EECS 16B –Spring 2022 — Homework 00

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Collaborators: None

1. Policy Quiz

The screenshot of my policy quiz results is attached at the end of this file.

2.

I understand how the Discord, Gradescope, and OH Queue tools work.

3.

(a) We essentially want to ensure that A is a positive semi-definite ma trix. So we condition that $det(A) = a^2 - b^2 > 0$.

(b) We know that $Ax = \lambda x$. For both eigenvaleus λ_1 and λ_2, we move λx over to the other side and take the determinant of both sides. Then, we take the system of equations and solve for a and b.

$$
\begin{aligned}
det(A - \frac{5}{2}) &= 0 \\
det(A - \frac{9}{2}) &= 0 \\
det\left(\begin{bmatrix} a - \frac{5}{2} & b \\ b & a - \frac{5}{2} \end{bmatrix}\right) &= 0 \\
det\left(\begin{bmatrix} a - \frac{9}{2} & b \\ b & a - \frac{9}{2} \end{bmatrix}\right) &= 0 \\
\left(a - \frac{5}{2}\right)^2 - b^2 &= 0 \\
\left(a - \frac{9}{2}\right)^2 - b^2 &= 0 \\
a^2 - \frac{20}{4}a + \frac{25}{4} - b^2 &= 0 \\
a^2 - \frac{36}{4}a + \frac{81}{4} - b^2 &= 0 \\
\frac{16}{4}a - \frac{56}{4} &= 0 \\
a &= \frac{7}{2} \\
b &= 1
\end{aligned}
$$

(c) Solve for the eigenvalues, then normalized eigenvectors, of $\widehat{H}$.

$$
\begin{aligned}
det(A - \lambda I) &= 0 \\
(3 - \lambda)^2 - 2^2 &= 0 \\
\lambda^2 - 6\lambda + 5 &= 0 \\
\lambda_1 = 1, \lambda_2 &= 5 \\
A - 1I = 0 \Longrightarrow \vec{v}_{\lambda_1} &= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -t6t\frac{\sqrt{2}}{2} \end{bmatrix} \\
A - 5I = 0 \Longrightarrow \vec{v}_{\lambda_2} &= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}
\end{aligned}
$$

Solve the system of equations $\vec{v}_s = \alpha\vec{v}_{\lambda_1} + \beta\vec{v}_{\lambda_2}$ for α and β. Then find the magnitude of α.

$$\begin{aligned}
0 &= \alpha\frac{\sqrt{2}}{2} + \beta\frac{\sqrt{2}}{2} \\
1 &= -\alpha\frac{\sqrt{2}}{2} + \beta\frac{\sqrt{2}}{2} \\
1 &= \beta\sqrt{2} \\
\beta = \frac{\sqrt{2}}{2}, \alpha &= -\frac{\sqrt{2}}{2} \\
|\alpha| &= \frac{\sqrt{2}}{2}
\end{aligned}$$

4.

(a) First, we test for symmetry.

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}^T (3\vec{x} + 3\vec{y}) = 6\vec{x} + 3\vec{y} = (3\vec{x} + 3\vec{y})^T \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Second, we test for linearity.

$$\begin{aligned} \langle 3\vec{x} + 3\vec{y}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rangle = 3\langle \vec{x}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rangle + 3\langle \vec{y}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rangle \\ = 3(2\vec{x}) \end{aligned}$$

5.

YOUR ANSWER GOES HERE