MA314 - Problem Set One

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Problem 1.1: The one-dimensional peak problem. Find the peak in the sequence, where the peak is defined as an element in the sequence which is greater than or equal to both of its neighbours in a sequence that is increasing and then decreasing.

Problem 1.2: A slow iterative algorithm. The sequence is increasing, then it is decreasing (I will assume strictly in both cases), so the peak will be the first number which is larger then the one directly following it. This is the check my algorithm will perform until it finds this case. The sequence is subsequently decreasing so, by definition, all following numbers will be smaller than the peak.

Running my code on the lists:

```
l1 = [1,1,1,1,1,2,3,2,1]
l2 = [3, 2, 1]
l3 = [1,2,3,4,5]
```

My code prints:

```
The peak is at index:  8
The peak is at index:  0
The peak is at index:  4
```

For length $n \geq 2$, the maximum number of comparisons is equal to the length of the list less one and the minimum is one.

Problem 1.3: A faster recursive algorithm. This algorithm will conduct an iterative binary search for the peak. First, it consider the middle two elements of the list, comparing them and then reconsidering the half of the list with the greatest of the two elements. It should be noted that in a list of n elements the 'middle' elements are the $\lfloor \frac{n}{2} \rfloor$ and $\lfloor \frac{n}{2} \rfloor + 1$ th elements.
Again, my code outputs:

```
The peak is at index:  8
The peak is at index:  0
The peak is at index:  4
```

The solution to the recurrence relation for $n = 2^i$ is $T(n) = i + 1$. If the 1s were to become threes, the answer would change to $T(n) = 3i + 1$.