The Formal Methods of Boreal v2.5

A Mathematical Framework for Neuro-Silicon Homeostasis

This document outlines the rigorous mathematical foundations of the **Boreal Neuro-Core**. The system operates by minimizing a single information-theoretic objective: **Variational Free Energy (F)**.

1. The Core Objective: Variational Free Energy

The Boreal Core treats the brain-machine interface as a problem of **Active Inference**. The goal is to minimize the "Surprise" (surprisal) between the internal model and external neural spikes.
The Variational Free Energy F is defined as:
Where:

- y is the observed neural data (EEG/EPOC X).
- ψ is the "Hidden State" or the user's intended manifold.
- $q(\psi)$ is the FPGA's internal "guess" (the recognition density).
- D_{KL} is the Kullback-Leibler divergence (the "Inference Gap").

To implement this in silicon, we simplify F into a quadratic **Prediction Error**:
Where:

- μ is the internal state (Manifold Position).
- W is the synaptic weight matrix stored in BRAM.
- σ is the non-linear activation function (Sigmoid).

2. State Estimation: Gradient Descent in Silicon

To update the manifold position μ in real-time, the FPGA performs a gradient descent on the Free Energy manifold F:
The discretized update rule executed in the Verilog fabric every clock cycle is:
Where:

- η is the learning rate (Inference Sensitivity).
- σ' is the derivative of the activation function (stored in the ROM LUT).
- λ is the "decay" or "prior" that prevents the manifold from drifting into instability.

3. Temporal Predictive Coding (Lag Cancellation)

To cancel the $\approx$ 30ms Bluetooth lag of the EPOC X, we model the temporal dynamics of the manifold as a second-order system:
Where:

- $\dot{\mu}_t = \mu_t - \mu_{t-1}$ (The velocity vector).
- k is the **Lead Factor** (The look-ahead constant).

By outputting $\hat{\mu}_{t+k}$ instead of μ_t, the FPGA "front-runs" the biological intent, achieving zero-perceived-latency control.

4. Robotic Inverse Kinematics (IK)

Mapping the 3D endpoint (μ_x, μ_y, μ_z) to joint angles $(\theta_1, \theta_2, \theta_3)$ requires solving the Law of Cosines in real-time. For a two-link arm with lengths L_1 and L_2:
The Elbow Angle θ_2 is derived as:
The Shoulder Angle θ_1 is derived as:
The Boreal Core utilizes **CORDIC (Coordinate Rotation Digital Computer)** algorithms to calculate these trigonometric functions using only bit-shifts and additions, ensuring the IK solution is found in < 100ns.

5. Autonomic Discordance & AD-Detection

To detect **Autonomic Dysreflexia (AD)**, the core calculates the **Correlation Coefficient (R)** between Heart Rate Variability (HRV) and Inference Error (ϵ):
When R diverges significantly from the "Baseline Manifold" (High Error + Low HRV), the system identifies **Autonomic Discordance** and triggers the Vagus Brake.

6. Stimulus Physics: Charge Balancing

To prevent tissue damage, the stimulation output must maintain **Electrochemical Neutrality**:
The Boreal Core ensures this by generating **Biphasic Square Waves**. Every positive pulse of current (+I) is followed by an identical negative pulse (-I), resulting in a net charge injection of zero coulombs.

7. Signal Conditioning: IIR DC-Blocker

To strip the 500mV DC offset from EEG electrodes without losing the $10\mu V$ neural signal, we use a single-pole Infinite Impulse Response (IIR) filter:
Where $\alpha \approx 0.995$. This acts as a high-pass filter with a cutoff frequency:
At $f_s = 100$MHz, this provides perfect isolation of the neural manifold from biological drift.
Summary: The Boreal Neuro-Core v2.5 is a hardware implementation of Variational Calculus, providing a mathematically rigorous bridge between the continuous dynamics of the brain and the discrete logic of silicon.