CS170–Spring 2022 — Homework 00

Dayne Tran, SID 3034839991

Collaborators: None

1. Course Syllabus

(a) None.

(b) Yes, but anonymously.

2.

(a) Midterm 1 is on February 23, 8pm-10pm. Midterm 2 is on April 5, 8pm-10pm. Final is on May 11, 11:30am-2:30pm.

(b) The course staff recommend having the homework finished by 10pm.

(c) "[Course Staff] accept absolutely no submissions after 11:59pm, even after technical issues or emergencies. No exceptions."

(d) Piazza

(e) I have read and understood the course syllabus and policies.

3.

(a) Not OK

(b) Not OK

(c) Not OK

(d) Not OK

4.

Refer to Course Notes page 3, where it is shown that applying limit tests to f and g asymptotics can be used for proofs. Rough draft: define f(n) as 2^n.
First, we want to prove that for all $c > 0$, $f(n) = \Omega(n^c)$, i.e. n^c dominates $f(n)$. We know from lectures and course notes that we can use limit tests to prove dominance.

$$(1^+)^n = \Omega(n^c) \iff \lim_{n\to\infty} \frac{(1^+)^n}{n^c} > 0 \text{ for all } c > 0$$

Apply L'Hopital's Rule because the immediate result is $\frac{\infty}{\infty}$:

$$\lim_{n\to\infty} \frac{\ln 1^+ \cdot (1^+)^n}{c \cdot n^{c-1}}$$

The result is still $\frac{\infty}{\infty}$, so apply L'Hopital's Rule repeatedly until the following expression appears:

$$\lim_{n\to\infty} \frac{(\ln 1^+)^c \cdot (1^+)^n}{c!} = \infty > 0$$

Second, we want to prove that for all $\alpha > 1$, $f = O(\alpha^n)$. Again, using our lecture and course notes, we can make use of limit tests for our proof.

$$(1^+)^n = O(\alpha^n) \iff \lim_{n\to\infty} \frac{(1^+)^n}{\alpha^n} < \infty \text{ for all } \alpha > 1$$

We can simplify the limit to get to a determinate result:

$$\begin{aligned}
\lim_{n\to\infty} \frac{(1^+)^n}{\alpha^n} &= \lim_{n\to\infty} \left(\frac{1^+ \cdot 1^+ \cdot \ldots \cdot 1^+}{\alpha \cdot \alpha \cdot \ldots \alpha} \right) \\
&= \lim_{n\to\infty} \left(\frac{1^+}{\alpha} \cdot \frac{1^+}{\alpha} \cdot \ldots \cdot \frac{1^+}{\alpha} \right) \\
&= \lim_{n\to\infty} \left(\frac{1^+}{\alpha} \right)^n \\
&\leq 1 \text{ since } \frac{1^+}{\alpha} \leq 1 \text{ for } \alpha > 1. \\
&< \infty
\end{aligned}$$

5.

We want to prove that for any $\epsilon > 0$, we have $\log x \in O(x^\epsilon)$.

$$log(x) = O(x^\epsilon) \iff \lim_{x\to\infty} \frac{log(x)}{x^\epsilon} < \infty \text{ for all } \epsilon > 0$$

Apply L'Hopital's Rule because the immediate result is $\frac{\infty}{\infty}$.

$$= \lim_{x\to\infty} \frac{1}{x} \cdot \frac{1}{\epsilon \cdot x^{\epsilon-1}}$$

Plug in 0^+ for ϵ since $\epsilon > 0$.

$$\begin{aligned}
&= \lim_{x\to\infty} \frac{1}{0^+ x^{0^+}} \\
&= \frac{1}{\infty} = 0 < \infty \\
&\iff log(x) = O(x^\epsilon) \text{for all } \epsilon > 0.
\end{aligned}$$