SEC 10 Exercise p101

- Computations

1. Find all cosets of the subgroup $4\mathbb{Z}$ of $\mathbb{Z}$

(pf) left coset 만 찾자. $4\mathbb{Z}$. $4\mathbb{Z}+1$. $4\mathbb{Z}+2$. $4\mathbb{Z}+3$

$=$ right coset 이므로 ··

2. $4\mathbb{Z}$ of $2\mathbb{Z}$

(pf) left coset ? $4\mathbb{Z} = \{\cdots, -8, -4, 0, 4, \cdots\}$

$4\mathbb{Z}+1 = \{\cdots, -7, -3, 1, 5, \cdots\}$

3. $\langle 2\rangle$ of $\mathbb{Z}_{12}$

(pf) left coset ? $\langle 2\rangle = \{0, 2, 4, 6, 8, 10\}$

$1+\langle 2\rangle = \{1, 3, 5, 7, 9, 11\}$

$\cancel{2+\langle 2\rangle = \{2, 4, 6, 8, 10, 0\}} = \langle 2\rangle$

4. $\langle 4\rangle$ of $\mathbb{Z}_{12}$

(pf) left coset ? $\langle 4\rangle = \{0, 4, 8\}$

$1+\langle 4\rangle = \{1, 5, 9\}$

$2+\langle 4\rangle = \{2, 6, 10\}$

$3+\langle 4\rangle = \{3, 7, 11\}$

5. $\langle 18\rangle$ of $\mathbb{Z}_{36}$

(pf) left coset ? $\langle 18\rangle = \{0, 18\}$

$1+\langle 18\rangle = \{1, 19\}$

$2+\langle 18\rangle = \{2, 20\}$

$\vdots$

$17+\langle 18\rangle = \{17, 35\}$

12. Find the index of $\langle 3\rangle$ in the group $\mathbb{Z}_{24}$

(pf) $(\mathbb{Z}_{24} : \langle 3\rangle) = ?$ 3.

left coset ? $\langle 3\rangle = \{0, 3, 6, \cdots, 21\}$

$1+\langle 3\rangle = \{1, 4, 7, \cdots, 22\}$

$2+\langle 3\rangle = \{2, 5, 8, \cdots, 23\}$ $\rightarrow 3$

$\cancel{3+\langle 3\rangle = \{3, 6, 9, \cdots, 24\}}$ $\overset{\shortparallel}{0}$

or, $\langle 3\rangle = \{0, 3, 6, \cdots, 21\}$ $|\langle 3\rangle| = 8$

$\Rightarrow |\mathbb{Z}_{24}| / |\langle 3\rangle| = 24/8 = 3$ ∎

19. Mark each of the following true or false.

T **a.** Every subgroup of every group has left cosets.

T **b.** The number of left cosets of a subgroup of a finite group divides the order of the group.

T **c.** Every group of prime order is abelian.

F **d.** One cannot have left cosets of a finite subgroup of an infinite group.

T **e.** A subgroup of a group is a left coset of itself.

F **f.** Only subgroups of finite groups can have left cosets.

T **g.** A_n is of index 2 in S_n for $n > 1$.

T **h.** The theorem of Lagrange is a nice result.

F **i.** Every finite group contains an element of every order that divides the order of the group.

T **j.** Every finite cyclic group contains an element of every order that divides the order of the group.

- Proof Synopsis

25. Give a one-sentance synopsis of the pf of Thm 10.10

(pf) The left cosets of the subgroup H form a partition of G

and each coset has the same number of elements as H has.

30. If $aH = bH$, then $Ha = Hb$?

(pf) No.

31. If $Ha = Hb$, then $b \in Ha$?

(pf) True. $b = eb$ and $e \in H$ so $b \in Hb$.

Because $Ha = Hb$. we have $b \in Ha$.

32. If $aH = bH$. then $Ha^{-1} = Hb^{-1}$?

(pf) True. Because H is subgroup. we have $\{h^{-1} \mid h \in H\} = H$.

Therefore $Ha^{-1} = \{ha^{-1} \mid h \in H\} = \{h^{-1}a^{-1} \mid h \in H\}$.

That is. Ha^{-1} consists of all inverse of elements in aH.

Similiarly. Hb^{-1} consists of all inverses of elements in bH.

Because $aH = bH$. we must have $Ha^{-1} = Hb^{-1}$.

33. If $aH = bH$, then $a^2H = b^2H$?

(pf) No.