1. *The selfish shopper.* Jim is gift shopping. He has to decide whether to shop online or go to the store. If he shops online he will be able to find a gift (g) or three things that he would like for himself (j_1, j_2, j_3); the menu of options he faces if he shops online is $\{g, j_1, j_2, j_3\}$. The stores have fewer things that interest Jim, so if he shops at the store he faces a menu $\{g, j_3\}$.

   When Jim chooses an item x from a menu A he derives utility

   $$U(A, x) = u(x) - s(A, x) - c_{store}. \tag{1}$$

   $s(A, x) = \max_{y \in A} v(y) - v(x)$ is the cost of self-control, which depends on the most tempting thing foregone. Since prices are a little higher at the store, $c_{store} = 0.5$ if Jim shops at the store and $c_{store} = 0$ if he shops online.

   Jim likes to buy things for himself, but he is very generous and so likes to give gifts more: $u(j_1) = 4$, $u(j_2) = 2$, $u(j_3) = 1$, $u(g) = 6$. However, he is often tempted to buy things for himself when he should be buying gifts: $v(j_i) = u(j_i)$, $v(g) = 0$.

   a) Where will Jim shop? What does he buy?

   For this question we need to figure out what utility Jim gets from each possible choice he might make. First let's check the store. $U(store, g) = u(g) - s(store, g) - c_{store} = 6 - 1 - 0.5 = 4.5$ **and** $U(store, j_3) = u(j_3) - s(store, j_3) - c_{store} = 1 - 0 - 0.5 = 0.5$. **So we know that if he shops at the store he will buy** g**.**

   Next, what if he shops online? $U(online, g) = u(g) - s(online, g) - c_{store} = 6 - 4 - 0 = 2$ **and** $U(online, j_1) = u(j_1) - s(online, j_1) - c_{store} = 4 - 0 - 0 = 4$. **Any other of the** j **goods will yield a smaller utility since their** u **value is smaller and there will be a self-control cost due to forgoing** j_1**. So from this we know he will buy** j_1 **if he shops online.**

   So: if he shops at the store he buys g **for utility** 4.5**, but if he shops online he buys** j_1 **for utility** 4**. The prediction: he shops at the store and buys** g**.**

   The store owner is concerned that he doesn't carry j_1, the most popular gadget of the season. He decides to add j_1 to his stock, so that the menu of options at the store is now $\{g, j_1, j_3\}$. Continue to assume that $c_{store} = 0.5$.

   b) Where will Jim shop? What does he buy?

   Let's run it back. Nothing has changed about the online option. But: at the store things are different. Now $U(store, g) = u(g) - s(store, g) - c_{store} = 6 - 4 - 0.5 = 1.5$ **and** $U(store, j_1) = u(j_1) - s(store, j_1) - c_{store} = 4 - 0 - 0.5 = 3.5$; **again since the other** j **good has a lower** u **and requires more self-control, it is definitely worse. So we know that if he shops at the store he will buy** j_1**.**

   But now the utility he gets from shopping online is higher than from shopping at the store (4 **from the earlier work in part a), and** 3.5 **now from the store). So the prediction: he shops online and buys** j_1**.**

A new store opens that sells only g, so it offers a menu $\{g\}$, but is also more expensive than online, so that $c_{newstore} = x$, $x > 0$.

c) How high can the new store make its price premium $c_{newstore}$ while still selling to Jim?

We can calculate that $U(newstore, g) = u(g) - s(store, g) - c_{store} = 6 - 0 - x = 6 - x$. For this to be the best option for Jim it must yield a higher payoff than his next best option. That was $U(online, g) = 4$, so x can be up to 2 and Jim will shop at the new gift-only store.

d) In this question, the menu of alternatives mattered for Jim's utility because he suffered a self-control cost that depended on the most tempting thing foregone. Briefly propose another, different way in which a DM's utility from a choice may depend on the menu from which the choice was made. What is a setting in which a model of your proposal would predict different choices than the standard model? [100-200 words]

There are lots and lots of options here. For example, in HW 1 we had a DM who always chose the average slice of cake—the menu matters there because what the average is depends on the menu of slices. In that case it's fear of looking greedy that motivated the DM's 'weird' behavior. Another possibility closer to the model we used in this question is that the self-control cost might depend on the temptingness of all of the foregone items, not just the most tempting one. A third possibility might be the choice overload example from topic 1—the size of a menu might decrease the DM's utility, leading them to walk away from menus that are too big where they might have chosen something from a smaller menu.

2. *Choice from menus.* In the previous question we used an example of a 'choice from menus' model: the menu from which a selection was made influences the DM's utility. In this example that was because the DM suffers a self-control cost: if there is another item that was more tempting than the one they chose, they must exercise self-control equal to the most tempting thing foregone. For this question: propose two different 'choice from menus' model of your own that you think capture plausible types of behavior. In each case, explain the behavior you are trying to capture and, in math and in words, what the DM's utility function looks like in your model and why. What choice patterns can your model rationalize that the standard model cannot?

Lots and lots of options here! In class I mentioned a very simple change: you could have self-control depend on how tempting *all* the options foregone are, rather than just the most tempting option. That would look like $U(A, x) = u(x) - s(A, x)$ with $s(A, x) = \sum y \neq x \in A v(y) - v(x)$. Now the self-control cost adds up the $v(\cdot)$ of every item in the choice menu, rather than just the most tempting one that wasn't picked. This decision maker would have a very extreme preference for smaller menus, perhaps to avoid the pain of having to choose. In particular, if we had two menus that both featured the item that had the highest value of $U(x, x)$, the decision maker would strictly prefer the smaller one.

I'll leave the other examples up to you! Looking forward to seeing a few of your suggestions.

3. *Risk aversion.* Person A has Bernoulli utility function $u_A = \sqrt{x_A}$ and person B has $u_B = \ln x_B$, where x_i is DM i's final wealth. Both are Expected Utility maximizers.

   a) Say that both people are offered a gamble with a \$50 stake. Can we say that if A rejects the gamble then B definitely also rejects it? Why or why not?

   For this we will need the coefficient of absolute risk aversion for each utility function. For A: $u'_A(x_A) = \frac{1}{2x_A^{\frac{1}{2}}}$ **and** $u''_A(x_A) = -\frac{1}{4x_A^{\frac{3}{2}}}$**. Therefore the coefficient of absolute risk aversion is** $r_A(x_A) = -\frac{u''_A(x_A)}{u'_A(x_A)} = \frac{1}{2x_A}$**.**

   For B: $u'_B(x_B) = \frac{1}{x_B}$ **and** $u''_B(x_B) = -\frac{1}{x_B^2}$**. Therefore the coefficient of absolute risk aversion is** $r_A(x_B) = -\frac{u''_B(x_B)}{u'_B(x_B)} = \frac{1}{x_B}$**.**

   Since we don't have information on the wealth of each DM, we don't know which of these coefficients of absolute risk aversion is bigger. That means that we can't say one way or the other whether the statement is true.

   b) Say instead that the gamble had been for a stake of 5% of current wealth. In this case, can we say that if A rejects the gamble then B definitely also rejects it? Why or why not?

   For this we will need the coefficient of relative risk aversion for each utility function. This is x **times the coefficient of absolute risk aversion, and so here for A it is** $r_R(x_A) = \frac{1}{2}$ **and for B it is** $r_R(x_B) = 1$**.**

   So we have that for gambles proportional to wealth, B is always more risk averse than A. That means that if A rejects such a gamble, it must be the case that B definitely rejects it too.

4. An expected utility maximizer has zero initial wealth, and has a Bernoulli utility function $u = x^2$, where x is 'final wealth'. He holds a lottery ticket that pays \$1 with probability $\frac{1}{2}$ and pays nothing with probability $\frac{1}{2}$.

   a) What is his expected utility?

   $EU = \frac{1}{2}u(0) + \frac{1}{2}u(1) = \frac{1}{2}0^2 + \frac{1}{2}1^2 = \frac{1}{2}$.

   b) What is the smallest amount of money he would accept to give up the ticket?

   We need an amount a **such that** $a^2 \geq \frac{1}{2}$**, so approximately \$0.71.**

   c) Calculate this individual's coefficient of absolute risk aversion. Interpret it in words.

   $u'(x) = 2x$ **and** $u''(x) = 2$**. Therefore the coefficient of absolute risk aversion is** $r_A(x) = -\frac{u''(x)}{u'(x)} = -\frac{1}{x}$**. This is negative: the DM is a risk-lover. They are less risk loving as their wealth increases, since as** x **increases the coefficient is getting less negative, increasing towards zero.**

d) Calculate this individual's coefficient of relative risk aversion. Interpret it in words.

Their coefficient of relative risk aversion is just x times $r_A(x)$, so $r_R(x) = -1$. Again we see a risk lover since this is negative. Since it's constant, this individual has the same tolerance for gambles of a given fraction of their wealth no matter what amount of wealth they initially have.

5. *Using Expected Utility.* DM A is a risk-averse expected utility maximizer has a Bernoulli utility function $u = \sqrt{x}$, where x is final wealth. They currently have \$1. They are offered a lottery ticket that costs \$1. It will pay \$y with probability $\frac{1}{100}$ and zero with the remaining probability.

a) What is the smallest acceptable prize y such that the DM is willing to buy the lottery ticket?

DM A's expected utility if they don't buy is equal to $\sqrt{1} = 1$. They will buy if

$$EU(buy) > EU(don't) \tag{2}$$

$$\frac{1}{100}\sqrt{y} + \frac{99}{100}\sqrt{0} > 1 \tag{3}$$

$$y > 10,000 \tag{4}$$

b) If the DM had \$100 in initial wealth instead of \$1, would the smallest acceptable prize be bigger, smaller, or the same as in b)?

We could check this mechanically. DM A's expected utility if they don't buy is equal to $\sqrt{100} = 10$.

$$EU(buy) > EU(don't) \tag{5}$$

$$\frac{1}{100}\sqrt{99+y} + \frac{99}{100}\sqrt{99} > 10 \tag{6}$$

and solve through for y (around \$125). However there is an easier way. We know that the prize would be smaller since the DM has decreasing absolute risk aversion (as we saw in the previous question with the same utility function) and so they are more willing to accept a given cash-stakes risk when they have more wealth. That is exactly what we have here!

DM B is an expected utility maximizer with a Bernoulli utility function $u = x^2$ and who currently has \$1.

c) What is the smallest prize y such that DM B would be willing to buy the \$1 lottery ticket?

DM B will buy if

$$EU(buy) > EU(don't) \tag{7}$$
$$\frac{1}{100}y^2 + \frac{99}{100}0^2 > 1^2 \tag{8}$$
$$y > 10 \tag{9}$$

d) If B rejects some gamble g, can we say whether A will accept or reject it? Explain how you know.

The easy answer to this one is that B is a risk lover and A is risk averse. So if a gamble is not acceptable to B then it is most definitely not acceptable to A. You can see that B is a risk lover because their Bernoulli function is convex—they prefer the expected utility of a gamble over the utility of its certainty equivalent. Conversely, A has a concave Bernoulli function so they are risk averse. You may have chosen a more elaborate way to answer this question—that's fine too!

Consider a different problem for the same two DMs. Assume each has \$100 in initial wealth, but with probability $\frac{1}{10}$ will lose it all. They are offered an insurance policy at a price $c \geq 0$ that will cover the whole loss should it occur.

e) What is the most that each DM would be willing to pay for the insurance policy?

For DM A:

$$EU(noinsurance) = [pr(noloss) * u(noloss)] + [pr(loss) * u(loss)] \tag{10}$$
$$= (\frac{9}{10} * \sqrt{100}) + (\frac{1}{10} * \sqrt{0}) \tag{11}$$
$$= 9 \tag{12}$$
$$EU(insurance) = \sqrt{100 - c} \tag{13}$$

So they will buy if

$$EU(insurance) > EU(noinsurance) \tag{14}$$
$$\sqrt{100 - c} > 9 \tag{15}$$
$$c < 19 \tag{16}$$

This, by the way, is more than the expected loss—this is one reason why insurance can be a profitable business. The risk averse DM is willing to pay a premium to shift some risk to the insurer and away from them. The trick for the insurer is to hold a portfolio of imperfectly correlated risks in order to be able to use the premia from no-loss cases more than cover the loss cases.

For DM B:

$$
\begin{aligned}
EU(noinsurance) &= [pr(noloss) * u(noloss)] + [pr(loss) * u(loss)] && (17)\\
&= (\frac{9}{10} * 100^2) + (\frac{1}{10} * 0^2) && (18)\\
&= 9,000 && (19)\\
EU(insurance) &= (100 - c)^2 && (20)
\end{aligned}
$$

So they will buy if

$$
\begin{aligned}
EU(insurance) &> EU(noinsurance) && (21)\\
(100 - c)^2 &> 9,000 && (22)\\
c &< 100 - \sqrt{9,000} && (23)
\end{aligned}
$$

This comes out to around \$5. Why? This DM is risk-loving so their maximum willingness to pay for the insurance is less than the expected loss.

6. *Consistency conditions under EUT.* In this question we will construct an example of a consistency condition on the choices of an Expected Utility maximizing DM. Consider the following two lotteries:

A. Win \$4000 with probability 0.8, else nothing.

B. Win \$3000 with probability 1, else nothing.

a) Assume that an 'outcome' is the lottery's money prize. Write an inequality that must hold if an Expected Utility maximizer prefers lottery B to lottery A.

The inequality is $u(3000) > 0.8u(4000) + 0.2u(0)$. (You may have normalized $u(0) = 0$; that's OK but remember to say so if you do.)

Consider the following two lotteries:

C. Win \$4000 with probability 0.2, else nothing.

D. Win \$3000 with probability 0.25, else nothing.

b) Using a degenerate lottery that pays nothing with probability 1 and lottery A, construct a compound lottery whose reduced lottery is equivalent to lottery C.

To get something equivalent to C, we need a mix of $\frac{1}{4}$ chance of lottery A and a $\frac{3}{4}$ chance of getting nothing.

c) Using a degenerate lottery that pays nothing with probability 1 and lottery B, construct a compound lottery whose reduced lottery is equivalent to lottery D.

To get something equivalent to D, we need a mix of $\frac{1}{4}$ chance of lottery B and a $\frac{3}{4}$ chance of getting nothing.

d) Demonstrate mathematically and explain why an Expected Utility maximizer who prefers lottery B to lottery A must also prefer lottery D to lottery C.

By the consequentialism assumption, the EUT maximizer cares about the ultimate probability distribution over final consequences. If a EUT maximizer prefers C to D, then $\frac{3}{4}u(0) + \frac{1}{4}u(3000) < \frac{3}{4}u(0) + \frac{1}{4}0.8u(4000)$**, which simplifies to** $u(3000) < 0.8u(4000) + 0.2u(0)$**, a contradiction with what we had in part a).**

This is an example of the independence axiom in action. In parts b) and c) we showed that you can mix in the same probability of a third lottery (here the degenerate lottery paying nothing) to A and B to construct C and D. The independence axiom says that an EUT maximizer's preference can't flip when we do that.