Real Business Cycle: A Case Study of Economic Growth

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Outline

Theoretical Model

Let us introduce the Real Business Model Theory as presented by Romer (1996) in his textbook *Advanced Macroeconomics*, where firms and households are the main decision makers in the economy.

Firms

The economy consists of identical, price-taking firms who are infinitely lived. Firms produce final goods by using capital, labor and technology. As Romer (1996) states in p.152 "The firm's output is divided among consumption (C_t), investment (I_t) ...". Following the example used by Romer (1996) in equation (4.1) we will use a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha}(A_t H_t)^{1-\alpha}, 0 < \alpha < 1 \tag{1}$$

Where:

$$K_t = \text{Capital}$$
$$A_t = \text{Technology}$$
$$H_t = \text{Worked Hours or Labor}$$
$$Y_t = \text{Output}$$

Following equations Romer (1996) 's (4.8) and (4.9), we can see that A_t follows a first order autoregressive process :

$$ln(A_{t+1}) = \beta ln(A_t) + \epsilon_{t+1}, \epsilon_{t+1} \sim N(0, \sigma^2), -1 < \beta < 1 \tag{2}$$

Where:

ϵ_t = white noise disturbances – a series of mean zero shocks that are uncorrelated with one another

Firms maximize their profits by optimizing both output (revenue) and their payments to workers wages (W_t) as well as the return (R_t) they pay on capital (cost):

$$max\ \Pi : K_t^{\alpha}(A_t H_t)^{1-\alpha} - W_t H_t - R_t K_t \tag{3}$$

By the First Order Conditions, we get the returns to capital:

$$\frac{\partial \Pi}{\partial K_t} = \alpha K_t^{\alpha-1}(H_t A_t)^{1-\alpha} - R_t = 0 \longrightarrow R_t = \alpha \frac{Y_t}{K_t} \tag{4}$$

And the labor demand:

$$\frac{\partial \Pi}{\partial H_t} = (1-\alpha) K_t^{\alpha} H_t^{-\alpha} A_t^{1-\alpha} - W_t = 0 \longrightarrow W_t = (1-\alpha)\frac{Y_t}{H_t} \tag{5}$$

Households

The economy consists of identical, price-taking households, who in turn are infinitely lived. The households are composed of infinitely lived members who optimize their consumption and leisure based on their utility maximization function. For this example, households will maximize a log-linear utility function similar to that in Romer (1996) Chapter 4, equation (4.7):

$$\max_{\{C_t,H_t\}} E_t\rho^t(\sum_{t=0}^{\infty} ln(c_t) + bln(1-Ł_t)), b > 0, \rho > 0 \tag{6}$$

Where:

$$ln(c_t) + bln(1-H_t)) = u(\cdot) = \text{instantaneous utility function of the representative member of the household}$$

$$1 - H_t = L_t = \text{Leisure} = \text{difference between time endowment per member (normalized to 1) and amount of hours each member works } \longrightarrow H_t + L_t = 1$$

$$\rho = \text{Utility Discount Rate}$$

$$C_t = \text{Consumption}$$

$$b = \text{Leisure share parameter}$$

$$\frac{\partial u(\cdot)}{\partial C_t} > 0 \longrightarrow u(\cdot) \text{ is increasing in } C_t$$

$$\frac{\partial u(\cdot)}{\partial L_t} > 0 \longrightarrow u(\cdot) \text{ is increasing in } L_t$$

$$\frac{\partial^2 u(\cdot)}{\partial C_t^2} < 0 \longrightarrow u(\cdot) \text{ has a decreasing rate of change in } C_t$$

$$\frac{\partial u^2(\cdot)}{\partial C_t^2} < 0 \longrightarrow u(\cdot) \text{ has a decreasing rate of change in } L_t$$

That is, household members increase their utility as they increase their consumption and leisure, but at a decreasing rate.

Let us also introduce a budget into this intertemporal utility maximization problem.

For this, we will assume that all household income comes from the member's work (H_t) and from their returns on capital. From there, they decide whether to spend (or consume) their income or save it for later. Therefore, we have an economy where households consume their money or save it to invest, eventually using it as capital. As an example, we will build our problem as explained in Romer (1996)'s *Chapter 4.4 Household Behavior*:

$$\max_{\{C_t,H_t\}} E_t\rho^t(\sum_{t=0}^{\infty} ln(c_t) + bln(1-H_t)),\ \text{s.t. } C_t + S_t = W_tH_t + R_tK_t \tag{7}$$

Where:

$$W_t = \text{ Wage (from work)}$$

$$H_t = \text{ Hours Worked}$$

$$R_t = \text{ Returns}$$

$$K_t = \text{ Capital}$$

$$S_t = \text{ Savings}$$

$$C_t + S_t = W_tH_t + R_tK_t \text{ [since households own capital and rent it to firms]}$$

$$K_{t+1} = I_t + (1-\delta)K_t \text{ [capital accumulation]}$$

$$S_t = I_t \text{ [savings are transformed into investments at no cost]}$$

After rewriting (2) with $S_t = I_t = (1-\delta)K_t - K_{t+1}$, we can outline the consumer problem as a Lagrangian in the following way:

$$\max_{\{C_t,H_t,K_t\}} \mathcal{L} : E_t(\sum_{t=0}^{\infty} \rho^t \{[ln(c_t) + bln(1-H_t))] - \lambda[C_t + K_{t+1} - W_t H_t - (1+R_t-\delta)K_t]\} \quad (8)$$

By First Order Condition:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \rho^t[\frac{1}{C_t} - \lambda_t] = 0 \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial H_t} = \rho^t[-\frac{b}{1-H_t} + \lambda_t W_t] = 0 \quad (10)$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \rho^{t+1}\lambda_{t+1}(1+R_{t+1}-\delta) - \rho^t \lambda_t = 0 \quad (11)$$

From (9) and (10) we get the labor supply:

$$H_t = \frac{W_t - bC_t}{W_t} \quad (12)$$

From (9) and (11) we get the Euler equation:

$$\frac{C_{t+1}}{C_t} = \rho^t[R_{t+1} + 1 - \delta] \quad (13)$$

Equilibrium

In summary, our economy operates under the following dynamics:

$$Y_t = C_t + I_t \tag{14}$$

$$K_{t+1} = I_t + (1 - \delta)K_t \tag{15}$$

$$\frac{C_{t+1}}{C_t} = \rho[R_{t+1} + 1 - \delta] \tag{16}$$

$$Y_t = K_t^{\alpha}(A_t H_t)^{1-\alpha} \tag{17}$$

$$H_t = \frac{W_t - bC_t}{W_t} \tag{18}$$

$$R_t = \alpha \frac{Y_t}{K_t} \tag{19}$$

$$W_t = (1 - \alpha)\frac{Y_t}{H_t} \tag{20}$$

$$ln(A_{t+1}) = \beta ln(A_t) + \epsilon_{t+1} \tag{21}$$

Equilibrium is achieved when our variables satisfy the following conditions:

1. Households optimize their level of consumption and leisure, given their wages and returns on capital.

2. Firms optimize their level of labor and capital, given their wages and returns on capital.

3. Markets clear.

Empirical Model

Data Sources

In this paper, the model is solved with data from Puerto Rico. The data frequency is annual, from 1976-2021. Gross Domestic Product (**GDP**) in current dollars, Total Gross Domestic Investment (**Investment**), and Personal Consumption Expenditure (**Consumption**) is taken from the *Economic Report to the Governor* (or *Apendice Estadistico: Informe Economico al Gobernador*) made publicly available by the *Puerto Rico Planning Board* (or Junta de Planificacion de Puerto Rico). These are all estimates, so they tend to vary by year of publishment. The *Economic Report to the Governor* was published in many years and the data retrieved varies by each published year: (1) published in 1989 (data retrieved for the years 1976-1989), (2) published in 2001 (data retrieved for the years 1990-1993), (3) published in 2003 (data retrieved for the years 1994-2002), (4) published in 2012 (data retrieved for the years 2003 - 2011), and (5) published in 2021 (data retrieved for the years 2012-2021). Additionally, the *Economic Report to the Governor* published in 1999 was used to retrieve data only for Total Gross Domestic Investment for the years 1990-1993 .

Moreover, the Average Weekly Hours of All Employees: Total Private in Puerto Rico (**Worked Hours**) for the years 2013-2021 is retrieved from the Federal Reserve Economic Data with data from the U.S. Bureau of Labor Statistics and Federal Reserve Bank of St. Louis. Similarly, the same statistic for the years 1990-2012 was shared upon request by Juan L. Gonzalez Figueroa, Economist & Director I, Worker Group and Special Studies Division from the Department of Labor and Human Resources. Due to lack of data, Worked Hours for the years 1976-1989 was obtained by linear extrapolation, using the USA's Quarterly Average Hours Worked as retrieved from the Bureau of Labor Statistics for the years 1976-2021 . Refer to Table 1 for summary statistics.

Calibration

All estimations are run on STATA. The Hodrick and Prescott (1980,1997) filter (HP filter) is commonly used to remove trends from time series data in macroeconomics. I use the $lambda = 6.25$ as proposed by Ravn and Uhlig (2002). After applying the HP filter, the trend component was obtained (refer to Figure 1). Additionally, the natural logarithm of each variable was taken before applying the HP filter to identify the cycle component (refer to Figure 2).

In this project, parameters are assigned in the following way (for a summary, refer to Table 2):

1. $\alpha = 0.33$. This represents the elasticity of output with respect to capital is set at 0.33, as per Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016)

2. $\rho = 0.99$. This is the utility discount factor, which represents how much household members discount future spending. That is, if it were 1, households would put all value on future spending, save all of their money and consume nothing today. For 0, households would put 0 value on future spending, do not save any money, and consume everything today. The average real return to equity from 1976-2021 is 8.01 % per year. That is, the stock market returns between 1976 and 2021 beats inflation during this period for an inflation-adjusted return of 8.01 % per year. Therefore, a standard value of rho of 0.99 is expected to give this value (as proposed by Fernandez-Villaverde et al. (2016)).

3. $\delta = 0.10$. This is the depreciation rate of capital. It is assumed to be 10 % per annum, as stated by Plosser (1989) in p. 75.

4. $b = 3.8$ This is the elasticity share of leisure. We can also define it as the share of time that people don't spend working (recall: $b = 1$ - Worked Hours). Since the average Worked Hours from 1976-2021 is 35.21686 hours. Therefore, we can estimate

that the average person has 20.9624 % of their time allocated to work (35.21686/168=0.209624) and the remaining 0.79 % goes to leisure activities (1-0.209624=0.790376). Therefore, we need to assign the $b = 3.8$ so as to get $H_t = 0.2$.

5. $\beta = 0.0829658$. This is the persistence of the technology shock estimated in with our data. It presents a standard deviation of 0.147016 . Its interpretation is that if we hold expected future technology constant, then a 1 percentage point increase in the output gap leads to a 0.0829658 percentage point increase in technology.

Results

Refer to Table 3 for Steady State results. With our model, we can notice that according to equation (14) we have around 70% of the economy's output dedicated to consumption and 30% dedicated to investment. Yet, we need to keep in mind that this is probably an overestimation because we are not accounting for Government Spending and Foreign Trade; in which case we would have $Y_t = C_t + I_t + G_t + (X_t + M_t)$. Similarly, the capital seems to be around three times as much as the economy's output, while worked hours are about 58% of the economy's output and 20% of a person's weekly time – as we had previously stated.

More over, we can refer to Table 4 for an overview of the model's policy and state-transition functions. Table 4 displays the impact effects of a change in a state variable on the control variables and on the expected future values of the state variables. We can interpret the values reported in the table as percentage deviations from the steady state. Therefore, we can observe that impact of A_t on Y_t, or productivity on output, is of 1.201626. Which implies that when productivity (A_t) rises by one percent, GDP (Y_t) rises by 1.2 percent. Similarly, the impact of capital expenditures (K_t) or the monetary value of physical assets necessary for production (i.e. buildings, machinery, and equipment) on output (Y_t) is of 0.15572. This implies that when capital (K_t) increases by one percent, GDP (Y_t) increases approximately by 0.16 percent. Interestingly, Capital (K_t) has a negative relationship with Worked Hours (H_t), Returns on Capital (R_t) and Wages (W_t). This makes sense, since according to our production function (refer to equation (17)), we have decreasing returns to scale with respect to Capital and increasing returns to scale with respect to Worked Hours or Labor. That is to say, that the marginal product of capital and labor decreases as more capital is added. However, as we previously pointed out, the output (Y_t) increases in a decreasing rate. Therefore, if we were to prioritize GDP, then an increase in productivity should be prioritized over capital.

On the other hand, we can also refer to Table 5 for the Transition Matrix, which reports what is expected to happen if a state variable changes by 1 percent in the current period. For instance, we can notice that for every additional 1 % increase in productivity (A_t), we can expect a 0.3522 % increase in the next period's capital expenditure (K_t). Similarly, for every 1% increase in capital (K_t), we have an increase of 0.8395 % in the next period's capital expenditure (K_{t+1}). Additionally, we can also observe that a 1% increase in the productivity or technology (A_t), generates a 0.083% increase in the next period's productivity (A_{t+1}). A takeway from this is that, the spillover between periods is much bigger for capital than for productivity.

Performance

As a performance check, we verify the endogenous variables standard deviation and correlation (refer to Figure 4) to see if the model makes sense. We can interpret the covariance as the tendency in the linear relationship between the variables. Where a tendency below 0 implies a negative linear relationship, where a coefficient above 0 implies a positive linear relationship and a coefficient near 0 implies no linear relationship. Similarly, we can get the correlation between all variables through the following equation $Corr(X_1, X_2) = \frac{Cov(X_1, X_2)}{\sigma_{X_1}\sigma_{X_2}}$, that is Correlation is equivalent to Covariance divided by the Standard Deviation. Therefore, we can observe that All variables have a high positive correlation (linear relationship) between themselves, which is to be expected from the macroeconomic models used in our model.

The weakest correlations can be observed from Consumption & Worked Hours (with a $Corr(C_t, W_t)$=2.96699) and Consumption & Output (with a $Corr(C_t, Y_t)$=3.353699) – which are still very strong correlations! This is very intuitive, because we know that when Consumption increases, Worked Hours and Output also tend to increase based on equations (14) and (18). On the other hand, we can also observe the standard deviations of the variables by taking the square root of the variance. Therefore, our model displays that Investment (I_t) is the most volatile of all the variables in the model, with a standard deviation of 3.6619, which makes it 2.9 times more volatile than Output (note: $\sigma_{Y_t} = 1.2572$) and 8.8 times more volatile than Consumption (note: $\sigma_{C_t} = 0.41727$).

Consumption, Wages and Return on Capital are the less volatile variables in our economy, which makes sense for a developed economy. We recognize that according to equation (16), consumption should be less volatile than output because consumers spread their consumption over time, which smooths the trend. Analogously, we can also view this as a sign that the economy has a small informal sector. On that same note, a low Wage volatility with respect to Output makes sense in a developed economy, because it implies that volatility in individual earnings (i.e. due to sudden job loss, pay cuts, or maybe salary bumps) is not a problem, because workers are able to maintain stable expectations in regards to their earnings, which leaps them into saving and consuming at a constant rate. Similarly, a low volatility in earnings is associated with low inequality, which is also to be expected from a developed economy. Rationally, a low volatility of Returns on Capital with respect to Output also boosts confidence in entrepreneurs to invest in companies in the current period, since they are able to estimate their expected return.

Impulse Response Functions

In order to answer policy relevant questions such as "What would happen if we increased productivity by 1%?" we can estimate impulse response functions. These functions can help us visualize the effect of an unexpected change in some state variable (i.e. returns on capital, GDP, etc.) in response to a shock on technology or productivity. An unexected change in this variable is is modeled as a shock to the A_t equation (21). An impulse is a series of values for the shock ϵ in (21): (1, 0, 0, 0, ...). The shock affects the state variables, leading to an increase in A_t. From there, the increase in A_t generates a change in all the other variables. Therefore, we can say that an impulse-response function displays the effect of a shock of one variable over the rest of the model, accounting for all the interrelationships in the model's equations.

Figure 5 displays the impulse–response graphs of each control variable to a one-standard-deviation shock in productivity. Therefore, we can interpret that a shock on Productivity (1 % increase) usually dissipates after four years and returns to its long-run value. That is to say, the effect of a new technological invention that increases productivity for one percent only increases productivity for the next four years in this economy. This same shock tends to increase Investment (I_t) the most (out of all the other variables), increasing it for about 4% for the first year, and then steeply decreasing it for the following year, until it stabilizes after eight years. A similar behavior is seen with respect to Returns on Capital (R_t), which peaks in the first year for about 1.3% and then declines steeply for the following years, until it stabilizes after sixteen years. A 1% increase in Productivity also seems to increase the GDP gap (Y_t) by about 1.3% stabilizing after four years. The positive effect of Productivity over the GDP gap promotes a positive effect over the rest of the control variables. This makes sense, since higher GDP implies a more extensive economy with higher Capital, Investment, Consumption, Wages and Worked Hours. Worked Hours increase after an increase in Productivity, and then decline, which could be interpreted as a decrease in labor effort after learning how to use the new technologies available. Additionally, the most lasting effects of Productivity seems to be on Capital (K_t) and Consumption (C_t) which stabilize after more than twenty years.

Forecast

The model is used to run a forecast for sixteen periods in the future and generate the graph displayed in Figure 6. The results reflect something similar to what we had already discusses before: the effect of a Productivity shock over the GDP gap extends for approximately four years. After four years, the shock generated by Productivity dissipates completely and GDP, and GDP returns to its long-run trend. Similarly, to check how well this model explains the past behavior of our economy, a One-Step ahead forecast is applied, Figure 7 presents the results of this estimation. This last forecast demonstrates that our model minimizes the shocks of the economy, and so we can view our results as conservative.

Conclusion

In this project, we have presented the basic set up of the RBC model, the steps required to calibrate it, and make relevant estimations to aid in policy-making decisions (i.e. impulse-response functions and forecasts). This model presents the Real Business Cycle for Puerto Rico and can aid in understanding the effects of a productivity shock in the island. Hopefully, this can help support the understanding of the technological spillover of a new innovation on the economy. Yet, many criticisms can arise from the model used in this project.

A shortcoming of the project is the limited amount of time periods included in the estimations. This is mainly due to the lack of data availability issued by the government of Puerto Rico. Additionally, one of the main assumptions used here is that technology generates business cycles. This may be why periods of increased productivity reflect periods of economic expansion, whole periods of low productivity generate economic contractions. Additionally, in this model, technology is the main driver of the business cycle. However, it is known that technology shocks do not account for the whole behavior of the economy. Therefore, to improve the estimations presented here, more variables should be included in the model. More on this, the model does not account for government and foreign trade. This is why the consumption to output ratio is high, since output is only affected by investment and consumption – something that does not hold in the real world.

Apendix

Table 1

Descriptive Statistics of Macroeconomic Variables (in Millions)

Descriptive Statistics					
Variable	Obs	Mean	Std.Dev.	Min	Max
Time	46	1998.5	13.423	1976	2021
Unemployment	46	14.565	3.866	8.075	23.375
GDP	46	58894.016	35531.078	8968.6	106525.7
Consumption	46	35938.47	20603.979	7490	71039
CPI	46	85.228	23.947	42.67	121.772
Worked Hours	46	35.205	0.398	34.1	36.15

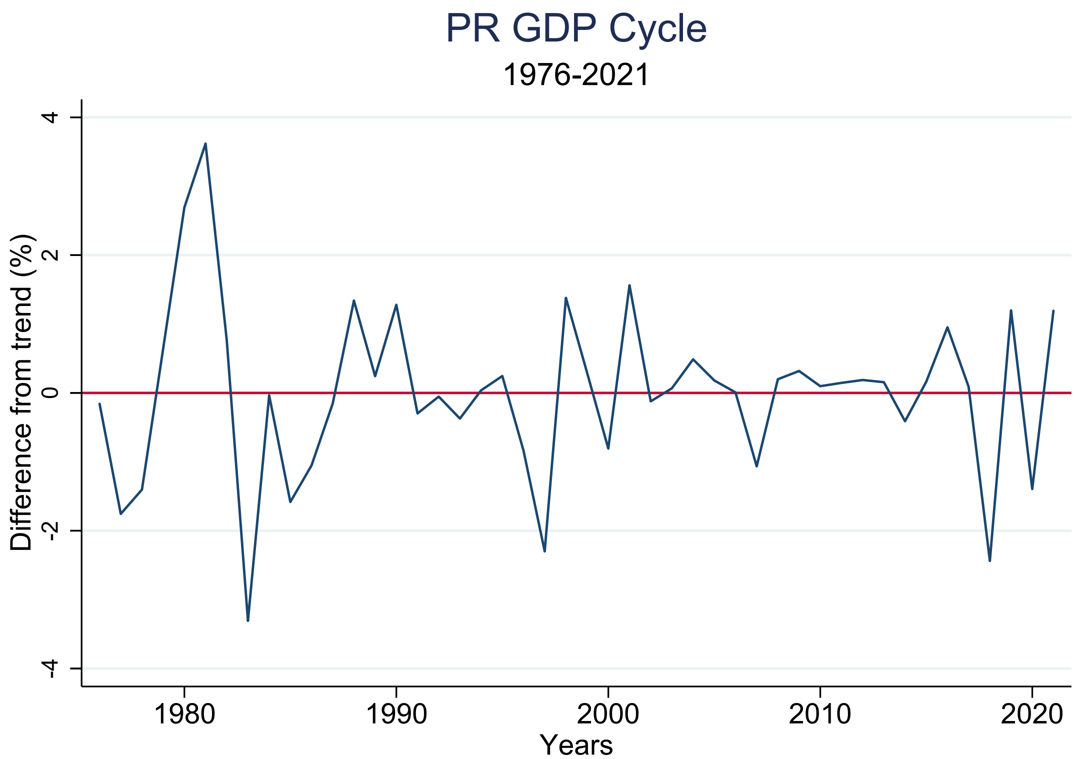

Figure 1. GDP trend from 1976-2021. Data Source: Economic Report to the Governor

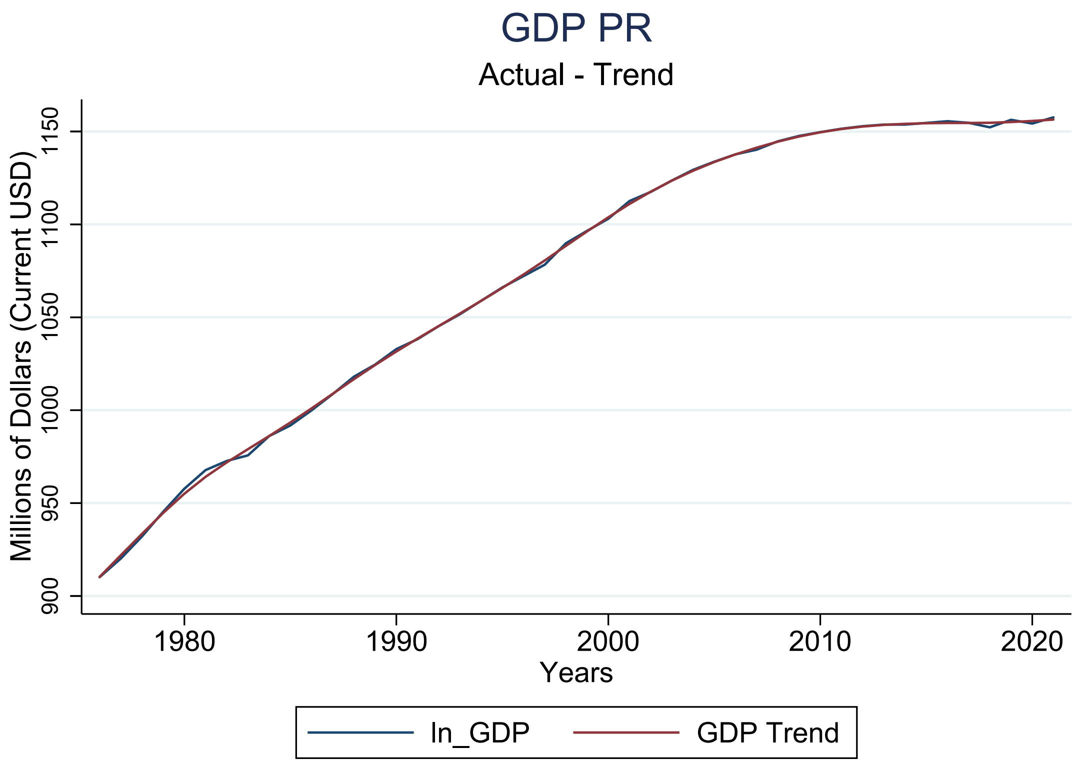

Figure 2. *GDP and its trend from 1976-2021. Data Source: Economic Report to the Governor*

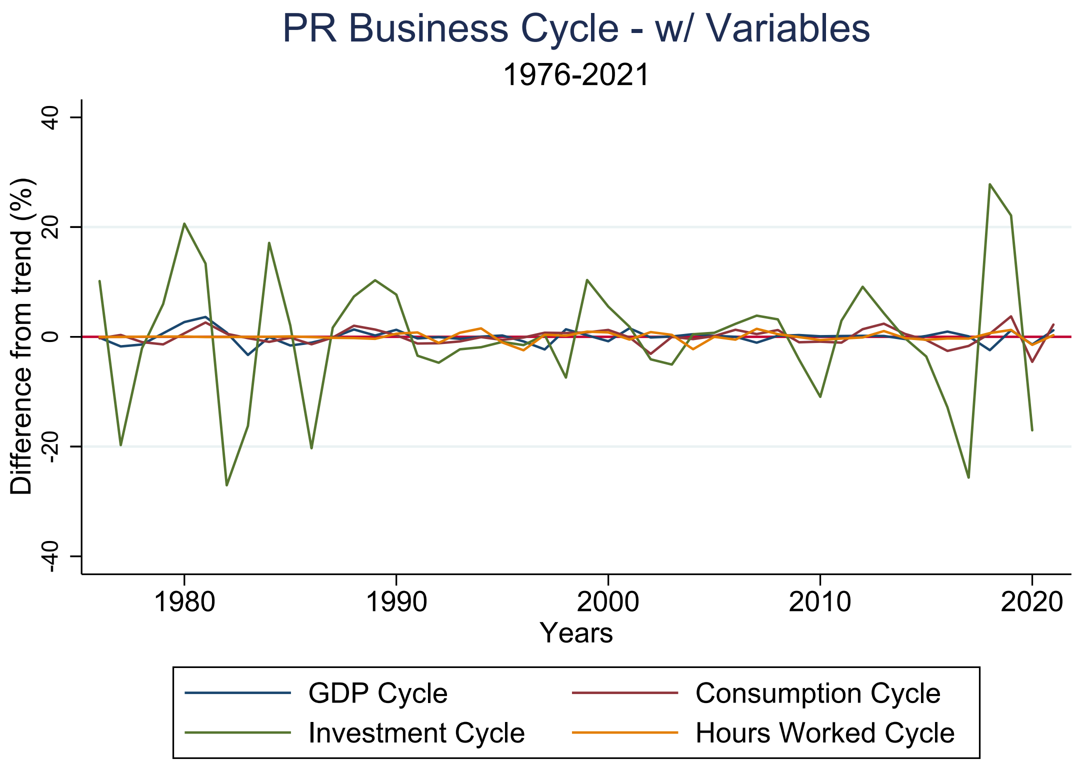

Figure 3. GDP and other variables from 1976-2021. Data Sources: Economic Report to the Governor

Table 2

Parameter Calibration and Results

Model Values		
Parameter	Interpretation	Value
α	Elasticity of Output with Respect to Capital	0.33
ρ	Utility Discount Factor	0.99
δ	Depreciation Rate of Capital	0.10
b	Elasticity Share of Leisure	4.3699
β	Persistence of a Technology Shock	0.0829658

Table 3

Steady State Results

Steady State	
Coefficient	Value
K_t	1.04
A_t	1
C_t	0.24
R_t	0.11
H_t	0.20
W_t	1.15
I_t	0.10
Y_t	0.35

Table 4

Policy Matrix Results

Steady State		
Variable	K_t	A_t
C_t	0.481333	0.208373
R_t	-0.84428	1.201626
H_t	-0.26012	0.793472
W_t	0.41584	0.408154
I_t	-0.60504	3.522258
Y_t	0.15572	1.201626

Table 5

Transition Matrix of State Variables Results

Transition Matrix		
Next Period	K_t	A_t
K_{t+1}	0.8394958	0.3522258
A_{t+1}	0	0.0829658

Variable	Delta-method Coefficient	std. err.	z	P>z	[95% conf. Int.]	
$\boldsymbol{C_t}$						
$Var(C_t)$	0.1741176	0.0621069	2.8	0.005	0.0523903	0.295845
$Cov(C_t, R_t)$	0.0718438	0.0170652	4.21	0	0.0383966	0.105291
$Cov(C_t, H_t)$	0.1240253	0.0303209	4.09	0	0.0645974	0.183453
$Cov(C_t, W_t)$	0.2053448	0.0692099	2.97	0.003	0.069696	0.340994
$Cov(C_t, I_t)$	0.692101	0.185468	3.73	0	0.3285905	1.055611
$Cov(C_t, Y_t)$	0.3293701	0.098211	3.35	0.001	0.13688	0.52186
$\boldsymbol{R_t}$						
$Var(R_t)$	1.857562	0.3873283	4.8	0	1.098413	2.616712
$Cov(R_t, H_t)$	1.108043	0.2319997	4.78	0	0.6533314	1.562754
$Cov(R_t, W_t)$	0.3508275	0.0743831	4.72	0	0.2050394	0.496616
$Cov(R_t, I_t)$	4.69951	0.9846152	4.77	0	2.7697	6.629321
$Cov(R_t, Y_t)$	1.45887	0.3061482	4.77	0	0.8588305	2.05891
$\boldsymbol{H_t}$						
$Var(H_t)$	0.6994048	0.146747	4.77	0	0.4117861	0.987024
$Cov(H_t, W_t)$	0.3001219	0.0636673	4.71	0	0.1753362	0.424908
$Cov(H_t, I_t)$	3.045043	0.63709	4.78	0	1.79637	4.293717
$Cov(H_t, Y_t)$	0.9995267	0.2084664	4.79	0	0.5909401	1.408113
$\boldsymbol{W_t}$						
$Var(W)$	0.2809097	0.0819581	3.43	0.001	0.1202749	0.441545
$Cov(W_t, I_t)$	1.458784	0.3212443	4.54	0	0.8291569	2.088411
$Cov(W_t, Y)$	0.5810316	0.1394461	4.17	0	0.3077223	0.854341
$\boldsymbol{I_t}$						
$Var(I_t)$	13.40952	2.797942	4.79	0	7.925658	18.89339
$Cov(I_t, Y_t)$	4.503827	0.9404929	4.79	0	2.660495	6.34716
$\boldsymbol{Y_t}$						
$Var(Y_t)$	1.580558	0.3358085	4.71	0	0.9223857	2.238731

Figure 4. Estimated Covariances of Model Variables

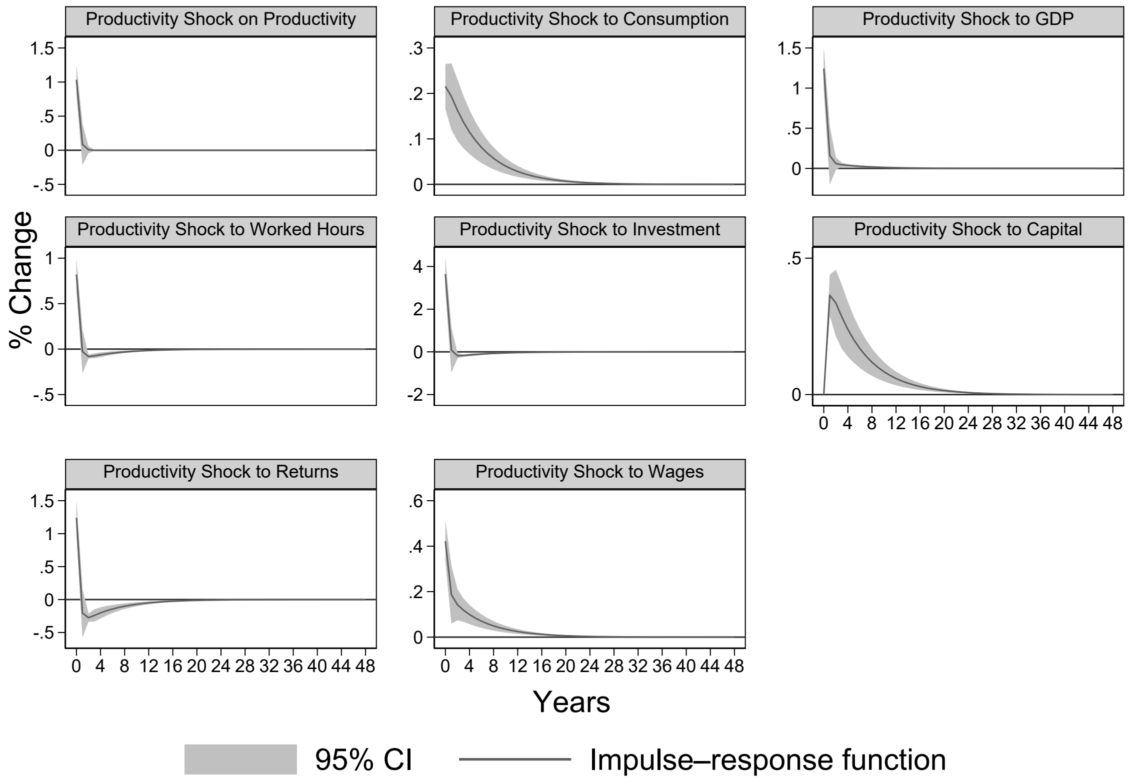

Figure 5. Impulse Response Functions. Impulse: A_t. *Response:* $A_t, C_t, H_t, I_t, K_t, R_t, W_t, Y_t$. *Magnitude of the shock: One Standard Deviation. Range: 1976-2021*

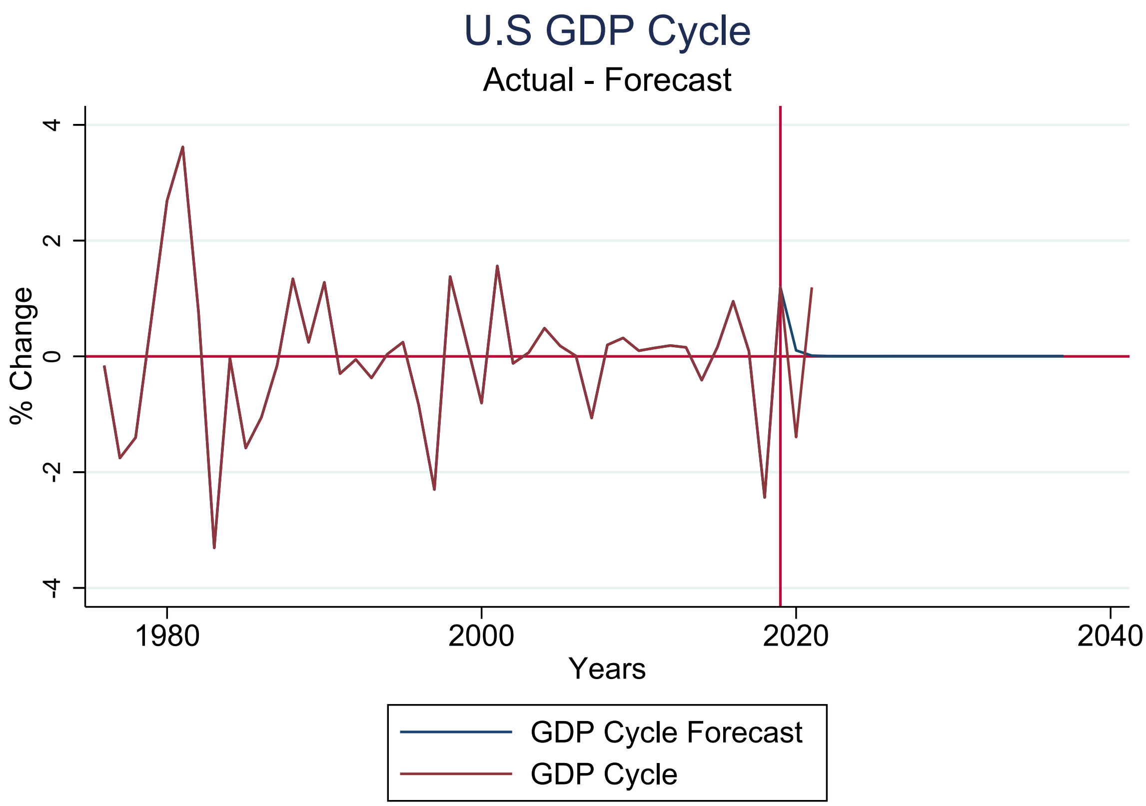

Figure 6. RBC Model Forecast. Range: 1976-2021

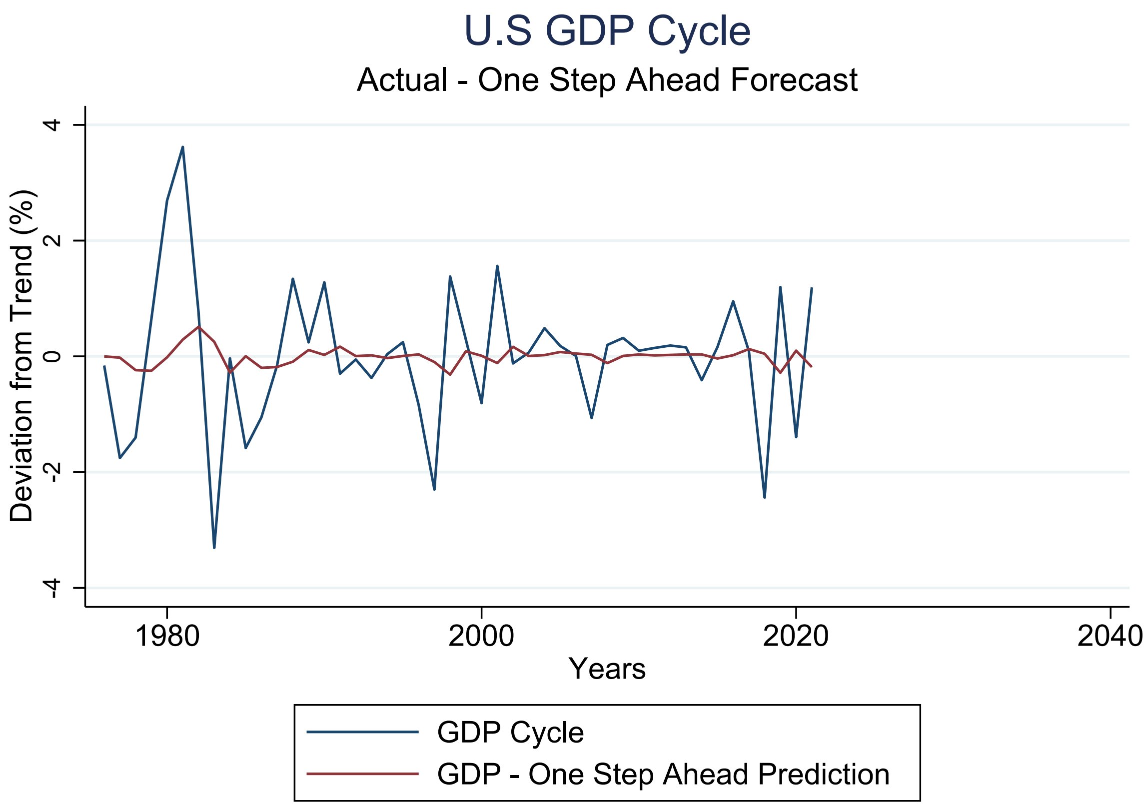

Figure 7. RBC Model Forecast Fitness. Range: 1976-2021

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