Assignment 03

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Problem 1
(a) Sample mean vector $\tilde{\bar{x}}$:

$$\tilde{\bar{x}} = \begin{pmatrix} 63.24528 \\ 252.50943 \\ 239.79245 \\ 165.43396 \\ 35.69811 \end{pmatrix} \tag{1}$$

Sample covariance matrix S_x:

$$S_x = \begin{bmatrix} 103.150217706821 & -250.511973875181 & -302.178882438316 & -156.089259796807 & -59.0399129172714 \\ -250.511973875181 & 38379.2162554427 & 10171.1654571843 & 19321.8708272859 & 315.791364296081 \\ -302.178882438316 & 10171.1654571843 & 33212.9753265602 & 4627.84179970972 & 1328.76306240929 \\ -156.089259796807 & 19321.8708272859 & 4627.84179970972 & 19363.9042089985 & 366.960449927431 \\ -59.0399129172714 & 315.791364296081 & 1328.76306240929 & 366.960449927431 & 369.176342525399 \end{bmatrix} \tag{2}$$

(b) Data matrix Y (scroll down, matrix too big):

$$Y = \begin{bmatrix} -65 & -140 & 302 \\ 183 & -20 & 80 \\ 148 & -124 & 185 \\ 108 & 27 & 30 \\ -325 & 232 & 244 \\ 191 & -91 & 162 \\ 70 & -253 & 350 \\ 6 & 18 & 95 \\ -108 & -215 & 413 \\ -449 & 71 & 297 \\ -541 & 266 & 185 \\ 74 & 9 & 105 \\ -624 & 327 & 152 \\ 166 & -64 & 104 \\ -744 & 352 & 156 \\ -30 & 198 & 61 \\ -38 & 24 & 191 \\ 110 & -50 & 151 \\ 131 & -26 & 114 \\ -13 & 125 & 48 \\ 41 & -109 & 294 \\ -102 & 50 & 136 \\ 156 & -50 & 114 \\ -271 & 154 & 101 \\ -54 & 38 & 127 \\ 129 & -243 & 354 \\ -243 & 57 & 253 \\ -42 & -44 & 310 \\ 74 & -115 & 522 \\ -231 & -273 & 527 \\ -100 & -114 & 351 \\ -864 & 393 & 160 \\ -854 & -124 & 668 \\ -30 & 154 & 322 \\ 18 & 258 & 165 \\ -502 & -276 & 683 \\ -510 & 335 & 65 \\ -138 & -36 & 230 \\ -18 & -53 & 182 \\ 191 & -629 & 682 \\ -434 & 121 & 242 \\ 44 & -237 & 460 \\ -227 & 397 & 92 \\ -703 & 324 & 613 \\ -2 & -25 & 153 \\ -40 & 256 & 142 \\ -234 & -404 & 725 \\ 197 & -127 & 423 \\ -3 & 161 & 424 \\ -227 & 56 & 182 \\ 31 & -204 & 534 \\ -349 & 571 & 318 \\ -433 & -254 & 622 \end{bmatrix} \tag{3}$$

Sample mean vector $\tilde{\bar{y}}$:

$$\tilde{\bar{y}} = \begin{pmatrix} -141.13208 \\ 12.71698 \\ 275.49057 \end{pmatrix} \tag{4}$$

Sample covariance matrix S_y:

$$S_y = \begin{bmatrix} 80257.0399129173 & -29233.057329463 & -11073.2608853411 \\ -29233.057329463 & 51249.8606676343 & -24054.7815674891 \\ -11073.2608853411 & -24054.7815674891 & 36239.6777939042 \end{bmatrix} \tag{5}$$

(c) The matrix A such that $A\tilde{X} = \tilde{Y}$:

$$A = \begin{bmatrix} 3 & 0 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{6}$$

Where
Row 1 = $(3, 0, 0, -2, 0)$ for $Y1 = 3X_1 + 0X_2 + 0X_3 - 2X_4 + 0X_5$.
Row 2 = $(0, 1, -1, -0, 0)$ for $Y2 = 0X_1 + 1X_2 - 1X_3 + 0X_4 + 0X_5$.
Row 3 = $(0, 0, 1, 0, 1)$ for $Y3 = 0X_1 + 0X_2 + 1X_3 + 0X_4 + 1X_5$.

(d) Sample mean vector $\tilde{\bar{y}} = A\tilde{\bar{x}}$:

$$\tilde{\bar{y}} = \begin{pmatrix} -141.13208 \\ 12.71698 \\ 275.49057 \end{pmatrix} \tag{7}$$

Sample covariance matrix $S_y = AS_xA^T$:

$$S_y = \begin{bmatrix} 80257.0399129173 & -29233.057329463 & -11073.2608853411 \\ -29233.057329463 & 51249.8606676343 & -24054.7815674891 \\ -11073.2608853411 & -24054.7815674891 & 36239.6777939042 \end{bmatrix} \tag{8}$$

Both $\tilde{\bar{y}}$ and S_y are the same compare to part (b).

Codes used to compute (a), (b), and (d):

```
library(readxl)
data<- read_excel("C:/Users/John/Desktop/STAT 445/Data/survival_data.xlsx", col_names = F)
```

```
## New names:
## * `` -> ...1
## * `` -> ...2
## * `` -> ...3
## * `` -> ...4
## * `` -> ...5
## * ...
```

```r
View(data)
X <- data[,2:6]
colnames(X) <- c("X1","X2","X3","X4","X5")
X
```

```
## # A tibble: 53 x 5
##       X1    X2    X3    X4    X5
##    <dbl> <dbl> <dbl> <dbl> <dbl>
##  1    61   124   264   124    38
##  2    69    42    62    12    18
##  3    62    25   149    19    36
##  4    66    45    18    45    12
##  5    63   412   180   257    64
##  6    79    51   142    23    20
##  7    54    46   299    46    51
##  8    62   103    85    90    10
##  9    46   146   361   123    52
## 10    57   340   269   310    28
## # ... with 43 more rows
```

```r
# a
x_bar <- colMeans(X)
x_bar
```

```
##        X1        X2        X3        X4        X5
##  63.24528 252.50943 239.79245 165.43396  35.69811
```

```r
Sx <- cov(X)
Sx
```

```
##            X1         X2         X3         X4         X5
## X1  103.15022  -250.5120  -302.1789  -156.0893  -59.03991
## X2 -250.51197 38379.2163 10171.1655 19321.8708  315.79136
## X3 -302.17888 10171.1655 33212.9753  4627.8418 1328.76306
## X4 -156.08926 19321.8708  4627.8418 19363.9042  366.96045
## X5  -59.03991   315.7914  1328.7631   366.9604  369.17634
```

```r
#b
Y1 <- 3*X$X1 - 2*X$X4
Y2 <- X$X2 - X$X3
Y3 <- X$X3 + X$X5

Y <- cbind(Y1,Y2,Y3)
Y
```

```
##         Y1   Y2  Y3
##  [1,]  -65 -140 302
##  [2,]  183  -20  80
##  [3,]  148 -124 185
##  [4,]  108   27  30
##  [5,] -325  232 244
```

```
##  [6,]  191  -91 162
##  [7,]   70 -253 350
##  [8,]    6   18  95
##  [9,] -108 -215 413
## [10,] -449   71 297
## [11,] -541  266 185
## [12,]   74    9 105
## [13,] -624  327 152
## [14,]  166  -64 104
## [15,] -744  352 156
## [16,]  -30  198  61
## [17,]  -38   24 191
## [18,]  110  -50 151
## [19,]  131  -26 114
## [20,]  -13  125  48
## [21,]   41 -109 294
## [22,] -102   50 136
## [23,]  156  -50 114
## [24,] -271  154 101
## [25,]  -54   38 127
## [26,]  129 -243 354
## [27,] -243   57 253
## [28,]  -42  -44 310
## [29,]   74 -115 522
## [30,] -231 -273 527
## [31,] -100 -114 351
## [32,] -864  393 160
## [33,] -854 -124 668
## [34,]  -30  154 322
## [35,]   18  258 165
## [36,] -502 -276 683
## [37,] -510  335  65
## [38,] -138  -36 230
## [39,]  -18  -53 182
## [40,]  191 -629 682
## [41,] -434  121 242
## [42,]   44 -237 460
## [43,] -227  397  92
## [44,] -703  324 613
## [45,]   -2  -25 153
## [46,]  -40  256 142
## [47,] -234 -404 725
## [48,]  197 -127 423
## [49,]   -3  161 424
## [50,] -227   56 182
## [51,]   31 -204 534
## [52,] -349  571 318
## [53,] -433 -254 622
```

```r
y_bar <- colMeans(Y)
y_bar
```

```
##          Y1          Y2          Y3
## -141.13208   12.71698  275.49057
```

```r
Sy <- cov(Y)
Sy
```

```
##           Y1        Y2        Y3
## Y1  80257.04 -29233.06 -11073.26
## Y2 -29233.06  51249.86 -24054.78
## Y3 -11073.26 -24054.78  36239.68
```

```r
# d
A <- matrix(c(3,0,0,-2,0,0,1,-1,0,0,0,0,1,0,1), ncol = 5, nrow = 3, byrow = T)
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    3    0    0   -2    0
## [2,]    0    1   -1    0    0
## [3,]    0    0    1    0    1
```

```r
A%*%x_bar
```

```
##            [,1]
## [1,] -141.13208
## [2,]   12.71698
## [3,]  275.49057
```

```r
y_bar
```

```
##         Y1         Y2         Y3
## -141.13208   12.71698  275.49057
```

```r
A%*%Sx%*%t(A)
```

```
##           [,1]      [,2]      [,3]
## [1,]  80257.04 -29233.06 -11073.26
## [2,] -29233.06  51249.86 -24054.78
## [3,] -11073.26 -24054.78  36239.68
```

```r
Sy
```

```
##           Y1        Y2        Y3
## Y1  80257.04 -29233.06 -11073.26
## Y2 -29233.06  51249.86 -24054.78
## Y3 -11073.26 -24054.78  36239.68
```

Problem 2
(a) The matrix A such that $A\tilde{X} = \tilde{Y}$:

$$A = \begin{bmatrix} 2 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix} \tag{9}$$

Where
Row 1 $= (2, 1, -3, 0)$ for $Y_1 = 2X_1 + 1X_2 - 3X_3 + 0X_4$.
Row 2 $= (0, 0, 0, 1)$ for $Y_2 = 0X_1 + 0X_2 + 0X_3 + 1X_4$.
Row 3 $= (0, -2, 2, 0)$ for $Y_3 = 0X_1 - 2X_2 + 2X_3 + 0X_4$.

The parameters for the distribution for $\tilde{Y}$ are:
Mean vector $\tilde{\mu_Y}$, where $\tilde{\mu_Y} = A\tilde{\mu}$:

$$\tilde{\mu_Y} = \begin{pmatrix} -2 \\ 2 \\ 8 \end{pmatrix} \tag{10}$$

Covariance matrix Σ_Y, where $\Sigma_Y = A\Sigma A$:

$$\Sigma_Y = \begin{bmatrix} 5 & -1 & -2 \\ -1 & 4 & 2 \\ -2 & 2 & 4 \end{bmatrix} \tag{11}$$

(b) Parameters for the distribution for $(X_1, X_4)^T$:
Mean vector $\tilde{\mu_{1,4}}$

$$\tilde{\mu_{1,4}} = \begin{pmatrix} \mu_1 \\ \mu_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{12}$$

Covariance matrix $\Sigma_{1,4}$, which is formed by the values on the four corners of matrix Σ

$$\Sigma_{1,4} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} \tag{13}$$

Codes used to solve (a) and (b):

```r
# 2
mu = c(2,-3,1,2)
mu
```

```
## [1]  2 -3  1  2
```

```r
Sigma <- matrix(c(1,1,1,1,1,3,2,0,1,2,2,1,1,0,1,4),nrow=4,ncol=4,byrow=T)
Sigma
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    1
## [2,]    1    3    2    0
## [3,]    1    2    2    1
## [4,]    1    0    1    4
```

```r
# a
A <- matrix(c(2,1,-3,0,0,0,0,1,0,-2,2,0), ncol = 4, nrow = 3, byrow = T)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    2    1   -3    0
## [2,]    0    0    0    1
## [3,]    0   -2    2    0
```

```r
mu_y <- A%*%mu
mu_y
```

```
##      [,1]
## [1,]   -2
## [2,]    2
## [3,]    8
```

```r
A%*%Sigma%*%t(A)
```

```
##      [,1] [,2] [,3]
## [1,]    5   -1   -2
## [2,]   -1    4    2
## [3,]   -2    2    4
```

```r
# b
new_mu <- mu[c(1,4)]
new_mu
```

```
## [1] 2 2
```

```r
new_Sigma <- Sigma[c(1,4,2,3),]
new_Sigma <- new_Sigma[,c(1,4,2,3)]
new_Sigma <- new_Sigma[c(1,2),c(1,2)]
new_Sigma
```

```
##      [,1] [,2]
## [1,]    1    1
## [2,]    1    4
```

Problem 3

(a) Reorder both columns and rows for the covariance matrix Σ in the order of $3, 6, 1, 2, 4, 5$, now the covariance between X_3 and X_6 are the partition of the first two columns and first two rows, the covariance between X_1, X_2, X_4 and X_5 are the partition of the last four columns and last four rows. We get the new Σ matrix:

$$\Sigma = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 2 & 0 \\ 0 & 0 & 1 & 8 & 2 & 1 \\ 0 & 0 & 2 & 2 & 5 & 3 \\ 0 & 0 & 0 & 1 & 3 & 6 \end{bmatrix} \tag{14}$$

As we can see, in this matrix the intersection between row 3 to 6 and column 1 to 2, row 1 to 2 and column 3 to 6 are all 0's. This means there is no correlation between $(X_3, X_6)^T$ and $(X_1, X_2, X_4, X_5)^T$. Therefore they are independent

(b) Let $\tilde{Y} = (Y_1, Y_2)$ where $Y_1 = X_1 + X_4$ and $Y_2 = X_2 + X_5$
The matrix A where $A\tilde{X} = \tilde{Y}$ is :

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{15}$$

Compute the parameter of the distribution of $\tilde{Y}$, where covariance matrix $\Sigma_Y = A\Sigma A^T$, get:

$$\Sigma_Y = \begin{bmatrix} 13 & 6 \\ 6 & 16 \end{bmatrix} \tag{16}$$

As we can see, the covariance between Y_1 and Y_2 are not all 0's.
Therefore $X_1 + X_4$ and $X_2 + X_5$ are dependent.

(c) Let $\tilde{Z} = (Z_1, Z_2)$ where $Z_1 = X_2$ and $Z_2 = 3X_1 - X_2 + 4X_3 + 2X_4 + X_5 + 5X_6$
The matrix A where $A\tilde{X} = \tilde{Z}$ is :

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & -1 & 4 & 2 & 1 & -5 \end{bmatrix} \tag{17}$$

Compute the parameter of the distribution of $\tilde{Z}$, where covariance matrix $\Sigma_Z = A\Sigma A^T$, get:

$$\Sigma_Z = \begin{bmatrix} 8 & 0 \\ 0 & 148 \end{bmatrix} \tag{18}$$

As we can see, the covariance between Z_1 and Z_2 are all 0's.
Therefore X_2 and $3X_1 - X_2 + 4X_3 + 2X_4 + X_5 + 5X_6$ are independent.

Codes used to solve for (a) to (c)

```
# 3
mu = c(3,-2,5,1,0,2)
mu
```

```
## [1]  3 -2  5  1  0  2
```

```
Sigma <- matrix(c(4,1,0,2,0,0,1,8,0,2,1,0,0,0,3,0,0,1,
                  2,2,0,5,3,0,0,1,0,3,6,0,0,0,1,0,0,2),nrow=6,ncol=6,byrow=T)
Sigma
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    4    1    0    2    0    0
## [2,]    1    8    0    2    1    0
## [3,]    0    0    3    0    0    1
## [4,]    2    2    0    5    3    0
## [5,]    0    1    0    3    6    0
## [6,]    0    0    1    0    0    2
```

```
# a
new_Sigma <- Sigma[c(3,6,1,2,4,5),]
new_Sigma <- new_Sigma[,c(3,6,1,2,4,5)]
new_Sigma
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    3    1    0    0    0    0
## [2,]    1    2    0    0    0    0
## [3,]    0    0    4    1    2    0
## [4,]    0    0    1    8    2    1
## [5,]    0    0    2    2    5    3
## [6,]    0    0    0    1    3    6
```

```
# b

A <- matrix(c(1,0,0,1,0,0,0,1,0,0,1,0), ncol = 6, nrow = 2, byrow = T)
A
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    0    0    1    0    0
## [2,]    0    1    0    0    1    0
```

```
A%*%Sigma%*%t(A)
```

```
##      [,1] [,2]
## [1,]   13    6
## [2,]    6   16
```

```
# dependent

# c
A <- matrix(c(0,1,0,0,0,0,3,-1,4,2,1,-5), ncol = 6, nrow = 2, byrow = T)
A
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    3   -1    4    2    1   -5
```

```
A%*%Sigma%*%t(A)
```

```
##      [,1] [,2]
## [1,]    8    0
## [2,]    0  148
```

```
# independent
```

```
# helper method for latex syntax
CJ <- function (matrix) {
  s = ""
  for (i in 1:nrow(matrix)) {
    p <- paste(paste(as.character(matrix[i,]), collapse = " & "), "\\ ")
    s <- paste(s,p)
  }
  s <- paste("\begin{align} Y &= \begin{bmatrix}", s, "\\end{bmatrix} \\end{align}")
  # editing required afterwards
  return (s)
}
CJ(A)
```

```
## [1] "\begin{align} Y &= \begin{bmatrix}  0 & 1 & 0 & 0 & 0 & 0 \\  3 & -1 & 4 & 2 & 1 & -5 \\  \\end
```