1. Jim Corp. has a brilliant invention but no-one's quite sure if it's going to work out. Jim Corp. creates an asset with price p that pays \$1000 if the invention succeeds and \$0 if the invention fails. Say that a DM is a Maxmin Expected Utility maximizer who has a Bernoulli utility function $u(x) = x$ where x is cash and believes that the probability of success is somewhere between $\frac{1}{10}$ and $\frac{1}{2}$. The DM can buy the asset (pay p and receive \$1000 in the success case) or short sell it (receive p and pay \$1000 in the success case).

   For what prices will the DM want to neither sell nor buy this asset? Why is it that this kind of thing can happen with Maxmin Expected Utility? What behavioral phenomenon is Maxmin Expected Utility particularly tailored to rationalize?

   First (and this is for your info not something you need to include in your answer necessarily) that since utility is linear in x it means we can look at *changes* in MEU. That is: wealth levels don't matter here.

   Anyway, let's calculate the MEU for this DM if they buy the asset. If it's bigger than zero then their MEU would increase if they bought the asset:

$$MEU(buy) = \frac{1}{10}(1000 - p) + \frac{9}{10}(-p) > 0 \tag{1}$$
$$p < 100 \tag{2}$$

   Next, let's calculate the MEU for this DM if they sell the asset. If it's bigger than zero then their MEU would increase if they sold the asset:

$$MEU(sell) = \frac{1}{2}(p - 1000) + \frac{1}{2}(p) > 0 \tag{3}$$
$$p > 500 \tag{4}$$

   So if the price is between 100 and 500 the DM wants to neither buy not sell. The deal here is that MEU is pessimistic: it evaluates lotteries according to the worst case scenario of the possible probabilities. That's why we used $\frac{1}{10}$ in the buy case (worst case there is the lowest possible probability of the asset being worth something) and $\frac{1}{2}$ in the sell case (worst case there is the highest chance of the asset being worth something). This is designed to capture ambiguity aversion: the DM dislikes not knowing for sure the probabilities in an uncertain situation, and so evaluates them pessimistically.

2. Consider a loss averse DM with utility function $u(x) = x$ (where x is dollars) and $\lambda \geq 1$. You offer them two possibilities:

   Option i. Two sequential 50-50 coin flips, where on each flip heads means you win $j > 1$ dollars and tails means you lose one dollar, and you learn the result of flip 1 before flip 2.

Option ii. Two sequential 50-50 coin flips, where on each flip heads means you win $k > 1$ dollars and tails means you lose one dollar, and you will not learn the result of flip 1 before flip 2.

a) As a function of λ, how much bigger must j be than k before this DM picks option i over option ii? Explain the relationship you found.

Let's calculate this DM's EU from each option (factoring in their loss aversion). The deal is that losses are amplified by λ in the utility function.

$$EU(i) = \frac{1}{4}(j + j) + \frac{1}{2}(j - \lambda) + \frac{1}{4}(-\lambda - \lambda) = j - \lambda \tag{5}$$

$$EU(ii) = \frac{1}{4}(2k) + \frac{1}{2}(k - \lambda) + \frac{1}{4}(-\lambda - \lambda) = k - \frac{1}{2} - \frac{1}{2}\lambda \tag{6}$$

So to get them to pick i we'd need $j - \lambda > k - \frac{1}{2} - \frac{1}{2}\lambda$, which with some algebra gives $j - k > \frac{1}{2}\lambda - \frac{1}{2}$. This makes sense because if $\lambda = 1$ we have a plain old EU maximizer with no loss aversion—in that case all that matters is the distribution of cash at the end of the day. But as λ gets bigger, the DM is more loss averse and so gets more and more upset to possibly see bad intermediate results in case i. That means we have to sweeten the pot more to induce them to prefer it.

b) Explain how the model of loss aversion is related to Prospect Theory and its motivating evidence from Kahneman & Tversky (1979) that we discussed in class.

This is related to Prospect Theory's reflection effect: the fact that DMs seem to fear and dislike losses more than they like equivalent gains. When something counts as a loss in the eyes of the DM it is chosen less; Problem 11 and 12 from K&T 1979 form a clean example of how the DM treats gains and losses differently, for example. This feeds in to the model's λ parameter which implements the idea of losses being more painful than equivalent gains.

3. Say that you're a marketer and you'd like to figure out if you can exploit people's loss aversion and/or reference dependent preferences to sell them more of your company's stuff or increase their engagement with your product. Think of a way in which you might be able to do that, and propose an experiment that you could run on your customers to test whether your idea would work. [200-300 words.]

Lots and lots of ways you could go with this one. Manipulating people's reference point is a tried and tested trick, and getting people attached to something to try to induce the endowment effect might work too. Some examples I thought of:

(a) **Try before you buy: e.g. Warby Parker sending you glasses; you'd be sad to not have them after you've experienced what they look like**

(b) **Artificially making it look like something's on sale: makes it seem like you're getting a deal relative to the reference sticker price**

(c) **'Limited time offer': you'll lose the chance it you don't act now**

(d) **'Here's how much electricity your neighbors used last month': manipulating the reference point**

(e) **Emphasizing risks when trying to sell you something like travel insurance**

You might have had other suggestions. The key thing here is how do you test it? One simple suggestion would be an experiment that basically mimics A/B testing: for example, a business could show half your customers a marketing email with 'limited time offer' in the subject line and half the customers the regular version, and see what happens in each group. As usual you would want to make sure that the groups are comparable—checking that the groups are similar in characteristics is important. The challenge for an economics experimenter will therefore be how to design a non-misleading experiment to achieve the same thing!

4. Consider a variation on the beauty contest game from class. Each player must simultaneously choose an integer between 0 and 100 (inclusive). The average of the chosen numbers will be calculated; call this average x. The player whose number is closest to x will win and earn a positive payoff. If two or more players are equally close to x, the winner will be chosen at random among them. Players who do not win earn a payoff of zero.

   a) What are the pure strategy Nash equilibria if there are two players?

   Any strategy profile is a NE. For any number selected by the opponent, the average is precisely half way between the guesses and so the player wins with probability $\frac{1}{2}$ no matter what number they pick. Thus any number is a best response to any other.

   b) What are the pure strategy Nash equilibria if there are three players?

   Any strategy profile in which all three select the same number is a NE. In that case all three win with probability $\frac{1}{3}$ while if any one chose a different number that player would not win. If there is any dispersion among the numbers, the player with the 'outlier' number will never win and so would have done better by moving closer to the other two.

   c) Let the number of players be 'large'. Let the level 0 strategy be to choose a number at random. Find the outcome when all players are level k reasoners with $k > 0$.

   L0 has a mean of 50. L1 best responds as if the other players were L0; the best response to others guessing 50 is to also guess 50 since this wins with probability $\frac{1}{3}$. Thus the L1 strategy is 50 and the same logic shows that L2 and higher play 50. All play 50.

5. Consider the simultaneous-move game represented by the following matrix:

		Column Player	
		L	R
Row Player	T	$2, 0$	$1, 3$
	B	$1, 1$	$3, 0$

a) If the L0 action places an equal probability on each pure strategy, derive actions for L1 through L4 of each player.

	Row player	**Column player**
L0	**Random**	**Random**
L1	**B**	**R**
L2	**B**	**L**
L3	**T**	**L**
L4	**T**	**R**

b) If all players are level $k > 0$ reasoners, does it always pay to think more deeply? That is: do players always do better when they have a higher level of reasoning than they would if they had a lower level of reasoning? If not, show with an example.

They do not always do better. Let's say that the Column player is L1, so that they play R. The L2 Row player plays B and so would earn a payoff of 3, but the L3 Row player plays T and so would earn a payoff of 1.

c) Say that the population was made up of a fraction α of L3 players and a fraction $(1-\alpha)$ of L4 players. Are there any values of α such that both players are better off (on average) than they would have been in the unique Nash equilibrium? If so, which?

First let's figure out what payoff each player earns in NE. To do this, we can compute the probability that we end up in each of the four possible cells and use that to compute the expected payoff to each player.

$$P1 : 2(\frac{2}{12}) + \frac{6}{12} + \frac{1}{12} + 3(\frac{3}{12}) = \frac{5}{3} \tag{7}$$

$$P2 : 3(\frac{1}{12}) + \frac{6}{12} = \frac{3}{4} \tag{8}$$

Next, if α are L3 and $1-\alpha$ are L4: both types of Row play T, while Column plays L if L3 and R if L4. That means we end up in the cell (T,L) with probability α and (T,R) with probability $1-\alpha$. We need to see if there are

values of α such that both types are better off than in the NE:

$$P1 : 2\alpha + (1 - \alpha) > \frac{5}{3} \tag{9}$$

$$\alpha > \frac{2}{3} \tag{10}$$

$$P2 : 3(1 - \alpha) > \frac{3}{4} \tag{11}$$

$$\alpha < \frac{3}{4} \tag{12}$$

It is possible. For α between $\frac{2}{3}$ and $\frac{3}{4}$, both players would do better than they did in the unique NE.

6. Consider the simultaneous-move game represented by the following matrix:

		Player 2		
		L	C	R
	T	$0, 1$	$2, 0$	$1, 3$
Player 1	M	$0, 2$	$1, 0$	$2, 1$
	B	$4, 0$	$3, 3$	$0, 0$

a) Find the unique Nash equilibrium in pure strategies.

Unique NE is at (B, C). By the way it is important that you specify NE (and other solution concepts) in terms of *strategies* not in terms of payoffs. The object of our interest in game theory is always strategies.

b) Let each player's L0 action be a uniform distribution over his set of pure strategies. Start to derive Lk actions for the two players. After how many levels of reasoning by each player does the level-k process 'stabilize' at the Nash equilibrium?

	Row player	**Column player**
L0	**Random**	**Random**
L1	**B**	**R**
L2	**M**	**C**
L3	**B**	**L**
L4	**B**	**C**

At $L4$ each player is playing the same strategy as in the NE, and so this is true of any level $k \geq 4$.