hw03

February 25, 2022

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

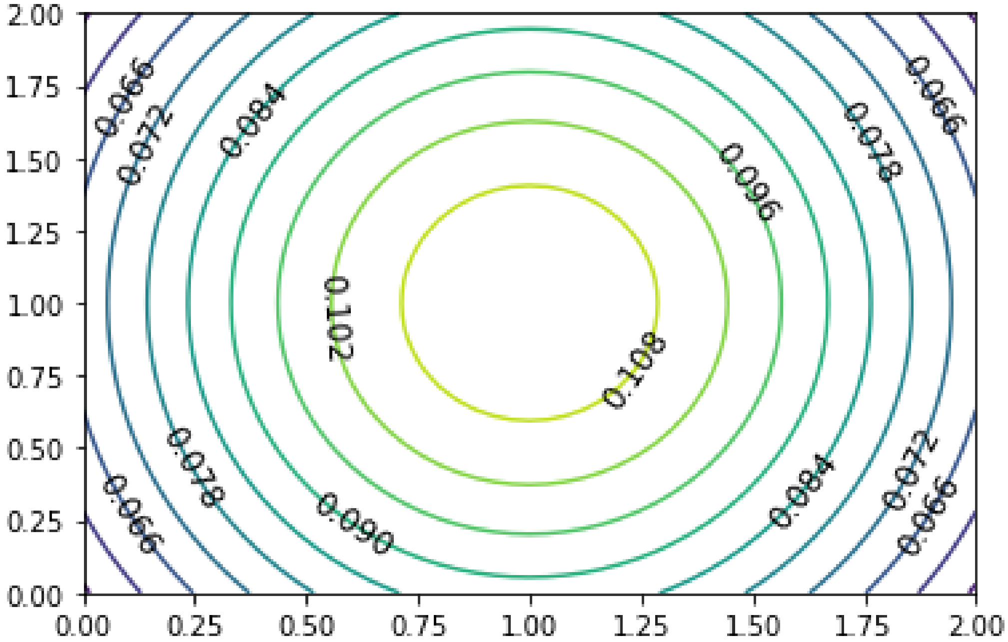
0.1 *3. Isocontours of Gaussian Distributions*

[ ]:
```
def compute_gauss_matrix(m,s,x,y):
    """Input: m and s are the Gaussian parameters. x and y compose a grid of␣
 ↪points."""
    """For each point (x_i,y_i), we compute the Gaussian pdf and return the␣
 ↪output z_i"""

    z = np.empty_like(x).astype(float)
    for i in range(x.shape[0]):
        for j in range(x.shape[1]):
            data_pt = np.array([x[i][j], y[i][j]])
            numer = np.exp(-0.5 * (data_pt-m).T @ (np.linalg.inv(s) @␣
 ↪(data_pt-m))) # e^( - 1/2 * (x-mu)T * S^-1 * (x-mu))
            denom = np.sqrt((2 * np.pi) ** 2 * np.linalg.det(s)) #␣
 ↪sqrt((2pi)**2 * det(sigma))
            z_pt = numer / denom
            z[i][j] = z_pt

    return z
```

1. $f(\mu, \Sigma)$**, where** $\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **and** $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

[ ]:
```
mu, sigma = np.asarray([1, 1]), np.array([[1, 0],[0, 2]])
X, Y = np.meshgrid(np.linspace(mu[0]-1, mu[0]+1, 51), np.linspace(mu[0]-1,␣
 ↪mu[1]+1, 51))

Z = compute_gauss_matrix(mu, sigma,X,Y)
p = plt.contour(X, Y, Z, 10)
plt.clabel(p, inline=False, fontsize=12, colors = 'black')
```

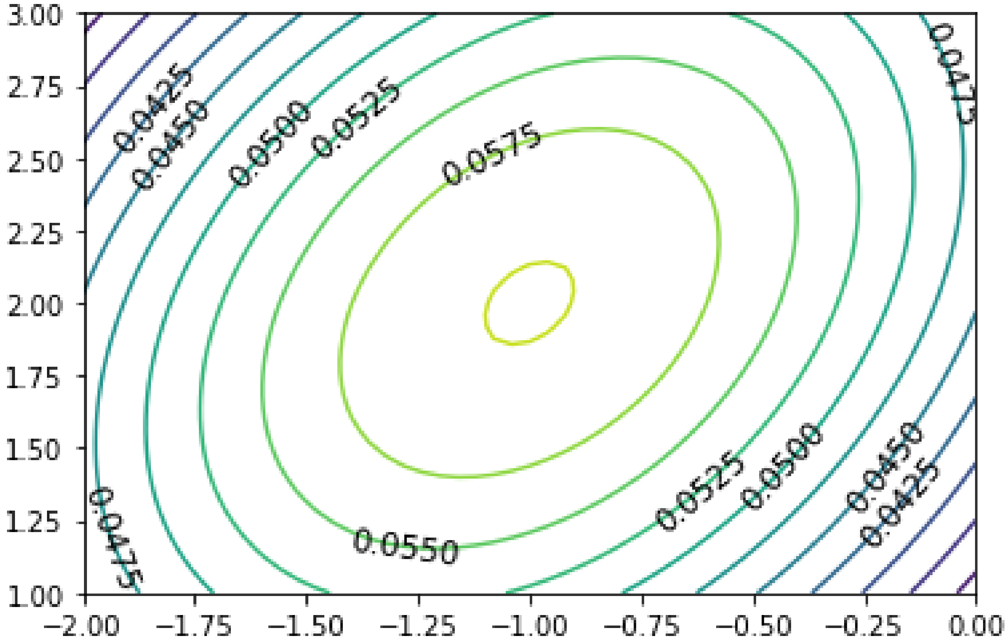
[ ]:
```
<a list of 14 text.Text objects>
```



2. $f(\mu, \Sigma)$**, where** $\mu = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ **and** $\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$**.**

[ ]:
```
mu, sigma = np.asarray([-1, 2]), np.array([[2, 1],[1, 4]])
X, Y = np.meshgrid(np.linspace(mu[0]-sigma[0][1], mu[0]+sigma[0][1], 51), np.
↪linspace(mu[1]-sigma[0][1], mu[1]+sigma[0][1], 51))

Z = compute_gauss_matrix(mu, sigma,X,Y)
p = plt.contour(X, Y, Z, 10)
plt.clabel(p, inline=False, fontsize=12, colors = 'black')
```

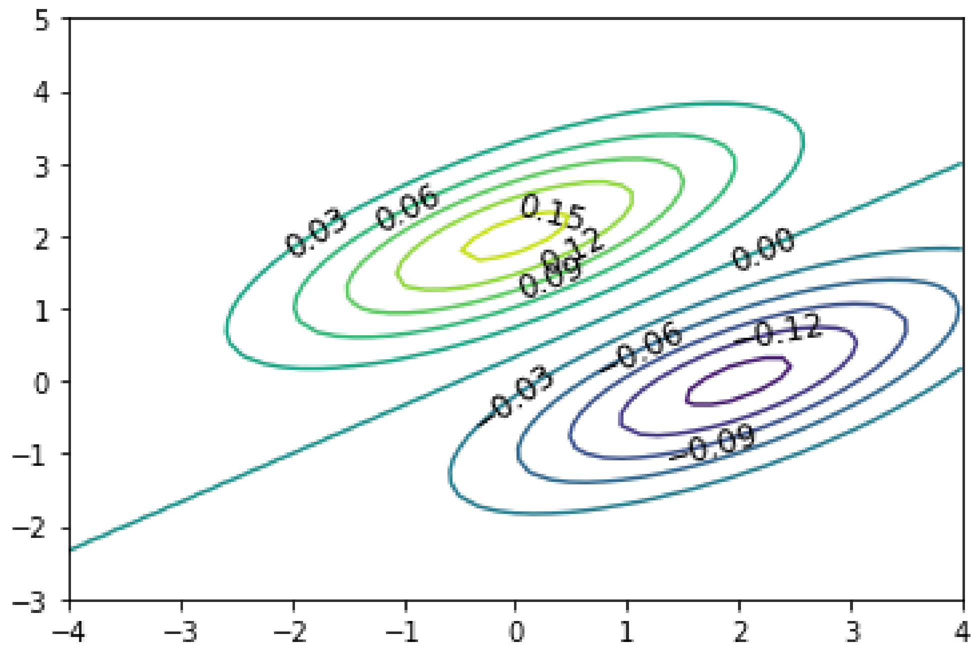
[ ]:
```
<a list of 12 text.Text objects>
```



3. $f(\mu_1, \Sigma) - f(\mu_2, \Sigma)$**, where** $\mu_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $\mu_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, **and** $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

[ ]:
```
mu_1 = np.asarray([0, 2])
mu_2 = np.asarray([2,0])
sigma = np.array([[2, 1],[1, 1]])
X, Y = np.meshgrid(np.linspace(mu[0]-3*sigma[0][1], mu[0]+5*sigma[0][1], 51),␣
 ↪np.linspace(mu[1]-5*sigma[0][1], mu[1]+3*sigma[0][1], 51))

Z_1 = compute_gauss_matrix(mu_1, sigma, X , Y)
Z_2 = compute_gauss_matrix(mu_2, sigma, X, Y)
p = plt.contour(X, Y, Z_1 - Z_2, 10)
plt.clabel(p, inline=False, fontsize=12, colors = 'black')
```

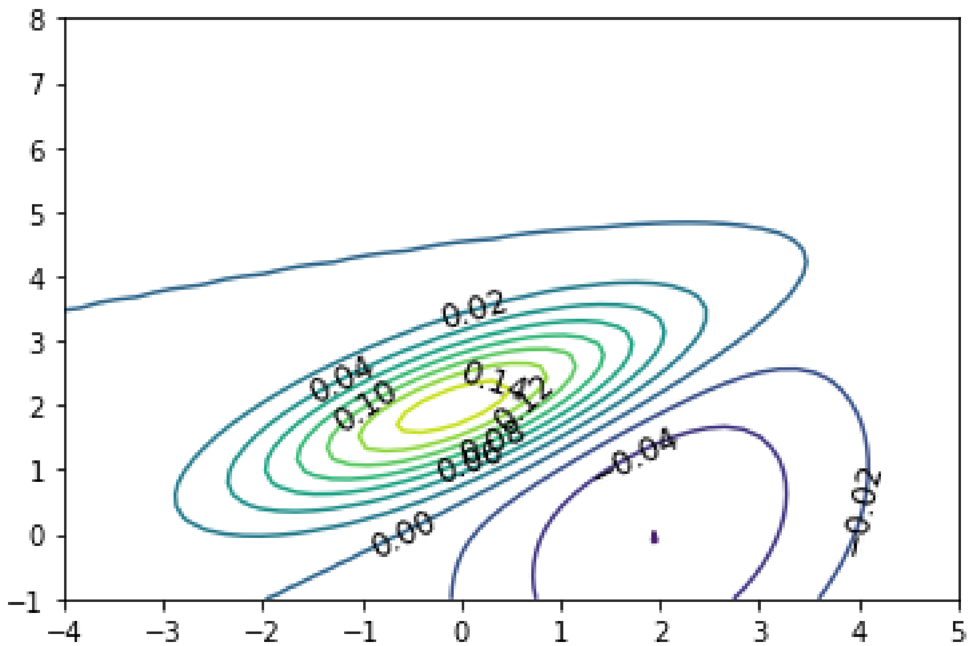
[ ]: <a list of 10 text.Text objects>



4. $f(\mu_1, \Sigma_1) - f(\mu_2, \Sigma_2)$, **where** $\mu_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $\mu_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\Sigma_1 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, **and** $\Sigma_2 = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$.

[ ]:
```
mu_1 = np.asarray([0, 2])
mu_2 = np.asarray([2,0])
sigma_1 = np.array([[2, 1],[1, 1]])
sigma_2 = np.array([[2,1],[1,4]])
X, Y = np.meshgrid(np.linspace(mu[0]-3*sigma[0][1], mu[0]+6*sigma[0][1], 51),␣
↪np.linspace(mu[1]-3*sigma[0][1], mu[1]+6*sigma[0][1], 51))

Z_1 = compute_gauss_matrix(mu_1, sigma_1, X , Y)
Z_2 = compute_gauss_matrix(mu_2, sigma_2, X, Y)
p = plt.contour(X, Y, Z_1 - Z_2, 10)
plt.clabel(p, inline=False, fontsize=12, colors = 'black')
```

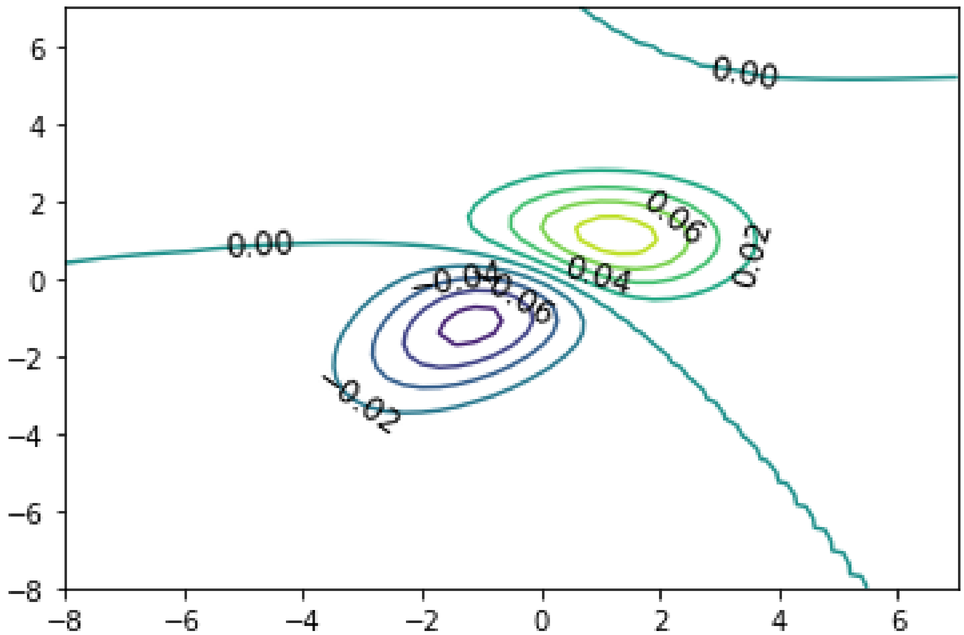
[ ]: <a list of 10 text.Text objects>



5. $f(\mu_1, \Sigma_1) - f(\mu_2, \Sigma_2)$, **where** $\mu_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mu_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$, $\Sigma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, **and** $\Sigma_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$**.**

[ ]:
```
mu_1 = np.asarray([1,1])
mu_2 = np.asarray([-1,-1])
sigma_1 = np.array([[2, 0],[0, 1]])
sigma_2 = np.array([[2,1],[1,2]])

X, Y = np.meshgrid(np.linspace(mu_1[0] - mu_2[0] - 10, mu_1[0] - mu_2[0] + 5, 51),
                   np.linspace(mu_1[1] - mu_2[1] - 10, mu_1[1] - mu_2[1] + 5, 51))

Z_1 = compute_gauss_matrix(mu_1, sigma_1, X , Y)
Z_2 = compute_gauss_matrix(mu_2, sigma_2, X, Y)
p = plt.contour(X, Y, Z_1 - Z_2, 10)
plt.clabel(p, inline=False, fontsize=12, colors = 'black')
```

[ ]:
```
<a list of 8 text.Text objects>
```



0.2 *4. Eigenvectors of the Gaussian Covariance Matrix*

[ ]:
```
from numpy.random import MT19937
from numpy.random import RandomState, SeedSequence

rs = RandomState(MT19937(SeedSequence(69420)))

X_1 = rs.normal(3, 3, 100)
X_2 = np.empty_like(X_1).astype(float)
for i in range(X_1.shape[0]):
    X_2 = rs.normal(4, 2, 100) + 0.5 * X_1

X_1 = X_1.reshape((X_1.shape[0], 1))
X_2 = X_2.reshape((X_2.shape[0], 1))

X = np.hstack((X_1, X_2))
```

1. Find the mean in (R^2) of the sample.

[ ]:
```
X_mu = np.mean(X,axis=0)
X_mu
```

[ ]:
```
array([2.76916189, 5.24308424])
```

2. Compute the 2x2 covariance matrix of the sample.

[ ]:
```
X_covar = np.cov(X.T)
X_covar
```

[ ]:
```
array([[8.73890363, 4.03970734],
       [4.03970734, 6.18340679]])
```

3. Compute the eigenvectors and eigenvalues of this covariance matrix.

[ ]:
```
covar_eigvals, covar_eigvecs = np.linalg.eig(X_covar)
print(f"Here are the eigenvalues of the covariance matrix: {covar_eigvals}")
print("Here is the eigenbasis (normalized eigenvectors) of the covariance␣
↪matrix:")
print(covar_eigvecs)
```

```
Here are the eigenvalues of the covariance matrix: [11.69812068  3.22418974]
Here is the eigenbasis (normalized eigenvectors) of the covariance matrix:
[[ 0.80671296 -0.59094349]
 [ 0.59094349  0.80671296]]
```

4. On a two-dimensional grid with a horizontal axis for X_1 with range [-15, 15] and a vertical axis for X_2 with range [-15, 15], plot

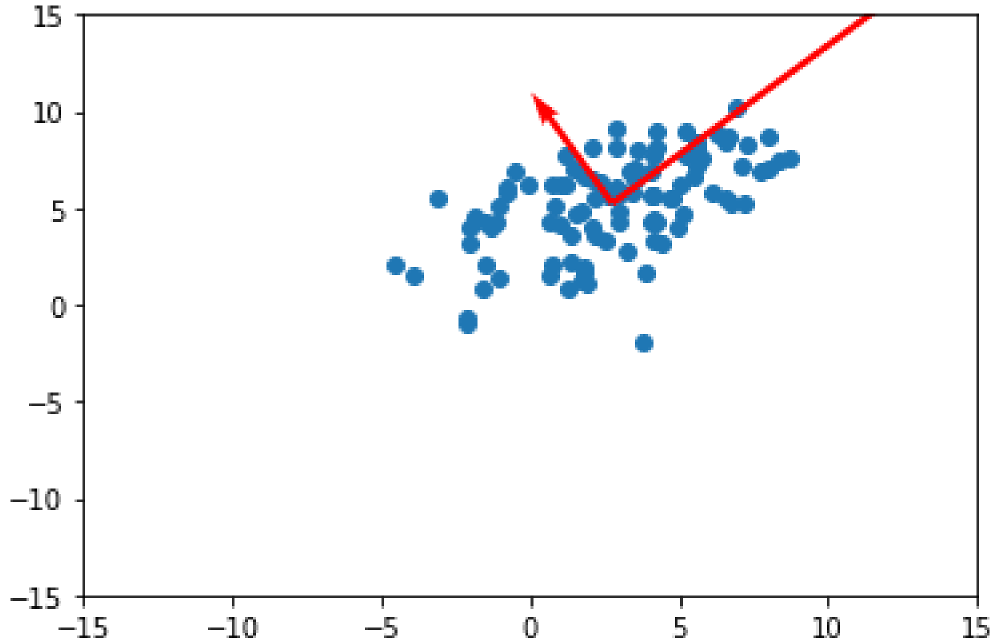
i. all $n = 100$ data points, and

ii. arrows representing both covariance eigenvectors. The eigenvector arrows should originate at the mean and have magnitudes equal to their corresponding eigenvalues.

[ ]:
```
plt.scatter(X_1, X_2)
plt.axis([-15, 15, -15, 15])

origin = X_mu
eigvec_1 = covar_eigvecs[:,0] * covar_eigvals[0]
eigvec_2 = covar_eigvecs[:,1] * covar_eigvals[1]

plt.quiver(*origin, *eigvec_1, color=['r'], scale=21)
plt.quiver(*origin, *eigvec_2, color=['r'], scale=21)

norm_1 = np.linalg.norm(covar_eigvecs[:,1] * covar_eigvals[0])
norm_2 = np.linalg.norm(covar_eigvecs[:,1] * covar_eigvals[1])
# check
print(f"The magnitude of eigenvector 1, {norm_1}, is equal to its corresponding␣
↪eigenvalue, {covar_eigvals[0]}")
print(f"The magnitude of eigenvector 2, {norm_2}, is equal to its corresponding␣
↪eigenvalue, {covar_eigvals[1]}")

plt.show()
```

```
The magnitude of eigenvector 1, 11.69812067707414, is equal to its corresponding
eigenvalue, 11.69812067707414
```

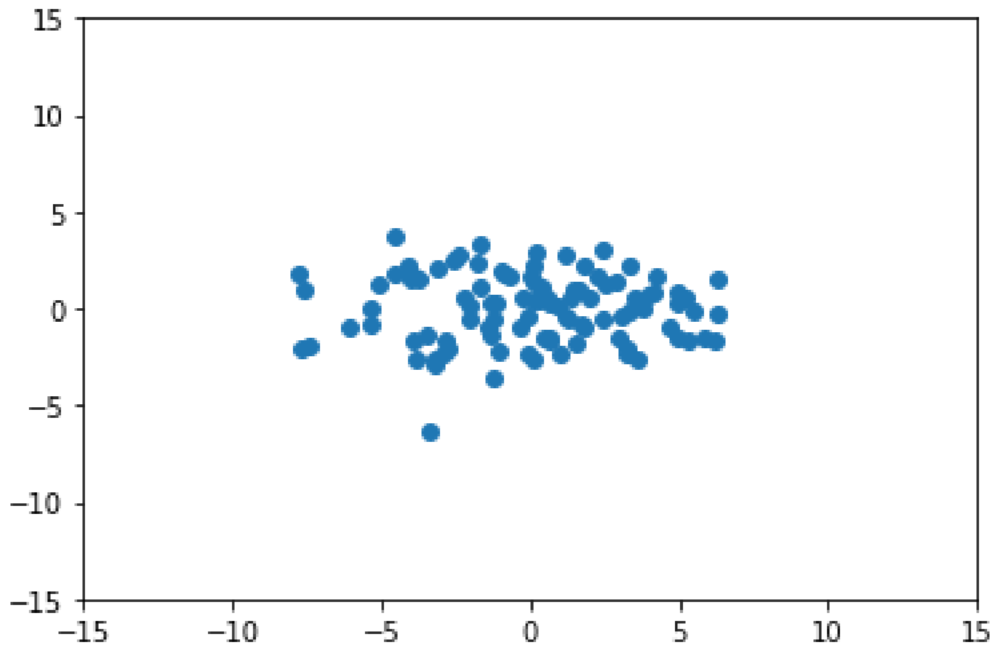
```
The magnitude of eigenvector 2, 3.224189736737576, is equal to its corresponding
eigenvalue, 3.224189736737576
```



5. Let $U = [v_1, v_2]$ be a 2 x 2 matrix whose columns are the unit eigenvectors of the covariance matrix, where v_1 is the eigenvector with the larger eigenvalue. We use U^T as a rotation matrix to rotate each sample point from the (X_1, X_2) coordinate system to a coordinate system aligned with the eigenvectors. Center your sample points by subtracting the mean μ from each point; then rotate each point by U^T, gaiving $x_{rotated} = U^T(x - \mu)$. Plot these rotated points on a new two-dimensional grid, again with both axes having range [-15, 15].

[ ]:
```
X_centered = np.empty_like(X).astype(float)
for i in range(100):
    X_centered[i][:] = X[i][:] - X_mu
X_centered.shape

X_rotated = X_centered @ covar_eigvecs
plt.scatter(X_rotated[:,0], X_rotated[:,1])
plt.axis([-15, 15, -15, 15])
```

[ ]:
```
(-15.0, 15.0, -15.0, 15.0)
```



0.3 *8. Gaussian Classifiers for Digits and Spam*

```
[ ]: import numpy as np
     import scipy as sp
     from scipy import io
     from scipy import stats
     import pandas as pd
     from sklearn.metrics import accuracy_score
```

First, we will load the MNIST data and hoild out 10,000 randomly chosen training points for a validation set.

```
[ ]: mnist = io.loadmat("data/%s_data.mat" % "mnist") # load the mnist data

     # separate into training data, training labels, and testing data
     mnist_raw_training_data = mnist["training_data"]
     mnist_training_labels = mnist["training_labels"]
     mnist_test_data = mnist["test_data"]

     mnist_tuples = np.append(mnist_raw_training_data, mnist_training_labels, axis=1)
     mnist_tuples_shuffled = np.random.permutation(mnist_tuples)

     # In order to partition our data, we will use pandas; we will convert our numpy␣
      ↪array into a temporary
```

```
# dataframe and set aside the first 10,000 shuffled tuples, which will then be␣
↪separated back into
# training data and training labels, to the validation set. Then we will assign␣
↪the remainder, again
# separating the training data and training labels to the training set.

mnist_validation_data = mnist_tuples_shuffled[0:10000, 0:-1]
mnist_validation_labels = mnist_tuples_shuffled[0:10000, -1]
mnist_training_data = mnist_tuples_shuffled[10000:, 0:-1]
mnist_training_labels = mnist_tuples_shuffled[10000:, -1]
```

For whatever reason (I didn't have enough time, lmao), I decided not to vectorize contrast_normalizing an image and apply it to the data. I just wrote two separate functions, one that contrast-normalizes the entire training set and one that contrast-normalizes one image at a time.

8.1. The mean and covariance matrix for each class is returned by the build_gaussians function.

8.2. The LDA function will output the covariance matrix for the digit 0. We can see that the diagonal terms are greater than the off-diagonal terms, but not by much. We can conclude that our eigenvalues are very small values, so we may be dealing with a singular covariance matrix.

[ ]:
```
def lda_predict(training_size, training_data, training_labels, testing_data,␣
↪num_c, mnist=True):
    num_classes = num_c
    def contrast_normalize_data(d):
        """Contrast-normalize an entire set of images"""
        c_n_data = np.empty_like(d).astype(float)
        for i in range(d.shape[0]):
            norm = np.linalg.norm(d[i, :])
            if norm == 0:
                c_n_data[i,:] = d[i,:]
            else:
                c_n_data[i,:] = d[i,:] / np.linalg.norm(d[i, :])
        return c_n_data

    def contrast_normalize_point(image):
        """Contrast-normalize a single image"""
        norm = np.linalg.norm(image)
        return image / norm

    def separate_data_by_digit(data,labels):
        """Returns an array where each index contains data corresponding to␣
↪index-matching digit
        Uses a pandas dataframe to neatly separate data by digits"""
```

```
        merged = pd.DataFrame(np.hstack((data, labels.reshape((data.shape[0],␣
↪1))))) # We will combine the training data and their corresponding training␣
↪labels
        data_by_digit = [merged[merged[merged.columns[-1]] == i].iloc[: , :-1].
↪to_numpy().astype(float) for i in range(num_classes)] # separate data by␣
↪their digit label, then remove the labels, then ensure types are float when␣
↪converting back to numpy
        return data_by_digit

    def build_gaussians(data_by_d):
        """Returns an array where each index contains a 2-element array␣
↪representing mu and Sigma,
        the parameters of the digit-conditional Gaussian distributions """
        gaussians = [0] * num_classes
        for i in range(num_classes):
            mu = np.mean(data_by_d[i], axis=0)
            sigma = np.cov(data_by_d[i].T)
            gaussians[i] = [mu, sigma]
        return gaussians

    def predict_point(image, gaussians, pinv):
        def compute_log_gaussian(x,mu, pinv):
            centered = x - mu
            product = np.dot(centered, pinv @ centered)
            return (-0.5 * product)

        discriminant_vals = [0] * num_classes
        if mnist==True:
            image = contrast_normalize_point(image)
        for i in range(num_classes):
            # discriminant_vals[i] = np.log(counts[i] / sum(counts)) + mv.
↪logpdf(x=c_n_image, mean=gaussians[i][0], cov=pooled_sigma,␣
↪allow_singular=True)
            discriminant_vals[i] = np.log(counts[i] / sum(counts)) +␣
↪compute_log_gaussian(image, gaussians[i][0], pinv)
        return np.argmax(discriminant_vals)


    # Training Phase ~ Model Gaussians
    training_set = training_data[:training_size,:]
    training_labels = training_labels[:training_size]
    if mnist==True:
        training_set = contrast_normalize_data(training_set)
    training_set_by_digits_arr = ␣
↪separate_data_by_digit(training_set,training_labels)
    gaussian_params_by_digits_arr = build_gaussians(training_set_by_digits_arr)
```

```
    plt.imshow(gaussian_params_by_digits_arr[0][1])

    # Testing Phase ~ Solve for Determinants
    counts = np.unique(training_labels, return_counts=True)[1] # arr to hold␣
↪counts of each digit using training labels
    pooled_sigma = sum(gaussian_params_by_digits_arr[i][1] for i in␣
↪range(num_classes)) / num_classes
    slogdet = np.linalg.slogdet(pooled_sigma)
    pseudo_inv_sigma = np.linalg.pinv(pooled_sigma)
    return [predict_point(testing_data[i,:
↪],gaussian_params_by_digits_arr,pseudo_inv_sigma) for i in␣
↪range(testing_data.shape[0])]
```

[ ]:
```
def qda_predict(training_size, training_data, training_labels, testing_data,␣
↪num_c, mnist=True):
    num_classes = num_c
    def contrast_normalize_data(d):
        """Contrast-normalize an entire set of images"""
        c_n_data = np.empty_like(d).astype(float)
        for i in range(d.shape[0]):
            norm = np.linalg.norm(d[i, :])
            if norm == 0:
                c_n_data[i,:] = d[i,:]
            else:
                c_n_data[i,:] = d[i,:] / np.linalg.norm(d[i, :])
        return c_n_data

    def contrast_normalize_point(image):
        """Contrast-normalize a single image"""
        norm = np.linalg.norm(image)
        return image / norm

    def separate_data_by_digit(data,labels):
        """Returns an array where each index contains data corresponding to␣
↪index-matching digit
        Uses a pandas dataframe to neatly separate data by digits"""
        merged = pd.DataFrame(np.hstack((data, labels.reshape((data.shape[0],␣
↪1))))) # We will combine the training data and their corresponding training␣
↪labels
        data_by_digit = [merged[merged[merged.columns[-1]] == i].iloc[: , :-1].
↪to_numpy().astype(float) for i in range(num_classes)] # separate data by␣
↪their digit label, then remove the labels, then ensure types are float when␣
↪converting back to numpy
        return data_by_digit

    def build_gaussians(data_by_d):
```

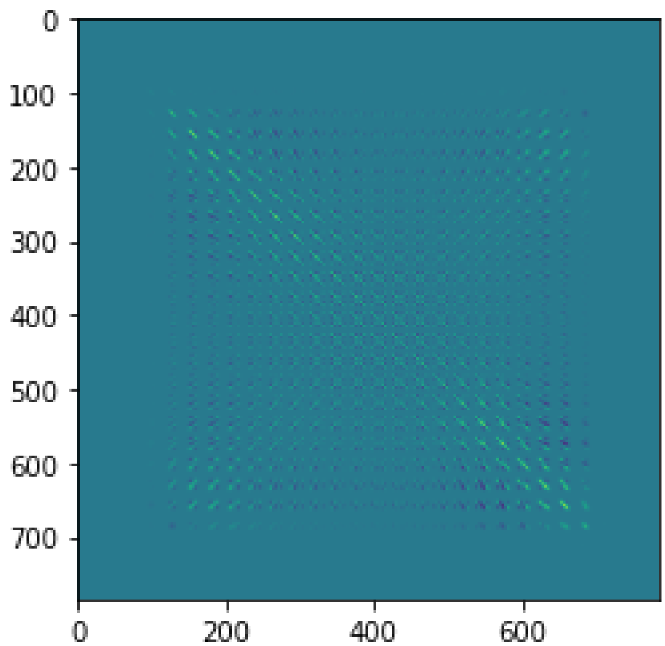
```
        """Returns an array where each index contains a 2-element array␣
↪representing mu and Sigma,
        the parameters of the digit-conditional Gaussian distributions """
        gaussians = [0] * num_classes
        for i in range(num_classes):
            mu = np.mean(data_by_d[i], axis=0)
            sigma = np.cov(data_by_d[i].T)
            gaussians[i] = [mu, sigma]
        return gaussians

    def predict_point(image, gaussians, pinvs):
        def compute_log_gaussian(x,mu, pinv):
            centered = x - mu
            product = np.dot(centered, pinv @ centered)
            return (-0.5 * product)

        discriminant_vals = [0] * num_classes
        if (mnist == True):
            image = contrast_normalize_point(image)
        for i in range(num_classes):
            # discriminant_vals[i] = np.log(counts[i] / sum(counts)) + mv.
↪logpdf(x=c_n_image, mean=gaussians[i][0], cov=pooled_sigma,␣
↪allow_singular=True)
            discriminant_vals[i] = np.log(counts[i] / sum(counts)) +␣
↪compute_log_gaussian(image, gaussians[i][0], pinvs[i])
        return np.argmax(discriminant_vals)


    # Training Phase ~ Model Gaussians
    training_set = training_data[:training_size,:]
    training_labels = training_labels[:training_size]
    if mnist == True:
        training_set = contrast_normalize_data(training_set)
    training_set_by_digits_arr = ␣
↪separate_data_by_digit(training_set,training_labels)
    gaussian_params_by_digits_arr = build_gaussians(training_set_by_digits_arr)
    # plt.imshow(gaussian_params_by_digits_arr[9][1])

    # Testing Phase ~ Solve for Determinants
    counts = np.unique(training_labels, return_counts=True)[1] # arr to hold␣
↪counts of each digit using training labels
    # pooled_sigma = sum(gaussian_params_by_digits_arr[i][1] for i in␣
↪range(10)) / 10
    # slogdet = np.linalg.slogdet(pooled_sigma)
    pseudo_inv_sigmas = [np.linalg.pinv(gaussian_params_by_digits_arr[i][1])␣
↪for i in range(num_classes)]
```

```
    return [predict_point(testing_data[i,:
↪],gaussian_params_by_digits_arr,pseudo_inv_sigmas) for i in␣
↪range(testing_data.shape[0])]
```

8.3(a) and (b). Below are the error rates over different numbers of randomly chosen training points.

3(c). LDA performed better. Over the larger training sizes, LDA consistently had lower error scores than QDA. Also, LDA had less variance over smaller training sizes than QDA did.

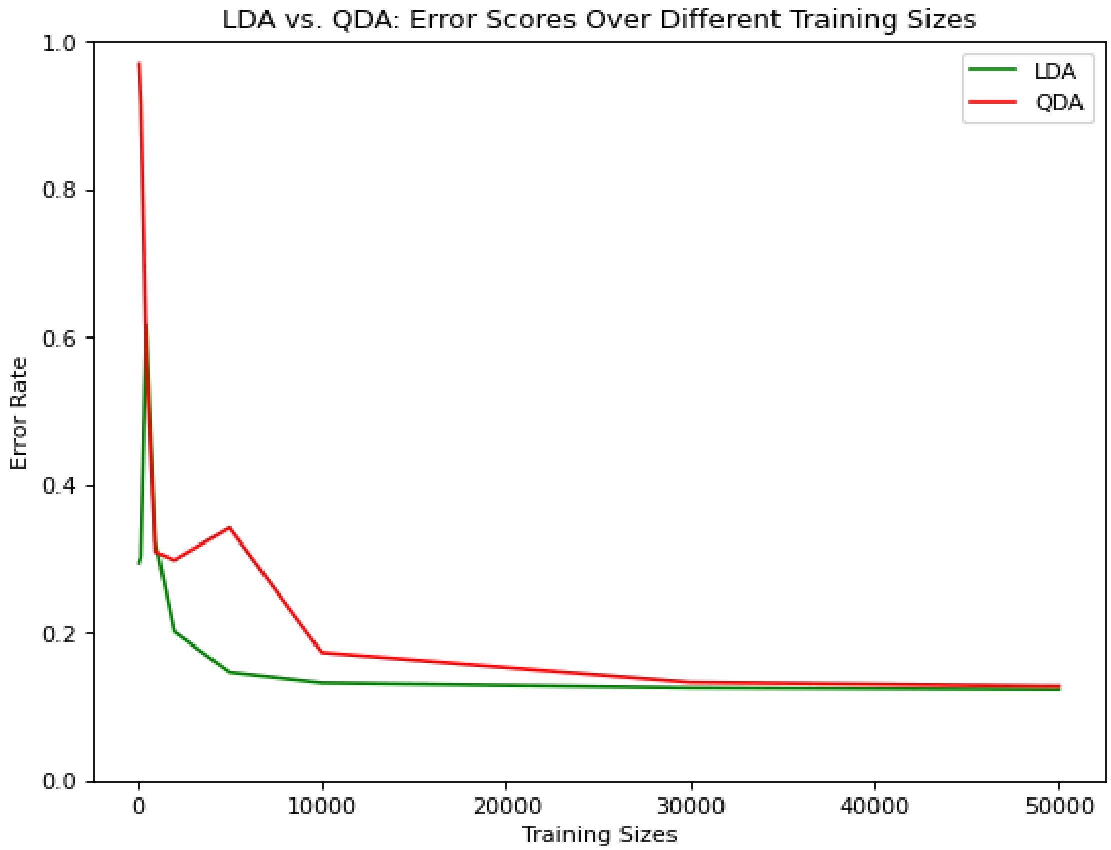
[ ]:
```
training_sizes = [100,200,500,1000,2000,5000,10000,30000,50000]
mnist_lda_error_rates = [1 - accuracy_score(mnist_validation_labels,␣
↪lda_predict(n, mnist_training_data, mnist_training_labels,␣
↪mnist_validation_data, 10)) for n in training_sizes]
mnist_qda_error_rates = [1 - accuracy_score(mnist_validation_labels,␣
↪qda_predict(i, mnist_training_data, mnist_training_labels,␣
↪mnist_validation_data, 10)) for i in training_sizes]
```



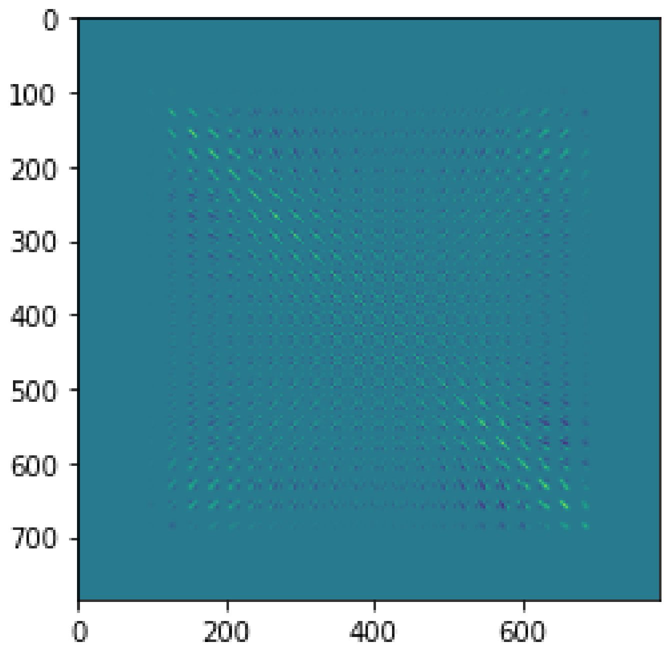
[ ]:
```
plt.figure(figsize=(8,6), dpi=80)
plt.plot(training_sizes, mnist_lda_error_rates, color='green', label='LDA')
plt.plot(training_sizes, mnist_qda_error_rates, color='red', label='QDA')
plt.legend(loc='upper right')
plt.title("LDA vs. QDA: Error Scores Over Different Training Sizes")
plt.ylim(0,1)
plt.ylabel("Error Rate")
```

```python
plt.xlabel("Training Sizes")
```

[ ]: Text(0.5, 0, 'Training Sizes')



[ ]:
```python
lda_predictions = [lda_predict(n, mnist_training_data, mnist_training_labels,␣
 ↪mnist_validation_data, 10) for n in training_sizes]
qda_predictions = [qda_predict(n, mnist_training_data, mnist_training_labels,␣
 ↪mnist_validation_data, 10) for n in training_sizes]
```

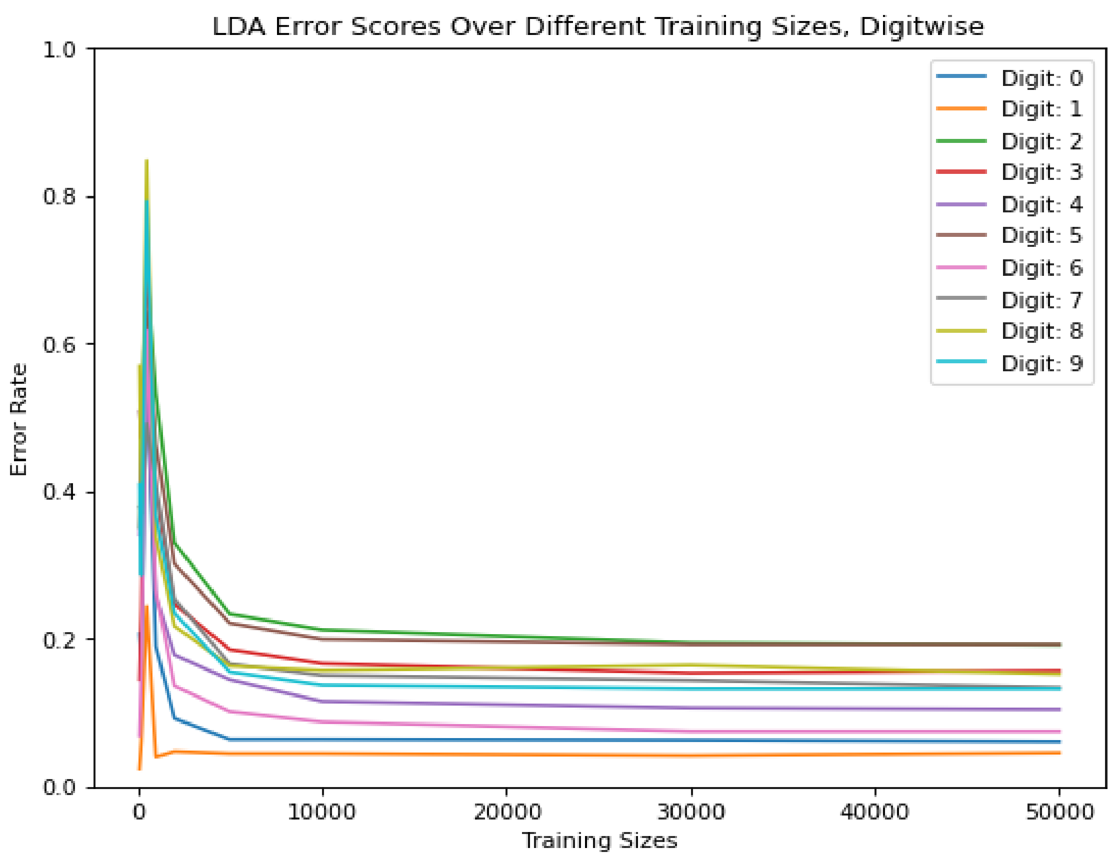



```
[ ]: def digitwise_error_score_by_training_data(predictions):
         tbl = np.empty((len(training_sizes), 10))
         for k in range(len(training_sizes)):
             digitwise = [0] * 10
             for i in range(10):
                 predictions_by_digit = np.empty((1,2))
                 for j in range(len(mnist_validation_labels)):
                     if mnist_validation_labels[j] == i:
                         pair = np.array([mnist_validation_labels[j],␣
     ↪predictions[k][j]]).reshape(1,2)
                         predictions_by_digit = np.vstack((predictions_by_digit,␣
     ↪pair))
                 digitwise[i] = predictions_by_digit[1:,:]
             accuracy_score_digitwise = [1 - accuracy_score(p[:,0], p[:,1]) for p in␣
     ↪digitwise]
             if k == 0:
                 tbl = np.asarray(accuracy_score_digitwise).reshape(1,10)
             else:
                 tbl = np.vstack((tbl, np.asarray(accuracy_score_digitwise).
     ↪reshape(1,10)))
         return tbl
```

3(d). Below are the plots of validation error vs. # of training points for each digit. It appears 0 is consistently the easiest to classify (#1 for LDA, #2 for QDA).

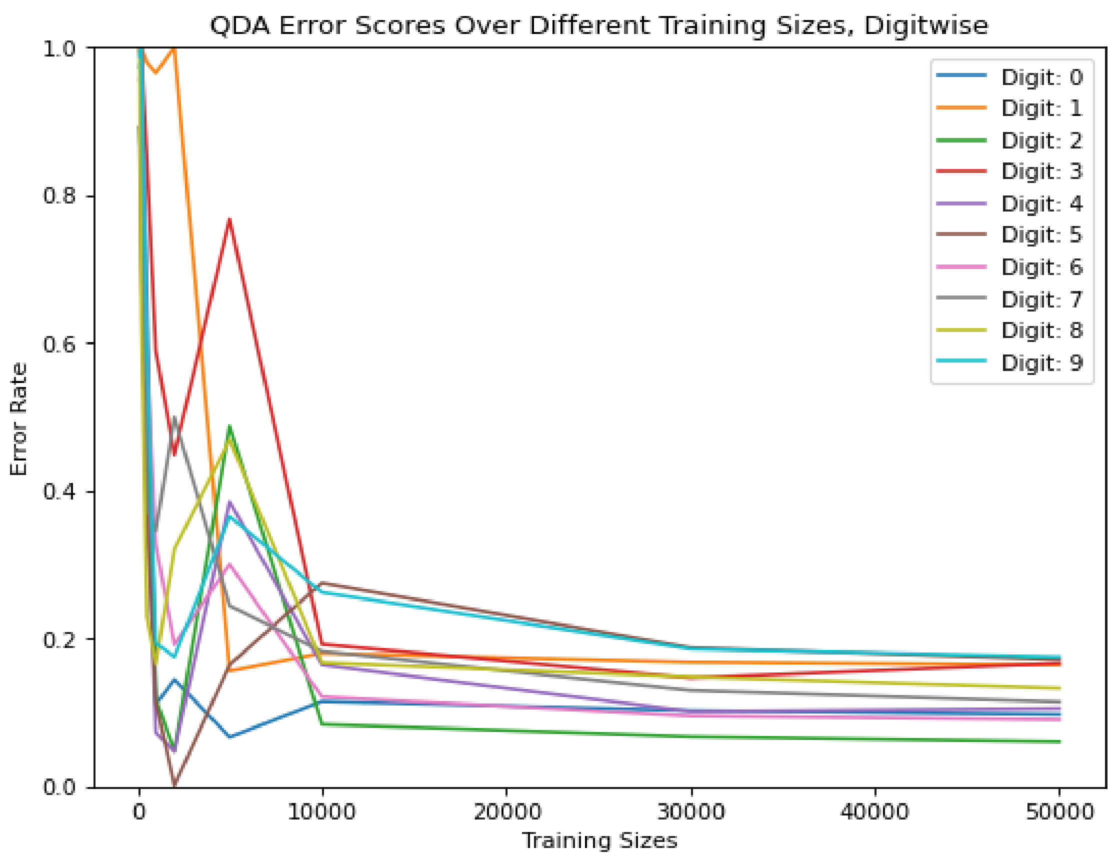
[ ]:
```
from matplotlib.pylab import figure

plt.figure(figsize=(8,6), dpi=80)
digitwise_sizewise_scores =␣
↪digitwise_error_score_by_training_data(lda_predictions)
for j in range(digitwise_sizewise_scores.shape[1]):
    size_column = np.asarray(training_sizes)
    pts = np.hstack((size_column.reshape(9,1), digitwise_sizewise_scores[:,j].
↪reshape(9,1)))
    plt.plot(pts[:,0], pts[:,1], label="Digit: " + str(j))
plt.legend(loc="upper right")
plt.title("LDA Error Scores Over Different Training Sizes, Digitwise")
plt.ylim(0,1)
plt.ylabel("Error Rate")
plt.xlabel("Training Sizes")
```

[ ]:
```
Text(0.5, 0, 'Training Sizes')
```



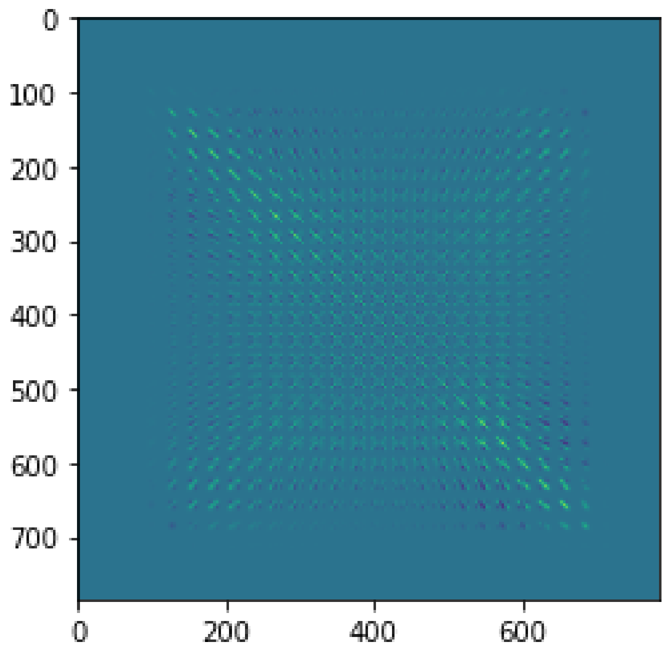
```
[ ]: plt.figure(figsize=(8,6), dpi=80)
     digitwise_sizewise_scores =␣
      ↪digitwise_error_score_by_training_data(qda_predictions)
     for j in range(digitwise_sizewise_scores.shape[1]):
         size_column = np.asarray(training_sizes)
         pts = np.hstack((size_column.reshape(9,1), digitwise_sizewise_scores[:,j].
      ↪reshape(9,1)))
         plt.plot(pts[:,0], pts[:,1], label="Digit: " + str(j))
     plt.legend(loc="upper right")
     plt.title("QDA Error Scores Over Different Training Sizes, Digitwise")
     plt.ylim(0,1)
     plt.ylabel("Error Rate")
     plt.xlabel("Training Sizes")
```

```
[ ]: Text(0.5, 0, 'Training Sizes')
```



8.3(d): Kaggle

```
[ ]: mnist_test_predictions = lda_predict(50000, mnist_training_data,␣
      ↪mnist_training_labels, mnist_test_data, 10, False)
```



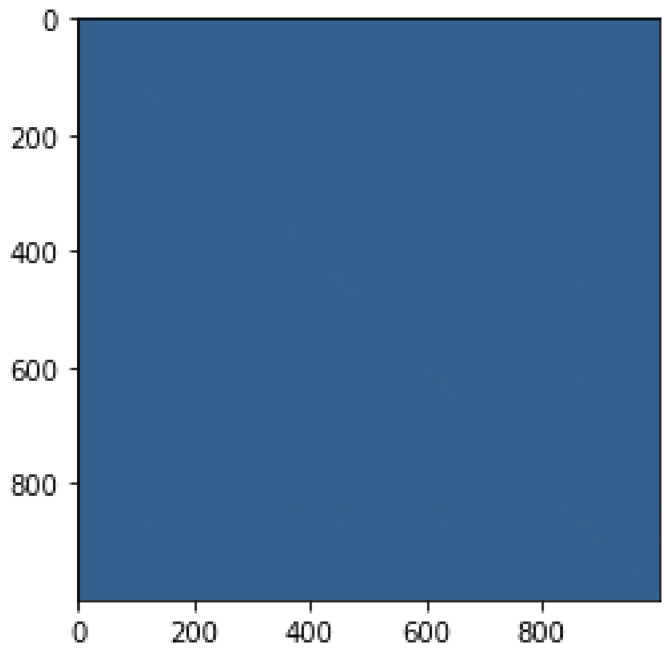
```
[ ]: mnist_pd = pd.DataFrame(np.int64(mnist_test_predictions),columns=['Category'])
     mnist_pd.index.name = 'Id'
     mnist_pd.index += 1
     mnist_pd.to_csv("mnist_test_predictions.csv")
```

8.4: Spam or Ham?

```
[ ]: # load in data
     spamham = io.loadmat("data/%s_data.mat" % "spam")
     spamham_raw_training_data = spamham["training_data"].toarray() # toarray()␣
      ↪necessary to get np array instead of sparse_ array
     spamham_training_labels = spamham["training_labels"]
     spamham_test_data = spamham["test_data"].toarray()

     # shuffle the data and the labels
     spamham_tuples = np.append(spamham_raw_training_data, spamham_training_labels,␣
      ↪axis=1)
     spamham_tuples_shuffled = np.random.permutation(spamham_tuples)

     # divide 80% of data into training data, then assign remaining data as␣
      ↪validation data
     eighty_perc_cutoff = int(spamham_raw_training_data.shape[0] * 0.8)
     spamham_training_data = spamham_tuples_shuffled[0:eighty_perc_cutoff, 0:-1]
     spamham_training_labels = spamham_tuples_shuffled[0:eighty_perc_cutoff, -1]
     spamham_validation_data = spamham_tuples_shuffled[eighty_perc_cutoff:, 0:-1]
```

```
spamham_validation_labels = spamham_tuples_shuffled[eighty_perc_cutoff:, -1]
```

[ ]:
```
lda_spam_predictions = lda_predict(spamham_training_data.shape[0],␣
↪spamham_training_data, spamham_training_labels, spamham_validation_data, 2)
qda_spam_predictions = qda_predict(spamham_training_data.shape[0],␣
↪spamham_training_data, spamham_training_labels, spamham_validation_data, 2)
```

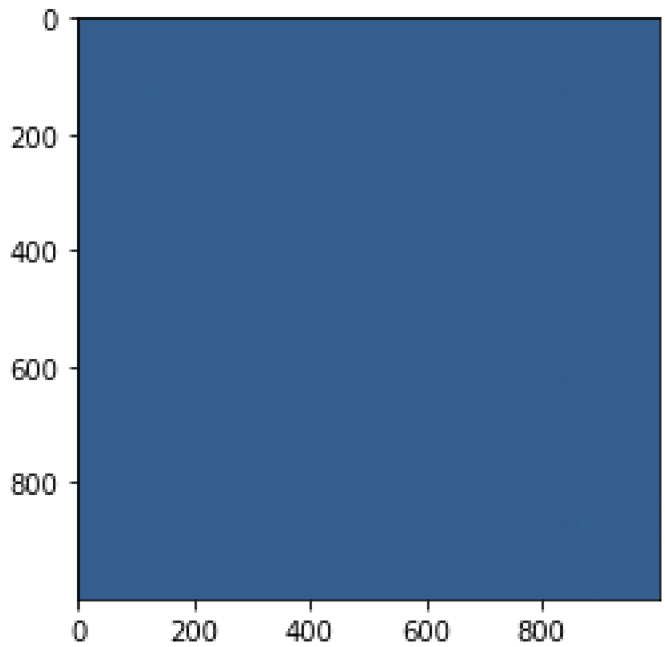


[ ]:
```
accuracy_score(spamham_validation_labels, lda_spam_predictions),␣
↪accuracy_score(spamham_validation_labels, qda_spam_predictions)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
/home/daynetran/yggdrasil/computer_science/upper_div/cs189/hw03/code/hw03.ipynb
↪Cell 47' in <module>
----> <a href='vscode-notebook-cell:/home/daynetran/yggdrasil/computer_science/
↪upper_div/cs189/hw03/code/hw03.ipynb#ch0000046?line=0'>1</a>␣
↪accuracy_score(spamham_validation_labels, lda_spam_predictions),␣
↪accuracy_score(spamham_validation_labels, qda_spam_predictions)

NameError: name 'accuracy_score' is not defined
```

[ ]:
```
spamham_test_predictions = lda_predict(spamham_training_data.shape[0],␣
↪spamham_training_data, spamham_training_labels, spamham_test_data, 2)
```

```
spamham_pd = pd.DataFrame(np.
↪int64(spamham_test_predictions),columns=['Category'])
spamham_pd['Category'] = spamham_pd['Category'].apply(lambda x: 1 if x == 0␣
↪else 0)
spamham_pd.index.name = 'Id'
spamham_pd.index += 1
spamham_pd.to_csv("spamham_test_predictions.csv")
```



[ ]:
```
mnist_pd
```

[ ]:
```
       Category
Id
1             0
2             5
3             4
4             9
5             6
…          …
9996          4
9997          1
9998          8
9999          3
10000         9

[10000 rows x 1 columns]
```

[ ]:
```
spamham_pd
```

[ ]:
```
      Category
Id
1            1
2            0
3            0
4            1
5            0
…          …
5853         0
5854         1
5855         0
5856         0
5857         1

[5857 rows x 1 columns]
```

KAGGLE SCORES

Kaggle Name: Dayne Tran

MNIST: 0.87230

Spam: 0.83472

For the code appendix, this is the featurize.py code

[ ]:
```
# # '''
# **************** PLEASE READ ***************

# Script that reads in spam and ham messages and converts each training example
# into a feature vector

# Code intended for UC Berkeley course CS 189/289A: Machine Learning

# Requirements:
# -scipy ('pip install scipy')

# To add your own features, create a function that takes in the raw text and
# word frequency dictionary and outputs a int or float. Then add your feature
# in the function 'def generate_feature_vector'

# The output of your file will be a .mat file. The data will be accessible using
# the following keys:
#     -'training_data'
#     -'training_labels'
#     -'test_data'
```

```
# Please direct any bugs to kevintee@berkeley.edu
# '''

# from collections import defaultdict
# import glob
# import re
# import scipy.io
# import numpy as np
# from sklearn.feature_extraction.text import CountVectorizer

# NUM_TRAINING_EXAMPLES = 5172
# NUM_TEST_EXAMPLES = 5857

# BASE_DIR = './'
# SPAM_DIR = 'spam/'
# HAM_DIR = 'ham/'
# TEST_DIR = 'test/'

# # ************* Features *************

# # Features that look for certain words
# def freq_pain_feature(text, freq):
#     return float(freq['pain'])

# def freq_private_feature(text, freq):
#     return float(freq['private'])

# def freq_bank_feature(text, freq):
#     return float(freq['bank'])

# def freq_money_feature(text, freq):
#     return float(freq['money'])

# def freq_drug_feature(text, freq):
#     return float(freq['drug'])

# def freq_spam_feature(text, freq):
#     return float(freq['spam'])

# def freq_prescription_feature(text, freq):
#     return float(freq['prescription'])

# def freq_creative_feature(text, freq):
#     return float(freq['creative'])

# def freq_height_feature(text, freq):
#     return float(freq['height'])
```

```
# def freq_featured_feature(text, freq):
#     return float(freq['featured'])

# def freq_differ_feature(text, freq):
#     return float(freq['differ'])

# def freq_width_feature(text, freq):
#     return float(freq['width'])

# def freq_other_feature(text, freq):
#     return float(freq['other'])

# def freq_energy_feature(text, freq):
#     return float(freq['energy'])

# def freq_business_feature(text, freq):
#     return float(freq['business'])

# def freq_message_feature(text, freq):
#     return float(freq['message'])

# def freq_volumes_feature(text, freq):
#     return float(freq['volumes'])

# def freq_revision_feature(text, freq):
#     return float(freq['revision'])

# def freq_path_feature(text, freq):
#     return float(freq['path'])

# def freq_meter_feature(text, freq):
#     return float(freq['meter'])

# def freq_memo_feature(text, freq):
#     return float(freq['memo'])

# def freq_planning_feature(text, freq):
#     return float(freq['planning'])

# def freq_pleased_feature(text, freq):
#     return float(freq['pleased'])

# def freq_record_feature(text, freq):
#     return float(freq['record'])

# def freq_out_feature(text, freq):
```

```
#     return float(freq['out'])

# # Features that look for certain characters
# def freq_semicolon_feature(text, freq):
#     return text.count(';')

# def freq_dollar_feature(text, freq):
#     return text.count('$')

# def freq_sharp_feature(text, freq):
#     return text.count('#')

# def freq_exclamation_feature(text, freq):
#     return text.count('!')

# def freq_para_feature(text, freq):
#     return text.count('(')

# def freq_bracket_feature(text, freq):
#     return text.count('[')

# def freq_and_feature(text, freq):
#     return text.count('&')

# # --------- Add your own feature methods ----------
# def example_feature(text, freq):
#     return int('example' in text)

# # Generates a feature vector
# def generate_feature_vector(text, freq):
#     feature = []
#     feature.append(freq_pain_feature(text, freq))
#     feature.append(freq_private_feature(text, freq))
#     feature.append(freq_bank_feature(text, freq))
#     feature.append(freq_money_feature(text, freq))
#     feature.append(freq_drug_feature(text, freq))
#     feature.append(freq_spam_feature(text, freq))
#     feature.append(freq_prescription_feature(text, freq))
#     feature.append(freq_creative_feature(text, freq))
#     feature.append(freq_height_feature(text, freq))
#     feature.append(freq_featured_feature(text, freq))
#     feature.append(freq_differ_feature(text, freq))
#     feature.append(freq_width_feature(text, freq))
#     feature.append(freq_other_feature(text, freq))
#     feature.append(freq_energy_feature(text, freq))
#     feature.append(freq_business_feature(text, freq))
#     feature.append(freq_message_feature(text, freq))
```

```
#     feature.append(freq_volumes_feature(text, freq))
#     feature.append(freq_revision_feature(text, freq))
#     feature.append(freq_path_feature(text, freq))
#     feature.append(freq_meter_feature(text, freq))
#     feature.append(freq_memo_feature(text, freq))
#     feature.append(freq_planning_feature(text, freq))
#     feature.append(freq_pleased_feature(text, freq))
#     feature.append(freq_record_feature(text, freq))
#     feature.append(freq_out_feature(text, freq))
#     feature.append(freq_semicolon_feature(text, freq))
#     feature.append(freq_dollar_feature(text, freq))
#     feature.append(freq_sharp_feature(text, freq))
#     feature.append(freq_exclamation_feature(text, freq))
#     feature.append(freq_para_feature(text, freq))
#     feature.append(freq_bracket_feature(text, freq))
#     feature.append(freq_and_feature(text, freq))

#     # --------- Add your own features here ---------
#     # Make sure type is int or float

#     return feature

# # This method generates a design matrix with a list of filenames
# # Each file is a single training example
# def generate_design_matrix(filenames):
#     corpus = []
#     design_matrix = []
#     for filename in filenames:
#         with open(filename, 'r', encoding='utf-8', errors='ignore') as f:
#             try:
#                 text = f.read() # Read in text from file
#             except Exception as e:
#                 # skip files we have trouble reading.
#                 continue
#             text = text.replace('\r\n', ' ') # Remove newline character
#             corpus.append(text)
#             # words = re.findall(r'\w+', text)
#             # word_freq = defaultdict(int) # Frequency of all words
#             # for word in words:
#             #     word_freq[word] += 1

#             # # Create a feature vector
#             # feature_vector = generate_feature_vector(text, word_freq)
#             # design_matrix.append(feature_vector)

#     vectorizer = CountVectorizer()
#     X = vectorizer.fit_transform(corpus)
```

```
#     design_matrix = X.toarray()
#     return design_matrix

# def build_corpus(filenames):
#     corpus = []
#     for filename in filenames:
#         with open(filename, 'r', encoding='utf-8', errors='ignore') as f:
#             try:
#                 text = f.read() # Read in text from file
#             except Exception as e:
#                 # skip files we have trouble reading.
#                 continue
#             text = text.replace('\r\n', ' ') # Remove newline character
#             corpus.append(text)
#     return corpus

# # ************** Script starts here **************
# # DO NOT MODIFY ANYTHING BELOW

# spam_filenames = glob.glob(BASE_DIR + SPAM_DIR + '*.txt')
# spam_corpus = build_corpus(spam_filenames)
# ham_filenames = glob.glob(BASE_DIR + HAM_DIR + '*.txt')
# ham_corpus = build_corpus(ham_filenames)

# training_corpus = ham_corpus + spam_corpus
# vectorizor = CountVectorizer(max_features=1000)
# X = vectorizor.fit_transform(training_corpus)
# training_design_matrix = X.toarray()

# # Important: the test_filenames must be in numerical order as that is the
# # order we will be evaluating your classifier
# test_filenames = [BASE_DIR + TEST_DIR + str(x) + '.txt' for x in
 ↪range(NUM_TEST_EXAMPLES)]
# test_corpus = build_corpus(test_filenames)
# test_design_matrix = vectorizor.transform(test_corpus)

# # X = spam_design_matrix + ham_design_matrix
# Y = np.array([1]*len(ham_corpus) + [0]*len(spam_corpus)).reshape((-1, 1))

# file_dict = {}
# file_dict['training_data'] = X
# file_dict['training_labels'] = Y
# file_dict['test_data'] = test_design_matrix
# scipy.io.savemat('spam_data.mat', file_dict)
```