2021 B형

2) 집합 $X = \{a, b, c\}$ $\mathcal{J}_1, \mathcal{J}_2$

(가) $|\mathcal{J}_1| = |\mathcal{J}_2| = 4$

(나) $(X, \mathcal{J}_1)$은 connected space

(다) $(X, \mathcal{J}_2)$는 disconnected space.

이 때 $\mathcal{J}_1, \mathcal{J}_2$ 각각 1개씩 ($|A|$는 A 원소 개수) [2점]

(pf) $\mathcal{J}_1 = \{\emptyset, X, \underset{\sim}{\{a\}}, \underset{\sim}{\{a,b\}}\}$

$\mathcal{J}_2 = \{\emptyset, X, \underset{\sim}{\{a\}}, \underset{\sim}{\{b,c\}}\}$

풀이 완벽..

6) $A = \begin{bmatrix} 3 & 2 & 2 \\ 0 & 3 & 1 \\ 0 & -2 & 0 \end{bmatrix}$ $A = PDP^{-1}$인 $D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$와 P?

또 A^n의 2행 3열 성분? [4점]

(pf) $\det|\lambda I - A| = \begin{vmatrix} \lambda-3 & -2 & -2 \\ 0 & \lambda-3 & -1 \\ 0 & 2 & \lambda \end{vmatrix} = (\lambda-3)(\lambda \times (\lambda-3)+2) = 0$

$+\cancel{2(0)} - \cancel{2(0)}$

$\lambda^2 - 3\lambda + 2$

$\Rightarrow (\lambda-3)(\lambda-2)(\lambda-1) = 0$

$\lambda = 3$ or 2 or 1

$\lambda = 3 \Rightarrow$ $\lambda I - A = \begin{bmatrix} 0 & -2 & -2 \\ 0 & 0 & -1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

?

$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ $x_3 = 0$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\lambda = 3$

$x_2 = 0$

$x_1 = t$

$\lambda=2$ 일때 $\lambda I - A = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

?

$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\quad x_1 = 0$, $\quad x_2 + x_3 = 0$ (t, $-t$) $\quad \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\lambda=2$

$\lambda=1$ 일때. $\begin{bmatrix} -2 & -2 & -2 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

?

$\begin{bmatrix} -2 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$+2\overset{-\frac{1}{2}t}{x_1} + \overset{t}{x_3} = 0$ put $x_3 = t$ ⇒ $x_1 = -\frac{1}{2}t$

$2\underset{-\frac{1}{2}t}{x_2} + x_3 = 0$ $\quad x_2 = -\frac{1}{2}t$

$\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$ ⇒ $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ $\lambda=1$

⇒ $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}$ $\quad P^{-1} = ?$

* P^{-1} 구하자.

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -2 & 0 & 0 & 1 \end{array}\right]$

$r_2 + r_3$, $-r_3$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & +1 & 0 & -1 & -1 \end{array}\right]$

$r_3 \times (-1) + r_2$, $r_3 \times (-1) + r_1$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right]$

$\therefore P^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & -1 \end{bmatrix}$

$-3+2$

$-3+2$

$2+3-2$

$2+1 \quad 6-2$

Then $D = P^{-1}AP = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 2 \\ 0 & 3 & 1 \\ 0 & -2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 & 3 \\ 0 & 4 & 2 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ⇒ D

2행 3열 ⇒ $2^n - 1$

$-1+2$

$A = PDP^{-1}$ ⇒ $A^n = PD^nP^{-1}$ $\quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}\begin{bmatrix} 3^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 1^n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 3^n & 0 & 1 \\ 0 & 2^n & 1 \\ 0 & -2^n & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$