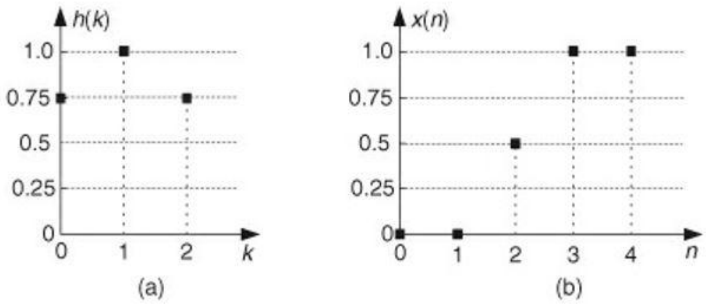
Problem 10

Consider an FIR filter whose impulse response is that shown in Figure P5-10(a). Given the x(n) filter input sequence shown in Figure P5-10(b), draw the filter's output sequence.

Figure P5-10

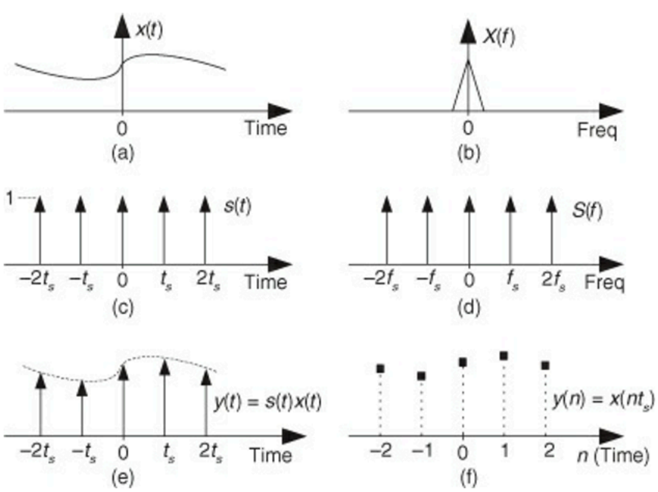


Answer:

Problem 11

Regarding the material in this chapter, it's educational to revisit the idea of periodic sampling that was presented in Chapter 2. Think about a continuous x(t) signal in Figure P5-11(a) whose spectrum is depicted in Figure P5-11(b). Also, consider the continuous periodic infinitely narrow impulses, s(t), shown in Figure P5-11(c). Reference [28] provides the algebraic acrobatics to show that the spectrum of s(t) is the continuous infinitely narrow impulses, S(f), shown in Figure P5-11(d). If we multiply the x(t) signal by the s(t) impulses, we obtain the continuous y(t) = s(t)x(t) impulse signal shown by the arrows in Figure P5-11(e).

Figure P5-11



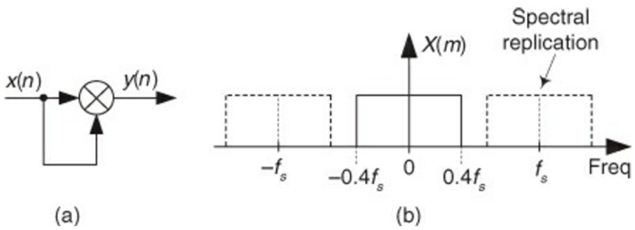
Now, if we use an analog-to-digital converter to represent those y(t) impulse values as a sequence of discrete samples, we obtain the y(n) sequence shown in Figure P5-11(f). Here is the problem: Briefly discuss what we learned in this Chapter 5 that tells us the spectrum of the y(n) samples comprises periodic replications of the X(f) in Figure P5-11(b). Your brief discussion should confirm the material in Chapter 2 which stated that discrete-time sequences have periodic (replicated) spectra.

Answer:

Problem 12

Now that we're familiar with the powerful convolution theorem, think about the discrete system shown in Figure P5-12(a).

Figure P5-12



Given that x(n)'s spectrum is the X(m) shown in Figure P5-12(b):

(a) Draw the Y(m) spectrum of sequence y(n). (We're not worried about the vertical axis scale here, merely the frequency axis and spectral shape of Y(m).)

(b) Will aliasing errors occur in the y(n) = x(n)2 output? (That is, will spectral replications in Y(m) overlap each other?)

(c) What is x(n)'s maximum one-sided bandwidth that will avoid aliasing errors in y(n)? (Stated in different words, what is the maximum one-sided bandwidth of x(n) that will avoid overlapped spectral replications in Y(m)?)

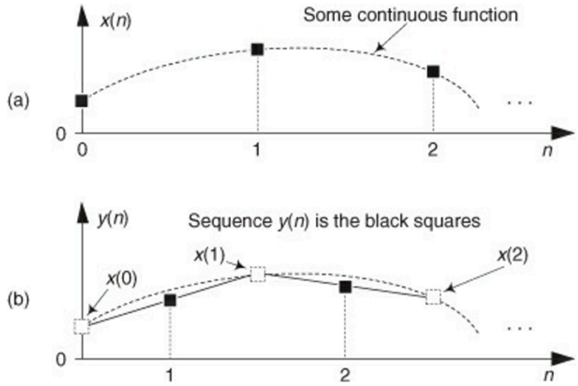
Problem 13

It's likely that you have heard of the process called linear interpolation. It's a computationally simple (but not terribly accurate) scheme for estimating sample values of a continuous function in between some given x(n) sample values of that function. For the x(n) time samples in Figure P513(a), linear interpolation is the process of computing the intermediate y(n) samples shown as the black squares in Figure P5-13(b). That is, the interpolated sample y(1) is the value lying on the center of the straight line connecting x(0) and x(1), the interpolated sample y(2) is the value lying on the center of the straight line connecting x(1) and x(2), and so on. Given this process of linear interpolation:

(a) What is the equation defining y(n) in terms of the x(n) samples?

(b) The implementation of linear interpolation is often called a filter because we build interpolators using tapped-delay line structures, just like standard FIR filter structures. Draw the block diagram of a linear interpolation filter that computes y(n) from the input x(n) sequence.

Figure P5-13



Problem 14

Consider a linear-phase lowpass FIR filter whose coefficients are

h1(k) = [−0.8, 1.6, 25.5, 47, 25.5, 1.6, −0.8],

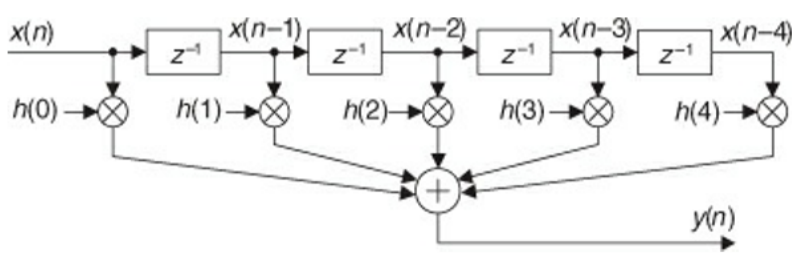
and whose DC gain, H1(0), is equal to 99.6. If we change those coefficients to

h2(k) = [−0.8, 1.6, Q, 47, Q, 1.6, −0.8],

we obtain a new DC gain equal to 103.6. What is the value of Q?

Problem 15

Figure P5-15 shows a linear-phase 5-tap FIR filter.



DSP engineers always seek to reduce the number of multipliers in their systems. Redesign the filter in Figure P5-15 to a form that reduces the number of necessary multiplications per output sample. Draw the block diagram of your new design.

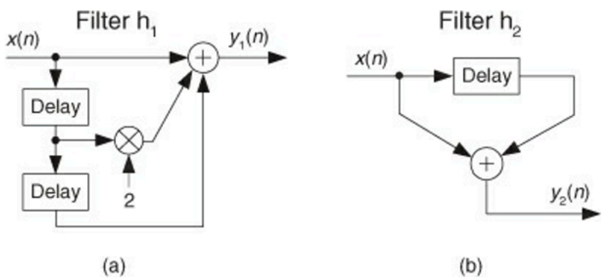
Hint: Write the difference equation for the y(n) output sequence, and recall the relationships between the filter's coefficients.

Problem 16

The two linear-phase lowpass filters in Figure P5-16 have very similar frequency responses, but those responses are not identical except at a single frequency. If we replaced Filter h1 with Filter h2 to reduce our filtering computational workload, determine the frequency, ωo, where the two H1(ω) and H2(ω) frequency responses are equal.

Hint: Begin by creating closed-form equations for H1(ω) and H2(ω) using the discrete-time Fourier transform (DTFT).

Figure P5-16



Problem 17

The following is a useful problem regarding the 3-tap nonrecursive FIR filter shown in Figure P5-17(a). The problem's solution shows us how to design computationally efficient narrowbandnoise reduction filters. If |h1| ≤ 2, the filter will have an |H(ω)| frequency magnitude response having two nulls at ±ωn as shown in Figure P5-17(b). (Here, the frequency axis value of π radians/sample corresponds to a cyclic frequency of half the sample rate, fs/2.)

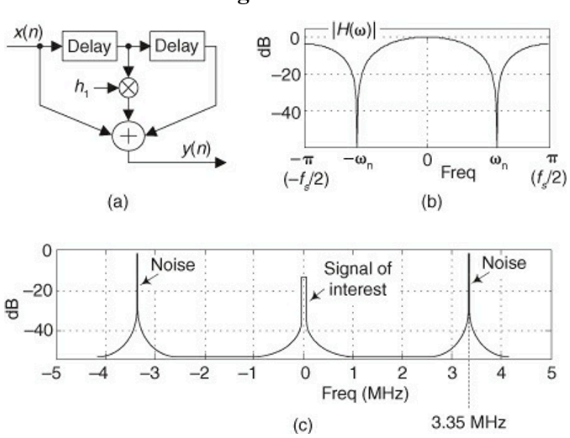
(a) Assume we have a low-frequency signal of interest that's contaminated with high-level narrowband noise located at ±3.35 MHz when the sample rate is fs = 8.25 MHz as shown in Figure P5-17(c). To attenuate that noise, for what value of h1 will the 3-tap FIR filter's nulls be located at the noise center frequency of ±3.35 MHz? Show your work.

Hint: Use the discrete-time Fourier transform (DTFT) of the filter's impulse response to create a closed-form equation for the filter's H(ω) frequency response in terms of the coefficient h1 and frequency ω. Next, obtain the expression for h1 in terms of the filter's null frequency ωn.

(b) What is the DC gain (gain at zero Hz) of our 3-tap FIR filter?

(c) Explain why the filter has a linear, or nonlinear, phase response.

Figure P5-17



Problem 18

What characteristic must the coefficients of an FIR filter have to ensure that its frequencydomain phase response is a linear function of frequency (i.e., linear phase)?

Problem 19

Quickfilter Technologies Inc. produces a tapped-delay line FIR filter chip (Part #QF1D512) that has an astounding N = 512 taps. When a new filter input sample is applied to the chip, how many addition operations must this chip perform to compute a single filter output sample?