1 Reading Assignment

2 Filtering Out The Troll

(a)

$$\begin{aligned}
\vec{m}_1 &= \cos(\alpha) \cdot \vec{s} + \cos(\beta) \cdot \vec{r} \\
&= \cos(\frac{\pi}{4}) \cdot \vec{s} + \cos(-\frac{\pi}{6}) \cdot \vec{r} \\
&= \frac{\sqrt{2}}{2} \cdot \vec{s} + \frac{\sqrt{3}}{2} \cdot \vec{r} \\
\vec{m}_2 &= \sin(\alpha) \cdot \vec{s} + \sin(\beta) \cdot \vec{r} \\
&= \sin(\frac{\pi}{4}) \cdot \vec{s} + \sin(-\frac{\pi}{6}) \cdot \vec{r} \\
&= \frac{\sqrt{2}}{2} \cdot \vec{s} - \frac{1}{2} \cdot \vec{r}
\end{aligned}$$

(b) We can substitute through $\vec{r}$ to get $\vec{s}$ as a weighted combination of $\vec{m}_1$ and $\vec{m}_2$.

$$\begin{aligned}
\vec{r} &= \frac{2}{\sqrt{3}}\vec{m}_1 - \frac{\sqrt{2}}{\sqrt{3}} \cdot \vec{s} \\
\vec{m}_2 &= \frac{\sqrt{2}}{2} \cdot \vec{s} - \frac{1}{2} \cdot (\frac{2}{\sqrt{3}}\vec{m}_1 - \frac{\sqrt{2}}{\sqrt{3}} \cdot \vec{s}) \\
\frac{\sqrt{6}+\sqrt{2}}{2\sqrt{3}} \cdot \vec{s} &= -\frac{1}{\sqrt{3}}\vec{m}_1 - \vec{m}_2
\end{aligned}$$

3 Multiply the Matrices

(a) Yes, it is a valid operation; the dimensions of **AB** is 3×4.

(b)

$$\begin{aligned}
AB &= \begin{bmatrix} 1 \cdot 1 + 0 \cdot -3 & 1 \cdot 2 + 0 \cdot 0 & 1 \cdot -1 + 0 \cdot 2 & 1 \cdot 0 + 0 \cdot -1 \\ 2 \cdot 1 + 1 \cdot -3 & 2 \cdot 2 + 1 \cdot 0 & 2 \cdot -1 + 1 \cdot 2 & 2 \cdot 0 + 1 \cdot -1 \\ 0 \cdot 1 + 1 \cdot -3 & 0 \cdot 2 + 1 \cdot 0 & 0 \cdot -1 + 1 \cdot 2 & 0 \cdot 0 + 1 \cdot -1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -3 & 0 & 2 & -1 \end{bmatrix}
\end{aligned}$$

BA is invalid because the middle dimensions (4 and 3) are not equal.

(c) goo goo ga ga

4 Linear Dependence

(a) The set of vectors is linearly independent.

(b) The set of vectors is linearly dependent: $-2\vec{v_1} + \vec{v_2} + 3\vec{v_3}$

(c)

$$
\left(\begin{array}{ccc|c} 2 & 0 & 2 & 0 \\ 2 & 1 & 4 & -1 \\ 0 & 1 & -1 & 1 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right)
$$
$$
= \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & 2 \end{array}\right)
$$
$$
= \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & -\frac{2}{3} \end{array}\right)
$$

The set is linearly dependent because the last vector is a linear combination of the first three vectors: $\frac{2}{3}\vec{v_1} + \frac{1}{3}\vec{v_2} + -\frac{2}{3}\vec{v_3} - \vec{v_4} = \tilde{\mathbf{0}}$

(d) The set is linearly independent.

5 Linear Dependence in a Square Matrix

6 Image Stitching

(a) **Step 1:** The two geometric transformations that get applied to $\vec{u}$ to get $\vec{v_1}$ are rotation (we set a new basis with the 2×2 matrix) and scaling (there is a factor of 2 that can drawn out of the matrix). **Step 2:** The addition of $\vec{w}$ applies translation/shifting to $\vec{v_1}$.