EECS 16B –Spring 2022 — Homework 00

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Collaborators: None

1. Policy Quiz

The screenshot of my policy quiz results is attached at the end of this file.

2.

I understand how the Discord, Gradescope, and OH Queue tools work.

3.

(a) We essentially want to ensure that A is a positive semi-definite ma trix. So we condition that $det(A) = a^2 - b^2 > 0$.

(b) We know that $Ax = \lambda x$. For both eigenvaleus λ_1 and λ_2, we move λx over to the other side and take the determinant of both sides. Then, we take the system of equations and solve for a and b.

$$
\begin{aligned}
det(A - \frac{5}{2}) &= 0 \\
det(A - \frac{9}{2}) &= 0 \\
det\left(\begin{bmatrix} a - \frac{5}{2} & b \\ b & a - \frac{5}{2} \end{bmatrix}\right) &= 0 \\
det\left(\begin{bmatrix} a - \frac{9}{2} & b \\ b & a - \frac{9}{2} \end{bmatrix}\right) &= 0 \\
\left(a - \frac{5}{2}\right)^2 - b^2 &= 0 \\
\left(a - \frac{9}{2}\right)^2 - b^2 &= 0 \\
a^2 - \frac{20}{4}a + \frac{25}{4} - b^2 &= 0 \\
a^2 - \frac{36}{4}a + \frac{81}{4} - b^2 &= 0 \\
\frac{16}{4}a - \frac{56}{4} &= 0 \\
a &= \frac{7}{2} \\
b &= 1
\end{aligned}
$$

(c) Solve for the eigenvalues, then normalized eigenvectors, of $\widehat{H}$.

$$
\begin{aligned}
det(A - \lambda I) &= 0 \\
(3 - \lambda)^2 - 2^2 &= 0 \\
\lambda^2 - 6\lambda + 5 &= 0 \\
\lambda_1 = 1, \lambda_2 &= 5 \\
A - 1I = 0 \Longrightarrow \vec{v}_{\lambda_1} &= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -t6t\frac{\sqrt{2}}{2} \end{bmatrix} \\
A - 5I = 0 \Longrightarrow \vec{v}_{\lambda_2} &= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}
\end{aligned}
$$

Solve the system of equations $\vec{v}_s = \alpha\vec{v}_{\lambda_1} + \beta\vec{v}_{\lambda_2}$ for α and β. Then find the magnitude of α.

$$
\begin{aligned}
0 &= \alpha\frac{\sqrt{2}}{2} + \beta\frac{\sqrt{2}}{2} \\
1 &= -\alpha\frac{\sqrt{2}}{2} + \beta\frac{\sqrt{2}}{2} \\
1 &= \beta\sqrt{2} \\
\beta = \frac{\sqrt{2}}{2}, \alpha &= -\frac{\sqrt{2}}{2} \\
|\alpha| &= \frac{\sqrt{2}}{2}
\end{aligned}
$$

4.

(a) First, we test for symmetry.

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}^T (3\vec{x} + 3\vec{y}) = 6x_1 + 3x_2 + 6y_1 + 3y_2 = (3\vec{x} + 3\vec{y})^T \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Second, we test for linearity.

$$\langle 3\vec{x} + 3\vec{y}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rangle = 6x_1 + 3x_2 + 6y_1 + 3y_2 = 3\langle \vec{x}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rangle + 3\langle \vec{y}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rangle$$

Third, we test for positive semi-definiteness.

$$\langle \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rangle = 5 > 0$$

$$\langle 3x + 3y, 3x + 3y \rangle = \langle 3 \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}, 3 \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix} \rangle = 9(x_1 + y_1)^2 + 9(x_2 + y_2)^2 > 0$$

All three innner product properties hold.

(b) $2 \cdot 2 + 5 \cdot 5 + 6 \cdot 6 + 2 \cdot 2 = 69$

5.

(a) Projecting $\vec{x}$ onto $\vec{y}$ means the following:

$$proj_{\vec{y}}\vec{x} = \frac{\langle \vec{y}, \vec{x} \rangle}{\langle \vec{y}, \vec{y} \rangle}\vec{y}$$

Project $\vec{x}_{sample}$ onto the footprint of Electric Love, $\vec{x}_1$:

$$proj_{\vec{x_1}}\vec{x}_{sample} = \frac{\langle \vec{x}_1, \vec{x}_{sample} \rangle}{\langle \vec{x}_1, \vec{x}_1 \rangle}\vec{x}_1 = \frac{(1 \cdot 2) + (-1 \cdot 0) + (1 \cdot -1) + (-1 \cdot 1)}{(1 \cdot 1) + (-1 \cdot -1) + (1 \cdot 1) + (-1 \cdot -1)}\vec{x}_1 = \frac{0}{4}\vec{x}_1 = \vec{0}$$

Project $\vec{x}_{sample}$ onto the footprint of She's Electric, $\vec{x}_2$:

$$proj_{\vec{x_2}}\vec{x}_{sample} = \frac{\langle \vec{x}_2, \vec{x}_{sample} \rangle}{\langle \vec{x}_2, \vec{x}_2 \rangle}\vec{x}_2 = \frac{(2 \cdot 2) + (-2 \cdot 0) + (-8 \cdot -1) + (7 \cdot 1)}{(2 \cdot 2) + (-2 \cdot -2) + (-8 \cdot -8) + (7 \cdot 7)}\vec{x}_2 = \frac{19}{128}\vec{x}_2$$

Project $\vec{x}_{sample}$ onto the footprint of Electric Feel, $\vec{x}_3$:

$$proj_{\vec{x_3}}\vec{x}_{sample} = \frac{\langle \vec{x}_3, \vec{x}_{sample} \rangle}{\langle \vec{x}_3, \vec{x}_3 \rangle}\vec{x}_3 = \frac{(4 \cdot 2) + (1 \cdot 0) + (-2 \cdot -1) + (2 \cdot 1)}{(4 \cdot 4) + (1 \cdot 1) + (-2 \cdot -2) + (2 \cdot 2)}\vec{x}_3 = \frac{12}{25}\vec{x}_3$$

Now, we find the error $\vec{e} = \vec{x}_{sample} - proj$.

$$\begin{aligned}
e_1 &= \vec{x}_{sample} - proj_{\vec{x_1}}\vec{x}_{sample} = \begin{bmatrix} 2 & 0 & -1 & 1 \end{bmatrix}^T \\
e_2 &= \vec{x}_{sample} - proj_{\vec{x_2}}\vec{x}_{sample} = \begin{bmatrix} 0.297 & 0 & -0.148 & 0.148 \end{bmatrix}^T \\
e_3 &= \vec{x}_{sample} - proj_{\vec{x_3}}\vec{x}_{sample} = \begin{bmatrix} 0.96 & 0 & -0.48 & 0.48 \end{bmatrix}^T
\end{aligned}$$

Now, we find the error vector which has the smallest magnitude; the song that corresponds with this minimum-difference vector is likeliest song of the three to be playing according to the sample footprint.

$$\begin{aligned}
||e_1|| &= 2.45 \\
||e_2|| &= 0.36 \\
||e_3|| &= 1.48
\end{aligned}$$

We conclude that the song playing must be Electric Feel.

(b) The cross-correlation plot is highest at the 180-second mark, so we believe that to be when the sample was taken.

(c) (ii) $\vec{a}_n^T(MM^T)^{-1}M\vec{b}_n$

(d) No.

(e) Three.

6.

(a) Gaussian: 6-second shift is $6s \cdot \frac{300m}{s} = 1800$ m away. NVA: 300 m away. Charge: 900 m away.

(b) $\vec{b}$ aligns most closely with the distances.

(c) We can find our location (x_1, x_2) using the following system of equations:

$$\begin{pmatrix} 2\frac{\vec{a}_1^T}{v\tau_1} - 2\frac{\vec{a}_2^T}{v\tau_2} \\ 2\frac{\vec{a}_1^T}{v\tau_1} - 2\frac{\vec{a}_3^T}{v\tau_3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} v\tau_2 - v\tau_1 + \frac{\vec{a}_1^T\vec{a}_1}{v\tau_1} - \frac{\vec{a}_2^T\vec{a}_2}{v\tau_2} \\ v\tau_3 - v\tau_1 + \frac{\vec{a}_1^T\vec{a}_1}{v\tau_1} - \frac{\vec{a}_3^T\vec{a}_3}{v\tau_3} \end{pmatrix}$$

where 1 corresponds to the Gaussian tower, 2 corresponds to the NVA tower, and 3 corresponds to the Charge tower. After plugging everything in and doing arithmetic, we arrive at the two following equations:

$$\frac{32}{\sqrt{72}}x_1 + \frac{22}{\sqrt{72}}x_2 = \sqrt{72} - \sqrt{18} + \frac{53}{\sqrt{18}} - \frac{53}{\sqrt{72}}$$

$$\frac{14}{\sqrt{18}}x_1 + (\frac{4}{\sqrt{18}} - \frac{18}{116})x_2 = \sqrt{116} - \sqrt{18} + \frac{53}{\sqrt{18}} - \frac{81}{\sqrt{116}}$$

We find that our coordinates are (3.314, -0.775).

7.

(a)