

# Operations Research, Spring 2020 (108-2)

## Modeling the Optimal Vehicle Scheduling for YouBike2.0 for NTU Campus Based on Integer Programming

Yu-Chieh Kuo B07611039, Yu-Hsuan Chang B05607004, Oliver Dawson F10815022, Benjamin Pfundstein T08701102, Chung-Chia Kuo B05703100, Hung-Wen Tsai B05705007, Cho-Ting Ying, B04705030

YouBike2.0, a bicycle-sharing system in Taiwan, launched its service on NTU campus in January 2020, where lots of students get around by their own bikes everyday. Having rental sites spread across the campus, YouBike2.0 provides an alternative option for students and tourists alike to get around conveniently. To encourage the use of the bike-sharing system, it is important to make sure that there are enough bikes for rent as well as vacancy for returning of bikes at each site.

Our goal is to find the best routes for YouBike2.0 dispatchers to travel between the rental sites on their trucks, within a certain time limit, considering the need for bike reallocation at each site.

We came up with two tasks. One is to maximize the number of bikes reallocated, and the other is to minimize the sum of the difference between actual and ideal number of bikes at each site. We formulate two integer programs and build models with AMPL. We obtain our data from Wei-Cheng Huang, the student representative from department of engineering, who has been devoted to the operation of YouBike2.0 on NTU campus, and from the YouBike official website. The result shows the optimal route to travel at different times of a day, where there are varying demands at each site. At the end, we analyze our results and give them reasonable explanations.

# 1 The Preliminary Task

On NTU campus, there are 54 YouBike2.0 rental sites with 984 racks and about 500 to 600 YouBike2.0s available every day. Pickup trucks accommodate YouBike2.0s at different times to ensure the availability at all spots. To find the distance between different spots, we made a proxy that the distance between each spot can be found by calculating the norm from their latitude and longitude directly.

We denote the starting and the finishing spot as  $O$ , the set of spots  $S$ , the total number of spots  $N$ , the number of racks  $q_i$  at spot  $i$ , the distance  $c_{ij}$  between spot  $i$  and  $j$ , the number of available YouBike2.0s  $a_i$  at spot  $i$ , the number of trucks  $K$ , the capacity of a truck  $Q$ , the average velocity of a truck  $v$ , the operation time of moving a YouBike2.0  $m$ , the time limit  $L$ . Our target is to maximize the number of reallocated YouBike2.0s within the given time limit. We calculate the difference between the number of available YouBike2.0s  $a_i$  and the number of racks  $q_i$  at spot  $i$  with a ratio  $r$ , which can be written as  $w_i = |q_ir - a_i|$ , to represent the demand at spot  $i$ . We assume  $r = 0.5$  is the ideal ratio.

Let spots be a complete directed graph  $G(V, E)$ ,  $x_{ij}^k$  be 1 if truck  $k$  travels directly from spot  $i$  to spot  $j$  or otherwise 0,  $y_i^k$  be a binary variable equal to 1 if spot  $i \in S$  is visited by the truck  $k$ 's route or 0 otherwise,  $u_i$  be the order of passing spots. Our objective function is

$$\max \sum_{i \in S} \sum_{k=1}^K w_i y_i^k$$

To ensure that one arc enters and one arc leaves each visited spot, we have

$$\begin{aligned} \sum_{i \in S} \sum_{k=1}^K x_{ij}^k &= y_j^k \quad \forall j \in S \\ \sum_{i \in S} \sum_{k=1}^K x_{ji}^k &= y_j^k \quad \forall j \in S \\ \sum_{k=1}^K x_{ii}^k &= 0 \quad \forall i \in S \end{aligned}$$

Thus, to ensure there is no repeated route, we have

$$\begin{aligned}\sum_{k=1}^K y_1^k &\leq K \\ \sum_{k=1}^K y_i^k &\leq 1 \quad \forall i \in S/\{1\}\end{aligned}$$

In order to eliminate subtours, we use the method taught in class, the part about vehicle routing problems. Let  $u_i$  represent the order of passing nodes. More precisely,  $u_i = n$  if node  $i$  is the  $j$ th node to be passed in a tour. There are  $N$  spots on NTU campus, therefore, we can add the following constraints

$$u_1^k = 1 \quad k = 1, \dots, K$$

$$2 \leq u_i^k \leq N \quad \forall i \in S/\{1\}, \quad k = 1, \dots, K$$

$$u_i^k - u_j^k + 1 \leq (N - 1)(1 - x_{ij}^k) \quad \forall (i, j) \in S, i \neq 1, j \neq 1, \quad k = 1, \dots, K$$

Also, the whole accommodation work should be finished in the given time limit.

$$\sum_{i,j \in S} \frac{c_{ij}x_{ij}^k}{v} + d_{ij}m \leq L$$

Finally, the integrality constraints defining the nature of the decision variables are

$$x_{ij}^k, y_i^k \in \{0, 1\} \quad \forall (i, j) \in S, k = 1, \dots, K$$

$$u_i^k \geq 0 \quad \forall i \in S, \quad k = 1, \dots, K$$

Via this integer programming, we can find the optimal path at NTU campus in the given time limit.

## 2 Another Task

Since the former formulation ignores the capacity limit of a pickup truck, we improve the formulation with adding the capacity constraint. In addition, the former formulation only shows the demand of spots, but it doesn't show whether the demand is to deliver or to pickup. Also, to maintain the best ratio between the amount of YouBike2.0 and racks. In this case, we assume the ratio  $r = 0.5$ . Therefore, we modify it in this program.

We modify the original notation by replacing the  $w_i$ , with the demand for delivery being  $d_i$ , the demand for pickup  $p_i$  for spot  $i$ , and adding a binary parameter  $z_i$  to determine whether the delivered demand or the pickup demand for spot  $i$ ,  $z_i$  is equal to 1 if there is the delivered demand and 0 otherwise. But in this version, we can only deal with one-truck problems due to the difficulty of expanding it to a multi-truck problem. Decision variables need to be modified as well. Let  $x_{ij}^n$  be 1 if the truck travels directly from spot  $i$  to spot  $j$  with  $n$  YouBike2.0s,  $y_i$  be a binary variable equal to 1 if spot  $i \in S$  is visited by the truck or 0 otherwise,  $u_i$  be the order of passing spots,  $t$  be used to linearize the absolute value. Our objective function is to minimize the difference between the amount of YouBike2.0 that are in the racks and half of racks that exist for spot  $i$ , that is

$$\begin{aligned} \min \quad & \sum_{j=2}^N [z_j(d_j - (\sum_{i=1}^N \sum_{n=0}^Q nx_{ij}^n - \sum_{k=1}^N \sum_{m=0}^Q mx_{jk}^m)) \\ & + (1 - z_j)(p_j - (\sum_{k \in S} \sum_{n=0}^Q nx_{jk}^n - \sum_{i=1}^N \sum_{m=0}^Q mx_{jk}^m))] \\ & + (1 - z_1)(p_1 - \sum_{k=2}^N \sum_{n=0}^Q nx_{1k}^n) + z_1 t \end{aligned}$$

To ensure that the amount of YouBike2.0 in our trucks after leaving a spot is greater or equal to the amount of YouBike2.0 that were contained in a truck before arriving to the spot (minus the demand on the spot, yet, not more than the amount of bikes on the truck upon arrival, plus, the surplus of bikes on the spot), we have to define the following demand constraints (so we always have enough bikes to distribute to the next spot).

$$\begin{aligned} \sum_{i=1}^N \sum_{n=0}^Q nx_{ij}^n - d_j &\leq \sum_{k=1}^N \sum_{m=0}^Q mx_{jk}^m \quad j = 2, \dots, N \\ \sum_{i=1}^N \sum_{n=0}^Q nx_{ij}^n + p_j &\geq \sum_{k=1}^N \sum_{m=0}^Q mx_{jk}^m \quad j = 2, \dots, N \end{aligned}$$

To check the amount of YouBike2.0 the truck can move from the starting spot and to ensure the demand for the starting spot is non-negative, let  $h = \min\{p_1, Q\}$ , then we have

$$\sum_{j=2}^N x_{ij}^h = 1$$

$$t \geq \sum_{i=2}^N \sum_{m=0}^Q mx_{i1}^m - d_1$$

$$t \geq d_1 - \sum_{i=2}^N \sum_{m=0}^Q mx_{i1}^m$$

Also, the whole accommodation work should be finished in the given time limit, and we need to consider the operation time of moving YouBike2.0s. Hence, we have

$$\begin{aligned} & \sum_{j=2}^N [z_j (\sum_{i=1}^N \sum_{n=0}^Q nx_{ij}^n - \sum_{k=1}^N \sum_{m=0}^Q mx_{jk}^m) + (1 - z_j) (\sum_{k=1}^N \sum_{n=0}^Q nx_{jk}^n - \sum_{i=1}^N \sum_{m=0}^Q mx_{ij}^m)] \\ & + \sum_{i=1}^N \sum_{j=1}^N \sum_{n=0}^Q \frac{c_{ij}x_{ij}^n}{v} + (1 - z_1) \sum_{k=2}^N \sum_{n=0}^Q nx_{1k}^n + z_1 \sum_{i=2}^N \sum_{m=0}^Q mx_{i1}^m \leq L \end{aligned}$$

To ensure that one arc enters and one arc leaves each visited spot, we have

$$\begin{aligned} & \sum_{i=1}^N \sum_{n=0}^Q x_{ij}^n = y_j \quad j = 1, \dots, N \\ & \sum_{i=1}^N \sum_{n=0}^Q x_{ji}^n = y_j \quad j = 1, \dots, N \\ & x_{ii}^n = 0 \quad i = 1, \dots, N \quad n = 1, \dots, Q \end{aligned}$$

In order to eliminate subtours, we use the method in the slide of IPapp, the part of vehicle routing problems. Let  $u_i$  represent the order of passing nodes. More precisely,  $u_i = n$  if node  $i$  is the  $j$ th node to be passed in a tour. There are  $N$  spots at NTU campus, therefore, we can add the following constraints

$$u_1 = 1$$

$$2 \leq u_i \leq N \quad \forall i \in S / \{1\}$$

$$u_i - u_j + 1 \leq (N - 1)(1 - x_{ij}^n) \quad \forall (i, j) \in S, i \neq 1, j \neq 1, n = 0, \dots, Q$$

Finally, the integrality constraints defining the nature of the decision variables are

$$x_{ij}^n, y_i \in \{0, 1\} \quad \forall (i, j) \in S, n = 0, \dots, Q$$

$$t \geq 0, \quad u_i \geq 0 \quad \forall i \in S$$

### 3 Data Acquisition

We scrap the data for YouBike2.0 from the official website. With the web crawler, we can acquire the amount of available YouBike2.0  $a_i$  for spot  $i$  directly. Also, the data from the official website shows the distance between each spot. Therefore, we own all necessary data sets for this project.

We select the amount of available YouBike2.0  $a_i$  for spot  $i$  in a different time to find a different demand and the truck route. The selecting criteria is based on the timing particularity (If it is a rush hour or not). For example, we select the data in the starting time for first class in the morning and in the afternoon, and the ending time for the last class in the afternoon. We finally select 7am to 9am, 12pm to 1pm, 5pm to 7pm as out data file. Besides, due to the huge loading time to execute the AMPL program in our computers, we eliminate spots with a rack count of 10 or less YouBike2.0 on the NTU campus, which decreases the number of spots from 54 to 32. The distance between selected spots is shown as below.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32		
1	0	64	1070	874	1046	611	718	430	786	1441	1571	1502	688	698	713	767	1471	941	463	898	570	620	474	883	1012	677	665	259	437	299	959		
2	64	0	1132	936	1110	688	780	302	489	839	1506	1638	1567	752	763	774	827	1538	995	504	963	620	680	537	945	1078	735	723	312	502	323	1015	
3	1070	1132	0	554	294	509	356	876	655	465	499	613	591	457	470	503	632	505	392	771	331	620	466	598	203	150	712	458	843	644	956	331	
4	874	936	554	0	305	638	499	634	633	761	644	764	678	267	239	193	125	685	803	843	225	752	561	518	540	406	237	629	763	490	926	761	
5	1046	1110	294	305	0	616	441	818	696	660	396	562	460	358	351	352	415	428	641	871	148	740	546	599	380	152	518	582	868	612	1015	586	
6	611	668	508	638	616	516	476	484	184	196	972	1097	1053	733	402	455	615	989	338	264	503	125	77	212	307	513	614	56	359	292	448	349	
7	718	780	356	499	441	176	0	542	299	263	798	925	878	255	281	338	817	336	437	335	300	114	259	166	339	530	143	487	313	608	315		
8	240	302	876	634	818	484	542	0	308	679	1213	1342	1271	468	468	477	528	1245	821	450	670	493	466	298	706	800	441	531	241	232	391	826	
9	430	489	655	633	696	184	299	308	0	373	1078	1207	1152	375	399	441	575	119	521	214	561	191	193	115	460	625	541	237	188	169	319	533	
10	786	839	465	761	660	196	263	679	373	0	961	1078	1051	513	542	591	763	968	158	366	582	221	242	407	285	525	781	161	528	486	575	192	
11	1441	1506	494	644	398	972	798	1213	1078	961	0	130	95	754	746	744	768	47	888	1235	543	1092	911	988	883	459	871	928	1257	1006	1396	826	
12	1571	1635	613	764	525	1097	925	1342	1207	1078	130	0	97	868	875	870	889	109	995	1361	673	1217	1038	1118	604	580	1000	1052	1386	1136	1525	934	
13	1502	1567	591	678	460	809	807	821	1271	1078	95	0	61	615	804	795	804	115	980	1315	605	1175	990	1058	540	910	1011	1321	1271	1471	919		
14	688	752	452	287	168	373	255	460	100	510	754	863	815	0	29	84	250	785	584	579	485	298	263	351	561	284	388	506	528	685	554		
15	998	1062	763	470	239	351	402	281	168	399	542	718	875	804	100	510	521	778	810	100	244	782	343	394	359	269	455	275	468	360			
16	1678	1730	781	836	351	455	336	477	441	397	742	810	795	84	559	0	168	771	809	660	209	564	379	326	422	380	193	453	378	303	741	632	
17	767	827	632	126	415	615	602	528	575	763	768	869	804	250	221	166	0	809	829	788	305	716	541	460	680	491	111	617	681	419	851	794	
18	1471	1536	503	685	428	980	817	1245	1101	969	47	109	115	785	778	777	809	0	888	1253	576	1108	930	1014	695	778	919	943	1282	1035	1418	827	
19	941	1045	504	771	843	871	264	437	450	214	366	1235	1361	1315	679	695	660	680	788	1253	576	1108	930	1014	695	778	919	943	1282	1035	1418	827	
20	463	504	771	843	871	264	437	450	214	366	1235	1361	1315	679	695	660	680	788	1253	576	1108	930	1014	695	778	919	943	1282	1035	1418	827		
21	898	963	331	255	148	503	336	670	561	582	543	673	605	211	203	305	375	598	747	0	627	429	484	329	187	398	478	777	466	870	551		
22	570	620	620	752	740	125	300	493	191	221	1027	1217	1176	484	513	513	564	716	110	379	154	627	0	198	275	418	635	704	165	312	348	357	408
23	820	680	488	561	544	114	468	192	242	911	1028	980	295	326	328	541	920	362	326	428	198	0	175	268	453	544	82	380	281	492	361		
24	474	537	598	518	599	212	250	294	115	407	988	1118	1058	283	286	328	460	1014	537	328	458	275	175	0	416	544	434	249	270	80	423	535	
25	883	945	203	540	380	307	166	705	460	285	683	804	770	351	374	422	580	629	569	329	418	268	416	0	241	633	256	648	475	754	222		
26	1012	1076	150	406	152	513	339	800	625	525	459	584	540	351	356	380	492	478	492	776	187	635	452	544	241	0	580	471	808	576	940	436	
27	577	735	712	237	518	614	530	441	541	781	879	1003	915	284	260	213	111	919	865	754	704	546	433	580	0	626	620	373	790	838			
28	665	723	458	629	582	56	143	531	237	161	928	1052	1011	388	399	453	617	943	290	313	478	165	82	249	256	471	626	0	414	327	502	297	
29	259	312	843	763	865	359	487	241	186	528	1257	1386	1327	526	545	576	681	128	683	221	727	312	380	270	648	808	620	414	0	273	170	704	
30	437	502	644	490	612	292	313	232	169	486	1006	1071	258	275	303	416	1024	614	827	62	557	551	408	361	535	222	436	838	297	704	610	765	
31	299	323	958	929	1015	448	606	391	319	575	1398	1525	1471	695	708	741	851	1418	732	210	375	492	423	754	940	790	502	107	439	610			
32	959	1015	331	761	586	349	315	826	533	192	826	934	919	556	580	632	794	827	62	557	551	408	361	535	222	436	838	297	704	610	765		

Figure 1: The distance in spots

### 4 Simulation with the Reality Situation

#### 4.1 Task1: Maximize the amount of accommodation

Now we formulate an integer program and acquire the reality data, and we are able to simulate the truck route with computer program. At first, we deal with the preliminary

task of the time limit  $L = 60$  mins, the operation time of moving a YouBike2.0  $m = 1$  mins, the average velocity of truck  $v = 200 \text{ m/s}$ , the amount of truck  $k = 1$ . The following figure shows the different routes for different times. In this case, the problem can be perfectly done by AMPL, and we can conclude that the result is reasonable.

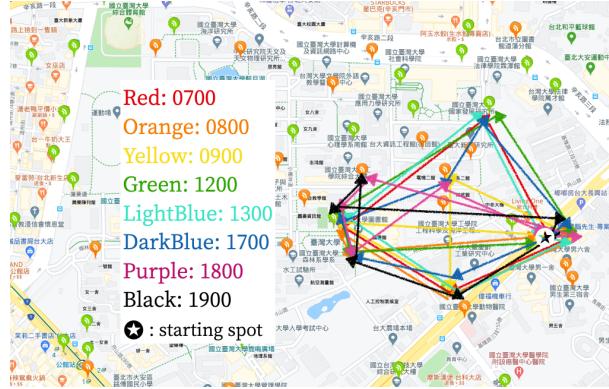


Figure 2: The visualized result

We are going to deal with the advanced case. In our observation, the YouBike2.0 staff can move a YouBike to a truck (and vice versa) in about 20 seconds. To simplify the model, we assume that the operation time of moving a YouBike2.0 is  $m = 0.5$  minutes. In this case, although we use the computer with i9 CPU and GTX-2080 GPU to compute the model, it still costs too much time to finish it. Hence, we find the approximate solution by AMPL. We use the same computer to simulate for 20 minutes, and we only test the data for 7am, 12pm and 5pm, to find the approximate optimal route. It's reasonable to merely use the data for three different time since these three data sets can simulate the situation of how YouBike2.0 company deals with the accommodation problem before the rush time.



Figure 3: The visualized result with  $k = 1, m = 0.5$

Also, our formulation can simulate the accommodation with multi-vehicles. Due to the restriction of computing resources, we only acquire the results for the amount of truck  $k = 1, 2$  with  $m = 0.5$  and  $k = 1, \dots, 3$  with  $m = 1$  for 7am, 12pm and 5pm. The figure below shows the 2 trucks with  $m = 0.5$ , which is one of our total results.

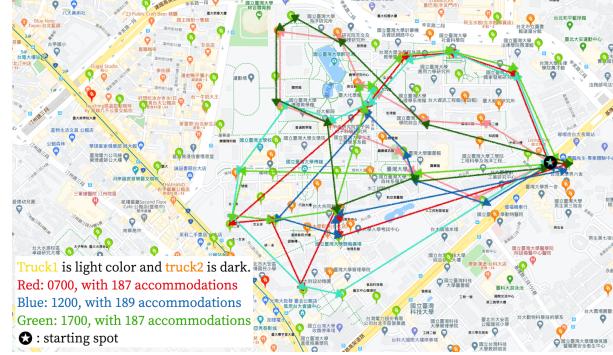


Figure 4: The visualized result with  $k = 2, m = 0.5$

The following table is to compare the differences between the total amount of accommodation with different parameters.

Comparison List				
Time	$k = 1$ and $m = 1$	$k = 1$ and $m = 0.5$	$k = 2$ and $m = 1$	$k = 2$ and $m = 0.5$
7am	53	99	103	187
8am	54	None	None	None
9am	53	None	None	None
12pm	53	88	103	189
1pm	54	None	None	None
5pm	53	98	102	189
6pm	54	None	None	None
7pm	54	None	None	None

## 4.2 Task2: Maintain the best ratio between the amount of YouBike2.0 racked and total racks

The second task is to maintain the ratio between the amount of YouBike2.0 in the racks and the total racks for each spot *i.e.* to minimize the difference between the amount of YouBike2.0 and racks with the ratio  $r = 0.5$ . In comparison with the difference after we apply our formulation, we can find the effectiveness of our formulation. The given parameters are time limit  $L = 60$  mins, the operation time of moving a YouBike2.0  $m = 0.5$  mins, the average velocity of truck  $v = 200$  m/s, the amount of truck  $k = 1$ , and the capacity of truck  $Q = 14$ .

The figure below shows the route and the amount of accommodation for each spot on the route in different times and the table shows the change of difference after accommodation. To keep the clear figure, we only draw the three routes for 7am, 12pm and 5pm, respectively. Note that the route is a directed weighted graph, the positive weight means from spot  $i$  to  $j$ , how many YouBike2.0 the truck delivers from spot  $i$  to  $j$  and the negative weight means how many YouBike2.0 the truck picks up from spot  $i$  to  $j$ .

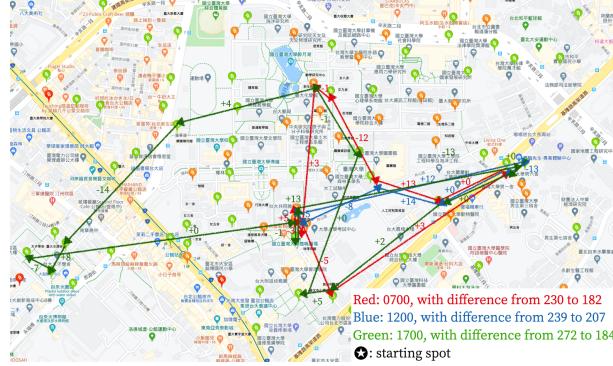


Figure 5: The visualized result to minimize the difference

Comparison List		
Time	Before	After
7am	230	182
8am	221	194
9am	235	200
12pm	239	207
1pm	257	205
5pm	272	184
6pm	256	208
7pm	237	187

## 5 Results Analysis

Instead of merely acquiring results from the computer programming, we want to analyze the result further and try to give the result a reasonable explanation, and find the insufficiency of our model.

### 5.1 The relationship between multi-vehicle and the amount of accommodation

In our results, the amount of accommodation is proportional to the number of trucks, However, The relationship between multi-vehicle and the amount of accommodation may be changed because of scaling problems when the amount of trucks increases. Without the simulation with enough trucks, we cannot determine whether the proportion will not change when the truck count is too high. The proportion may be saturated when there are many operating trucks, and it's difficult for us to execute adequate simulations to find the relationship between the number of truck and the amount of accommodation.

## 5.2 Single starting position

In our model, all trucks start from the same spot. But in reality, it is possible for trucks to start from different spots. Since it is too difficult for us to deal with the multi-starting-position problem, which is more like the real situation, we have no choice but to add the single starting position constraints. In the multi-starting-position problem, trucks can go through more spots to accommodate the YouBike2.0, which may result in a more effective route.

## 6 Conclusion

With the rise of the sharing economy, people gradually change their way of commuting. Although it aroused controversy when YouBike2.0 began to operate at NTU campus, there is no denying that the campus life becomes more convenient due to YouBike2.0. As an NTU student, the most important thing for us is to have a good environment for studying, for attending the college events, and contributing to the development of society. We believe that YouBike2.0 will be more accessible and convenient for us, and that's the main reason we select this topic and do our best to solve it. In this project, we successfully constructed the integer program, and we use the computer program to help deal with solving it, and finally find the reasonable optimal solutions. Although it is not close enough to the reality situation, we believe that this project still provides valuable insights for contemporary issues related to the accessibility of YouBike2.0 commuting.