hw04_code

March 9, 2022

```
[ ]: import numpy as np
     import scipy as sp
     from scipy import io
     from scipy import stats
     import pandas as pd
     from sklearn.metrics import accuracy_score
     import matplotlib.pyplot as plt
     from numpy.random import MT19937
     from numpy.random import RandomState, SeedSequence
```

0.0.1 3. Wine Classification with Logistic Regression

```
[ ]: # set random state to replicate random paritioning of training and validation␣
      ↪data
     rs = RandomState(MT19937(SeedSequence(694200)))

     wine = io.loadmat("data_wine.mat")
     # separate into training data, training labels, testing data, and feature␣
      ↪descriptions
     wine_raw_training_data = wine["X"]
     wine_training_labels = wine["y"]
     wine_test_data = wine["X_test"]
     wine_descriptions = wine["description"]

     # attach labels to corresponding data, then shuffle
     wine_tuples = np.hstack((wine_raw_training_data, wine_training_labels))
     wine_tuples_shuffled = rs.permutation(wine_tuples)

     # assign 500 tuples from randomly shuffled data to validation, the rest to␣
      ↪training
     wine_validation_data = wine_tuples_shuffled[0:500, 0:-1]
     wine_validation_labels = wine_tuples_shuffled[0:500, -1]
     wine_training_data = wine_tuples_shuffled[500:, 0:-1]
     wine_training_labels = wine_tuples_shuffled[500:, -1]

     # Standardize Data By Feature and Add Fictitious Dimension
     training_features_means = np.mean(wine_training_data, axis=0)
```

```
training_features_sd = np.std(wine_training_data, axis=0)
wine_standardized_training_data = (wine_training_data -␣
↪training_features_means) / training_features_sd
wine_training_data = np.hstack((wine_standardized_training_data, np.
↪ones((wine_standardized_training_data.shape[0],1))))

# standardize validation and testing data by the training data's mean and std,␣
↪add fictitious dimension to both as well
wine_validation_data = (wine_validation_data - training_features_means) /␣
↪training_features_sd
wine_validation_data = np.hstack((wine_validation_data, np.
↪ones((wine_validation_data.shape[0],1))))

wine_test_data = (wine_test_data - training_features_means) /␣
↪training_features_sd
wine_test_data = np.hstack((wine_test_data, np.ones((wine_test_data.
↪shape[0],1))))
```

[ ]:
```
def logistic_cost(data,labels,w):
    """
    Computes the logistic cost function

    :param data: the training data; for wine, expect (5500,13) matrix
    :param labels: the training labels; for wine, expect (5500,) vector
    :param w: the training weights; for wine, expect (13,) vector

    :return: scalar logistic cost
    """

    sigmoid = sp.special.expit(data@w) # s = 1 / (1 + e^(-Xw))

    first_component = -1 * np.dot(labels, np.log(sigmoid)) # -y . ln(s)
    second_component_sub_a = np.ones((data.shape[0], 1)).reshape(-1) - labels #␣
↪1-y
    second_component_sub_b = np.ones((data.shape[0], 1)).reshape(-1) - sigmoid␣
↪#  1-s
    second_component = -1 * np.dot(second_component_sub_a, np.
↪log(second_component_sub_b)) # -(1-y) . ln(1-s)

    # - y . ln(s) - (1-y) . ln(1-s)
    return first_component + second_component
```

1. The batch gradient descent update law for logistic regression with l2 regularization: we use the gradient over the entire sample of training examples and use it to update our weights. We do this for each and every step of gradient descent and we move towards an optimum solution (the algo converges when either the movement of the cost function is extremely minimal or we reach a maximum number of steps).

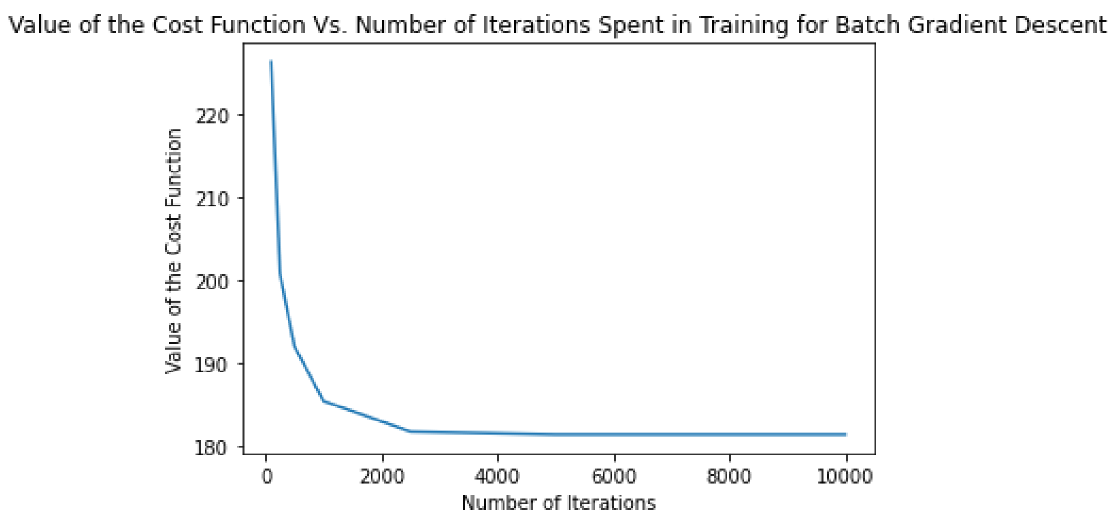
2. The code for batch gradient descent is below, as is the plot of the cost function vs. the number of iterations spent in training.

```
[ ]: def descend_bgd(alpha, data=wine_training_data, labels=wine_training_labels, ep␣
     ↪= 0.0001, max_iters=10000, l2=0):
         """
         Operationalizes batch gradient descent in order to find the weights that␣
     ↪best minimize the logistic cost function

         :param alpha: the learning rate (hyperparameter)
         :param data: the training data
         :param labels: the training labels
         :param ep: desired difference in current and previous cost, indicator of␣
     ↪convergence
         :param max_iters: maximum number of interactions (batches)
         :param l2: regularization parameter in L2

         :return: most recent gradient vector, weights vector, logistic cost
         """

         converged = False
         w_i = rs.random((13,))
         iter = 0
         J = logistic_cost(data,labels, w_i)

         while not converged:
             # compute the gradient
             residual = sp.special.expit(data @ w_i) - labels
             gradient = (data.T @ (residual)) + (2 * l2 * w_i)

             # update the weights
             w_i = w_i - alpha * gradient

             # find new cost
             e = logistic_cost(data,labels, w_i)
             if abs(J-e) <= ep:
                 print(f"Converged, iterations: {iter}")
                 converged =  True

             J = e # update error
             iter += 1

             if iter == max_iters:
                 print(f"Max iteractions {max_iters} exceeded")
                 converged = True

         return gradient, w_i, J
```

[ ]:
```
rs = RandomState(MT19937(SeedSequence(42069)))

num_iters = [100,250,500,1000,2500,5000,7500,10000]
cost_function_values = [descend_bgd(0.0005, wine_training_data,␣
↪wine_training_labels, max_iters=i)[2] for i in num_iters]
plt.plot(num_iters, cost_function_values)
plt.title("Value of the Cost Function Vs. Number of Iterations Spent in␣
↪Training for Batch Gradient Descent")
plt.xlabel("Number of Iterations")
plt.ylabel("Value of the Cost Function")
```

```
Max iteractions 100 exceeded
Max iteractions 250 exceeded
Max iteractions 500 exceeded
Max iteractions 1000 exceeded
Max iteractions 2500 exceeded
Converged, iterations: 3736
Converged, iterations: 3736
Converged, iterations: 3749
```

[ ]:
```
Text(0, 0.5, 'Value of the Cost Function')
```



3. The batch gradient descent update law for logistic regression with l2 regularization: we use the gradient over the entire sample of training examples and use it to update our weights. We do this for each and every step of gradient descent and we move towards an optimum solution (the algo converges when either the movement of the cost function is extremely minimal or we reach a maximum number of steps).

4. The stochastic gradient descent code is below, as is the plot of the cost function vs. the number of iterations spent in training. Stochastic gradient descent converges more quickly

but the cost function (a measure of inaccuracy) is much higher for stochastic gradient descent than for batch gradient descent.

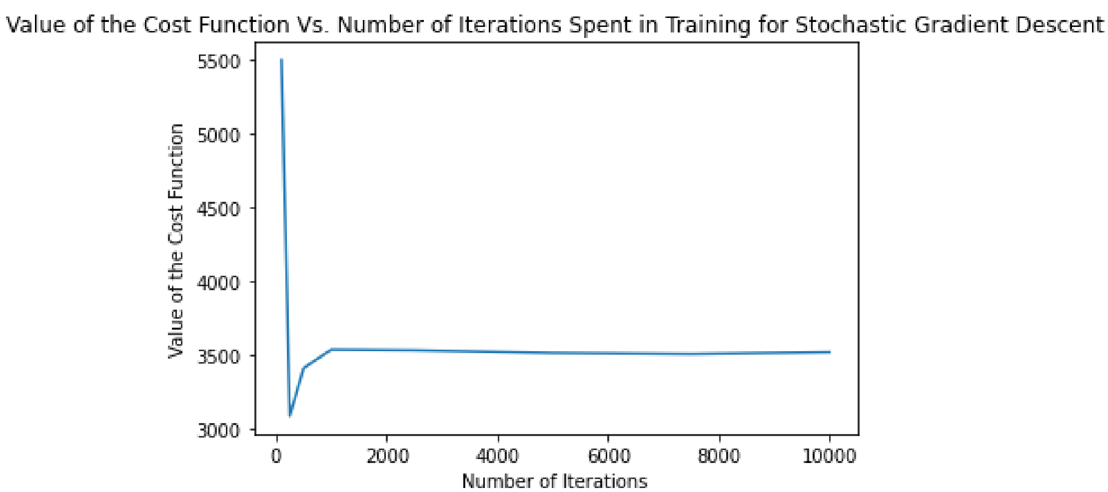
```
[ ]: def descend_sgd(alpha, data=wine_training_data, labels=wine_training_labels, ep␣
    ↪= 0.0001, max_iters=10000, l2=4):
        """
        Operationalizes stochastic gradient descent in order to find the weights␣
    ↪that best minimize the logistic cost function

        :param alpha: the learning rate (hyperparameter)
        :param data: the training data
        :param labels: the training labels
        :param ep: desired difference in current and previous cost, indicator of␣
    ↪convergence
        :param max_iters: maximum number of interactions (batches)
        :param l2: regularization parameter in L2

        :return: most recent gradient vector, weights vector, logistic cost
        """

        converged = False
        w_i = rs.random((13,))
        iter = 0
        J = logistic_cost(data,labels, w_i)

        while not converged:
            # compute the gradient
            random_int = rs.randint(0, data.shape[0])
            sample = data[random_int, :].reshape((1,13))
            residual = sp.special.expit(sample @ w_i) - labels[random_int]
            gradient = (sample.T @ residual) + (2 * l2 * w_i)

            # update the weights
            w_i = w_i - alpha * gradient

            # find new cost
            e = logistic_cost(data,labels, w_i)
            if abs(J-e) <= ep:
                print(f"Converged, iterations: {iter}")
                converged =  True

            J = e # update error
            iter += 1

            if iter == max_iters:
                print(f"Max iteractions {max_iters} exceeded")
                converged = True
```

```
    return gradient, w_i, J
```

[ ]:
```
rs = RandomState(MT19937(SeedSequence(69696969)))

num_iters = [100,250,500,1000,2500,5000,7500,10000]
cost_function_values = [descend_sgd(0.0005, wine_training_data,␣
 ↪wine_training_labels, max_iters=i, l2=4)[2] for i in num_iters]
plt.plot(num_iters, cost_function_values)
plt.title("Value of the Cost Function Vs. Number of Iterations Spent in␣
 ↪Training for Stochastic Gradient Descent")
plt.xlabel("Number of Iterations")
plt.ylabel("Value of the Cost Function")
```

```
Max iteractions 100 exceeded
Max iteractions 250 exceeded
Max iteractions 500 exceeded
Max iteractions 1000 exceeded
Max iteractions 2500 exceeded
Max iteractions 5000 exceeded
Max iteractions 7500 exceeded
Max iteractions 10000 exceeded
```

[ ]:
```
Text(0, 0.5, 'Value of the Cost Function')
```



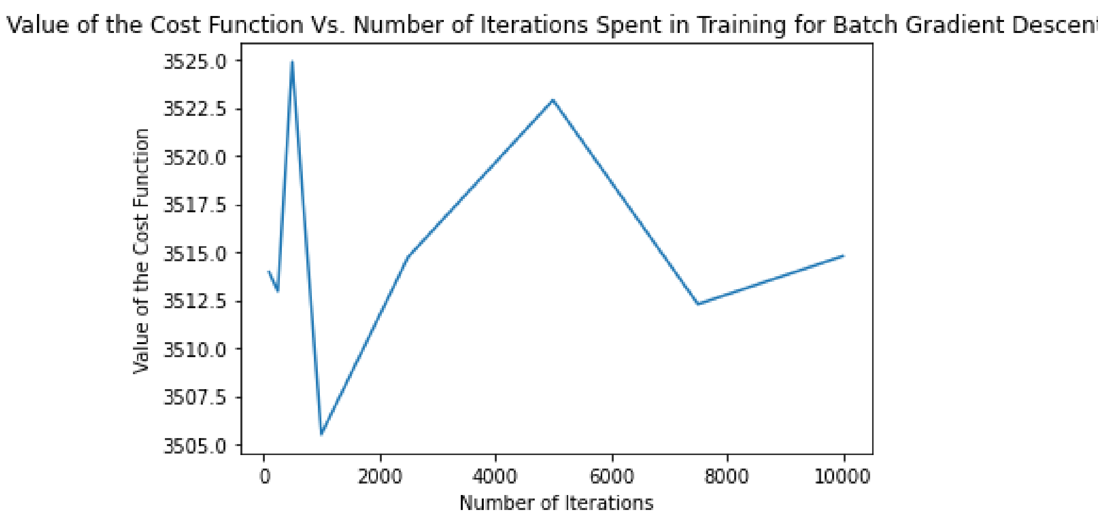
5. Below is the code for the SGD with a decreasing learning rate. I chose the value of 0.1 for the hyperparameter. I noticed that while it may appear more erratic than vanilla SGD, this new SGD is actually operating very smoothly around 3500; this is actually really close to the scores for vanilla SGD, while also converging incredibly more quickly. Basically, we can speed up the time to convergence without sacrificing an inordinate amount of accuracy.

[ ]:
```python
def descend_sgd_variable_alpha(alpha, data=wine_training_data,␣
↪labels=wine_training_labels, ep = 0.0001, max_iters=10000, l2=4):
    """
    Operationalizes stochastic gradient descent in order to find the weights␣
↪that best minimize the logistic cost function

    :param alpha: the learning rate (hyperparameter)
    :param data: the training data
    :param labels: the training labels
    :param ep: desired difference in current and previous cost, indicator of␣
↪convergence
    :param max_iters: maximum number of interactions (batches)
    :param l2: regularization parameter in L2

    :return: most recent gradient vector, weights vector, logistic cost
    """

    converged = False
    w_i = rs.random((13,))
    iter = 1
    J = logistic_cost(data,labels, w_i)

    while not converged:
        # compute the gradient
        random_int = rs.randint(0, data.shape[0])
        sample = data[random_int, :].reshape((1,13))
        residual = sp.special.expit(sample @ w_i) - labels[random_int]
        gradient = (sample.T @ residual) + (2 * l2 * w_i)

        # update the weights
        w_i = w_i - alpha/iter * gradient

        # find new cost
        e = logistic_cost(data,labels, w_i)
        if abs(J-e) <= ep:
            print(f"Converged, iterations: {iter}")
            converged =  True

        J = e # update error
        iter += 1

        if iter == max_iters:
            print(f"Max iteractions {max_iters} exceeded")
            converged = True

    return gradient, w_i, J
```

[ ]:
```
rs = RandomState(MT19937(SeedSequence(69696969)))

num_iters = [100,250,500,1000,2500,5000,7500,10000]
cost_function_values = [descend_sgd_variable_alpha(0.1, wine_training_data,␣
 ↪wine_training_labels, max_iters=i, l2=4)[2] for i in num_iters]
plt.plot(num_iters, cost_function_values)
plt.title("Value of the Cost Function Vs. Number of Iterations Spent in␣
 ↪Training for Batch Gradient Descent")
plt.xlabel("Number of Iterations")
plt.ylabel("Value of the Cost Function")
```

```
Max iteractions 100 exceeded
Max iteractions 250 exceeded
Max iteractions 500 exceeded
Converged, iterations: 739
Converged, iterations: 1503
Converged, iterations: 1959
Converged, iterations: 1622
Converged, iterations: 926
```

[ ]:
```
Text(0, 0.5, 'Value of the Cost Function')
```



[ ]:
```
bgd = descend_bgd(0.0005, wine_training_data, wine_training_labels)
test_predictions = np.round(sp.special.expit(wine_test_data @ bgd[1]))

wine_pd = pd.DataFrame(np.int64(test_predictions),columns=['Category'])
wine_pd.index.name = 'Id'
wine_pd.index += 1
wine_pd.to_csv("wine_test_predictions.csv")
```

```
Converged, iterations: 3739
```

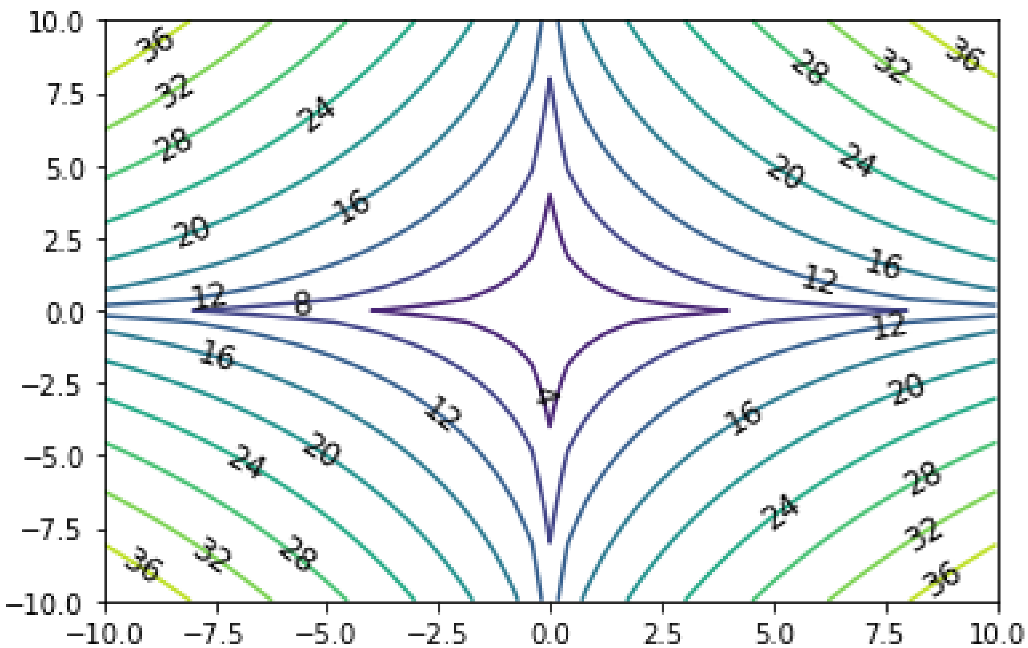
6. Kaggle:

Kaggle Username: Dayne Tran

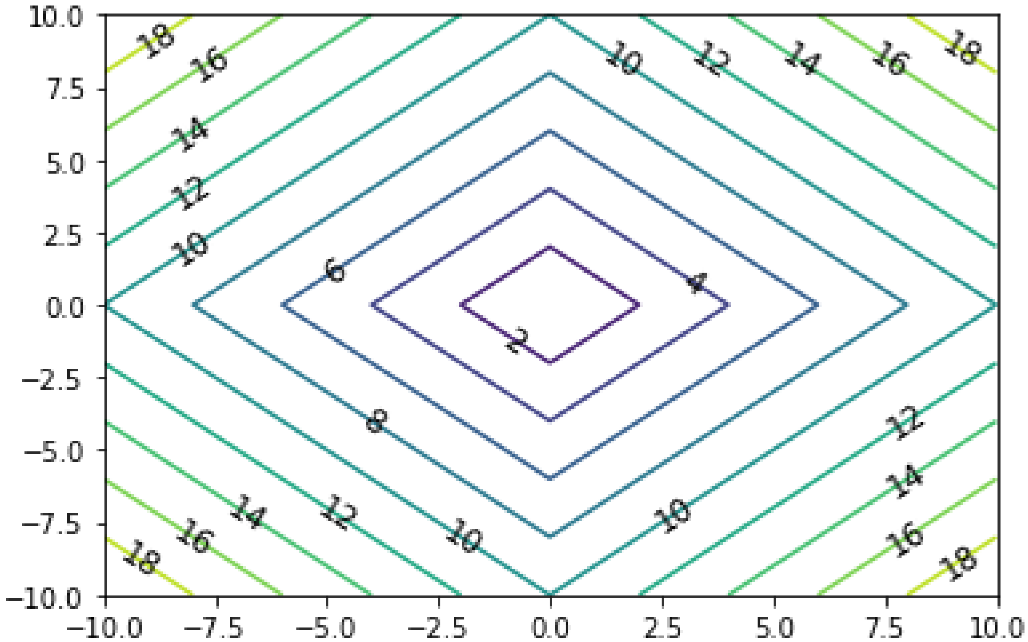
Score: 0.98792

[ ]:
```
def plot_norm_contours(l=2):
    """
    plots isocontours for the l-norms of 2D weights

    :param l: the norm type (l=1 means lasso, l=2 means ridge, etc.)

    """

    X, Y = np.meshgrid(np.linspace(-10,10, 51), np.linspace(-10,10, 51))
    Z = np.empty_like(X).astype(float)
    for i in range(X.shape[0]):
        for j in range(X.shape[1]):
            norm_pt = (np.abs(X[i][j])**l + np.abs(Y[i][j])**l)**(1/l)
            Z[i][j] = norm_pt
    p = plt.contour(X, Y, Z, 10)
    plt.clabel(p, inline=False, fontsize=12, colors = 'black')
    return
```

[ ]:
```
plot_norm_contours(0.5)
```



```
[ ]: plot_norm_contours(1)
```




```
[ ]: plot_norm_contours(2)
```