Homework #4

1.) $f(x,y) = 3x^2 - y^2 = 0$
$g(x,y) = 3xy^2 - x^3 - 1 = 0$

Want to find sol. near $(x,y) = (1,1)$

$x_0 = y_0 = 1$

a.) With a tolerance 1×10^{-10} the iteration converged in 33 iterations

b.) The reason for the numerical 2x2 matrix is it is the inverse of the jacobian at x_0, y_0 therefore setting up the iteration as lazy newton

$$\text{Jacobian} = \begin{bmatrix} 6x & -2y \\ 3y^2-3x^2 & 6xy \end{bmatrix}$$

$$J^{-1} = \frac{1}{36x^2y + 6y^3}\begin{bmatrix} 6xy & 2y \\ 3x^2-3y^2 & 6x \end{bmatrix}$$

$$J^{-1}(1,1) = \frac{1}{36}\begin{bmatrix} 6 & 2 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 1/6 & 1/18 \\ 0 & 1/6 \end{bmatrix}$$

c.) With tolerance 1×10^{-10} the iteration converged in 5 iterations

d.) Numerical = (.5, .8660254)

$$x = \sqrt{\frac{y^2}{3}} = \frac{y}{\sqrt{3}}$$

$$3\frac{y}{\sqrt{3}}y^2 - \left(\frac{y}{\sqrt{3}}\right)^3 - 1 = 0$$

$$3\frac{y^3}{\sqrt{3}} - \frac{1}{3}\frac{y^3}{\sqrt{3}} - 1 = 0$$

$$\frac{8}{3}\frac{y^3}{\sqrt{3}} = 1$$

$$y = \sqrt[3]{y^3} = \sqrt[3]{\frac{3\sqrt{3}}{8}} = \frac{\sqrt{3}}{2}$$

$$x = \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}} = \frac{1}{2}$$

$$f\left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right) = 3\left(\tfrac{1}{2}\right)^2 - \left(\tfrac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} - \frac{3}{4} = 0$$

$$g\left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right) = 3\tfrac{1}{2}\left(\tfrac{\sqrt{3}}{2}\right)^2 - \left(\tfrac{1}{2}\right)^3 - 1 = \frac{9}{8} - \frac{1}{8} - 1 = 0$$

2.) Using $x_0 = (0,0,0)$ and tol $= 10^{-6}$
my algo for steepest descent converged to $[-2.552..\times10^{-8}, 9.99998..\times10^{-2}, 9.99...\times10^{-1}]$
in 5 iterations.
My algo for newton converged in 3 iterations to $[0., 0.1, 1.]$.
Using steepest descent as an intial guess for newton converged in two iterations.
Newton was clearly faster here with this initial guess, however it does take more computing work because of calculating the inverse of the jacobian, but this gives it the ability to converge in fewer iterations.