BAYES ESTIMATION

Practice 2

Analytical demonstration of μ recursive and iterative formula.

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September 2023

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1 Demonstration

Analytical demonstration of μ iterative and recursive formula equivalence.

1.1 Definitions

Suppose you have a sequence of numbers $x_1, x_2, x_3, \ldots, x_n$ that you want to average, where n is the number of elements in the sequence.

1.2 Iterative formula

The iterative formula for calculating the average of a sequence of numbers is:

$$\mu_n = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{1}$$

or

$$\mu_n = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \tag{2}$$

1.3 Recursive formula

The recursive formula for calculating the average of a sequence of numbers is:

$$\mu_n = \begin{cases} x_n & \text{if } n = 1 \\ \frac{(n-1)\mu_{n-1} + x_n}{n} & \text{if } n > 1 \end{cases} \tag{3}$$

1.4 Induction

Prove that these two formulas are equivalent using mathematical induction.

1.5 Base step $n = 1$

Prove that the recursive formula is valid for the base case, i.e. when $n = 1$.

Iterative Formula and Recursive Formula when $n = 1$

$$\frac{x_1}{n} = x_n \tag{4}$$

Substitute $n = 1$ in both formulas

$$\frac{x_1}{1} = x_1 \tag{5}$$

Verify

$$x_1 = x_1 \tag{6}$$

1.6 Inductive step $n > 1$

Prove that the recursive formula is valid for the inductive step, i.e. when $n > 1$.

Suppose the iterative and recursive formulas are equivalent for $n = k$, i.e. that:

$$\mu_k = \frac{1}{k}\sum_{i=1}^{k} x_i \tag{7}$$

$$\mu_k = \frac{x_1 + x_2 + x_3 + \cdots + x_k}{k} \tag{8}$$

$$\tag{9}$$

Prove that the iterative and recursive formulas are equivalent for $n = k + 1$, i.e. that:

$$\mu_{k+1} = \frac{1}{k+1}\sum_{i=1}^{k+1} x_i \tag{10}$$

$$\mu_{k+1} = \frac{x_1 + x_2 + x_3 + \cdots + x_k + x_{k+1}}{k+1} \tag{11}$$

$$\tag{12}$$

Iterative Formula and Recursive Formula when $n = k + 1$

$$\frac{x_1 + x_2 + x_3 + \cdots + x_k + x_{k+1}}{k+1} = \frac{k \cdot \mu_k + x_{k+1}}{k+1} \tag{13}$$

Since we assume that the iterative and recursive formulas are equivalent for $n = k$, then we can substitute the iterative formula of μ_k in the recursive formula to obtain:

$$\frac{x_1 + x_2 + x_3 + \cdots + x_k + x_{k+1}}{k+1} = \frac{k \cdot \frac{x_1 + x_2 + x_3 + \cdots + x_k}{k} + x_{k+1}}{k+1} \tag{14}$$

Simplify

$$\frac{x_1 + x_2 + x_3 + \cdots + x_k + x_{k+1}}{k+1} = \frac{\cancel{k} \cdot \frac{x_1 + x_2 + x_3 + \cdots + x_k}{\cancel{k}} + x_{k+1}}{k+1} \tag{15}$$

Verify

$$\frac{x_1 + x_2 + x_3 + \cdots + x_k + x_{k+1}}{k+1} = \frac{x_1 + x_2 + x_3 + \cdots + x_k + x_{k+1}}{k+1} \tag{16}$$

1.7 Conclusion

It has been demonstrated using the method of mathematical induction. We have shown that if the formulas are equivalent for $n = k$, then they are also equivalent for $n = k + 1$. Since we have already shown that they are equivalent for $n = 1$ (base step), we can conclude that the iterative and recursive formulas for calculating μ are equivalent for any value of n, therefore, they are equally valid.