Elementary Differential Equations and Boundary Value Problems

Twelfth Edition

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Chapter 1

Introduction

Section 1.1
Basic Mathematical Models and Direction Fields

Applications for Differential Equations

- **Differential equations** are equations containing derivatives.
- The following are examples of physical phenomena involving rates of change:
  - Motion of fluids
  - Motion of mechanical systems
  - Flow of current in electrical circuits
  - Dissipation of heat in solid objects
  - Seismic waves
  - Population dynamics
- A differential equation that describes a physical process is often called a **mathematical model.**

Example 1.1.1: Free Fall

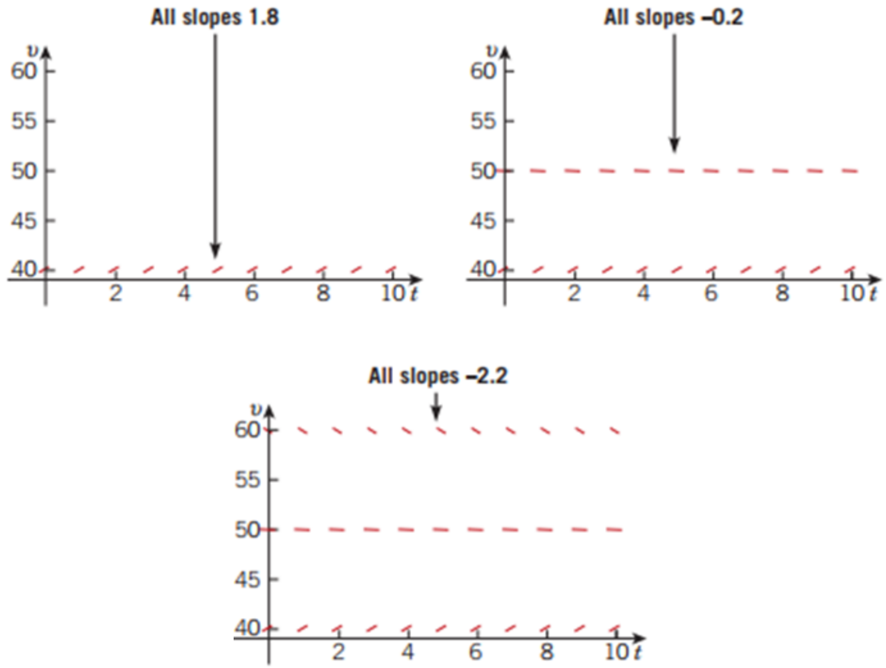
- Formulate a differential equation describing motion of an object falling in the atmosphere near sea level.
- Variables: time t, velocity v
- Newton's 2nd Law: $F = ma = m\left(\frac{dv}{dt}\right)$ (net force)
- Force of gravity: $F = mg$ (downward force)
- Force of air resistance: $F = \gamma v$ (upward force)
- Then $m\frac{dv}{dt} = mg - \gamma v$
- Taking $g = 9.8$ m/s², $m = 10$ kg, $\gamma = 2$ kg/s,

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}$$

Example 1.1.2: Sketching Direction Field for Velocity vs. Time

- Using differential equation and table, plot slopes (estimates) on axes below. The resulting graph is called a **direction field**. (Note that values of *v* do not depend on *t*.)

v	v'
0	9.8
5	8.8
10	7.8
15	6.8
20	5.8
25	4.8
30	3.8
35	2.8
40	1.8
45	0.8
50	–0.2
55	–1.2
60	–2.2

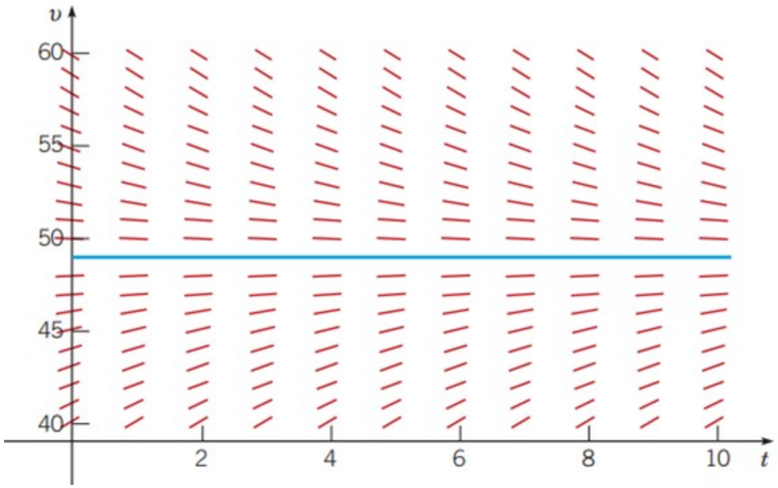


Example 1.1.2: Plotting Direction Field Using Maple Software

- Sample Maple commands for graphing a direction field:
  - with(DEtools):
  - DEplot(diff(v(t),t)=9.8-v(t)/5,v(t), t = 0..10,v = 0..80, stepsize = .1, color = blue);
- When graphing direction fields, be sure to use an appropriate window, in order to display all equilibrium solutions and relevant solution behavior.

Example 1.1.2: Including an Equilibrium Solution in the Direction Field

- Arrows give tangent lines to solution curves, and indicate where the solution is increasing & decreasing (and by how much).
- Horizontal solution curves are called **equilibrium solutions**.
- Use the graph below to solve for equilibrium solution, and then determine analytically by setting $v' = 0$.



Set $v' = 0$:

$$\Leftrightarrow 9.8 - 0.2v = 0$$

$$\Leftrightarrow v = (5)(9.8)$$

$$\Leftrightarrow v = 49$$

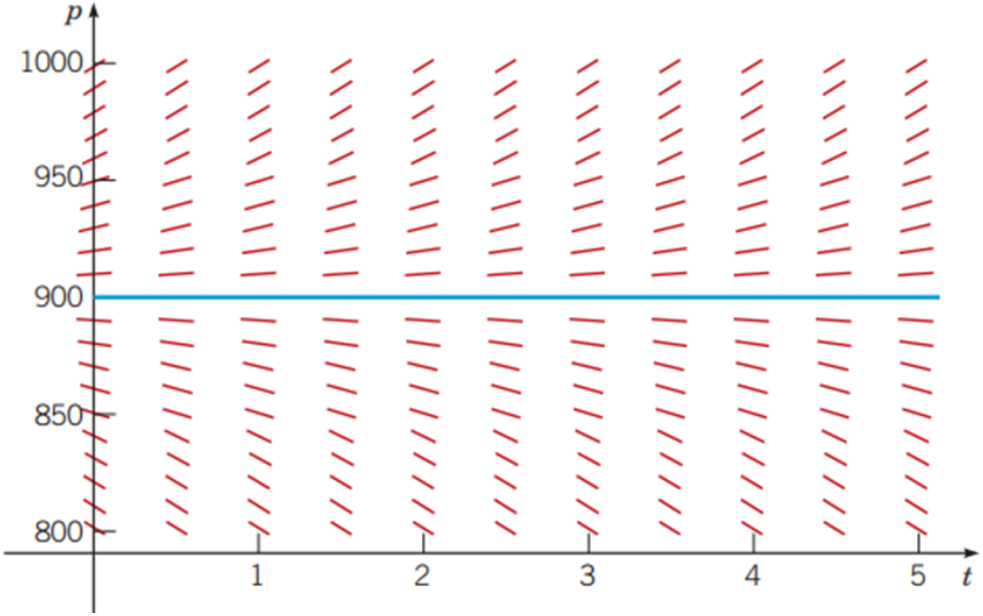
Example 1.1.3: Mice and Owls Population

- Consider a mouse population that reproduces at a rate proportional to the current population, with a rate constant equal to 0.5 mice/month (assuming no owls present).
- When owls are present, they eat the mice. Suppose that the owls eat 15 per day (average). Write a differential equation describing mouse population in the presence of owls. (Assume that there are 30 days in a month.)
- Solution:

$$\frac{dp}{dt} = \frac{p}{2} - 450$$

Example 1.1.3: Direction Field and Equilibrium Solution for Mice Population

Discuss the solution curve behavior, and find the equilibrium solution.



$$\frac{dp}{dt} = \frac{p}{2} - 450$$

Steps in Constructing Mathematical Models Using Differential Equations

- Identify independent and dependent variables and assign letters to represent them.
- Choose the units of measure for each variable.
- Articulate the basic principle that underlies or governs the problem you are investigating.
  - This requires your being familiar with the field in which the problem originates.
- Express the principle or law in the previous step in terms of the variables identified at the start.
  - This may involve the use of intermediate variables related to the primary variables.
- Make sure each term of your equation has the same physical units.
- The result may involve one or more differential equations.

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