Bayes Estimation

Practice 2

Ing. Dayan Bravo Fraga[1]
Ing. Mario Herrera Almira[2]

Teacher:
Dr. Arturo Espinosa Romero[3]

MCC
Facultad de Matemáticas
Universidad Autónoma de Yucatán

September 2023

Table of contents

Expected Value

Definition

The expected value of a random variable X is the weighted average of the possible values that X can take, each value being weighted according to the probability of that event occurring.

Definition

In probability theory, the expected value (also called expectation, expectancy, expectation operator, mathematical expectation, mean, average, or first moment) is a generalization of the weighted average. Informally, the expected value is the arithmetic mean of a large number of independently selected outcomes of a random variable.

Notations

The expected value of a random variable X is often denoted by $E[X]$, $E(X)$, or EX

- $E[X]$: Expected value of X.
- $E[X|Y]$: Expected value of X given Y.
- $E[X|Y = y]$: Expected value of X given $Y = y$.

Equation: Discrete case

For a discrete random variable X with probability function $P[X = x_i]$, with $i = 1, 2, \ldots, n$, the expected value of X is defined as:

$$E[X] = \sum_{i=1}^{n} x_i P[X = x_i] \tag{1}$$

Example

Let X represent the outcome of a roll of a fair six-sided die. More specifically, X will be the number of pips showing on the top face of the die after the toss. The possible values for X are 1, 2, 3, 4, 5, and 6, all of which are equally likely with a probability of $\frac{1}{6}$.
The expectation of X is "3.5".

$$E[X] = \sum_{i=1}^{6} x_i \frac{1}{6} = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5 \tag{2}$$

Proof

An illustration of the convergence of sequence averages of rolls of a die to the expected value of 3.5 as the number of rolls (trials) grows.

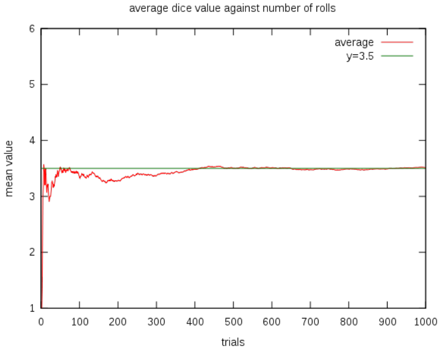

Figure: Average of 1000 rolls of a die.

Equation: Continuous case

For a continuous random variable X with probability density function $f(x)$, the expected value of X is defined as:

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx \tag{3}$$

Sum of Expectations

If X and Y are independent random variables, then the expected value of the sum of X and Y is equal to the sum of their expected values:

$$E[X+Y] = E[X] + E[Y] \tag{4}$$

Conditional Probability

Definition

The conditional probability of an event A given that another event B has occurred is the probability of A given B:

$$P[A|B] = \frac{P[A \cap B]}{P[B]} \tag{5}$$