

CS170–Spring 2022 — Homework 00

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Collaborators: None

1. Course Syllabus

- (a) None.
- (b) Yes, but anonymously.

2.

- (a) Midterm 1 is on February 23, 8pm-10pm. Midterm 2 is on April 5, 8pm-10pm. Final is on May 11, 11:30am-2:30pm.
- (b) The course staff recommend having the homework finished by 10pm.
- (c) “[Course Staff] accept absolutely no submissions after 11:59pm, even after technical issues or emergencies. No exceptions.”
- (d) Piazza
- (e) I have read and understood the course syllabus and policies.

3.

- (a) Not OK
- (b) Not OK
- (c) Not OK
- (d) Not OK

4.

Refer to Course Notes page 3, where it is shown that applying limit tests to f and g asymptotics can be used for proofs. Rough draft: define $f(n)$ as 2^n .

First, we want to prove that for all $c > 0$, $f(n) = \Omega(n^c)$, i.e. n^c dominates $f(n)$. We know from lectures and course notes that we can use limit tests to prove dominance.

$$(1^+)^n = \Omega(n^c) \iff \lim_{n \rightarrow \infty} \frac{(1^+)^n}{n^c} > 0 \text{ for all } c > 0$$

Apply L'Hopital's Rule because the immediate result is $\frac{\infty}{\infty}$:

$$\lim_{n \rightarrow \infty} \frac{\ln 1^+ \cdot (1^+)^n}{c \cdot n^{c-1}}$$

The result is still $\frac{\infty}{\infty}$, so apply L'Hopital's Rule repeatedly until the following expression appears:

$$\lim_{n \rightarrow \infty} \frac{(\ln 1^+)^c \cdot (1^+)^n}{c!} = \infty > 0$$

Second, we want to prove that for all $\alpha > 1$, $f = O(\alpha^n)$. Again, using our lecture and course notes, we can make use of limit tests for our proof.

$$(1^+)^n = O(\alpha^n) \iff \lim_{n \rightarrow \infty} \frac{(1^+)^n}{\alpha^n} < \infty \text{ for all } \alpha > 1$$

We can simplify the limit to get to a determinate result:

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{(1^+)^n}{\alpha^n} &= \lim_{n \rightarrow \infty} \left(\frac{1^+ \cdot 1^+ \cdot \dots \cdot 1^+}{\alpha \cdot \alpha \cdot \dots \cdot \alpha} \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{1^+}{\alpha} \cdot \frac{1^+}{\alpha} \cdot \dots \cdot \frac{1^+}{\alpha} \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{1^+}{\alpha} \right)^n \\ &\leq 1 \text{ since } \frac{1^+}{\alpha} \leq 1 \text{ for } \alpha > 1. \\ &< \infty \end{aligned}$$

5.

YOUR ANSWER GOES HERE