Summary: Vectors and vector space.

If $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$

1. Vector equality : $x = y \Leftrightarrow x_i = y_i$ $(i=1,2,3)$
2. Vector addition : $x + y = z \Leftrightarrow x_i + y_i = z_i$ (")
3. Scalar multiplication : $ax \Leftrightarrow (ax_1, ax_2, ax_3)$ $a \in \mathbb{R}$
4. Negative of vector : $-x = (-1)x \Leftrightarrow (-x_1, -x_2, -x_3)$
5. Null vector : There exists a null vector $0 = (0,0,0)$

Since our vector components are real num (or complex), the followings hold.

1. addition of vector is commutative : $x+y = y+x$
2. " associative : $(x+y)+z = x+(y+z)$
3. Scalar multiplication is distribute : $a(x+y) = ax+ay$
4. Scalar multiplication is associative : $(ab)x = a(bx)$

Def 1.6) Fundamental Unit Vectors.

$e_1 = (1,0,\cdots,0)$, $e_2 = (0,1,0,\cdots,0)$ $\cdots$ $e_n = (0,\cdots,1)$

→ Fundamental unit vector of vector space $\mathbb{R}^n$.

Thm 1.3) Linear combination

Any vector of $\mathbb{R}^n$ can be expressed by Linear combination of e_i ,

$\Rightarrow$ $v = (v_1, \cdots, v_n) \in \mathbb{R}^n$, $v = \sum_{i=1}^{n} v_i e_i$

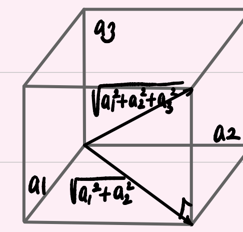
1.2 Inner product (내적)

Thm 1.5) Cosine Rule

세 변 길이 a, b, c $\angle C = \theta$ 인 임의의 삼각형 $\triangle ABC$

$c^2 = a^2 + b^2 - 2ab\cos\theta$

Thm 1.6) Norm of a vector



Vector of $\mathbb{R}^3$, $a = (a_1, a_2, a_3)$

length of a $\|a\| := \sqrt{a_1^2 + a_2^2 + a_3^2}$

Thm 1.7) Norm Property (a, b : vector, α : scalar)

(1) $\|a\| = 0 \Leftrightarrow a = 0$

(2) $\|\alpha a\| = |\alpha| \, \|a\|$

(3) $\|a+b\| \le \|a\| + \|b\|$

Def 1.7) Inner product

$\angle(a,b) = \theta$ $(0 \le \theta \le \pi)$,

define $a \cdot b = \|a\|\|b\| \cos\theta$

$= 0 \Leftrightarrow$ a or b is zero vector

Thm 1.8) Cosine Rule

$\forall a, b$: vector, $a \cdot b = \frac{1}{2}\left(\|a\|^2 + \|b\|^2 - \|b-a\|^2\right)$

Thm 1.9) Parallel and orthogonal Property

a, b are not zero vector, e_i : unit vector

(1) $e_i \cdot e_j = \delta_{ij}$, $\delta_{ij} = \begin{cases} 0, & i \ne j \\ 1, & i = j \end{cases}$

(2) $a \parallel b \Leftrightarrow a \cdot b = \pm \|a\| \|b\|$

(3) $a \perp b \Leftrightarrow a \cdot b = 0$

Thm 1.10) Inner product

$a = (a_1, a_2, a_3)$ $b = (b_1, b_2, b_3)$

$\Rightarrow a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$

Thm 1.11) Inner Product Property

(1) $a \cdot b = b \cdot a$ (2) $a \cdot (b+c) = a \cdot b + a \cdot c$

(3) $(\alpha a) \cdot b = a \cdot (\alpha b) = \alpha(a \cdot b)$ (4) $\|a\| = \sqrt{a \cdot a}$