Assignment06

4 factors and correlation matrix R are used for all four analyses. We assume the data is normal in order to make maximum likelihood method work. There are 25 observations for each of the 9 variables. Results have been round to 4 decimal places.

(a) Principal components method without rotation.

i. Display the table of results

```
##                  load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.6057 -0.0719 -0.3160  0.6317  0.8709    0.1291
## White meat       0.6216 -0.3029  0.6626 -0.0361  0.9185    0.0815
## Eggs             0.8540 -0.0452  0.1928  0.3060  0.8622    0.1378
## Milk             0.7561 -0.2360 -0.4096 -0.0032  0.7951    0.2049
## Fish             0.2715  0.8271 -0.3412 -0.2110  0.9187    0.0813
## Cereals         -0.8762 -0.2986  0.1019 -0.0061  0.8673    0.1327
## Starchy foods    0.5950  0.4511  0.2580 -0.3290  0.7323    0.2677
## Nuts            -0.8413  0.1832 -0.0578  0.3227  0.8489    0.1511
## Fruits/Veg      -0.2210  0.6856  0.4328  0.4515  0.9101    0.0899
## Var. Acc. For    4.0064  1.6350  1.1279  0.9547      NA        NA
## Prop. Tot. Var.  0.4452  0.1817  0.1253  0.1061      NA        NA
```

ii. Show the estimate $\hat{L}\hat{L}^T + \hat{\Psi}$

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1661  0.6529  0.6023  0.0795 -0.5453  0.0386 -0.3007 -0.0348
##  [2,]  0.1661  1.0000  0.6613  0.2702 -0.3002 -0.3865  0.4161 -0.6284 -0.0745
##  [3,]  0.6529  0.6613  1.0000  0.5764  0.0642 -0.7170  0.4368 -0.6392  0.0018
##  [4,]  0.6023  0.2702  0.5764  1.0000  0.1505 -0.6337  0.2387 -0.6567 -0.5077
##  [5,]  0.0795 -0.3002  0.0642  0.1505  1.0000 -0.5183  0.5160 -0.1253  0.2641
##  [6,] -0.5453 -0.3865 -0.7170 -0.6337 -0.5183  1.0000 -0.6277  0.6746  0.0303
##  [7,]  0.0386  0.4161  0.4368  0.2387  0.5160 -0.6277  1.0000 -0.5390  0.1410
##  [8,] -0.3007 -0.6284 -0.6392 -0.6567 -0.1253  0.6746 -0.5390  1.0000  0.4323
##  [9,] -0.0348 -0.0745  0.0018 -0.5077  0.2641  0.0303  0.1410  0.4323  1.0000
```

iii. Show the error matrix $E = R - (\hat{L}\hat{L}^T + \hat{\Psi})$ and $\|E\|$

E =

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0131 -0.0673 -0.0994 -0.0186  0.0454  0.0968 -0.0488 -0.0394
##  [2,] -0.0131  0.0000 -0.0408  0.0113  0.0662 -0.0273 -0.1023 -0.0065  0.0132
##  [3,] -0.0673 -0.0408  0.0000 -0.0009  0.0014  0.0046  0.0154  0.0794 -0.0474
##  [4,] -0.0994  0.0113 -0.0009  0.0000 -0.0126  0.0410 -0.0163  0.0357  0.0993
##  [5,] -0.0186  0.0662  0.0014 -0.0126  0.0000 -0.0059 -0.1122 -0.0219  0.0020
##  [6,]  0.0454 -0.0273  0.0046  0.0410 -0.0059  0.0000  0.0945 -0.0236  0.0162
##  [7,]  0.0968 -0.1023  0.0154 -0.0163 -0.1122  0.0945  0.0000  0.0647 -0.0566
##  [8,] -0.0488 -0.0065  0.0794  0.0357 -0.0219 -0.0236  0.0647  0.0000 -0.0573
##  [9,] -0.0394  0.0132 -0.0474  0.0993  0.0020  0.0162 -0.0566 -0.0573  0.0000
```

$\|E\| =$

```
## [1] 0.4549
```

(b) Principal components method with varimax rotation.

i. Display the table of results

```
##                  load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.0506  0.9310  0.0136 -0.0365  0.8709    0.1291
## White meat       0.9435  0.1265 -0.0989 -0.0501  0.9185    0.0815
## Eggs             0.6279  0.6639  0.1637 -0.0203  0.8622    0.1378
## Milk             0.1968  0.6103  0.2194 -0.5795  0.7951    0.2049
## Fish            -0.2280  0.0885  0.9209  0.1046  0.9187    0.0813
## Cereals         -0.3937 -0.5494 -0.6240  0.1450  0.8673    0.1327
## Starchy foods    0.5135  0.0045  0.6841  0.0261  0.7323    0.2677
## Nuts            -0.6377 -0.2633 -0.3272  0.5157  0.8489    0.1511
## Fruits/Veg      -0.0103 -0.0027  0.1780  0.9372  0.9101    0.0899
## Var. Acc. For    2.2030  2.0751  1.9289  1.5169      NA        NA
## Prop. Tot. Var.  0.2448  0.2306  0.2143  0.1685      NA        NA
```

ii. Show the estimate $\hat{L}\hat{L}^T + \hat{\Psi}$

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1661  0.6529  0.6023  0.0795 -0.5453  0.0386 -0.3007 -0.0348
##  [2,]  0.1661  1.0000  0.6613  0.2702 -0.3002 -0.3865  0.4161 -0.6284 -0.0745
##  [3,]  0.6529  0.6613  1.0000  0.5764  0.0642 -0.7170  0.4368 -0.6392  0.0018
##  [4,]  0.6023  0.2702  0.5764  1.0000  0.1505 -0.6337  0.2387 -0.6567 -0.5077
##  [5,]  0.0795 -0.3002  0.0642  0.1505  1.0000 -0.5183  0.5160 -0.1253  0.2641
##  [6,] -0.5453 -0.3865 -0.7170 -0.6337 -0.5183  1.0000 -0.6277  0.6746  0.0303
##  [7,]  0.0386  0.4161  0.4368  0.2387  0.5160 -0.6277  1.0000 -0.5390  0.1410
##  [8,] -0.3007 -0.6284 -0.6392 -0.6567 -0.1253  0.6746 -0.5390  1.0000  0.4323
##  [9,] -0.0348 -0.0745  0.0018 -0.5077  0.2641  0.0303  0.1410  0.4323  1.0000
```

iii. Show the error matrix $E = R - (\hat{L}\hat{L}^T + \hat{\Psi})$ and $\|E\|$

E =

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0131 -0.0673 -0.0994 -0.0186  0.0454  0.0968 -0.0488 -0.0394
##  [2,] -0.0131  0.0000 -0.0408  0.0113  0.0662 -0.0273 -0.1023 -0.0065  0.0132
##  [3,] -0.0673 -0.0408  0.0000 -0.0009  0.0014  0.0046  0.0154  0.0794 -0.0474
##  [4,] -0.0994  0.0113 -0.0009  0.0000 -0.0126  0.0410 -0.0163  0.0357  0.0993
##  [5,] -0.0186  0.0662  0.0014 -0.0126  0.0000 -0.0059 -0.1122 -0.0219  0.0020
##  [6,]  0.0454 -0.0273  0.0046  0.0410 -0.0059  0.0000  0.0945 -0.0236  0.0162
##  [7,]  0.0968 -0.1023  0.0154 -0.0163 -0.1122  0.0945  0.0000  0.0647 -0.0566
##  [8,] -0.0488 -0.0065  0.0794  0.0357 -0.0219 -0.0236  0.0647  0.0000 -0.0573
##  [9,] -0.0394  0.0132 -0.0474  0.0993  0.0020  0.0162 -0.0566 -0.0573  0.0000
```

$\|E\| =$

```
## [1] 0.4549
```

(c) Maximum likelihood method without varimax rotation.
i. Display the table of results

```
##                  load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.4122  0.0885  0.5747  0.1970  0.5468    0.4532
## White meat       0.8864 -0.2478 -0.3221  0.0066  0.9509    0.0491
## Eggs             0.8308  0.0813  0.3065  0.3292  0.8991    0.1009
## Milk             0.5573  0.1607  0.5409 -0.2224  0.6784    0.3216
## Fish            -0.0018  0.9921 -0.0416 -0.0024  0.9859    0.0141
## Cereals         -0.7026 -0.5396 -0.2392 -0.0418  0.8438    0.1562
## Starchy foods    0.4717  0.4076 -0.0436  0.0218  0.3910    0.6090
## Nuts            -0.8007 -0.1536 -0.1064  0.4364  0.8665    0.1335
## Fruits/Veg      -0.1503  0.2524 -0.3966  0.5829  0.5834    0.4166
## Var. Acc. For    3.3362  1.6304  1.0501  0.7292      NA        NA
## Prop. Tot. Var.  0.3707  0.1812  0.1167  0.0810      NA        NA
```

ii. Show the estimate $\hat{L}\hat{L}^T + \hat{\Psi}$

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1596  0.5906  0.5110  0.0626 -0.4831  0.2097 -0.3188 -0.1527
##  [2,]  0.1596  1.0000  0.6197  0.2784 -0.2341 -0.4123  0.3313 -0.6346 -0.0642
##  [3,]  0.5906  0.6197  1.0000  0.5686  0.0656 -0.7147  0.4188 -0.5666 -0.0340
##  [4,]  0.5110  0.2784  0.5686  1.0000  0.1365 -0.5984  0.2999 -0.6255 -0.3873
##  [5,]  0.0626 -0.2341  0.0656  0.1365  1.0000 -0.5239  0.4053 -0.1475  0.2658
##  [6,] -0.4831 -0.4123 -0.7147 -0.5984 -0.5239  1.0000 -0.5418  0.6527  0.0399
##  [7,]  0.2097  0.3313  0.4188  0.2999  0.4053 -0.5418  1.0000 -0.4261  0.0620
##  [8,] -0.3188 -0.6346 -0.5666 -0.6255 -0.1475  0.6527 -0.4261  1.0000  0.3782
##  [9,] -0.1527 -0.0642 -0.0340 -0.3873  0.2658  0.0399  0.0620  0.3782  1.0000
```

iii. Show the error matrix $E = R - (\hat{L}\hat{L}^T + \hat{\Psi})$ and $\|E\|$

E =

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0066 -0.0050 -0.0080 -0.0017 -0.0168 -0.0743 -0.0307  0.0785
##  [2,] -0.0066  0.0000  0.0007  0.0030  0.0000 -0.0015 -0.0175 -0.0004  0.0029
##  [3,] -0.0050  0.0007  0.0000  0.0069  0.0000  0.0022  0.0334  0.0068 -0.0115
##  [4,] -0.0080  0.0030  0.0069  0.0000  0.0014  0.0056 -0.0775  0.0044 -0.0210
##  [5,] -0.0017  0.0000  0.0000  0.0014  0.0000 -0.0003 -0.0014  0.0004  0.0003
##  [6,] -0.0168 -0.0015  0.0022  0.0056 -0.0003  0.0000  0.0086 -0.0017  0.0066
##  [7,] -0.0743 -0.0175  0.0334 -0.0775 -0.0014  0.0086  0.0000 -0.0482  0.0224
##  [8,] -0.0307 -0.0004  0.0068  0.0044  0.0004 -0.0017 -0.0482  0.0000 -0.0032
##  [9,]  0.0785  0.0029 -0.0115 -0.0210  0.0003  0.0066  0.0224 -0.0032  0.0000
```

$\|E\| =$

```
## [1] 0.2199
```

(d) Maximum likelihood method with varimax rotation.
i. Display the table of results

```
##                  load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.7203  0.0375  0.0700 -0.1472  0.5468    0.4532
## White meat       0.1676  0.9571 -0.0650 -0.0511  0.9509    0.0491
## Eggs             0.7805  0.5205  0.1381  0.0051  0.8991    0.1009
## Milk             0.5723  0.1787  0.2285 -0.5165  0.6784    0.3216
## Fish             0.0298 -0.1768  0.9672  0.1355  0.9859    0.0141
## Cereals         -0.5643 -0.3662 -0.6104  0.1365  0.8438    0.1562
## Starchy foods    0.2269  0.3385  0.4743 -0.0024  0.3910    0.6090
## Nuts            -0.2639 -0.6097 -0.3343  0.5599  0.8665    0.1335
## Fruits/Veg      -0.0775 -0.0023  0.1731  0.7398  0.5834    0.4166
## Var. Acc. For    1.9301  1.8719  1.7551  1.1889      NA        NA
## Prop. Tot. Var.  0.2145  0.2080  0.1950  0.1321      NA        NA
```

ii. Show the estimate $\hat{L}\hat{L}^T + \hat{\Psi}$

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1596  0.5906  0.5110  0.0626 -0.4831  0.2097 -0.3188 -0.1527
##  [2,]  0.1596  1.0000  0.6197  0.2784 -0.2341 -0.4123  0.3313 -0.6346 -0.0642
##  [3,]  0.5906  0.6197  1.0000  0.5686  0.0656 -0.7147  0.4188 -0.5666 -0.0340
##  [4,]  0.5110  0.2784  0.5686  1.0000  0.1365 -0.5984  0.2999 -0.6255 -0.3873
##  [5,]  0.0626 -0.2341  0.0656  0.1365  1.0000 -0.5239  0.4053 -0.1475  0.2658
##  [6,] -0.4831 -0.4123 -0.7147 -0.5984 -0.5239  1.0000 -0.5418  0.6527  0.0399
##  [7,]  0.2097  0.3313  0.4188  0.2999  0.4053 -0.5418  1.0000 -0.4261  0.0620
##  [8,] -0.3188 -0.6346 -0.5666 -0.6255 -0.1475  0.6527 -0.4261  1.0000  0.3782
##  [9,] -0.1527 -0.0642 -0.0340 -0.3873  0.2658  0.0399  0.0620  0.3782  1.0000
```

iii. Show the error matrix $E = R - (\hat{L}\hat{L}^T + \hat{\Psi})$ and $\|E\|$

E =

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0066 -0.0050 -0.0080 -0.0017 -0.0168 -0.0743 -0.0307  0.0785
##  [2,] -0.0066  0.0000  0.0007  0.0030  0.0000 -0.0015 -0.0175 -0.0004  0.0029
##  [3,] -0.0050  0.0007  0.0000  0.0069  0.0000  0.0022  0.0334  0.0068 -0.0115
##  [4,] -0.0080  0.0030  0.0069  0.0000  0.0014  0.0056 -0.0775  0.0044 -0.0210
##  [5,] -0.0017  0.0000  0.0000  0.0014  0.0000 -0.0003 -0.0014  0.0004  0.0003
##  [6,] -0.0168 -0.0015  0.0022  0.0056 -0.0003  0.0000  0.0086 -0.0017  0.0066
##  [7,] -0.0743 -0.0175  0.0334 -0.0775 -0.0014  0.0086  0.0000 -0.0482  0.0224
##  [8,] -0.0307 -0.0004  0.0068  0.0044  0.0004 -0.0017 -0.0482  0.0000 -0.0032
##  [9,]  0.0785  0.0029 -0.0115 -0.0210  0.0003  0.0066  0.0224 -0.0032  0.0000
```

$\|E\| =$

```
## [1] 0.2199
```

(e)

i. **Both** methods produce a reasonable good approximation. Since rotation doesn't affect the error, there are only two error matrices, one for each method.

Three criteria are used to check the errors:

1. Check $\|E\|$ and compare to $\|R\|$,where $\|R\| = 4.618$.

2. Check the entries in error matrix and their relative error proportion, which is calculated by dividing each entry in the error matrix by its corresponding entry in the R matrix, then take absolute value of it.

3. Check the average (mean) of the relative error proportion.

For principal components method error matrix:

1. $\|E\|$ is relatively small compared to $\|R\|$.

2. **53** out of **81** entries has an error **below 15%**.

3. Average relative error is **17.12%**, still reasonable.

For maximum likelihood method error matrix:

1. $\|E\|$ is relatively small compared to $\|R\|$.

2. **71** out of **81** entries has an error **below 15%**

3. Average relative error is **8.07%**, which is pretty reasonable.

Therefore, **both** methods produce a reasonable good approximation.

ii. **Maximum likelihood method** produces the better approximation here. It has the smaller $\|E\| \approx 0.22$ and average relative error %. Most of the entries in the maximum likelihood error matrix have **smaller maginitudes** than their counter part in the principal components error matrix. We can observe this by **subtracting the absolute values** of the matrices:
abs(E(principal components)) - abs(E(maximum likelihood)) =

```
##          [,1]   [,2]    [,3]    [,4]   [,5]   [,6]    [,7]   [,8]    [,9]
##  [1,]  0.0000 0.0065  0.0623  0.0913 0.0169 0.0286  0.0226 0.0181 -0.0391
##  [2,]  0.0065 0.0000  0.0401  0.0083 0.0661 0.0258  0.0848 0.0061  0.0103
##  [3,]  0.0623 0.0401  0.0000 -0.0060 0.0014 0.0024 -0.0181 0.0726  0.0359
##  [4,]  0.0913 0.0083 -0.0060  0.0000 0.0112 0.0353 -0.0612 0.0312  0.0783
##  [5,]  0.0169 0.0661  0.0014  0.0112 0.0000 0.0056  0.1107 0.0215  0.0017
##  [6,]  0.0286 0.0258  0.0024  0.0353 0.0056 0.0000  0.0859 0.0220  0.0096
##  [7,]  0.0226 0.0848 -0.0181 -0.0612 0.1107 0.0859  0.0000 0.0165  0.0341
##  [8,]  0.0181 0.0061  0.0726  0.0312 0.0215 0.0220  0.0165 0.0000  0.0541
##  [9,] -0.0391 0.0103  0.0359  0.0783 0.0017 0.0096  0.0341 0.0541  0.0000
```

Nearly all the entries of the result are positive, meaning the entries in maximum likelihood error matrix have **smaller errors** in general.

Code used to solve the questions:

```
rm(list = ls())
library(readxl)
library(psych)
protein <- read_excel("C:/Users/John/Desktop/STAT 445/Data/europe_protein_data.xls")[,-1]
R <- cor(protein)
maketable <- function(){
  table <- matrix(NA, nrow=11,ncol=6)
  rownames(table) <- c(colnames(R), "Var. Acc. For", "Prop. Tot. Var.")
  colnames(table) <- c("load 1", "load 2", "load 3", "load 4", "commun.", "spec.var.")
  return(table)
}

filltable <- function(t, x) {
  for(i in 1:4){
    t[1:9,i] <- x$loadings[,i]
    t[10:11,i] <- x$Vaccounted[1:2,i]
  }
  t[1:9,5] <- x$communality
  t[1:9,6] <- x$uniquenesses
  return(t)
}

# a
# i
a <- principal(R,nfactors=4, n.obs=25, rotate="none")
t1 <- maketable()
t1 <- filltable(t1,a)
round(t1, 4)
```

```
##                  load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.6057 -0.0719 -0.3160  0.6317  0.8709    0.1291
## White meat       0.6216 -0.3029  0.6626 -0.0361  0.9185    0.0815
## Eggs             0.8540 -0.0452  0.1928  0.3060  0.8622    0.1378
## Milk             0.7561 -0.2360 -0.4096 -0.0032  0.7951    0.2049
## Fish             0.2715  0.8271 -0.3412 -0.2110  0.9187    0.0813
## Cereals         -0.8762 -0.2986  0.1019 -0.0061  0.8673    0.1327
## Starchy foods    0.5950  0.4511  0.2580 -0.3290  0.7323    0.2677
## Nuts            -0.8413  0.1832 -0.0578  0.3227  0.8489    0.1511
## Fruits/Veg      -0.2210  0.6856  0.4328  0.4515  0.9101    0.0899
## Var. Acc. For    4.0064  1.6350  1.1279  0.9547      NA        NA
## Prop. Tot. Var.  0.4452  0.1817  0.1253  0.1061      NA        NA
```

```
# ii
estimate1 <-a$loadings%*%t(a$loadings)+diag(a$uniquenesses)
colnames(estimate1)=NULL; rownames(estimate1)=NULL
round(estimate1,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1661  0.6529  0.6023  0.0795 -0.5453  0.0386 -0.3007 -0.0348
##  [2,]  0.1661  1.0000  0.6613  0.2702 -0.3002 -0.3865  0.4161 -0.6284 -0.0745
##  [3,]  0.6529  0.6613  1.0000  0.5764  0.0642 -0.7170  0.4368 -0.6392  0.0018
```

```
##  [4,]  0.6023  0.2702  0.5764  1.0000  0.1505 -0.6337  0.2387 -0.6567 -0.5077
##  [5,]  0.0795 -0.3002  0.0642  0.1505  1.0000 -0.5183  0.5160 -0.1253  0.2641
##  [6,] -0.5453 -0.3865 -0.7170 -0.6337 -0.5183  1.0000 -0.6277  0.6746  0.0303
##  [7,]  0.0386  0.4161  0.4368  0.2387  0.5160 -0.6277  1.0000 -0.5390  0.1410
##  [8,] -0.3007 -0.6284 -0.6392 -0.6567 -0.1253  0.6746 -0.5390  1.0000  0.4323
##  [9,] -0.0348 -0.0745  0.0018 -0.5077  0.2641  0.0303  0.1410  0.4323  1.0000
```

```
# iii
error1 <- R-estimate1
colnames(error1)=NULL; rownames(error1)=NULL
round(error1,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0131 -0.0673 -0.0994 -0.0186  0.0454  0.0968 -0.0488 -0.0394
##  [2,] -0.0131  0.0000 -0.0408  0.0113  0.0662 -0.0273 -0.1023 -0.0065  0.0132
##  [3,] -0.0673 -0.0408  0.0000 -0.0009  0.0014  0.0046  0.0154  0.0794 -0.0474
##  [4,] -0.0994  0.0113 -0.0009  0.0000 -0.0126  0.0410 -0.0163  0.0357  0.0993
##  [5,] -0.0186  0.0662  0.0014 -0.0126  0.0000 -0.0059 -0.1122 -0.0219  0.0020
##  [6,]  0.0454 -0.0273  0.0046  0.0410 -0.0059  0.0000  0.0945 -0.0236  0.0162
##  [7,]  0.0968 -0.1023  0.0154 -0.0163 -0.1122  0.0945  0.0000  0.0647 -0.0566
##  [8,] -0.0488 -0.0065  0.0794  0.0357 -0.0219 -0.0236  0.0647  0.0000 -0.0573
##  [9,] -0.0394  0.0132 -0.0474  0.0993  0.0020  0.0162 -0.0566 -0.0573  0.0000
```

```
round(sqrt(sum(error1^2)),4)
```

```
## [1] 0.4549
```

```
# b
# i
b <- principal(R,nfactors=4, n.obs=25, rotate="varimax")
t2 <- maketable()
t2 <- filltable(t2,b)
round(t2, 4)
```

```
##                  load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.0506  0.9310  0.0136 -0.0365  0.8709    0.1291
## White meat       0.9435  0.1265 -0.0989 -0.0501  0.9185    0.0815
## Eggs             0.6279  0.6639  0.1637 -0.0203  0.8622    0.1378
## Milk             0.1968  0.6103  0.2194 -0.5795  0.7951    0.2049
## Fish            -0.2280  0.0885  0.9209  0.1046  0.9187    0.0813
## Cereals         -0.3937 -0.5494 -0.6240  0.1450  0.8673    0.1327
## Starchy foods    0.5135  0.0045  0.6841  0.0261  0.7323    0.2677
## Nuts            -0.6377 -0.2633 -0.3272  0.5157  0.8489    0.1511
## Fruits/Veg      -0.0103 -0.0027  0.1780  0.9372  0.9101    0.0899
## Var. Acc. For    2.2030  2.0751  1.9289  1.5169      NA        NA
## Prop. Tot. Var.  0.2448  0.2306  0.2143  0.1685      NA        NA
```

```
# ii
estimate2 <-b$loadings%*%t(b$loadings)+diag(b$uniquenesses)
colnames(estimate2)=NULL; rownames(estimate2)=NULL
round(estimate2,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1661  0.6529  0.6023  0.0795 -0.5453  0.0386 -0.3007 -0.0348
##  [2,]  0.1661  1.0000  0.6613  0.2702 -0.3002 -0.3865  0.4161 -0.6284 -0.0745
##  [3,]  0.6529  0.6613  1.0000  0.5764  0.0642 -0.7170  0.4368 -0.6392  0.0018
##  [4,]  0.6023  0.2702  0.5764  1.0000  0.1505 -0.6337  0.2387 -0.6567 -0.5077
##  [5,]  0.0795 -0.3002  0.0642  0.1505  1.0000 -0.5183  0.5160 -0.1253  0.2641
##  [6,] -0.5453 -0.3865 -0.7170 -0.6337 -0.5183  1.0000 -0.6277  0.6746  0.0303
##  [7,]  0.0386  0.4161  0.4368  0.2387  0.5160 -0.6277  1.0000 -0.5390  0.1410
##  [8,] -0.3007 -0.6284 -0.6392 -0.6567 -0.1253  0.6746 -0.5390  1.0000  0.4323
##  [9,] -0.0348 -0.0745  0.0018 -0.5077  0.2641  0.0303  0.1410  0.4323  1.0000
```

```
# iii
error2 <- R-estimate2
colnames(error2)=NULL; rownames(error2)=NULL
round(error2,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0131 -0.0673 -0.0994 -0.0186  0.0454  0.0968 -0.0488 -0.0394
##  [2,] -0.0131  0.0000 -0.0408  0.0113  0.0662 -0.0273 -0.1023 -0.0065  0.0132
##  [3,] -0.0673 -0.0408  0.0000 -0.0009  0.0014  0.0046  0.0154  0.0794 -0.0474
##  [4,] -0.0994  0.0113 -0.0009  0.0000 -0.0126  0.0410 -0.0163  0.0357  0.0993
##  [5,] -0.0186  0.0662  0.0014 -0.0126  0.0000 -0.0059 -0.1122 -0.0219  0.0020
##  [6,]  0.0454 -0.0273  0.0046  0.0410 -0.0059  0.0000  0.0945 -0.0236  0.0162
##  [7,]  0.0968 -0.1023  0.0154 -0.0163 -0.1122  0.0945  0.0000  0.0647 -0.0566
##  [8,] -0.0488 -0.0065  0.0794  0.0357 -0.0219 -0.0236  0.0647  0.0000 -0.0573
##  [9,] -0.0394  0.0132 -0.0474  0.0993  0.0020  0.0162 -0.0566 -0.0573  0.0000
```

```
round(sqrt(sum(error2^2)),4)
```

```
## [1] 0.4549
```

```
# c
# i
c <- fa(R,nfactors=4, n.obs=25, rotate="none", fm="ml")
t3 <- maketable()
t3 <- filltable(t3,c)
round(t3, 4)
```

```
##                 load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.4122  0.0885  0.5747  0.1970  0.5468    0.4532
## White meat       0.8864 -0.2478 -0.3221  0.0066  0.9509    0.0491
## Eggs             0.8308  0.0813  0.3065  0.3292  0.8991    0.1009
## Milk             0.5573  0.1607  0.5409 -0.2224  0.6784    0.3216
## Fish            -0.0018  0.9921 -0.0416 -0.0024  0.9859    0.0141
## Cereals         -0.7026 -0.5396 -0.2392 -0.0418  0.8438    0.1562
## Starchy foods    0.4717  0.4076 -0.0436  0.0218  0.3910    0.6090
## Nuts            -0.8007 -0.1536 -0.1064  0.4364  0.8665    0.1335
## Fruits/Veg      -0.1503  0.2524 -0.3966  0.5829  0.5834    0.4166
## Var. Acc. For    3.3362  1.6304  1.0501  0.7292      NA        NA
## Prop. Tot. Var.  0.3707  0.1812  0.1167  0.0810      NA        NA
```

```r
# ii
estimate3 <-c$loadings%*%t(c$loadings)+diag(c$uniquenesses)
colnames(estimate3)=NULL; rownames(estimate3)=NULL
round(estimate3,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1596  0.5906  0.5110  0.0626 -0.4831  0.2097 -0.3188 -0.1527
##  [2,]  0.1596  1.0000  0.6197  0.2784 -0.2341 -0.4123  0.3313 -0.6346 -0.0642
##  [3,]  0.5906  0.6197  1.0000  0.5686  0.0656 -0.7147  0.4188 -0.5666 -0.0340
##  [4,]  0.5110  0.2784  0.5686  1.0000  0.1365 -0.5984  0.2999 -0.6255 -0.3873
##  [5,]  0.0626 -0.2341  0.0656  0.1365  1.0000 -0.5239  0.4053 -0.1475  0.2658
##  [6,] -0.4831 -0.4123 -0.7147 -0.5984 -0.5239  1.0000 -0.5418  0.6527  0.0399
##  [7,]  0.2097  0.3313  0.4188  0.2999  0.4053 -0.5418  1.0000 -0.4261  0.0620
##  [8,] -0.3188 -0.6346 -0.5666 -0.6255 -0.1475  0.6527 -0.4261  1.0000  0.3782
##  [9,] -0.1527 -0.0642 -0.0340 -0.3873  0.2658  0.0399  0.0620  0.3782  1.0000
```

```r
# iii
error3 <- R-estimate3
colnames(error3)=NULL; rownames(error3)=NULL
round(error3,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0066 -0.0050 -0.0080 -0.0017 -0.0168 -0.0743 -0.0307  0.0785
##  [2,] -0.0066  0.0000  0.0007  0.0030  0.0000 -0.0015 -0.0175 -0.0004  0.0029
##  [3,] -0.0050  0.0007  0.0000  0.0069  0.0000  0.0022  0.0334  0.0068 -0.0115
##  [4,] -0.0080  0.0030  0.0069  0.0000  0.0014  0.0056 -0.0775  0.0044 -0.0210
##  [5,] -0.0017  0.0000  0.0000  0.0014  0.0000 -0.0003 -0.0014  0.0004  0.0003
##  [6,] -0.0168 -0.0015  0.0022  0.0056 -0.0003  0.0000  0.0086 -0.0017  0.0066
##  [7,] -0.0743 -0.0175  0.0334 -0.0775 -0.0014  0.0086  0.0000 -0.0482  0.0224
##  [8,] -0.0307 -0.0004  0.0068  0.0044  0.0004 -0.0017 -0.0482  0.0000 -0.0032
##  [9,]  0.0785  0.0029 -0.0115 -0.0210  0.0003  0.0066  0.0224 -0.0032  0.0000
```

```r
round(sqrt(sum(error3^2)),4)
```

```
## [1] 0.2199
```

```r
# d
# i
d <- fa(R,nfactors=4, n.obs=25, rotate="varimax", fm="ml")
t4 <- maketable()
t4 <- filltable(t4,d)
round(t4, 4)
```

```
##                  load 1  load 2  load 3  load 4 commun. spec.var.
## Red meat         0.7203  0.0375  0.0700 -0.1472  0.5468    0.4532
## White meat       0.1676  0.9571 -0.0650 -0.0511  0.9509    0.0491
## Eggs             0.7805  0.5205  0.1381  0.0051  0.8991    0.1009
## Milk             0.5723  0.1787  0.2285 -0.5165  0.6784    0.3216
## Fish             0.0298 -0.1768  0.9672  0.1355  0.9859    0.0141
## Cereals         -0.5643 -0.3662 -0.6104  0.1365  0.8438    0.1562
## Starchy foods    0.2269  0.3385  0.4743 -0.0024  0.3910    0.6090
```

```
## Nuts            -0.2639 -0.6097 -0.3343  0.5599  0.8665    0.1335
## Fruits/Veg      -0.0775 -0.0023  0.1731  0.7398  0.5834    0.4166
## Var. Acc. For    1.9301  1.8719  1.7551  1.1889      NA        NA
## Prop. Tot. Var.  0.2145  0.2080  0.1950  0.1321      NA        NA
```

```
# ii
estimate4 <-d$loadings%*%t(d$loadings)+diag(d$uniquenesses)
colnames(estimate4)=NULL; rownames(estimate4)=NULL
round(estimate4,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  1.0000  0.1596  0.5906  0.5110  0.0626 -0.4831  0.2097 -0.3188 -0.1527
##  [2,]  0.1596  1.0000  0.6197  0.2784 -0.2341 -0.4123  0.3313 -0.6346 -0.0642
##  [3,]  0.5906  0.6197  1.0000  0.5686  0.0656 -0.7147  0.4188 -0.5666 -0.0340
##  [4,]  0.5110  0.2784  0.5686  1.0000  0.1365 -0.5984  0.2999 -0.6255 -0.3873
##  [5,]  0.0626 -0.2341  0.0656  0.1365  1.0000 -0.5239  0.4053 -0.1475  0.2658
##  [6,] -0.4831 -0.4123 -0.7147 -0.5984 -0.5239  1.0000 -0.5418  0.6527  0.0399
##  [7,]  0.2097  0.3313  0.4188  0.2999  0.4053 -0.5418  1.0000 -0.4261  0.0620
##  [8,] -0.3188 -0.6346 -0.5666 -0.6255 -0.1475  0.6527 -0.4261  1.0000  0.3782
##  [9,] -0.1527 -0.0642 -0.0340 -0.3873  0.2658  0.0399  0.0620  0.3782  1.0000
```

```
# iii
error4 <- R-estimate4
colnames(error4)=NULL; rownames(error4)=NULL
round(error4,4)
```

```
##          [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]    [,9]
##  [1,]  0.0000 -0.0066 -0.0050 -0.0080 -0.0017 -0.0168 -0.0743 -0.0307  0.0785
##  [2,] -0.0066  0.0000  0.0007  0.0030  0.0000 -0.0015 -0.0175 -0.0004  0.0029
##  [3,] -0.0050  0.0007  0.0000  0.0069  0.0000  0.0022  0.0334  0.0068 -0.0115
##  [4,] -0.0080  0.0030  0.0069  0.0000  0.0014  0.0056 -0.0775  0.0044 -0.0210
##  [5,] -0.0017  0.0000  0.0000  0.0014  0.0000 -0.0003 -0.0014  0.0004  0.0003
##  [6,] -0.0168 -0.0015  0.0022  0.0056 -0.0003  0.0000  0.0086 -0.0017  0.0066
##  [7,] -0.0743 -0.0175  0.0334 -0.0775 -0.0014  0.0086  0.0000 -0.0482  0.0224
##  [8,] -0.0307 -0.0004  0.0068  0.0044  0.0004 -0.0017 -0.0482  0.0000 -0.0032
##  [9,]  0.0785  0.0029 -0.0115 -0.0210  0.0003  0.0066  0.0224 -0.0032  0.0000
```

```
round(sqrt(sum(error4^2)),4)
```

```
## [1] 0.2199
```

```
# e
# i
length(which(abs(error1/R)<0.15))
```

```
## [1] 53
```

```
length(which(abs(error3/R)<0.15))
```

```
## [1] 71
```

```r
round(sqrt(sum(R^2)),4)
```

```
## [1] 4.618
```

```r
mean(abs(error1/R))
```

```
## [1] 0.1711873
```

```r
mean(abs(error3/R))
```

```
## [1] 0.08068495
```

```r
# ii
round(abs(error1)-abs(error3),4)
```

```
##          [,1]   [,2]    [,3]    [,4]   [,5]   [,6]    [,7]   [,8]    [,9]
##  [1,]  0.0000 0.0065  0.0623  0.0913 0.0169 0.0286  0.0226 0.0181 -0.0391
##  [2,]  0.0065 0.0000  0.0401  0.0083 0.0661 0.0258  0.0848 0.0061  0.0103
##  [3,]  0.0623 0.0401  0.0000 -0.0060 0.0014 0.0024 -0.0181 0.0726  0.0359
##  [4,]  0.0913 0.0083 -0.0060  0.0000 0.0112 0.0353 -0.0612 0.0312  0.0783
##  [5,]  0.0169 0.0661  0.0014  0.0112 0.0000 0.0056  0.1107 0.0215  0.0017
##  [6,]  0.0286 0.0258  0.0024  0.0353 0.0056 0.0000  0.0859 0.0220  0.0096
##  [7,]  0.0226 0.0848 -0.0181 -0.0612 0.1107 0.0859  0.0000 0.0165  0.0341
##  [8,]  0.0181 0.0061  0.0726  0.0312 0.0215 0.0220  0.0165 0.0000  0.0541
##  [9,] -0.0391 0.0103  0.0359  0.0783 0.0017 0.0096  0.0341 0.0541  0.0000
```