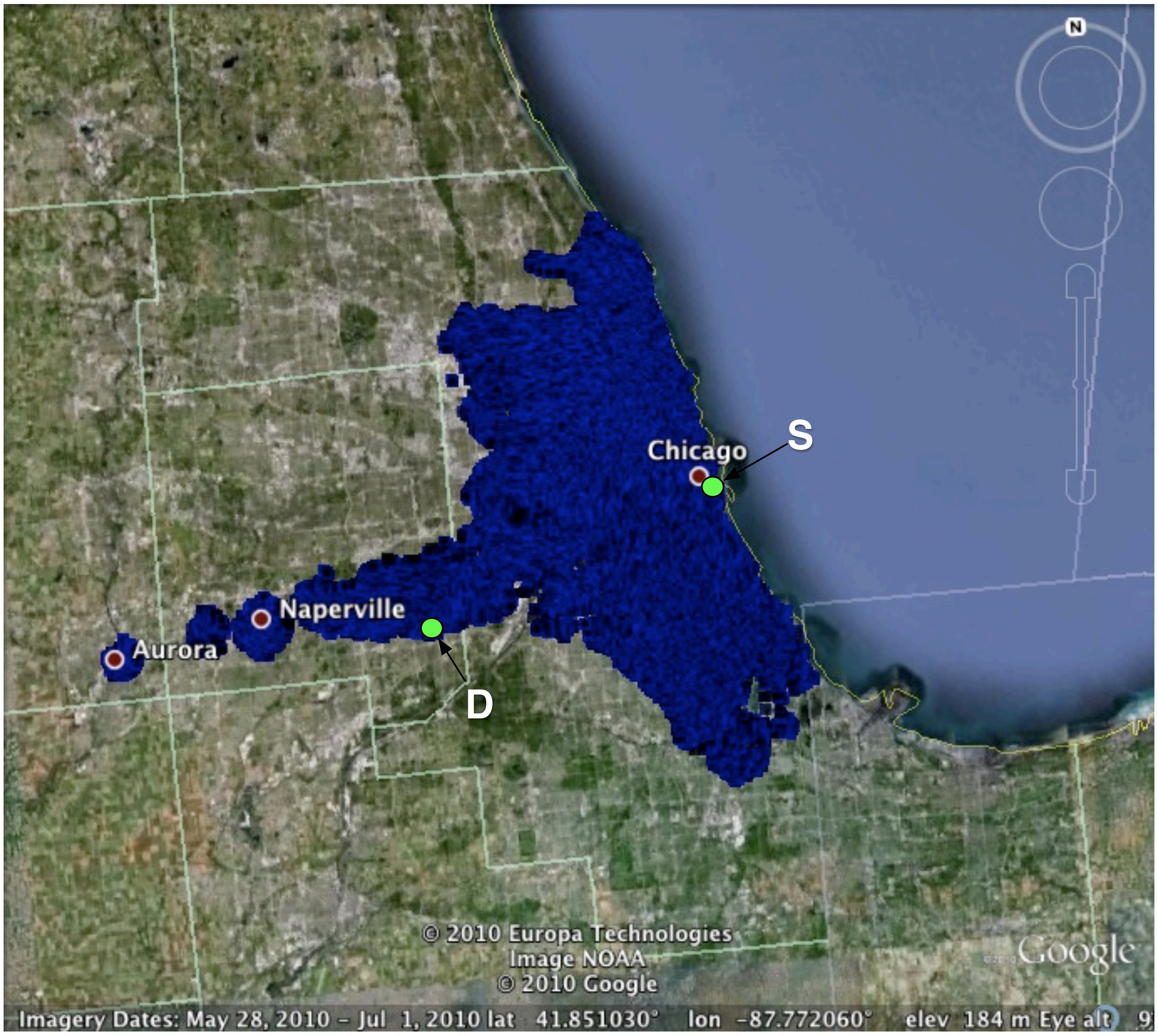

S
D
Chicago
Naperville
Aurora
© 2010 Europa Technologies
Image NOAA
© 2010 Google
Google
Imagery Dates: May 28, 2010 – Jul 1, 2010 lat 41.851030° lon -87.772060° elev 184 m Eye alt

Figure 1. A single-source single-destination search using Dijkstra's algorithm.

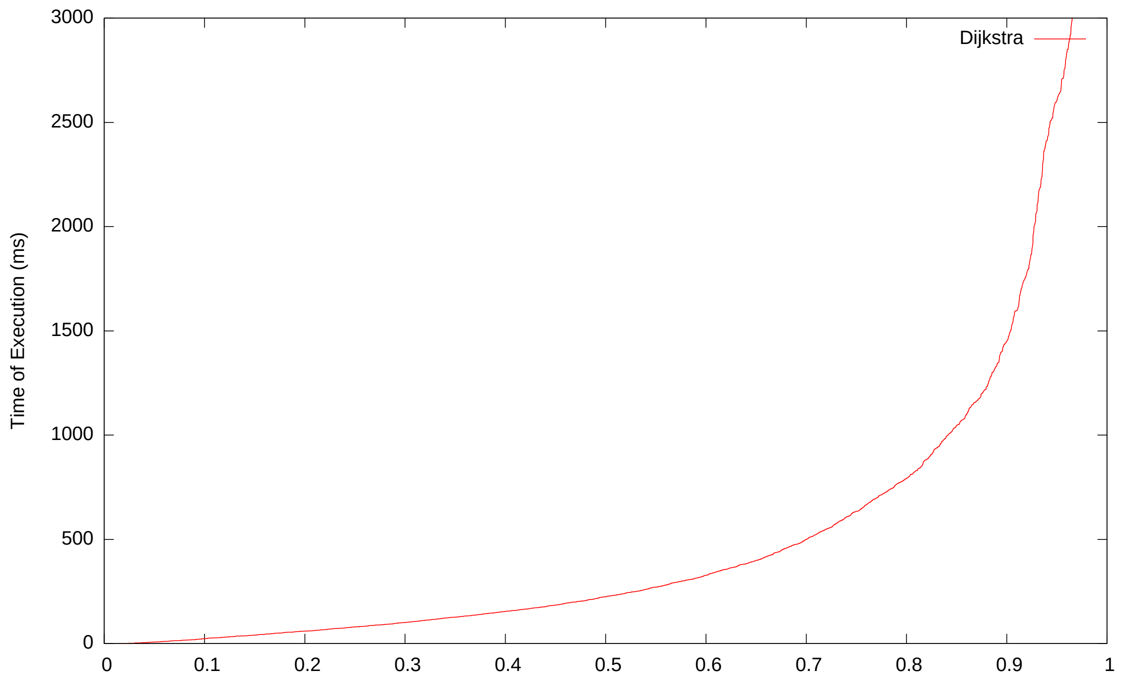


Figure 2. A CDF of the execution time of 5,000 queries taken from the TransitGenie logs.

CHAPTER 2

PRELIMINARIES AND RELATED WORK

We begin with some common definitions and classical methods followed by more recent work in shortest path algorithms.

2.1 Graph Preliminaries and Definitions

A *simple directed graph* G is comprised of $\{V, E\}$ s.t. V is a set of *vertices* and E is a set of *edges*. Edges are binary relationships between vertices (u, v) such that u, v in V and $u \neq v$. A graph is also sometimes referred to as a *network*. A *cost* function $c(e)$ associates positive *weights* with all edges $e \in E$. Two vertices are *adjacent* if there exists some edge connecting them.

A *path* between a *source* vertex v_1 and a *destination* vertex v_n in V is a sequence of adjacent vertices $\{v_1, v_2, ..., v_{n-1}, v_n\}$. If a graph G is *connected* then $\forall v_i, v_j$ in V there exists a path between v_i and v_j. If a single path exists then there also exists an *optimal shortest path* with respect to the cost function c. The *shortest path problem* is the problem associated with finding the shortest path between two vertices.

An algorithm that solves the shortest path problem can be analyzed by observing its *search space*. The search space can be defined as the number of vertices that must be explored from s before the shortest path to d is discovered.

2.2 Classical Methods in Solving the Shortest Path Problem

The following section describes several classical methods in solving the shortest path problem given a source vertex s and a destination vertex d. Further information can be found in (2)(3)(4).

2.3 Dijkstra's Algorithm

The canonical method for computing *optimal* shortest paths beginning from some source vertex $s \in V$ with non-negative edge weights is Dijkstra's algorithm. The algorithm maintains the following state $\forall$ $u \in V$: $\mathsf{cost}(u)$ which is the current best known cost (the total cost of best known path) to u from s, $\mathsf{settled}(u)$ which is a boolean function for whether the lowest cost path has been found to vertex u, and $\mathsf{parent}(u)$ which is the parent of u along the shortest path. Initially, $\forall$ $u \in V$ s.t. $v \neq s$, $\mathsf{cost}(u)$ is set to ∞ and $\mathsf{settled}(u)$ is false. For s, $\mathsf{cost}(s)$ is initialized to 0.

We start at s and set $\mathsf{settled}(s)$ to true and explore all of its adjacent vertices u_1 to u_n. If the cost from s to some vertex u_i is less than $\mathsf{cost}(u_i)$ then we update $\mathsf{cost}(u_i)$ with the new cost and set $\mathsf{parent}(u_i)$ to vertex s. The next vertex to settle is chosen by finding the vertex with the lowest known cost that has not yet been settled. The algorithm is terminated when all vertices have been visited.

If the shortest path to only a single destination vertex d is desired then Dijkstras algorithm can be modified to terminate when $\mathsf{settled}(d)$ is true. In chapter 5 this variation will be used in comparison with our heuristic search.

Here it is important to note that similar to the previously discussed methods Dijkstra's algorithm will make no attempt to direct the search towards our destination vertex d. The search space will expand outwards in all directions around the source vertex with no direction. To minimize the search space we must look to other methods.

2.3.1 Bidirectional Dijkstra Search

One method of reducing the search space in the shortest path problem is to execute a Dijkstra search from both s and d simultaneously. We can determine when the frontiers meet by checking whether a vertex has been settled by both searches. If it has we have found the shortest path and we can terminate the algorithm. This technique is optimal and can reduce the size of the search space by as much as half.

2.3.2 Best-First Search and Heuristic Functions

Best-first search algorithms attempt to provide some direction to the search process and subsequently reduce the search space. They are referred to as *best-first* because the best vertices are chosen for exploration based on some function $f(v)$. This class of algorithms is also sometimes referred to as *informed* search strategies. A crucial component to this function $f(v)$ is a *heuristic* function $h(v, d)$ which provides a lower bound on the cost of the route between some vertex v and the destination d. A well known heuristic function is the Euclidean distance between v and d.

A *greedy best-first search* is simply when $f(v)=h(v, d)$. This means that we choose vertices for exploration based solely on our heuristic function. This method can reduce the search space dramatically but is not guaranteed to find the optimal path.(3)

2.3.3 The A* Algorithm and Admissible Heuristic Functions

The A* algorithm provides a nice compromise between the reduced search space of greedy-best-first search and the optimality of Dijkstra's search. It essentially performs Dijkstra's search but it chooses vertices for expansion based on the function $f(v) = \mathsf{cost}(v) + h(v, d)$. A* is guaranteed to find optimal solutions if the heuristic function is *admissible*. An admissible heuristic function never *overestimates* the cost to the destination vertex. An extreme example would be for $h(v, d)$=0 for all v. In this case A* would perform exactly like Dijkstra's algorithm. However, if $h(v, d)$ can provide accurate lower bounds to d then A* can reduce the search space while still providing optimal paths.(4)(3)

2.4 Definitions Related to Public Transportation Networks

A *time-dependent* graph G' utilizes a cost function $\mathsf{cost}(e, t)$ where t is the current time. Therefore, the cost of taking an edge varies depending on the time at which the query is executed. A time-dependent graph can also be referred to as a *dynamic* graph. A public transportation network is a time-dependent graph due to the schedules for buses and trains. Also, because there are varying modes of transportation in these networks we can describe them as *multi-modal* networks. The alternative would be a single mode network such as a road network where the only mode of transportation is a car.

The time-dependency and multi-modal nature of these networks adds additional complexity to the shortest path problem. In Chapter 3 we will discuss how this complexity impacts some of the techniques described in this chapter.

2.5 Contemporary Strategies

2.5.1 Arc Flags

Arc flags (6)(7) is a technique where we keep track of whether or not an arc is on the shortest path to a destination. Since storing a flag at every edge for every vertex is not possible for large graphs, the graph is instead partitioned into regions. If an edge is on the shortest path to any vertex in that region then we set the flag to true. Because arc weights are dynamic in public transportation networks it would be difficult to apply this approach. However, the heuristic presented in this thesis makes use of many techniques used in arc flags.

2.5.2 Hierarchical Approaches

In (9) the intuition of routing in road networks is reduced to local search and highway search. A local search explores edges at the end points (source and destination) of the search while highway edges connect the local searches. The precomputation algorithm can be applied recursively forming multi-level contracted graphs. Contraction Hierarchies (10) further expand the notion of contraction with a more sophisticated contraction technique.

2.5.3 SHARC

SHARC (8) combines hierarchical approaches and arc-flags into a unidirectional algorithm that can support time dependency. It was shown to improve upon Dijkstra in schedule based rail networks by approximately 100 times.

2.5.4 ALT

ALT (5) is a heuristic search that uses a set of landmarks to provide direction to the search. The distance to and from the landmarks are precomputed for all vertices in the graph. The technique works in such a way that if a segment is shared on the shortest path to the landmark and the destination, then it will be explored first. This works well in a road network where normally a shortest path will involve a few local edges, then a highway and then more local edges. In a road network a landmark can be very far away from the destination and yet share many edges along the shortest path (the first set of local edges and a portion of the highway for instance).

2.5.5 Our Contribution

Many of the techniques described above are designed for static road networks. Those that can be applied to public transportation networks perform contraction or precomputation that is based on schedules. Our contribution is a partitioning based heuristic that is schedule independent and works in dynamic networks.

CHAPTER 3

A SEARCH HEURISTIC FOR PUBLIC TRANSPORTATION NETWORKS

We begin this chapter by describing some of the difficulties associated with finding shortest paths in a public transportation network. We then present a novel heuristic that works in this class of networks.

3.1 A* Search using Euclidean distance in multi-modal networks

As discussed earlier, an informed search technique (such as A*) can direct the search towards the destination using a heuristic function that approximates lower bounds. In order to satisfy the criteria of an admissible heuristic, a function $h(v, d)$ must never overestimate the cost to the destination vertex.

In a multi-modal network we may have several possibilities for transport including walking, bus, subway and high-speed trains. If our heuristic function is the Euclidean distance between a given vertex and the destination, then the speed with which we travel along that straight-line distance must be the velocity of our fastest mode of transportation. For the Chicago transit graph, the heuristic then must use the speed of an express train. However, a large portion of routes will not include this mode of transportation; therefore the heuristic provides an inaccurate lower bound on the distance to the destination.

In Figure 3 we see an example of the issue in a simplistic graph topology. The explored search space is shown by stars next to the vertices. Figure Figure 3(a) shows a simple Dijkstra search while Figure Figure 3(b) illustrates the reduced search space using A* with a Euclidean distance heuristic. However, when we add a single high-speed edge in Figure 3(c) we see that the search space is once again increased by the loose lower bounds generated by the Euclidean distance heuristic. As our lower-bounds become less accurate the number of vertices in our search space begins to approach Dijkstra.

3.2 A Note on Bidirectional Dijkstra in Public Transportation Networks

A traditional bidirectional search begins at the destination vertex $\mathtt{d}$ and the source vertex $\mathtt{s}$ at the same time and meets somewhere in the middle. However, in a public transportation network we cannot execute a search starting at $\mathtt{d}$ because we do not know at what time to begin (a necessary parameter when searching a transit network). We will not know that time until we have executed a forward search and determined at which time we will arrive at $\mathtt{d}$. Only then can we begin executing the search in the backward direction. This makes a bidirectional Dijkstra search very difficult to implement unless we permit estimation of the arrival time which removes optimality. This is unfortunate because many nice techniques have been developed using bidirectional search.

3.3 Case Study: Dijkstra, Euclidean Distance and Our Heuristic

Here we look at how our heuristic compares to Dijkstra's algorithm and the Euclidean distance heuristic for a single search within the Chicago public transportation graph. A description of our heuristic can be found in the next section and a more detailed evaluation of these techniques will be discussed in Chapter 4.

Figure 4 shows three maps which detail the search space for a single query from downtown Chicago to a suburb. It was noted earlier in this chapter that A* with a Euclidean distance heuristic performs poorly in multi-modal networks. In 4(a) and 4(b) we see that the search space (the dark region) is nearly identical for the two techniques. Dijkstra's search space consists of 46,307 vertices while the Euclidean distance heuristic explores 19,516 vertices for this search. While it is an improvement, it is not a dramatic one. Contrast this with the results of 4(c) using our heuristic. The search space here is 800 vertices which yields approximately a 57x improvement over Dijkstra and approximately a 24x improvement over the Euclidean distance heuristic.

3.4 A Heuristic for Public Transportation Systems

It is well known that the best possible heuristic from some vertex v to a destination vertex d would be the cost of the actual shortest path from v to d. If this information was known, then a best-first search algorithm (such as Dijkstra's) would only expand nodes on the shortest path to the destination. Unfortunately, storing and computing this data for all-pairs is $O(n^2)$ and is therefore out of the question for any graph with a large number of vertices. Also, the dynamic nature of a public transportation network adds additional complexity because the cost of the shortest path from v to d can vary based on the departure time from v.

We can reduce the storage and computational complexity of all-pairs by partitioning our graph into a set of n cells, $\{c_0 ... c_n\}$. The most obvious way of performing the partitioning would be to conceptually draw a uniform two-dimensional grid on top of your map as shown in Figure 5. We then only have to compute (and store) the cost of the shortest-path between cells instead of vertices. Because the cost of the shortest path from any two vertices $\nu_{i,j},\nu_{i,k} \in c_i$ to any vertex $\nu_{n,m} \in c_n$ may be different, the cost of the shortest path between cells is the minimum cost from any vertex in c_i to any vertex in c_n. This will provide a lower-bounds on the distance from one cell to another and thus an admissible heuristic.

More intuitively, imagine the scenario in Figure 6(a) where we are trying to find the shortest path from S to D. In the figure, all edges have a cost of 10 with the exception of the edge with a cost of 3 on the right. The actual cost from each vertex to D is shown in the figure. By looking at the graph it is obvious that the fastest way to get from S to D is to move toward the vertex that connects to the destination with a cost of 3. If we partition the graph into cells and perform the all-pairs shortest path precomputation on cells, we now know the fastest possible paths from all cells to cell a where our destination vertex resides. These values are displayed in Figure 6(a) in the top left corner of each cell. A* can then use this heuristic to find the shortest path to D.

When the A* search begins and expands S it has three possible options of which vertex to expand next since all edges out of S have the same cost of 10. Two of those vertices reside in cell c where the heuristic cost is 23 while the other is in cell e where the heuristic cost is 3. A* will expand the vertex in e because of the lower heuristic cost. It then expands the next vertex in e because the total cost (heuristic and actual) is still less than the total cost of the vertices in c. Intuitively, the heuristic is a hint to A* that a fast way to our destination can be found in cell e.

3.4.1 The Importance of Transitional Edges

As we saw in the previous section, the heuristic value applies to all the vertices in a given cell. Another way to say that is that given any v_i, v_j in cell c_k and a destination vertex d, $h(v_i, d){=}h(v_j, d)$. For this reason, our heuristic does not help us unless we are considering vertices that are not in the same cell. Therefore, it is important to create a partition that maximizes the total number of cells and the total number of *transitional edges*. A transitional edge can be defined as an edge connecting two adjacent vertices that reside in different cells.

3.5 Grid Partitions

The following section describes the strategies used in creating the grid partitions. As we will see later, the choice of a grid partition is critical to the performance of the heuristic.

3.5.1 Uniform Grid Partitions

The most simplistic way of partitioning the map is using a uniform 2-dimensional grid. We define the *resolution* of the grid to be a value n s.t. the grid has n rows and n columns and $n * n$ total cells. As discussed earlier, the heuristic is only taken into account when encountering a transitional edge. Thus, a higher resolution grid will provide a larger number of transitional edges. Of course, this is a trade-off between storage/precomputation time and performance. A higher resolution grid will take longer to precompute and store but will yield better performance. In Chapter 4 we will experiment with partitions of resolution 50 and 100.

3.5.2 Partitioning with Kd-Trees

A kd-tree(1) is a simple binary tree data structure designed to store multi-dimensional spatial data. The 'k' in the name denotes the structures ability to handle an arbitrary number of dimensions. The partitioning algorithm works by alternating between dimensions and in our case the tree will be 2d with our dimensions being latitude and longitude. It works recursively by finding the median of a set of vertices in one dimension and then partitioning the cell into two new cells based on that median. The two new cells are then split again by the same method, except this time the other dimension is used. The recursion stops when the number of vertices in a cell drops below a configurable threshold. Figure 8 shows a simple example of how a kd-tree would partition a small number of points in 2-d Euclidean space. In our experiments we will use a kd-tree with approximately 8,000 cells (150 vertex per cell threshold).

The kd-tree provides a nice partition based on the density of vertices in the graph. Intuitively, the graph will be partitioned such that the area with a high vertex density will be very high resolution (small cell size) and the areas with a low vertex density will have low resolution (large cell size). Figure 9 demonstrates the partition executed on the Chicago graph. Not surprisingly, the area around downtown has a much higher resolution.

3.6 All Pairs Precomputation

The result of the precomputation step is that we determine the shortest path from all cells to all other cells. Here $\mathtt{min\_cost}(c_i, c_j)$ is the minimum cost from any vertex in c_i to any vertex in c_j. The simplest way to compute this is to run Dijkstra's algorithm starting at each vertex in the graph. If we begin from some source vertex in c_i, any time that the search expands a vertex

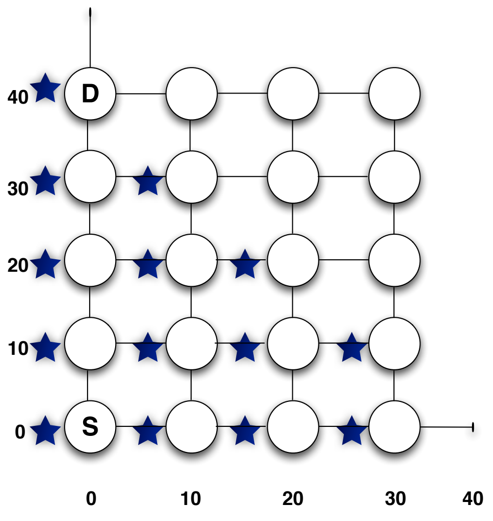
$v_{j,k}$ in c_j we check to see if the $\mathsf{cost}(v_{j,k})$ is less than $\mathsf{min\_cost}(c_i, c_j)$. If so, then we replace $\mathsf{min\_cost}(c_i, c_j)$ with $\mathsf{cost}(v_{j,k})$ and continue on with the search. Once Dijkstra's algorithm has been run from each vertex in c_i, $\mathsf{min\_cost}$ has been determined from c_i to all other cells in the graph. Further more, once the search has been run starting at each vertex in the graph, $\mathsf{min\_cost}$ has been determined from all cells to all other cells.

3.6.1 Speeding up the Precomputation

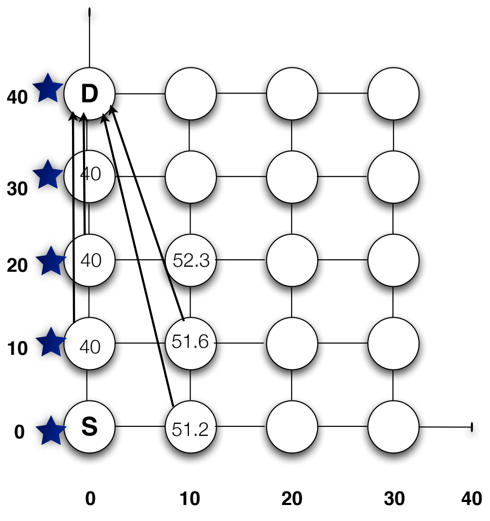
```
foreach cell $c_i$ in $C$ do
    foreach vertex $v_{i,j}$ in $c_i$ do
        AddToPriorityQueue($v_{i,j}$, 0)
    end
    while PriorityQueue is not empty do
        $v_{m,n}$ = GetMinFromPriorityQueue();
        Continue executing Dijkstra's...;
        $c_m$ = GetVertexCell($v_{m,n}$);
        if $\mathsf{cost}(v_{m,n}) < \mathsf{min\_cost}(c_i, c_m)$ then
            $\mathsf{min\_cost}(c_i, c_m) = \mathsf{cost}(v_{m,n})$;
        end
    end
end
```

Algorithm 1: Precomputing the shortest path from all cells to all other cells

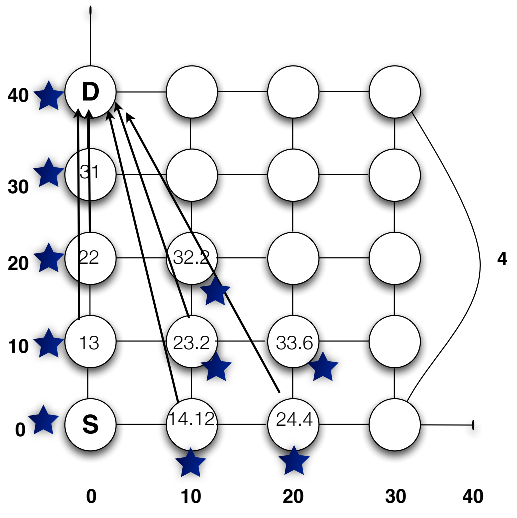
For large graphs with millions of vertices the precomputation described above is not practical. Thankfully, a technique was described in (6) that helps to greatly speedup the precomputation step. Instead of running Dijkstra's algorithm from each vertex in the graph we only have to run the algorithm once per cell. If we are computing $\mathtt{min\_cost}$ for c_i then we begin the Dijkstra's search by placing all vertices in c_i into the priority queue with a cost of 0. This effectively starts the search from every vertex in c_i. This simple technique improves the run time of the precomputation from $O(|V|)$ to $O(|C|)$ where $|C|$ is the number of cells in the graph. The full algorithm is presented in Algorithm 1.



(a) Dijkstra Search without heuristic

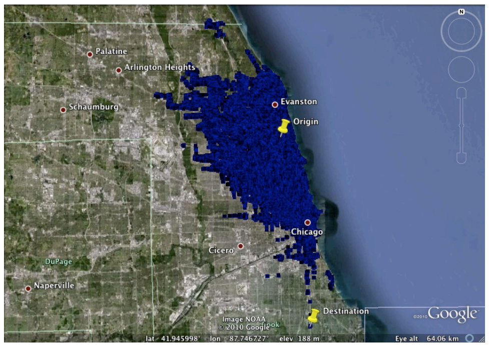


(b) A* with Euclidean distance

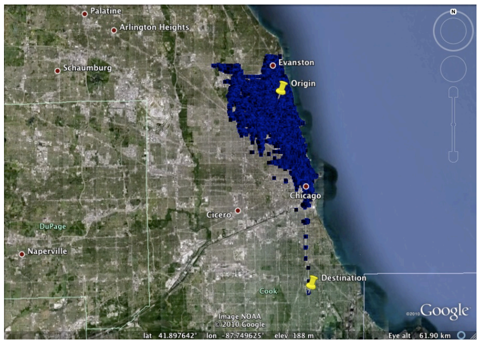


(c) The effect of a single high-speed edge on A*

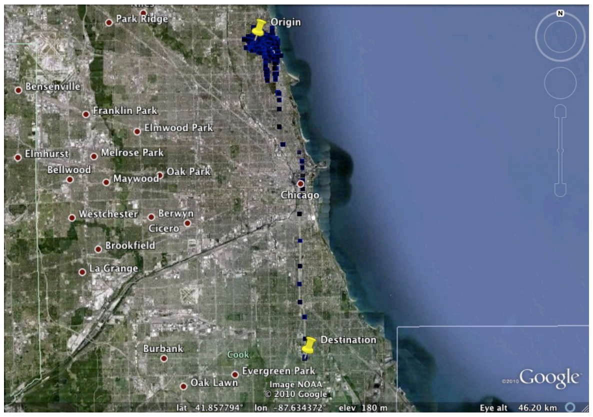
Figure 3. The effects of inaccurate lower bounds on A*



(a) Djikstra Search



(b) A* with Euclidean distance heuristic



(c) A* with our heuristic

Figure 4. A search conducted using Dijkstra, A* with Euclidean distance heuristic and A* with our heuristic.

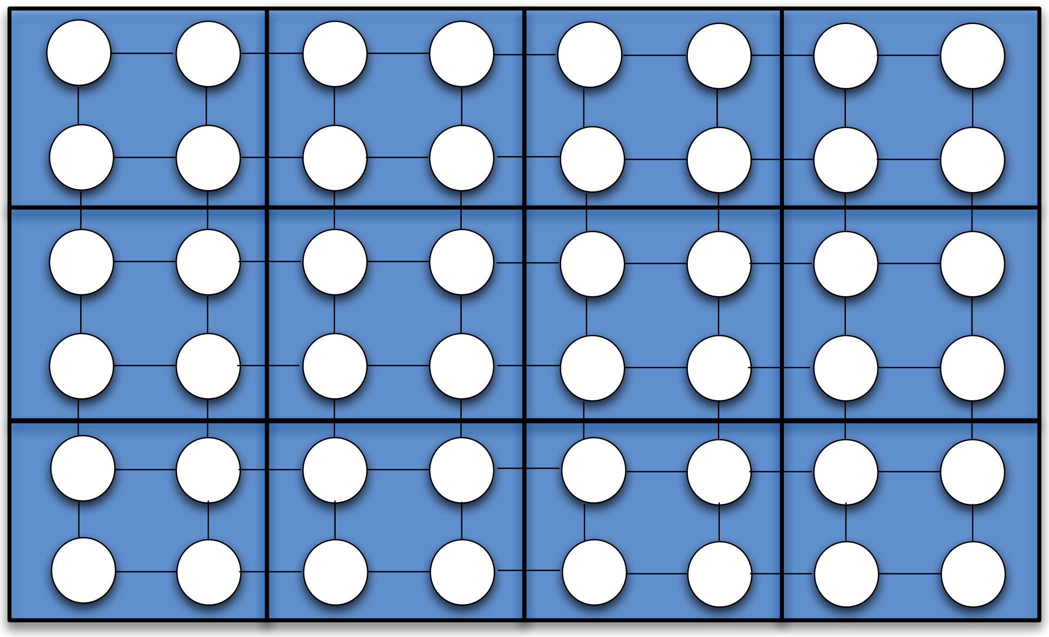
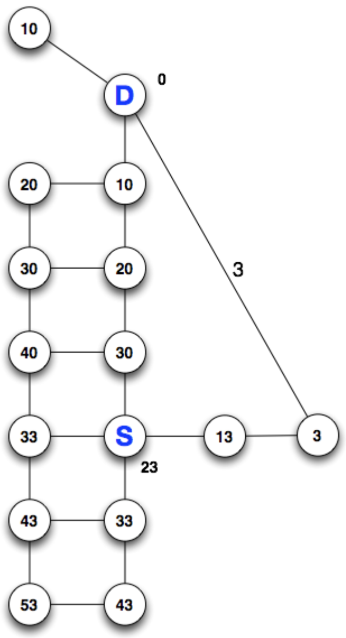
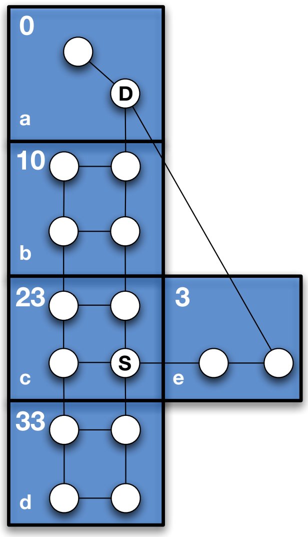


Figure 5. A uniform grid partitioning the graph into 12 cells.



(a) A graph with a high-speed line similar to a train in public transportation networks



(b) Grid drawn over graph where the letter and number in each cell are the cell identifier and the shortest path to cell a respectively

Figure 6. The effects of inaccurate lower bounds on A*

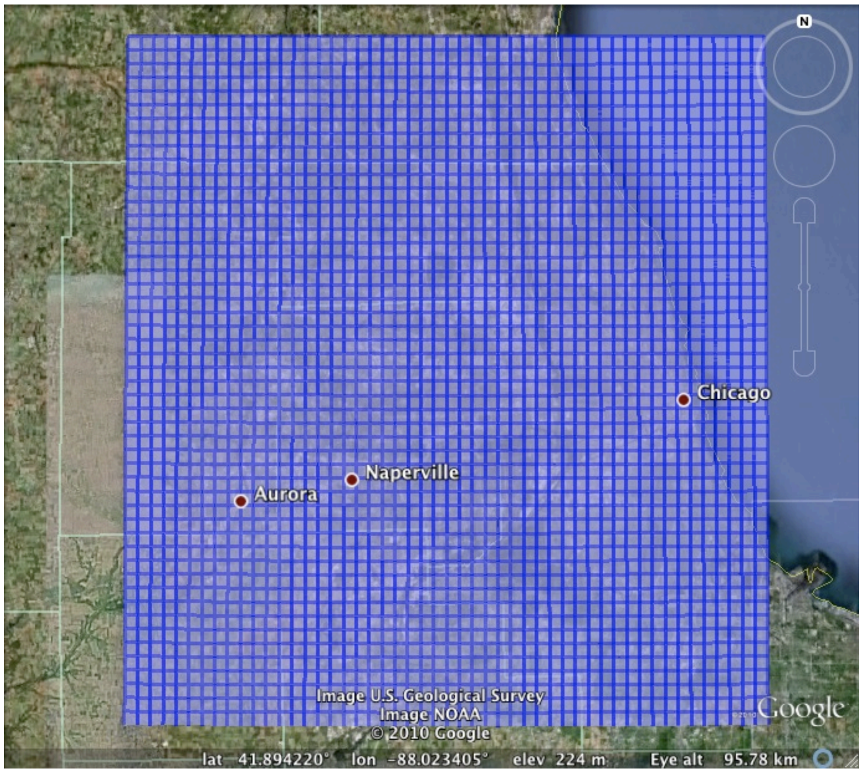

Chicago
Naperville
Aurora
Image U.S. Geological Survey
Image NOAA
© 2010 Google
Google
lat 41.894220° lon -88.023405° elev 224 m Eye alt 95.78 km
N

Figure 7. A 50x50 uniform grid partition.

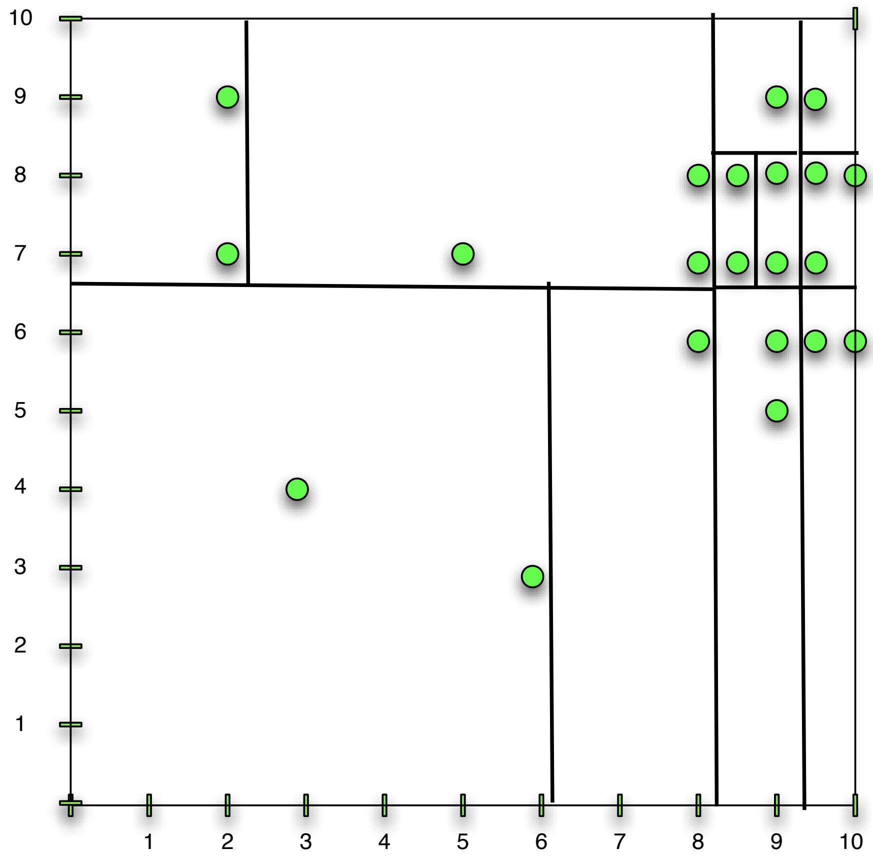


Figure 8. A simple kd-tree example and the cells it generates.

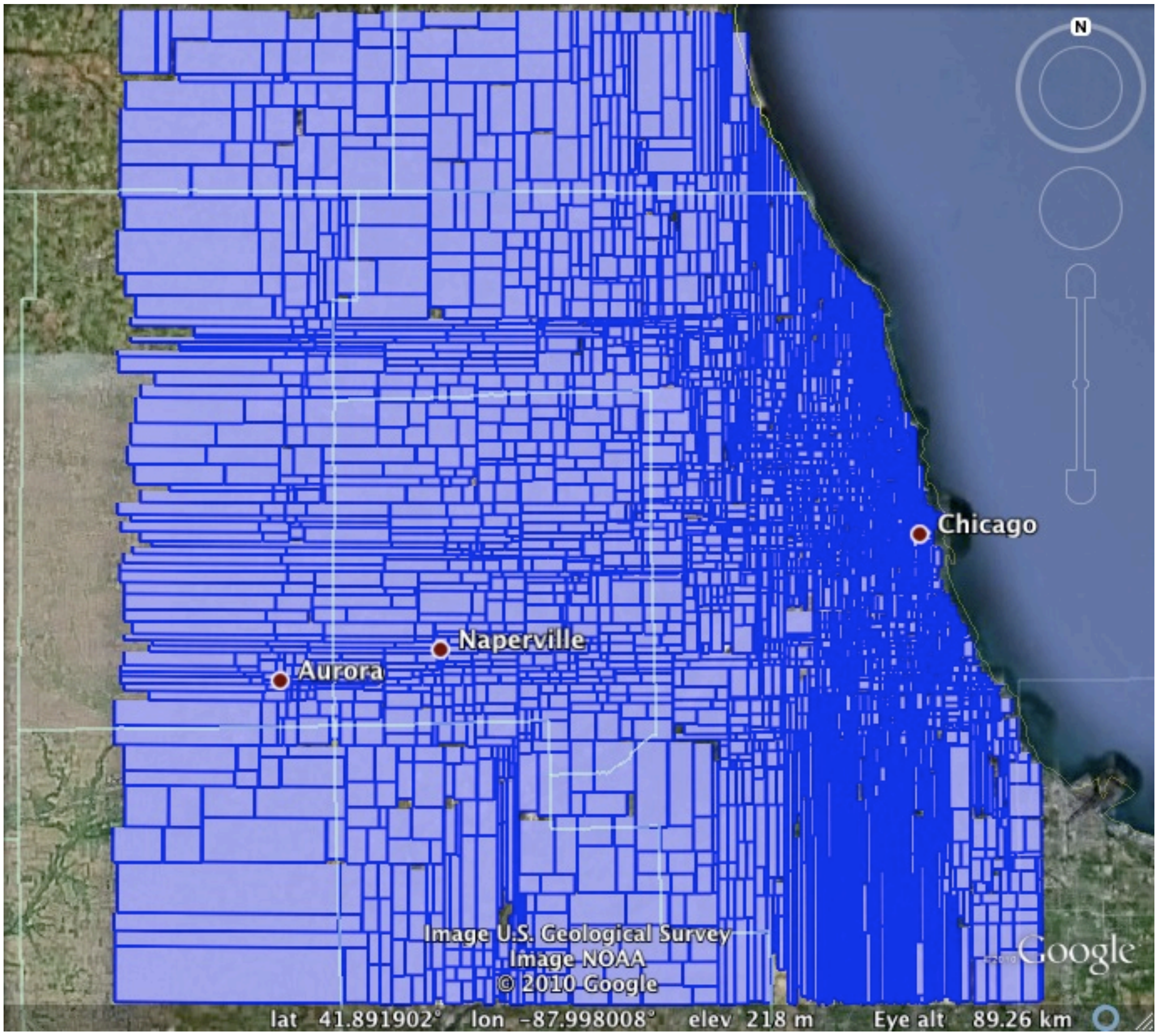

Chicago
Naperville
Aurora
Image U.S. Geological Survey
Image NOAA
© 2010 Google
Google
lat 41.891902° lon -87.998008° elev 218 m Eye alt 89.26 km

Figure 9. A kd-tree partition with approximately 8,000 cells.

CHAPTER 4

EVALUATION

In this chapter we will describe the evaluation methodology used and the results of that evaluation. Several types of queries are tested including both real-world queries (pulled from the TransitGenie query logs) and synthetic queries subdivided into several different types.

4.1 Methodology

We evaluate all the grid types described above and compare them with Dijkstra's and A* with a Euclidean distance heuristic. Below is a table describing the abbreviations used in the figures.

To evaluate the system we start with real world queries extracted from the TransitGenie logs. We use the same 5,000 real world searches that were used in Chapter 1 to evaluate the performance of different approaches. To get a more detailed view of the results we also use

TABLE I

ABBREVIATIONS USED IN THE EVALUATION.

Dijkstra's algorithm (terminated at destination)	D
A* with Euclidean distance	ED
A* with 50x50 grid	G50
A* with 100x100 grid	G100
A* with kdtree	KD

TABLE II

AVERAGE PERFORMANCE IMPROVEMENT OVER DIJKSTRA

Method	Real Searches	Short City	Long City	City to Suburbs	Suburbs to City
ED	1.41x	1.29x	1.43x	1.17x	1.44x
G50	2.42x	1.49x	3.89x	1.53x	2.57x
G100	3.47x	2.22x	5.90x	1.83x	3.19x
KD	6.09x	5.15x	10.56x	1.57x	3.40x

synthetic queries we generate ourselves that fall into several different types of searches. The synthetic queries are broken up into the following classes: short searches within the city, long searches within the city, searches from the city to the suburbs, and from the suburbs to the city.

4.2 Real-World Searches Extracted from the TransitGenie Logs

Using the same set of queries shown in Chapter 1 we evaluate the different techniques. As we can see in Figure 10 the success of the techniques is dependent on the number of transitional edges that factor into the problem space. For this reason the kd-tree's higher resolution does very well with this problem set. Also as expected, the 100x100 grid is consistently better than the 50x50 grid. The Euclidean distance heuristic provides marginal gains over the Dijkstra search.

While there is improvement (the kd-tree does over 6 times better than Dijkstra on average), there are still searches that do very poorly. This can be seen in the 85'th percentile of the log scale plot in Figure 11. To better understand these performance numbers we generated our own synthetic queries for a more in depth analysis.

4.3 Searches within the City

We tested both searches that covered large portions of the city and searches that covered a relatively short distances (just several blocks in some cases). CDF plots of these two data sets can be found in Figure 12 and Figure 13. Looking at Table II we can see that a kd-tree partition does very well for queries within the city, resulting with an average 10x performance improvement over Dijkstra for longer city queries. The 100x100 grid shows a non-trivial improvement over the Dijkstra search as well.

4.4 Longer Searches Involving Suburban Areas

We also performed searches covering a larger portion of the Chicagoland area including suburban regions. These searches tend to utilize the higher speed commuter rail lines that operate between the city and the suburbs. Examining the numbers in Table II we can see that the speedup from the city to the suburbs is disappointing for all techniques. CDF's for both city to suburbs and suburbs to city searches are found in Figure 14 and Figure 15. Both the kd-tree and grid partitions struggle to even improve the Dijkstra search by 2x. We speculate that the 100x100 grid does the best because it has a more constant resolution over the entire graph. In Chapter 5 we will discuss in detail why the speedup is so moderate on these longer searches.

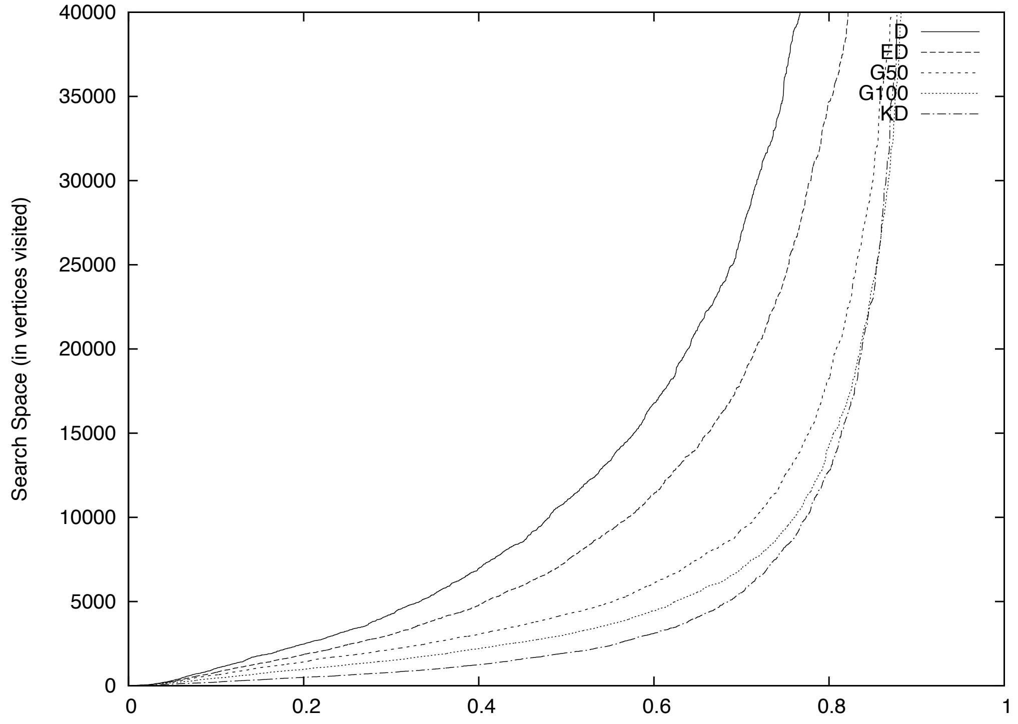


Figure 10. A CDF of the search space of several techniques tested against 5,000 queries taken from the TransitGenie logs.

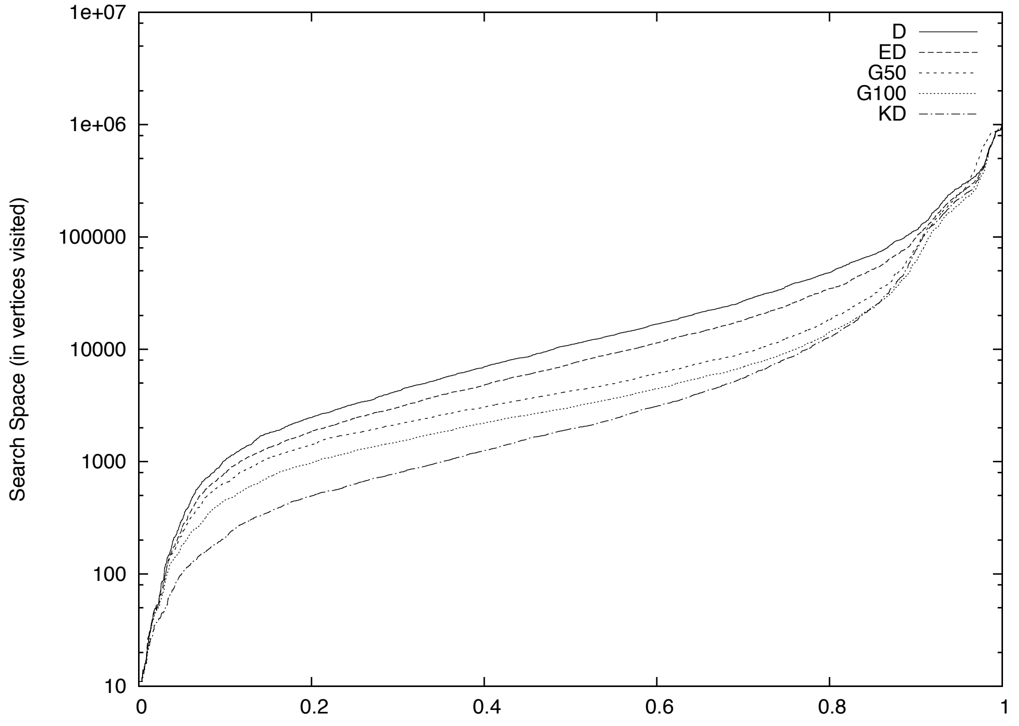


Figure 11. A CDF of the search space of our 5,000 queries using a log scale.

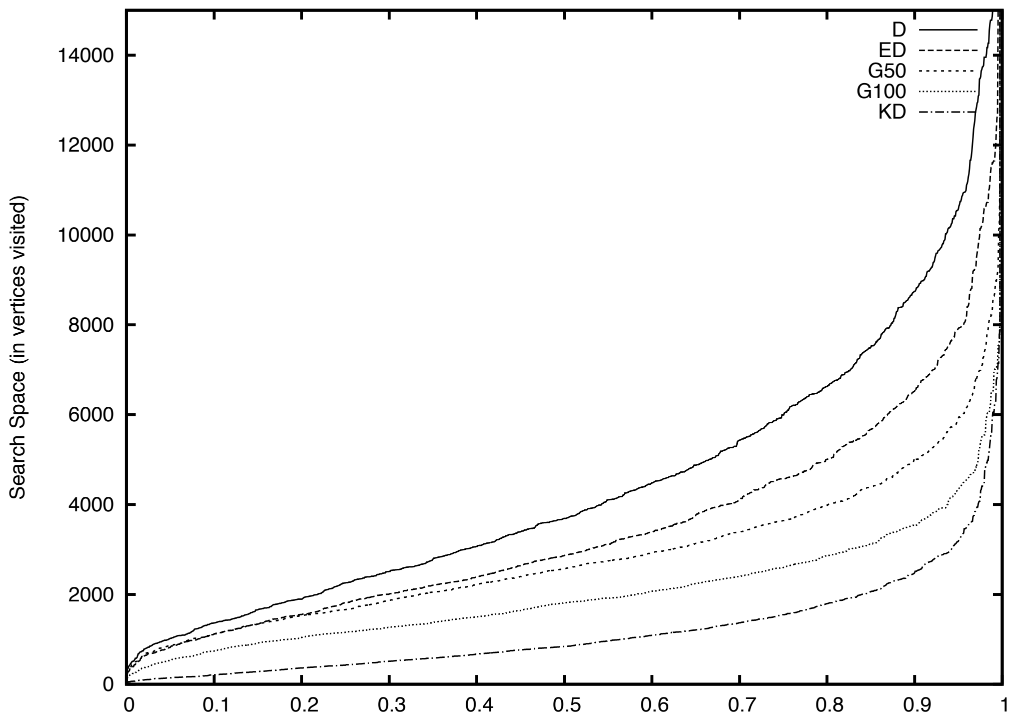


Figure 12. CDF of short searches within the downtown area of Chicago

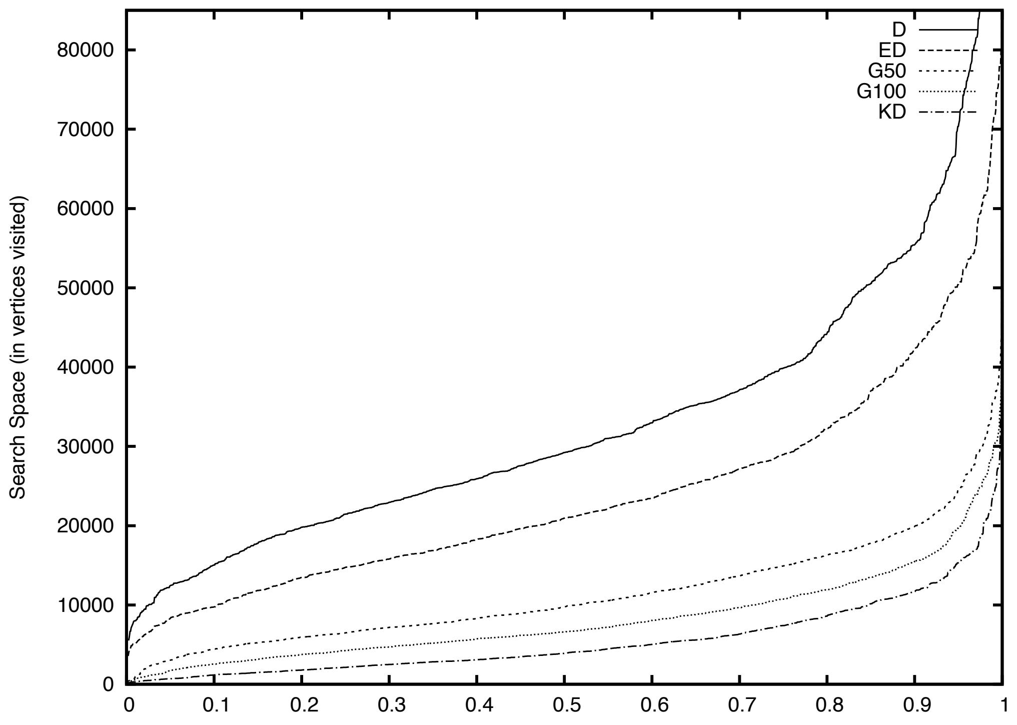


Figure 13. CDF of long searches within the city of Chicago

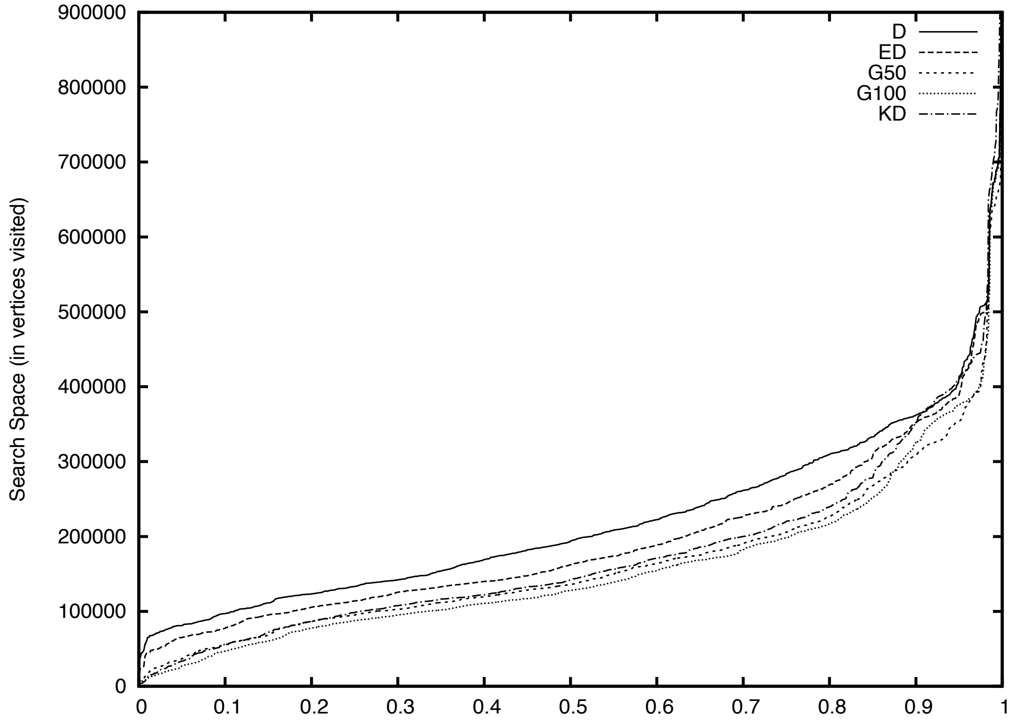


Figure 14. CDF of searches from the downtown area of Chicago to suburban areas.

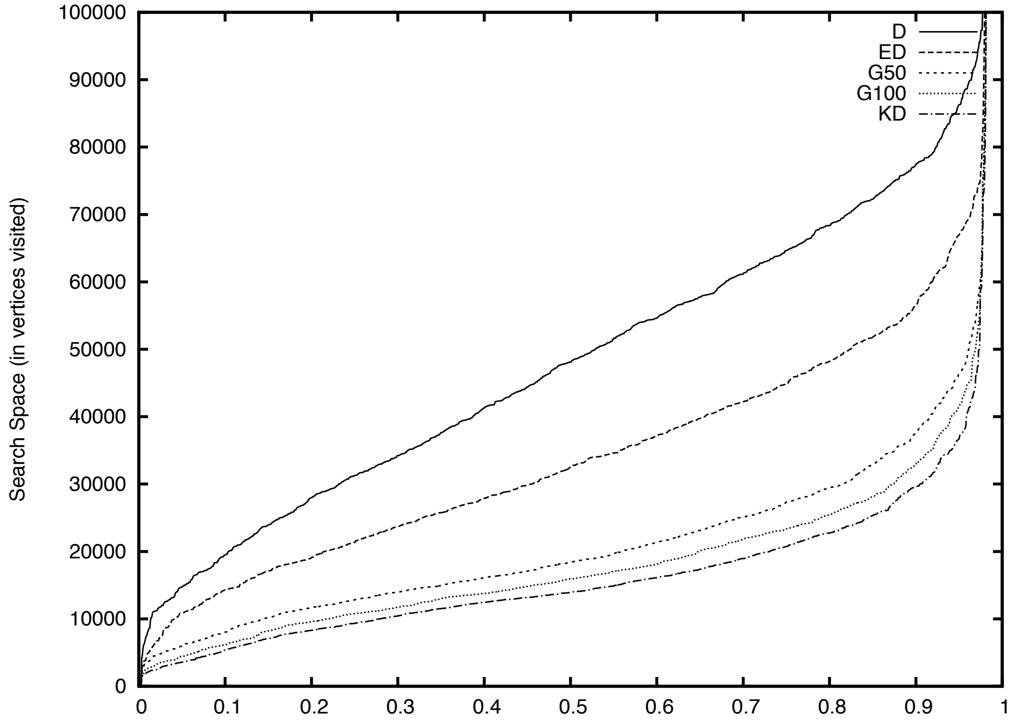


Figure 15. CDF of searches from suburban areas to the downtown area of Chicago.

CHAPTER 5

SELECTED ISSUES FOR FURTHER ANALYSIS

In the previous chapter we discussed the implementation, the evaluation methodology and the results of the evaluation. However, several of the results were non-intuitive. In this chapter we will attempt to shed some light on some of those issues.

5.1 Performance Challenges in City to Suburban Searches

In the previous chapter we saw that the city to suburban searches tended to still yield very high search spaces for all of our heuristic techniques. The reason for this is simple, these routes tend to use the higher speed commuter rail system that has relatively infrequent departure times when compared to other modes of transit. For instance, a common pattern is for a train to leave from a given station once every hour. When we perform our precomputation the shortest path from a city cell to a suburban cell will almost certainly use this mode of transit. However, the precomputation ignores all delay and provides us with the fastest possible route between cells. Intuitively, this means that the heuristic value is based on the notion that the train is sitting there waiting for you to arrive and then immediately departs.

When a search query is executed the heuristic directs the search towards the train station. However, in the worst case condition we have just missed the train and have to wait as much as 55 minutes for the next train. Of course, our algorithm does not spend this 55 minutes waiting around. It will continue to search for faster routes during this time. Figure 16 demonstrates this phenomenon. Here a single query between the city and the suburbs is executed several times over the course of an hour. For the kd-tree, in the best case it improves over Dijkstra by 2.25x and in the worst case 1.25x. The best case occurs when the search arrives at the train station just in time to catch the train and the worst case occurs when we just missed the train.

This issue was described in (12) as a problem of *bad connectivity* and is major problem with all heuristic searches in public transportation networks. When there is higher connectivity our heuristic can make some improvements however in this worst case it begins to approach Dijkstra.

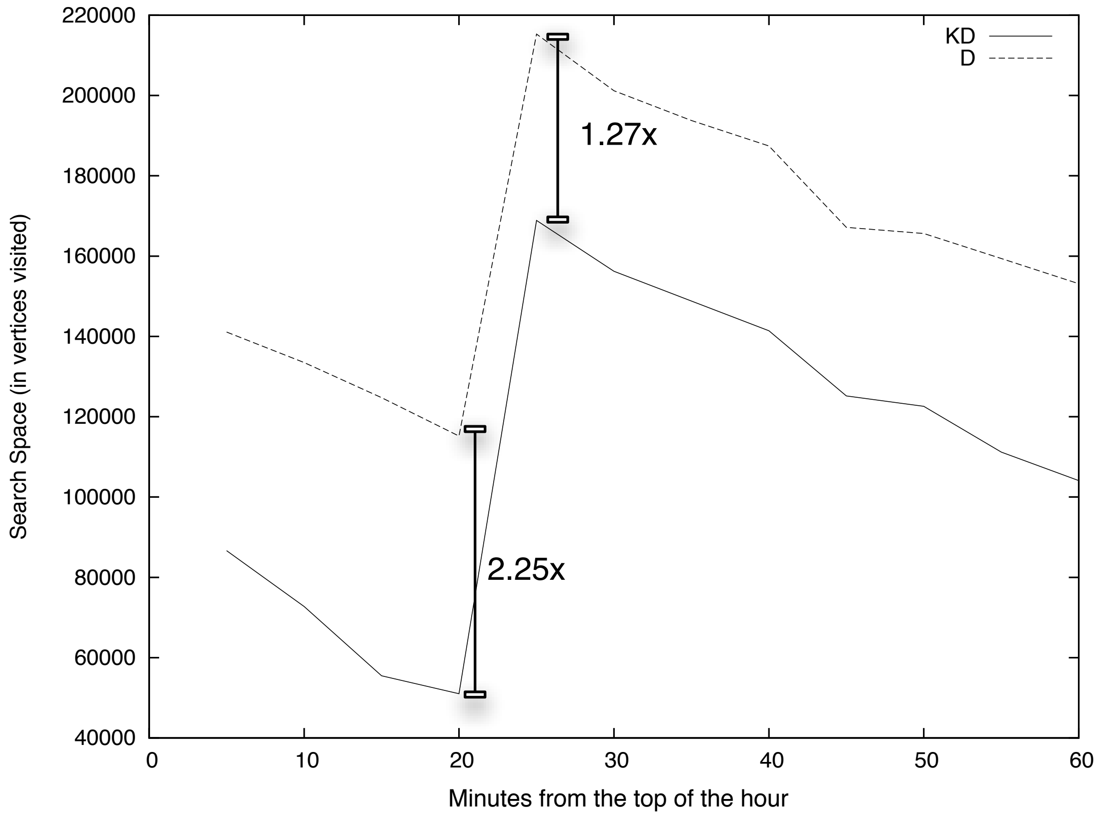


Figure 16. A single city to suburbs query executed at various times over the course of an hour.

CHAPTER 6

FUTURE WORK

We have shown that a simple precomputed heuristic can speed up certain classes of searches several times over Dijkstra. While this is a significant improvement, it is troubling that the issue of disconnectivity causes our optimization to perform sub-optimally on the class of searches (city to suburbs) that result in the highest query times. So the issue becomes, how can we improve our scheme to handle disconnectivity?

To solve the issue we plan to add an additional dimension to our heuristic, *time*. In Chapter 5 we discussed how the start time of our search is critical to minimizing the search space, particularly for searches that are relatively disconnected. This is because our heuristic was precomputed ignoring delay and storing the lowest possible cost (over all times) to a destination cell. If we create a partition with an additional time range property, we could compute the lowest cost for just that time range. We expect precomputation values based on certain ranges to produce more accurate lower bounds and reduce the search space.

It my appear that adding the time range property would cause the cell count to grow tremendously but it may not be so bad. Imagine a high-speed commuter rail line with trains leaving every hour. Each hour interval can be divided into 15 minute intervals and the common intervals can be attached to the same cell. For example, the following intervals: 12:00-12:15, 1:00-1:15, 2:00-2:15 could be associated with one cell because they should have very similar lower bounds.

CHAPTER 7

CONCLUSION

We have shown that given a suitable heuristic and a moderate amount of precomputation, we can see a tremendous speedup for certain classes of queries while maintaining optimality. Performing experiments on the Chicagoland graph, our heurisitic does 6x better on average than Dijkstra and 10x better on certain classes of queries. In the future, we look forward to continuing to improve the performance particularly on queries that involve a high level of disconnectivity.

CITED LITERATURE

1. JL Bentley: Multidimensional Binary Search Trees Used for Associative Searching Comm. ACM, pages 509–517, 1975.

2. RE Korf: Depth-First Iterative Deepening: An Optimal Admissable Tree Search Artificial Intelligence 27 (1), Pages 97-109, 1985 .

3. SJ Russell, P Norvig: Artificial Intelligence: A Modern Approach Englewood Cliffs, NJ: Prentice Hall, 1995

4. PE Hart, NJ Nilsson, and B Raphael: A Formal Basis for the Heuristic Determination of Minimum Cost Paths. IEEE Transactions on System Science and Cybernetics 4(2) 1968.

5. AV Goldberg, C Harrelson Computing the shortest path: A* search meets graph theory In Proc. 16th ACM-SIAM Symposium on Discrete Algorithms Pages 156-165 2005.

6. M Hilger, E Kohler, R Mohring, and H Schilling.: Fast Point-to-Point Shortest Path Computations with Arc-Flags. MobiSys, 2008.

7. R Mohring, H Schilling, B Schutz, D Wagner and T Willhalm. Partitioning Graphs to Speedup Dijkstras Algorithm. ACM Journal of Experimental Algorithmics11(2), 2006.

8. R Bauer, D Delling SHARC: Fast and robust unidirectional routing ACM Journal of Experimental Algorithmics14(2), 2009.

9. P. Sanders and D. Schultes Fast and Exact Shortest Path Queries Using Highway Hierarchies. Proc. 13th Annual European Symposium Algorithms, 2005.

10. R. Geisberger Contraction hierarchies: Faster and simpler hierarchical routing in road networks, 2008. Diploma Thesis, Universitat Karlsruhe.

11. J. Biagioni, A. Agresta, T. Gerlich, and J. Eriksson.

    Transitgenie: A real-time, context-aware transit navigator (demo abstract). SenSys, ACM, 2009 pages 329-330.

12. H Best. Car or Public Transport–Two Worlds Lecture Notes In Computer Science(5760), 2009.

13. Graphserver, the open-source multi-modal trip planner. `http://graphserver.github.com/graphserver/`.

VITA

Timothy Merrifield

.

Education

Bachelor of Science in Computer Science - Ohio University, Athens Ohio (2004))

Research Experience

- **BITS Networking Lab, UIC**: Research Assistant (May 2009 - Present)

Professional Experience

- **Argonne National Laboratory, Decision and Information Sciences Division**: Software Engineer (January 2010 - Present)
- **ShopLocal LLC**: Software Engineer (March 2005 - December 2009)