Problem 1:

Thinking about the FFT:

(a) How do the results differ between performing an N-point FFT and performing an N-point discrete Fourier transform (DFT) on the same set of time samples?

(b) What is the restriction on the number of time samples, N, in performing an N-point radix-2 FFT?

Answer:

A.) The results are exactly the same for the FFT and DFT

B.) In a FFT the number of time samples are half of the DFT.

Problem 2:

Assume we want to compute an N-point FFT of an x(n) audio signal from a compact disc (CD), with the FFT's output frequency-domain sample spacing no greater than 1 Hz. If x(n)'s sample rate is fs = 44.1 kHz, what is the number of necessary time samples, N, applied to the FFT?

Answer:

The number of necessary time samples is 44100

Problem 3:

Assume we have an x(n) time-domain sequence, whose length is 3800 samples, on which we want to perform an FFT. The 3800 time samples represent a total signal collection-interval duration of 2 seconds.

(a) How many zero-valued samples must be appended (zero padding) to x(n) in order to implement an FFT?

(b) After the FFT is performed, what is the spacing, measured in Hz, between the frequencydomain FFT samples?

(c) In the case of lowpass sampling, what is the highest-frequency spectral component permitted in the original analog x(t) signal such that no aliasing errors occur in x(n)?

Answer:

A.) The amount of zero value samples must be 296

B.) The spacinf between the FFT samples is .000263 Hz

C.) The highest frequency is 1Hz in the original signal so no aliasing errors occurs in x(n)

Problem 4:

This problem illustrates the computational savings afforded by the FFT over that of the discrete Fourier transform (DFT). Suppose we wanted to perform a spectrum analysis on a time-domain sequence whose length is 32768 (2 15) samples. Estimate the ratio of the number of complex multiplications needed by a 32768-point DFT over the number of complex multiplies needed by a 32768-point FFT.

Answer:

The computational saving by using a FFT over a DFT for a time domain sequence whose length is 32768 is less my a ratio of 4369.066

Problem 5:

Think about the system in Figure P4-5 using an FFT to measure the amplitude of the p(t) signal. The output of the mixer, the product p(t)q(t), contains the sum of two sinusoids whose amplitudes are proportional to the peak value of p(t). The frequencies of those sinusoids are 50 Hz and 2050 Hz. The lowpass filter rejects the 2050 Hz signal. Due to imperfections in the mixer, signal p(t)q(t) is riding on a constant DC (zero Hz) bias represented as value D. This scenario results in an x(n) time sequence whose average value is 17.

(a) What is the minimum value for the analog-to-digital converter's fs sample rate to satisfy the Nyquist criterion?

(b) If we collect 2048 filter output samples and perform a 2048-point FFT, what will be the magnitude of the FFT's X(0) sample?

Answer:

A.) The minimim value for the ADC fs sample rate to satisy the nyquist criterion is 34Hz

B.) The maginute of a FFT is N^2 so at X(0) the magnitude is 0 because N = 0.

Problem 6:

Assume you've purchased a high-performance commercial real-time spectrum analyzer that contains an analog-to-digital converter so that the analyzer can accept analog (continuous) x(t) input signals. The analyzer can perform a 1024-point FFT in 50 microseconds and has two banks of memory in which the analog-to-digital converter samples are stored as shown in Figure P4-6(a). An FFT is performed on 1024 x(n) signal samples stored in Memory Bank 1 while 1024 new x(n) time samples are being loaded into Memory Bank 2. Figure P4-6 At the completion of the first FFT, the analyzer waits until Memory Bank 2 is filled with 1024 samples and then begins performing an FFT on the data in that second memory. During the second FFT computation still newer x(n) time samples are loaded into Memory Bank 1. Thus the analyzer can compute 1024 FFT results as often as once every 50 microseconds, and that is the meaning of the phrase "real-time spectrum analyzer." Here's your problem: In a lowpass sampling scenario what is the maximum one-sided bandwidth Bmax of the analog x(t) input signal for which the analyzer can perform real-time FFTs without discarding (ignoring) any discrete x(n) samples?

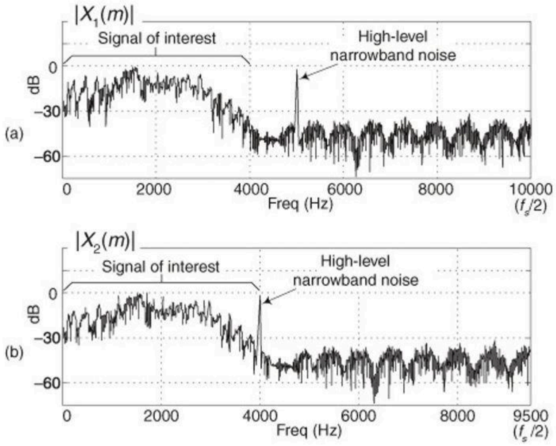
Answer:

The Bmax for the analyzer is 1.24 x 10^-5 Hz

Problem 7:

Here's an interesting problem. Assume we performed lowpass sampling of an analog x(t) signal, at a sample rate of fs = 20 kHz, obtaining a discrete sequence x1(n). Next we perform an FFT on x1(n) to obtain the |X1(m)| FFT magnitude results presented in Figure P4-7(a). There we see our signal of interest in the range of 0 to 4 kHz, but we detect a high-magnitude narrowband spectral noise signal centered at 5 kHz.

Figure P4-7



Experimenting, as every good engineer should, we change the sampling rate to f′s = 19 kHz, obtaining a new discrete sequence x2(n). Performing an FFT on x2(n), we obtain the |X2(m)| FFT magnitude results presented in Figure P4-7(b). In our new spectral results we see our signal of interest remains in the frequency range of 0 to 4 kHz, but the narrowband spectral noise signal is now centered near 4 kHz! (If this ever happens to you in practice, to quote Veronica in the 1986 movie The Fly, "Be afraid. Be very afraid.") Describe the characteristic of the analog x(t) that would account for the unexpected shift in center frequency of the narrowband noise in the |X2(m)| FFT results.

Answer:

The unexpected shift in center frequency of the narrowband noise was due to the analog x(t) signal frequency since there are spectral components that depend on sampling frequency. Ideally, we'd like to work with signals whose spectral amplitudes decrease with increasing frequency. We'll have to use an analog lowpass filter prior to A/D conversion. The cutoff frequency of the lowpass filter must, of course, be greater than the frequency band of interest but less than half the sample rate.

Problem 8:

In the text's derivation of the radix-2 FFT, to simplify the algebraic notation we represented unitymagnitude complex numbers (what we called "twiddle factors") in the following form:

$$\alpha = W_n^k$$

If k = 3 and N = 16:

(a) Express α as a complex number in polar (complex exponential) form.

(b) Express α as a complex number in rectangular form.

Answer:

a) $e^{-j2\frac{\pi 3}{16}}$

b) $\cos\left(6\frac{\pi}{16}\right) + j\sin\left(6\frac{\pi}{16}\right)$

Problem 9:

Reviewing the 8-point FFT signal-flow diagram in the text's Figure 4-5:

(a) Which x(n) input samples affect the value of the FFT's X(2) output sample?

Input samples affecting Output X(2):

x(2) + x(0) + x(3) + x(1) = X(2)

(b) Which x(n) input samples affect the value of the FFT's X(5) output sample?

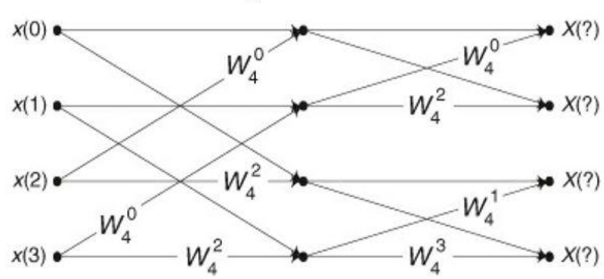
Input samples affecting Output X(5):

x(2) + x(0) + x(3) + x(1) = X(2)

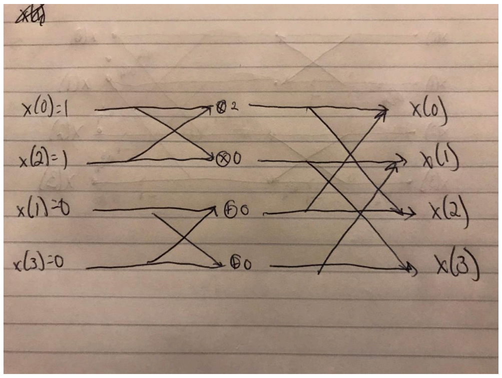
Problem 10:

Figure P4-10 shows a 4-point FFT using standard decimation-in-time butterflies. Redraw that FFT using optimized decimation-in-time butterflies as shown in the text's Figure 4-14(c). In your drawing provide the correct indices for the X(m) output samples

Figure P4-10

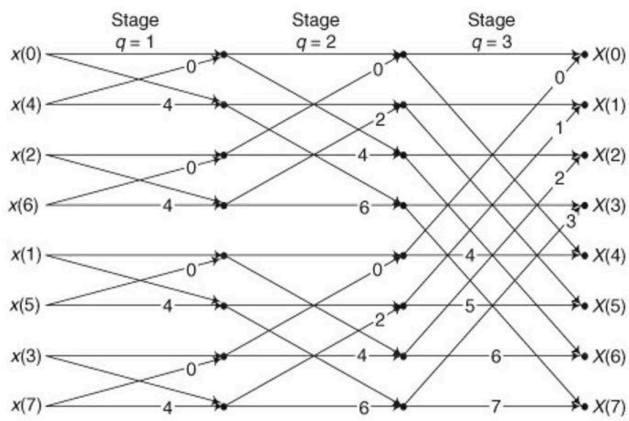


Answer:



Problem 11:

Being able to compute individual twiddle factors within an FFT can be important when implementing specialized FFTs, such as pruned FFTs. (Pruned FFTs are FFTs where we need not compute all N FFT output samples[Pruned FFT–1-Pruned FFT 4]). Figure P4-11 shows the signalflow diagram of a standard 8-point decimation-in-time (DIT) FFT with bit-reversed inputs. As in the text's Figure 4-8, the number on an arrow is the integer k of a butterfly's



twiddle factor. Notice that the number of unique twiddle factors is different in each of the three stages. The values of the R unique twiddle factors in the qth stage of a general N-point DIT FFT are given by

kth twiddle factor of qth stage = $W_N^{\frac{kN}{P}}$, for k = 0,1,2,..., R − 1. What are the expressions for the above R and P factors in terms of the FFT's q stage number?

Hint: Use the 8-point FFT in Figure P4-11 as a guide to find R and P.

Answer:

R = 2^q

P = 2^q

Problem 12:

4.12

$W_N = e^{-2j\pi/N}$

Twiddles $= e^{-2\pi j\, k/8}$

$W_0^{k=0} = e^0 = 1$

$W_1^{k=1} = e^{-j2\pi\left(\frac{1}{8}\right)} = e^{-j\left(\frac{\pi}{4}\right)} = \cos(\pi/4) - j\sin\left(\frac{\pi}{4}\right)$

$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j = \boxed{\frac{1}{\sqrt{2}}(1-j)}$

$W_2^{k=2} = e^{-j(2\pi)\left(\frac{2}{8}\right)} = e^{-j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)$

$\boxed{= -j}$

$W_3^{k=3} = e^{-j(2\pi)\left(\frac{3}{8}\right)} = e^{-j\frac{3\pi}{4}} = \cos\left(\frac{3\pi}{4}\right) - j\sin\left(\frac{3\pi}{4}\right)$

$W_3^{3/8} = \boxed{= -\frac{1}{\sqrt{2}}(1+j)}$

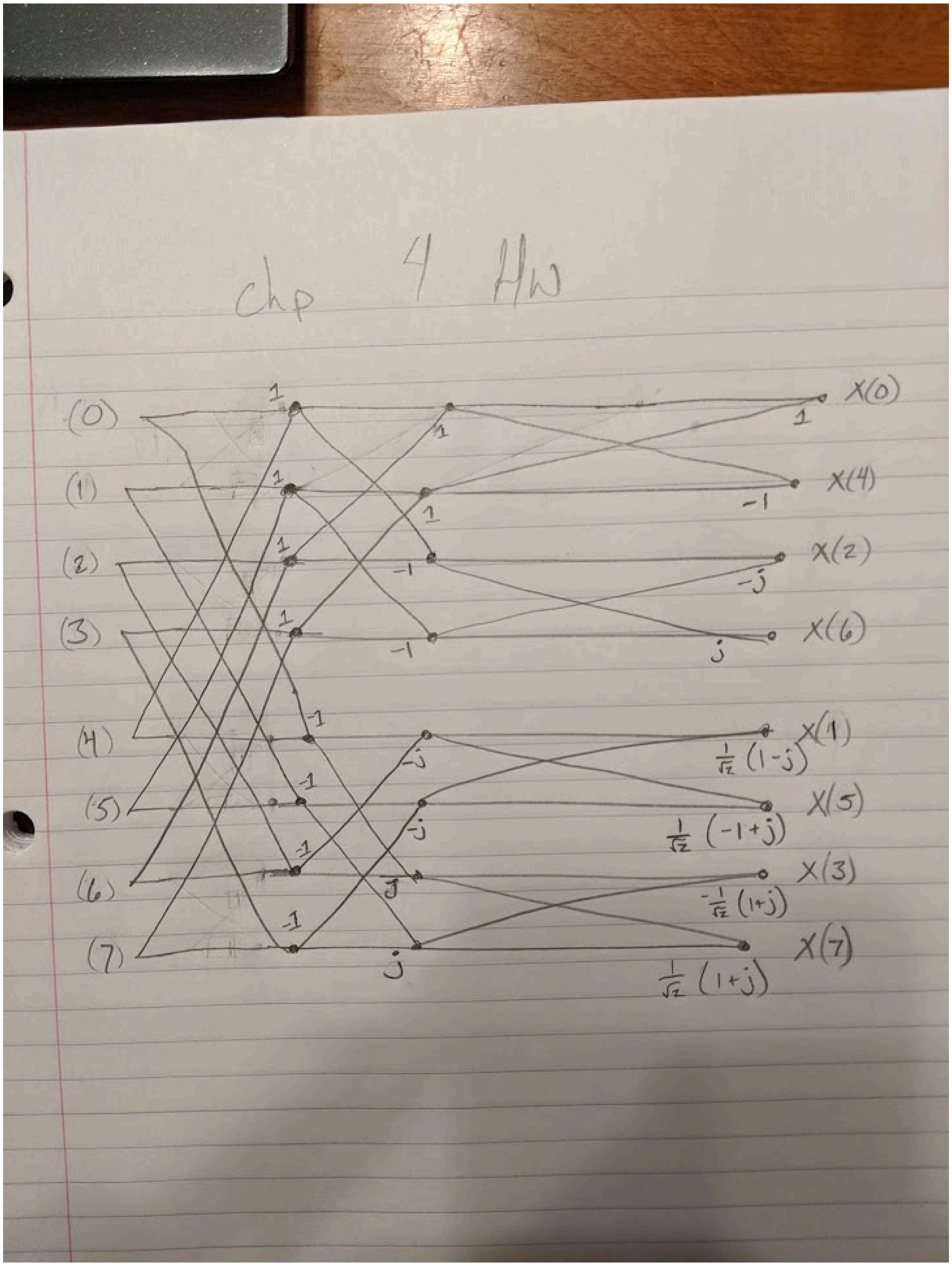
$W_4^{k=4} = e^{-j\pi} = \boxed{-1}$

$W_5^{k=5} = e^{-j2\pi\left(\frac{5}{8}\right)} = \boxed{\frac{1}{\sqrt{2}}(-1+j)}$

$W_6^{k=6} = e^{-j2\pi\left(\frac{6}{8}\right)} = \boxed{j}$

$W_7^{k=7} = e^{-2\pi\left(\frac{7}{8}\right)} = \boxed{\frac{1}{\sqrt{2}}(1+j)}$

$W_8^{k=8} = \boxed{1}$



Down the line each twiddle factors are square roots of eachother from left to right in each of the sequential stages.

Problem 13:

4.13

$A = x(0) + x(4)(1)$

$B = x(2) + x(6)(1)$

$C = x(1) + x(5)(1)$

$D = x(3) + x(7)(1)$

$E = A + B(-1)$

$= x(0) + x(4) - x(2) - x(6) = x(0) - x(2) + x(4) - x(6)$

$F = C + D(-1)$

$= x(1) - x(3) + x(5) - x(7)$

$X(2) = E + F(-j)$

$E \rightarrow x(0) - x(2) + x(4) - x(6)$

$F(j) \rightarrow -jx(1) + jx(3) - jx(5) + jx(7)$

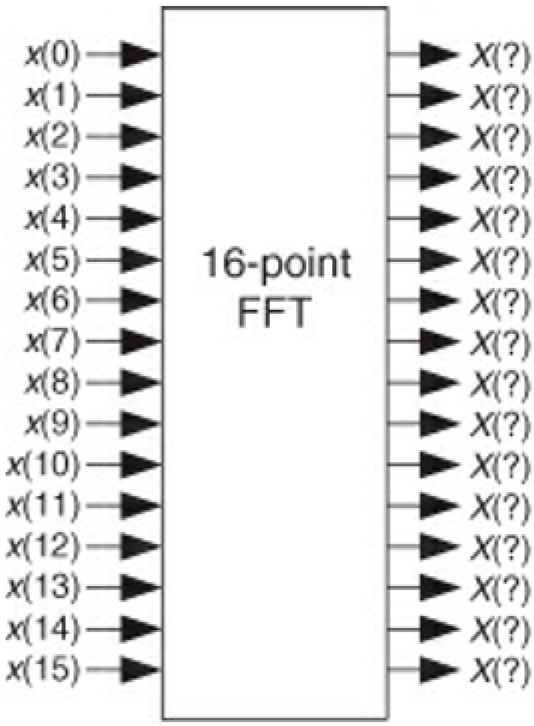
FFT $\rightarrow X(2) = x(0) - jx(1) - x(2) + jx(3) + x(4) - jx(5) - x(6) + jx(7)$

DFT $= X(2) = \sum_{m=0}^{7} x(m)\, e^{-j2\pi(2)m/8}$

$X(2) = x(0) - jx(1) - x(2) + jx(3) + x(4) - jx(5) - x(6) + jx(7)$

$\boxed{\text{FFT } X(2) = \text{DFT } X(2)}$

Problem 14:



X(0)

X(8)

X(2)

X(10)

X(4)

X(12)

X(6)

X(14)

X(1)

X(9)

X(3)

X(11)

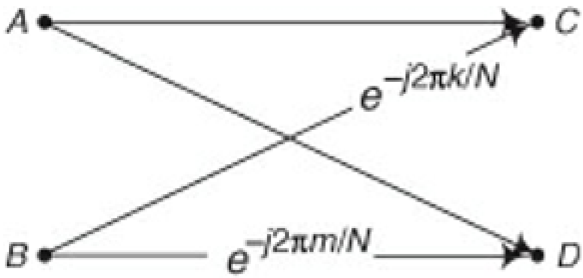
X(5)

X(13)

X(7)

X(15)

Problem 15:

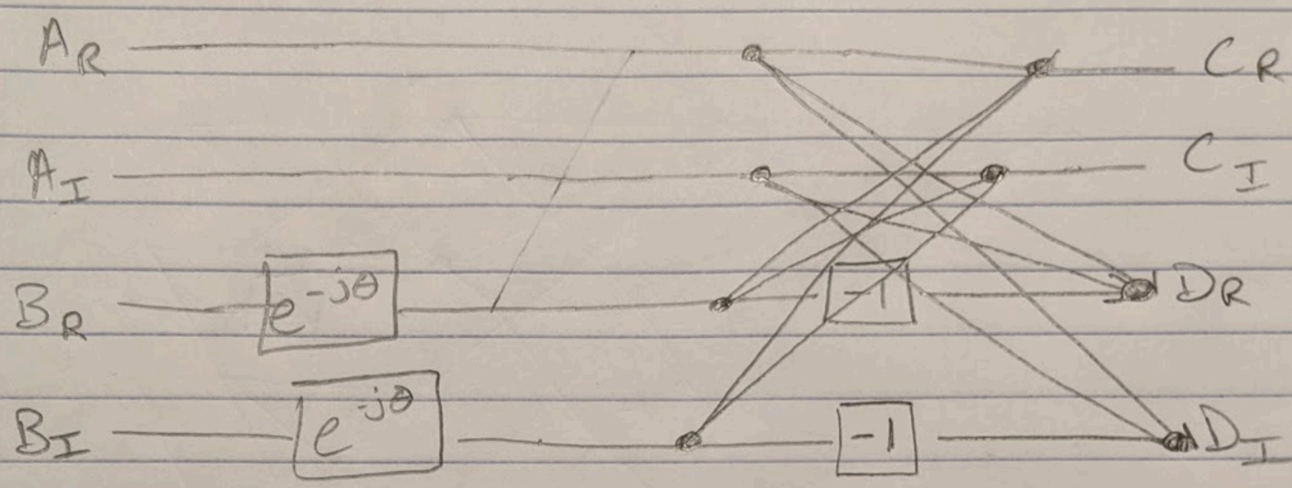


It is a Decimation in Time Butterfly (DIT) this is due to the textbook defining a DIT butterfly to have a twiddle factor from B --> D and B--> C

Whereas, Decimation in Frequency butterfly (FIT) has the same B-->D twiddle factor, although the difference is the twiddle factor A-->D

Problem 16:

4.16



$$C_R = A_R + B_R(e^{-j\theta}) + B_I(e^{-j\theta})$$

$$C_I = A_I + B_R(e^{-j\theta}) + B_I(e^{-j\theta})$$

$$D_R = -B_R(e^{-j\theta}) + A_R + A_I$$

$$D_I = -B_I(e^{-j\theta}) + A_R + A_I$$