PGE 338 Data Analytics and Geostatistics

Lecture 6: Heterogeneity

Lecture outline . . .

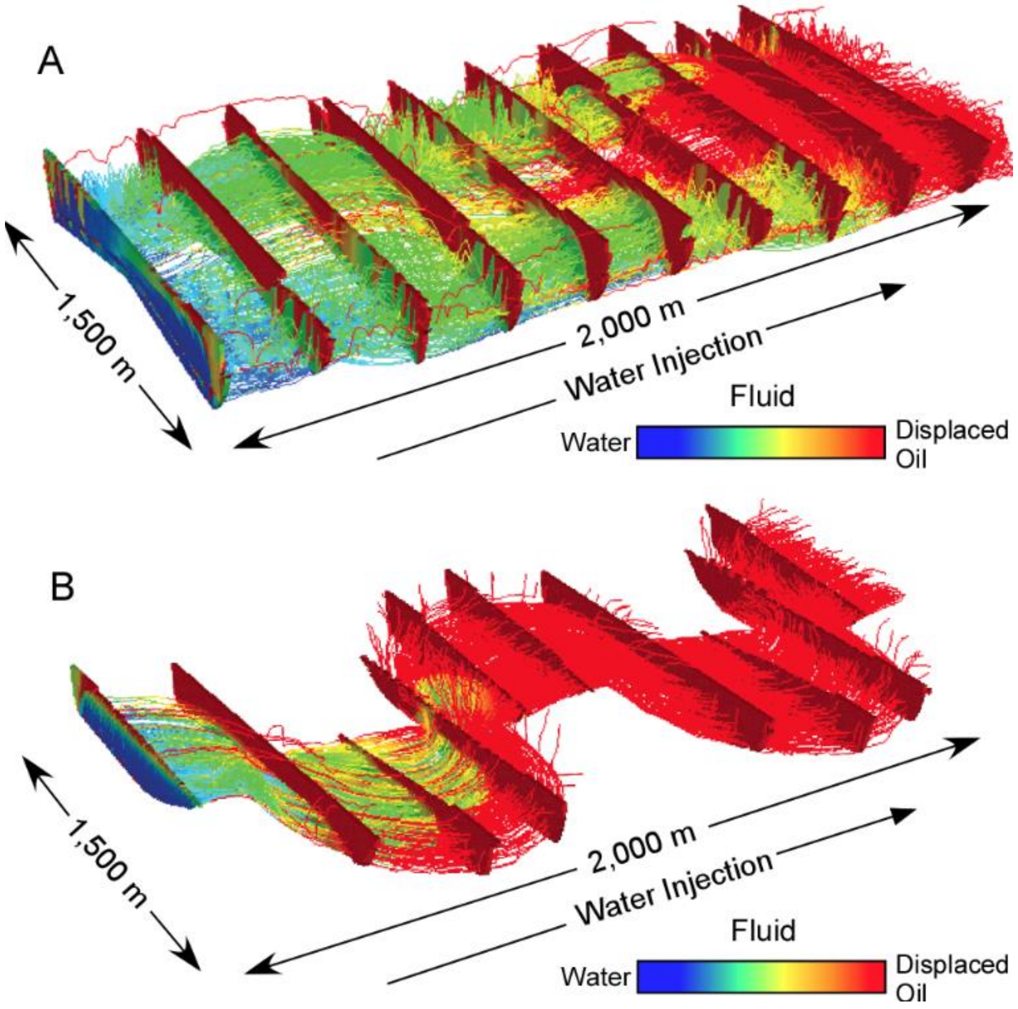
- Static Measures of Heterogeneity
- Lorenz Coefficient

Michael Pyrcz, The University of Texas at Austin

Motivation

There is a vast difference in heterogeneity of reservoirs

- **Heterogeneity** is the change in features over location or time.
- Conversely, **homogeneity** is the consistency of variables over location or time.
- Heterogeneity impacts the recovery of hydrocarbons from the reservoir.
- We need metrics to quantify the heterogeneity of our models.



Streamline simulation for two distinct fluvial reservoir styles Willis and Tang (2010).

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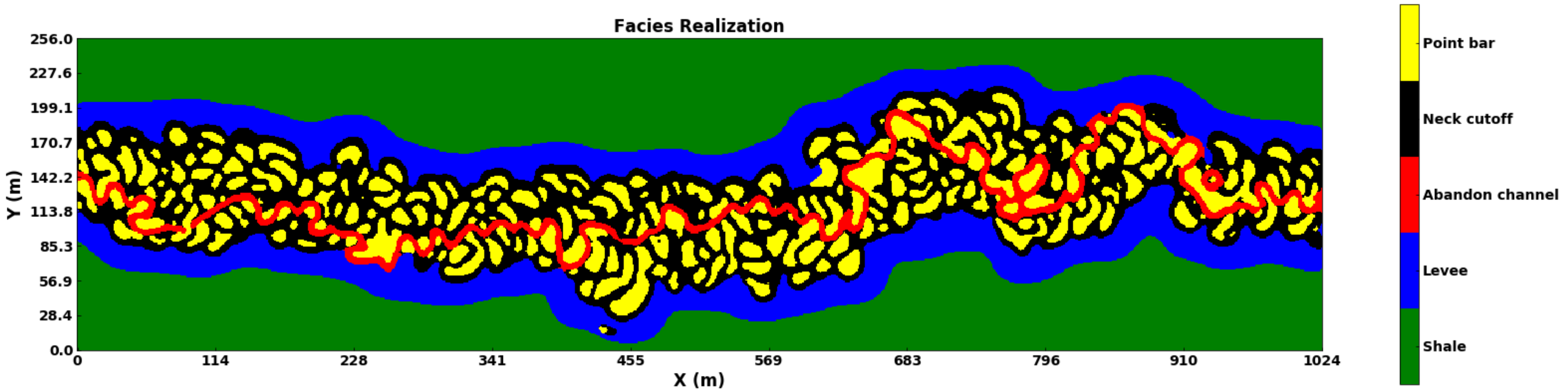
Heterogeneity Definition

The feature changes over a space or time study interval.



Deep learning, machine learning generated fluvial reservoir heterogeneity (Pan et al., 2022).

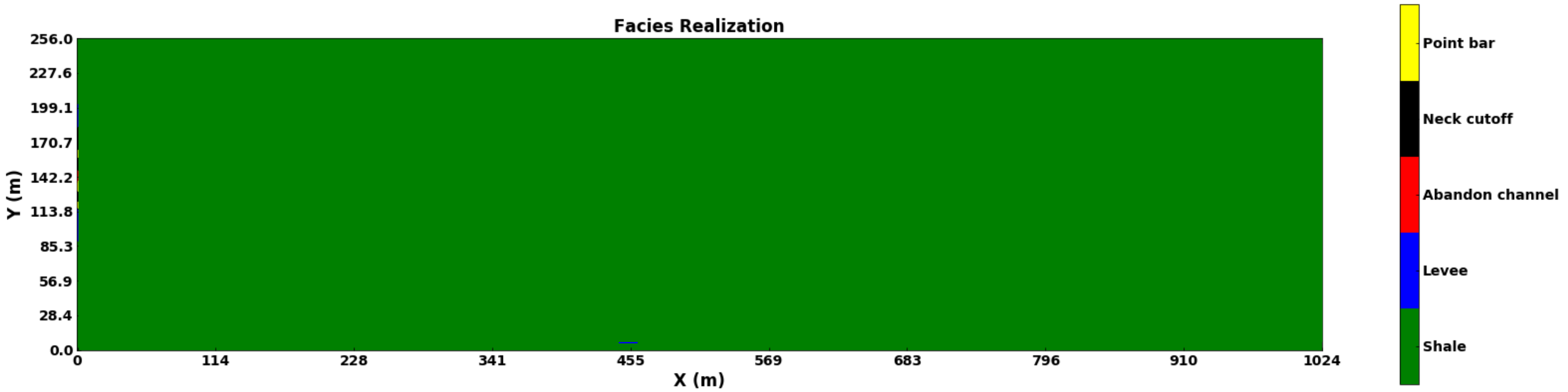
- many possible forms, scales, orientations / anisotropy etc.
- we calculate heterogeneity metrics to characterize, rank, and group the subsurface:

'This reservoir has high heterogeneity and we will need to drill more wells to reach production targets.'

'Reservoir A is less heterogeneous than Reservoir B and will be a better target for CO_2 sequestration.'

Homogeneity Definition

The feature does not change over a space or time study interval.



An example of homogeneous facies.

- homogeneity is extremely rare in the subsurface, heterogeneity is the default!

'Change is the only constant in life.' – Heraclitus (500 BC)

"Heraclitus, I believe, says that all things pass and nothing stays, and comparing existing things to the flow of a river, he says you could not step twice into the same river." – Plato (423 – 347 BC)

Measures of Heterogeneity Comments

Applications with Measures of Heterogeneity

- Measures of heterogeneity are often applied proxy, approximate measures to indicate reservoir production / performance.

- These measures may be applied to compare and rank reservoirs or reservoir model realizations for a single reservoir.

Best Practice

None of these measures are perfect.

The **best result possible from rigorous flow forecasting applied to good, full 3D reservoir models**.
Use the physics when possible!

- integrating all relevant information

- at sufficient scale to resolve important features

Measures of Heterogeneity Comments

Caution

Use of simple heterogeneity measures for ranking reservoir and reservoir models can be dangerous.

- Inaccuracy can result in incorrect rank estimates; therefore, incorrect business decisions.

Other Measures

We just consider simple, static measures here

Got your own measure? You may develop a new metric.

- Novel methods for quantifying heterogeneity within reservoirs is a currently active area of research.

Measures of Heterogeneity

Common Measures of Heterogeneity

- Static Measures
  - **Variance**
  - **The Coefficient of Variation**
  - **Dykstra-Parsons Coefficient of Variation**
  - **Lorenz Coefficient**
  - Gelhar-Axness Coefficient – $\sigma^2_{\ln(k)}$ x autocorrelation length

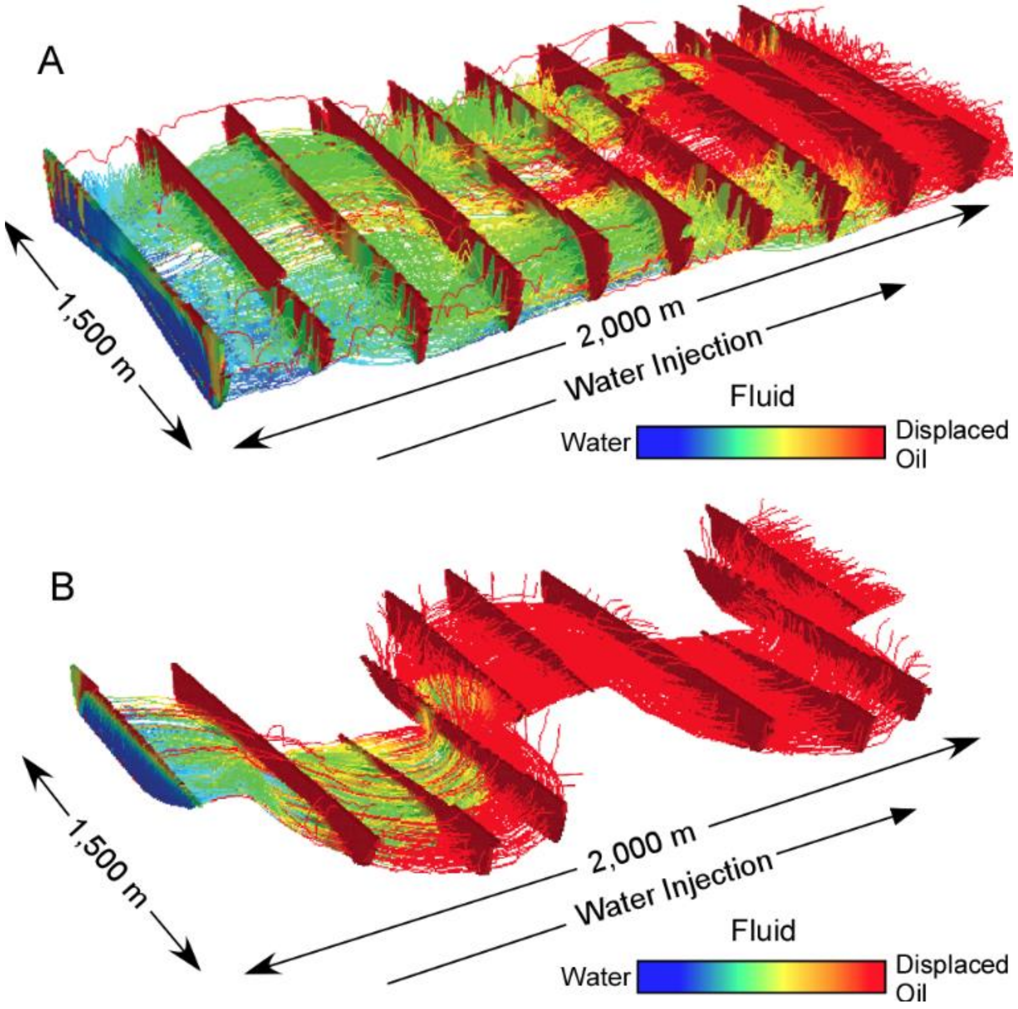
- Dynamic Measures
  - Koval Factor = total volume swept / pore volume at sweepout
  - Dispersion – longitudinal dispersivity

More information from Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier.

Measures of Heterogeneity

Motivation for the Use of Measures of Heterogeneity

- Heterogeneity matters
  - Direct impact on recovery factor, number of well required, production volumes and rates
- Heterogeneity varies greatly
  - DW proximal lobes vs. isolated DW channels
  - Even varies with the same net-to-gross
- Quantification is Beneficial
  - Decision of analogous fields
  - Comparison and decision making
  - Initial forecasts



Streamline simulation for two distinct fluvial reservoir styles Willis and Tang (2010).

Measures of Dispersion

Measures of Dispersion

- Variance of permeability
  - Population variance average squared difference from the mean

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

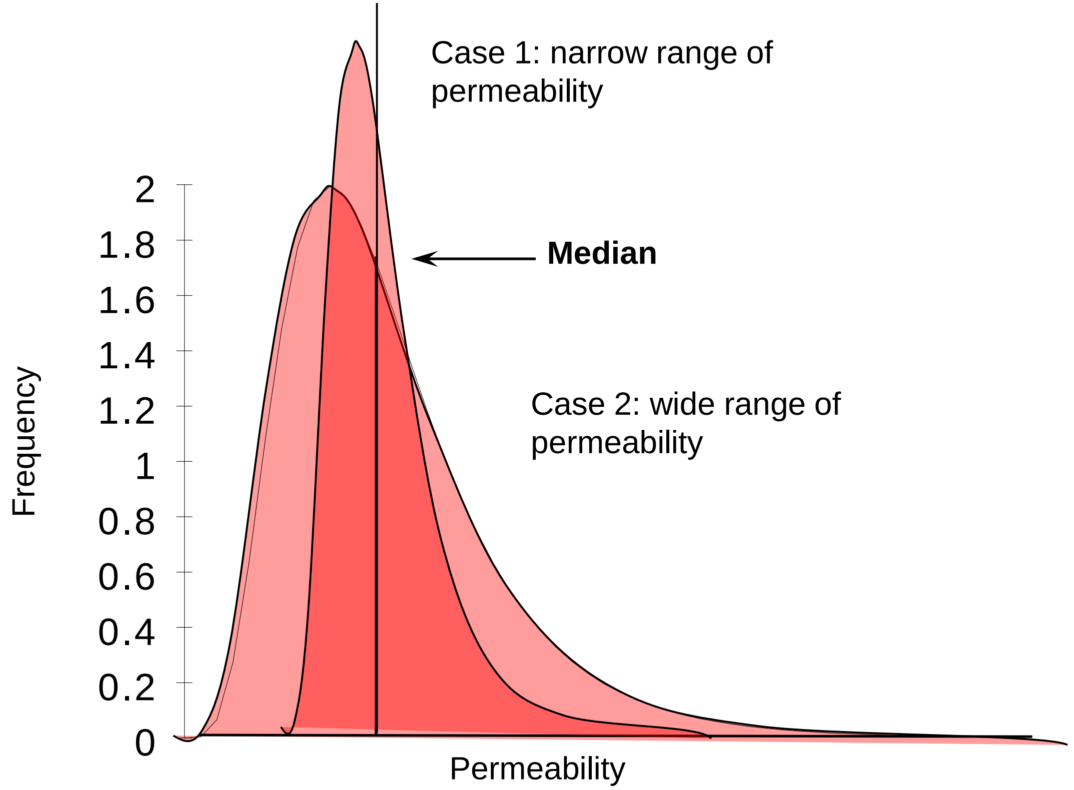
  - Unbiased estimate of population variance from sample set

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

- Standard Deviation of permeability

$$\sigma = \sqrt{\sigma^2}$$

- Low dispersion = Low Heterogeneity



Lognormal distributions with different dispersions. Note for lognormal distribution the median = geometric mean, the value that a lognormal collapses to as variance goes to 0.0.

Coefficient of Variation

Standardized Measures of Dispersion

Coefficient of Variation (CV), $C_v = \frac{\sigma}{\mu}$, or relative standard deviation is a standardized measure of dispersion, two common CV-based heterogeneity measures.

Coefficient of variation of permeability:

standard deviation

coefficient of variation of permeability

$$C_v = \frac{\sigma}{\mu} = \frac{\sqrt{\sigma_K^2}}{E[K]}$$

arithmetic mean

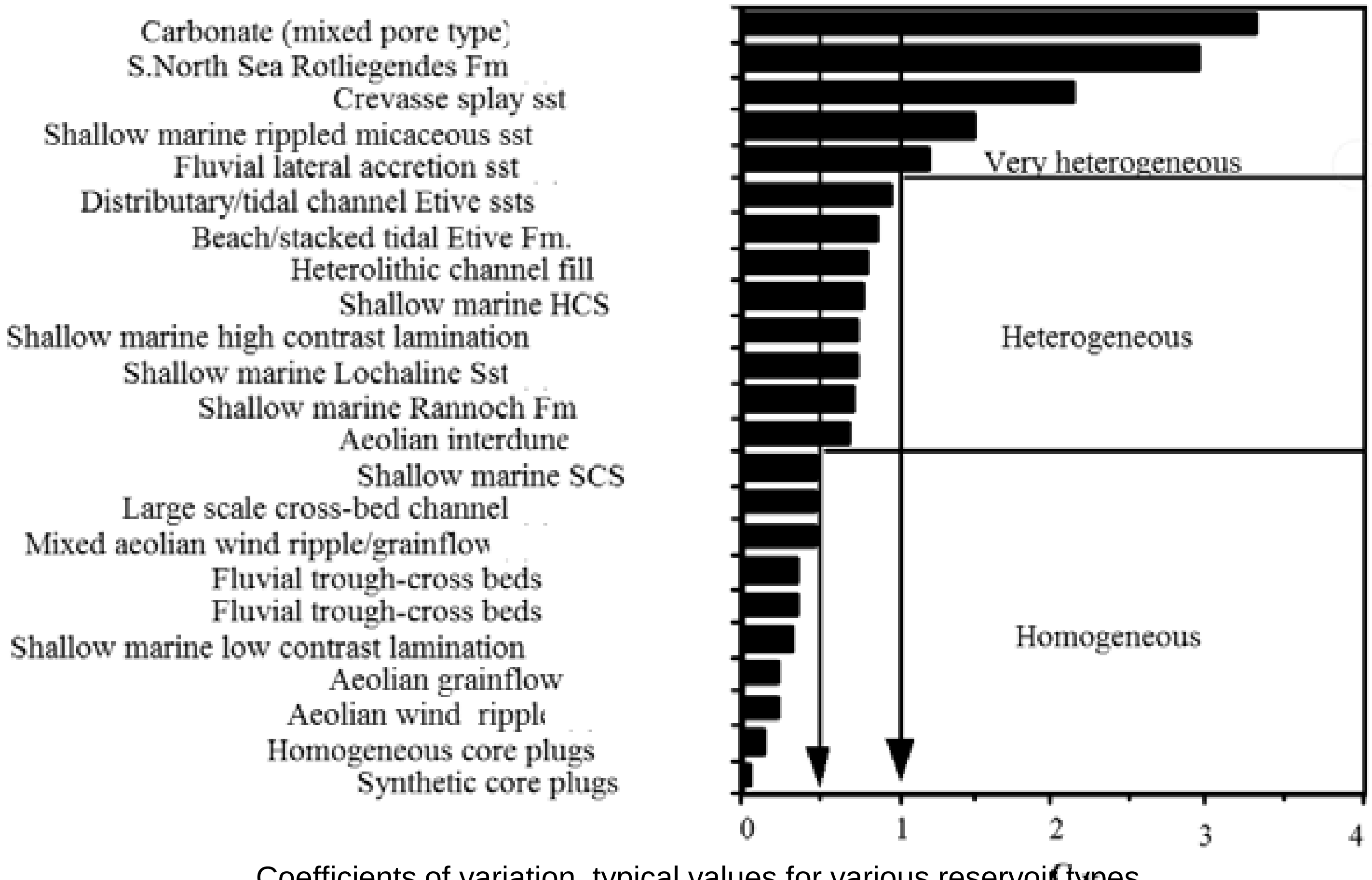
Coefficient of variance of the ratio of permeability / porosity:

coefficient of variation of permeability / porosity

$$C_{v'} = \frac{\sqrt{\sigma_{K/\phi}^2}}{E\left[\frac{K}{\phi}\right]}$$

Coefficient of Variation Examples

Standardized Measures of Dispersion



Coefficients of variation, typical values for various reservoir types.

Source: Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier.

Dykstra-Parsons Coefficent

Dykstra-Parsons Coefficient

Dykstra-Parsons Coefficient is a popular method for assessment of permeability variation (another dispersion measure):

permeability median

$\boldsymbol{F_K^{-1}}$**(0.16)**

Dykstra-Parsons

$$V = \frac{k_{50} - k_{16}}{k_{50}}$$

Practical interpretation:

Homogeneous Reservoir $\quad 0.0 \leq V \leq 1.0 \quad$ **Very Heterogeneous Reservoir**

Consider end members:

- If $\sigma_k^2 = 0$, then $k_{50} = k_{16}$ and $V = 0.0$
- $V \leq 1.0$ since $k_{16} \geq 0.0$.

Measures of Heterogeneity Exercise

Dykstra-Parsons Coefficient

Here's the data and fit lognormal distribution CDF for a more reliable P50 and P16 assessment.

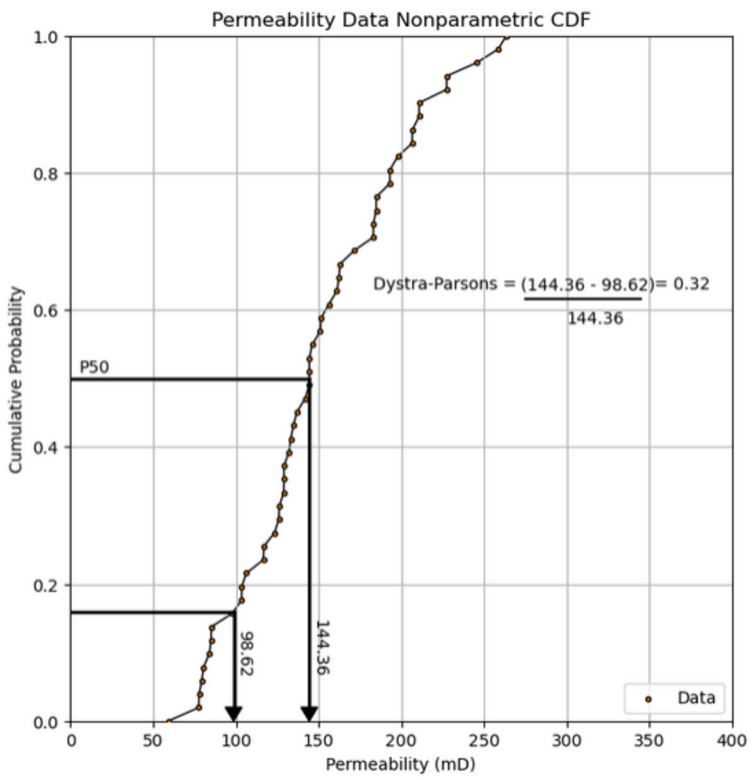
Fitting the lognormal distribution,

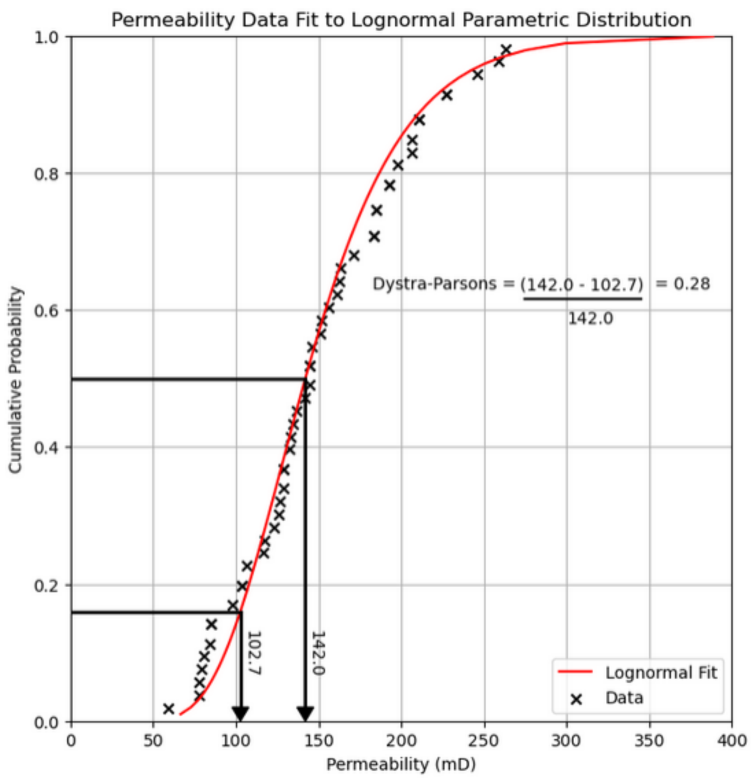
from the data calculate, mu:

$$\mu = ln\left(\frac{E[K]^2}{\sqrt{var[K] + (E[K])^2}}\right)$$

and calculate, sigma:

$$\sigma = \sqrt{ln\left(\frac{var[K]}{(E[K])^2} + 1\right)}$$



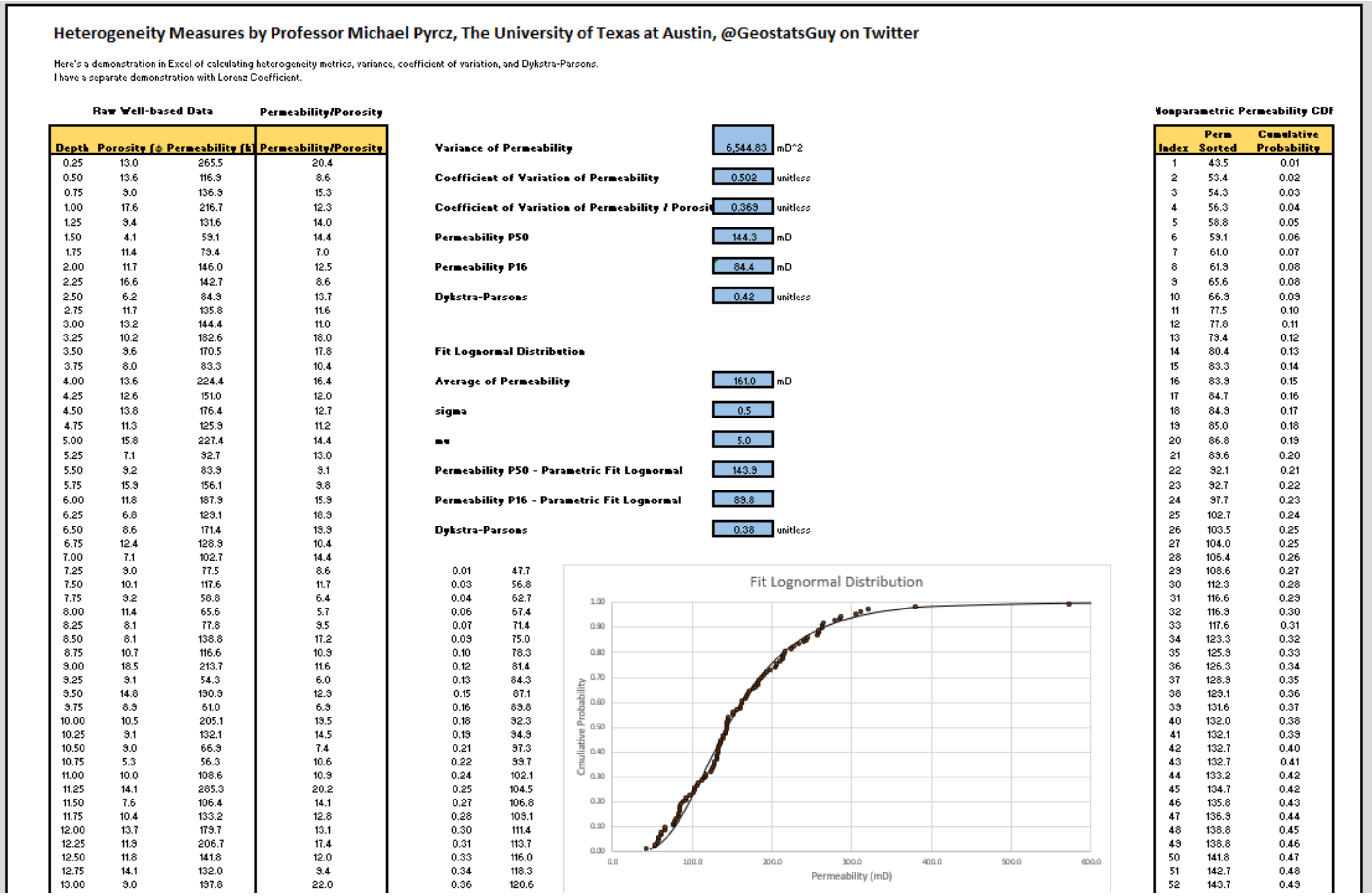


P50 and P16 of permeability from data CDF (left) and fit lognormal distribution for a more reliable calculation of Dykstra-Parsons (file is PythonDatBasics_Heterogeneity_Metrics.ipynb)

Heterogeneity Measures Demonstration in Excel

Here's a complete workflow in Excel to calculate heterogeneity metrics:

- permeability variance
- coefficient of variation
- Dykstra-Parsons (including fitting a lognormal parametric distribution)



Heterogeneity Measures by Professor Michael Pyrcz, The University of Texas at Austin, @GeostatsGuy on Twitter

Here's a demonstration in Excel of calculating heterogeneity metrics, variance, coefficient of variation, and Dykstra-Parsons.
I have a separate demonstration with Lorenz Coefficient.

Raw Well-based Data			Permeability/Porosity
Depth	Porosity (ϕ	Permeability (k	Permeability/Porosity
0.25	13.0	265.5	20.4
0.50	13.6	116.9	8.6
0.75	9.0	136.9	15.3
1.00	17.6	216.7	12.3
1.25	9.4	131.6	14.0
1.50	4.1	59.1	14.4
1.75	11.4	79.4	7.0
2.00	11.7	146.0	12.5
2.25	16.6	142.7	8.6
2.50	6.2	84.9	13.7
2.75	11.7	135.8	11.6
3.00	13.2	144.4	11.0
3.25	10.2	182.6	18.0
3.50	9.6	170.5	17.8
3.75	8.0	83.3	10.4
4.00	13.6	224.4	16.4
4.25	12.6	151.0	12.0
4.50	13.8	176.4	12.7
4.75	11.3	125.9	11.2
5.00	15.8	227.4	14.4
5.25	7.1	92.7	13.0
5.50	9.2	83.9	9.1
5.75	15.9	156.1	9.8
6.00	11.8	187.9	15.9
6.25	6.8	129.1	18.9
6.50	8.6	171.4	19.9
6.75	12.4	128.9	10.4
7.00	7.1	102.7	14.4
7.25	9.0	77.5	8.6
7.50	10.1	117.6	11.7
7.75	9.2	58.8	6.4
8.00	11.4	65.6	5.7
8.25	8.1	77.8	9.5
8.50	8.1	138.8	17.2
8.75	10.7	116.6	10.9
9.00	18.5	213.7	11.6
9.25	9.1	54.3	6.0
9.50	14.8	190.9	12.9
9.75	8.9	61.0	6.9
10.00	10.5	205.1	19.5
10.25	9.1	132.1	14.5
10.50	9.0	66.9	7.4
10.75	5.3	56.3	10.6
11.00	10.0	108.6	10.9
11.25	14.1	285.3	20.2
11.50	7.6	106.4	14.1
11.75	10.4	133.2	12.8
12.00	13.7	179.7	13.1
12.25	11.9	206.7	17.4
12.50	11.8	141.8	12.0
12.75	14.1	132.0	9.4
13.00	9.0	197.8	22.0

Variance of Permeability	6,544.83	mD^2
Coefficient of Variation of Permeability	0.502	unitless
Coefficient of Variation of Permeability / Porosit	0.369	unitless
Permeability P50	144.3	mD
Permeability P16	84.4	mD
Dykstra-Parsons	0.42	unitless
Fit Lognormal Distribution		
Average of Permeability	161.0	mD
sigma	0.5	
mu	5.0	
Permeability P50 - Parametric Fit Lognormal	143.9	
Permeability P16 - Parametric Fit Lognormal	89.8	
Dykstra-Parsons	0.38	unitless

0.01	47.7
0.03	56.8
0.04	62.7
0.06	67.4
0.07	71.4
0.09	75.0
0.10	78.3
0.12	81.4
0.13	84.3
0.15	87.1
0.16	89.8
0.18	92.3
0.19	94.9
0.21	97.3
0.22	99.7
0.24	102.1
0.25	104.5
0.27	106.8
0.28	109.1
0.30	111.4
0.31	113.7
0.33	116.0
0.34	118.3
0.36	120.6

Nonparametric Permeability CDF

Index	Perm Sorted	Cumulative Probability
1	43.5	0.01
2	53.4	0.02
3	54.3	0.03
4	56.3	0.04
5	58.8	0.05
6	59.1	0.06
7	61.0	0.07
8	61.9	0.08
9	65.6	0.08
10	66.9	0.09
11	77.5	0.10
12	77.8	0.11
13	79.4	0.12
14	80.4	0.13
15	83.3	0.14
16	83.9	0.15
17	84.7	0.16
18	84.9	0.17
19	85.0	0.18
20	86.8	0.19
21	89.6	0.20
22	92.1	0.21
23	92.7	0.22
24	97.7	0.23
25	102.7	0.24
26	103.5	0.25
27	104.0	0.25
28	106.4	0.26
29	108.6	0.27
30	112.3	0.28
31	116.6	0.29
32	116.9	0.30
33	117.6	0.31
34	123.3	0.32
35	125.9	0.33
36	126.3	0.34
37	128.9	0.35
38	129.1	0.36
39	131.6	0.37
40	132.0	0.38
41	132.1	0.39
42	132.7	0.40
43	132.7	0.41
44	133.2	0.42
45	134.7	0.42
46	135.8	0.43
47	136.9	0.44
48	138.8	0.45
49	138.8	0.46
50	141.8	0.47
51	142.7	0.48
52	143.7	0.49

Heterogeneity metrics demonstration in Excel, file is Heterogeneity_Metrics.xlsx.

Heterogeneity Measures Demonstration in Python

Here's a complete workflow in Excel to calculate heterogeneity metrics:

- permeability variance
- coefficient of variation
- Dykstra-Parsons (including fitting a lognormal parametric distribution)

Heterogeneity Metrics Demonstration

Calculating Heterogeneity Metrics to Summarize Subsurface Heterogeneity Tutorial

- demonstrate various mathematical operations for statistical expectation with distributions
- the same workflow is and data is demonstrated in Excel Heterogeneity Metrics.

Michael Pyrcz, Associate Professor, University of Texas at Austin

Twitter | *GitHub* | *Website* | *GoogleScholar* | *Book* | *YouTube* | *LinkedIn* | *GeostatsPy*

Heterogeneity Metrics

Applications with Measures of Heterogeneity

- Measures of heterogeneity are often applied proxy, approximate measures to indicate reservoir production / performance.
- These measures may be applied to compare and rank reservoirs or reservoir model realizations for a single reservoir.

Best Practice

- None of these metrics are perfect.
- The best result possible from rigorous flow forecasting applied to good, full 3D reservoir models, use the physics when possible!
- Integrate all relevant information, at sufficient scale to resolve important features

Caution

- Use of simple heterogeneity measures for ranking reservoir and reservoir models can be dangerous.
- Inaccuracy can result in incorrect rank estimates; therefore, incorrect business decisions.

Other Measures

- We just consider simple, static measures here
- I also have a Python demonstration for Lorenz coefficient
- Got your own measure? You may develop a new metric. Novel methods for quantifying heterogeneity within reservoirs is a currently active area of research.

Objective

Let's demonstate these heterogeneity metrics on a subsurface dataset.

Heterogeneity metrics demonstration in Python, file is PythonDataBasics_Heterogeneity_Metrics.ipynb.

PGE 338 Data Analytics and Geostatistics

Lecture 6: Heterogeneity

Lecture outline . . .

- Lorenz Coefficient

Michael Pyrcz, The University of Texas at Austin

Lorenz Coefficient

Lorenz Coefficient (also known as Static Lorenz in contrast to Dynamic Lorenz, not covered here)

Method was taken from **Economics, Lorenz curve for distribution of income**.

Order sample set from poorest to richest.

Calculate cumulative share of population and cumulative share of income earned.

A metric could be the area A as a proportion of area B.

- 1.0 – perfect inequality
- 0.0 – perfect equality

This is like heterogeneity if we think of rock and contribution to flow in a reservoir.

- we apply Lorenz to calculate heterogeneity over a well by comparing individual beds / geologic units.

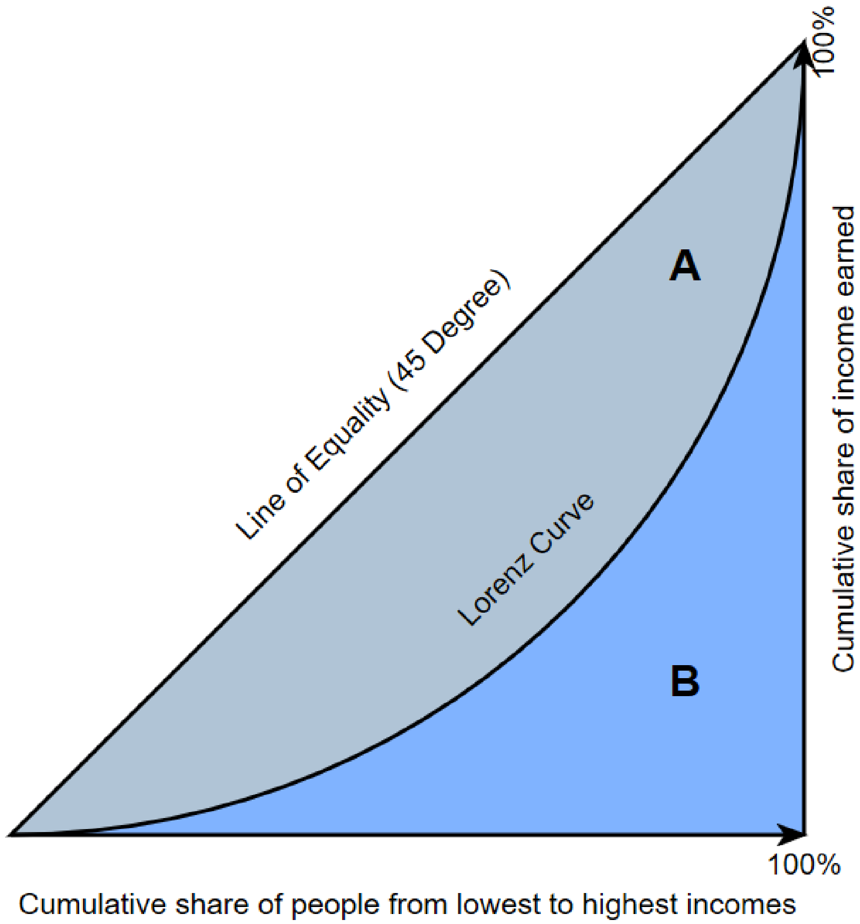


Illustration of Lorenz curve from economics.

Image from https://en.wikipedia.org/wiki/Lorenz_curve

Lorenz Coefficient

To calculate the Lorenz curve for a reservoir unit

1. sort the data in decreasing order of k/ϕ.
2. for each layer calculate flow capacity ($k \times h$) (permeability x layer thickness)
3. for each layer calculate storage capacity ($\varphi \times h$) (porosity x layer thickness)
4. for each layer calculate cumulative flow and storage capacity (sum all the layers $\sum_{i=1}^{current\ layer} \varphi h, \sum_{i=1}^{current\ layer} kh$
5. for each layer calculate the cumulative fraction of flow and storage capacity (divide the cumulative by the sum of all layers so the values are [0,1] (between 0 and 1).
6. plot cumulative fraction of flow capacity vs. storage capacity
7. fit polynomial function and calculate the area between function and straight line
8. Lorenz = this area / ½ (1/2 is the area of the triangle)

Steps 4 and 5 can be reversed. My Excel demo does step 5 and then step 4.

Lorenz Coefficient

To calculate the Lorenz curve for a set of reservoir units sort the data in decreasing order of k/ϕ then calculate as:

					Flow Capacity		Storage Capacity	Cumulative Fraction of Storage and Flow Capacity	
n	h_n	ϕ_n	k_n	$k_n h_n$	$\sum_{i=1}^{n} k_i h_i$	$\phi_n h_n$	$\sum_{i=1}^{n} \phi_i h_i$	$\frac{\sum_{i=1}^{n} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{n} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$
1	h_1	ϕ_1	k_1	$k_1 h_1$	$\sum_{i=1}^{1} k_i h_i$	$\phi_1 h_1$	$\sum_{i=1}^{1} \phi_i h_i$	$\frac{\sum_{i=1}^{1} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{1} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$
2	h_2	ϕ_2	k_2	$k_2 h_2$	$\sum_{i=1}^{2} k_i h_i$	$\phi_2 h_2$	$\sum_{i=1}^{2} \phi_i h_i$	$\frac{\sum_{i=1}^{2} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{2} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
N	h_N	ϕ_N	k_N	$k_N h_N$	$\sum_{i=1}^{N} k_i h_i$	$\phi_N h_N$	$\sum_{i=1}^{N} \phi_i h_i$	$\frac{\sum_{i=1}^{N} \phi_i h_i}{\sum_{i=1}^{N} \phi_i h_i}$	$\frac{\sum_{i=1}^{N} k_i h_i}{\sum_{i=1}^{N} k_i h_i}$
								X-axis	Y-axis

Steps for calculating the Lorenz curve from sorted well data (each row is a reservoir unit).

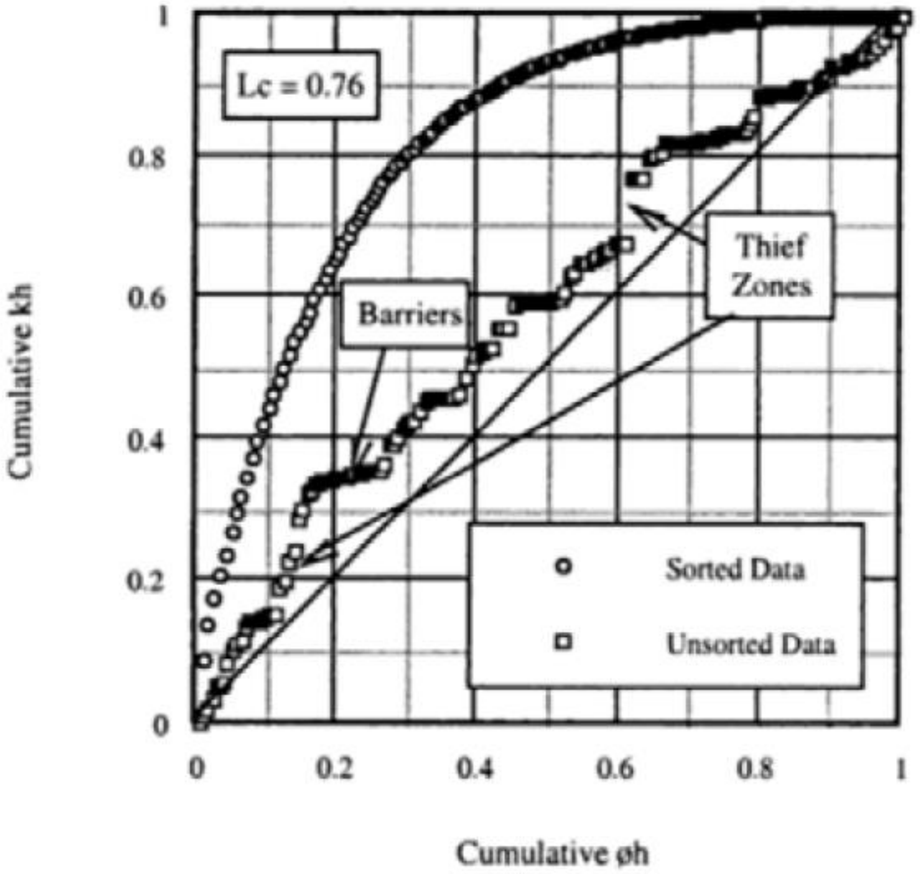
Lorenz Coefficient

There is an alternative to the Lorenz curve with unsorted data.

- sorted data for Lorenz coefficient calculation

  General Rule:
  - $L_c > 0.6$ – high
  - $L_c < 0.3$ - low

- unsorted to identify barriers and thief zones along the well.



Sorted and unsorted from Jensen, J. R., Lake, L. W., Corbett P. M. W., and Goggin, D. J., 2000, Statistics for Petroleum Engineers and Geoscientists, Elsevier.

Note: Lorenz curve in Oil and Gas is above the homogeneous, $y = x$, line. Lorenz curve in Economics is below the $y = x$ line.

Lorenz Coefficient Example

There is an example of Lorenz coefficient calculated from regularly sampled porosity and permeability on a well.

1. Raw Well-based Data

Depth	Porosity (ϕ)	Permeability (k)
0.25	13.0	265.5
0.50	13.6	116.9
0.75	9.0	136.9
1.00	17.6	216.7
1.25	9.4	131.6
1.50	4.1	59.1
1.75	11.4	79.4
2.00	11.7	146.0
2.25	16.6	142.7
2.50	6.2	84.9
2.75	11.7	135.8
3.00	13.2	144.4
3.25	10.2	182.6
3.50	9.6	170.5
3.75	8.0	83.3
4.00	13.6	224.4

2. Ratio Perm. / Por.

k/ϕ
20.4
8.6
15.3
12.3
14.0
14.4
7.0
12.5
8.6
13.7
11.6
11.0
18.0
17.8
10.4
16.4

3. Sorted by Descending Ratio of Perm. / Por

k/ϕ	Porosity	Permeability
33.3	17.2	573.5
28.6	9.2	263.5
27.7	11.0	305.1
27.4	8.9	244.1
22.1	17.2	380.1
22.0	9.0	197.8
20.4	13.0	265.5
20.2	14.1	285.3
19.9	8.6	171.4
19.5	10.5	205.1
19.4	13.4	258.6
19.3	11.1	214.9
19.0	16.9	321.1
18.9	6.8	129.1
18.9	14.0	264.2
18.4	13.1	240.6

4. Calculate kh and phi h

Phi h	k h
8.6	286.7
4.6	131.7
5.5	152.6
4.4	122.0
8.6	190.0
4.5	98.9
6.5	132.8
7.1	142.7
4.3	85.7
5.3	102.5
6.7	129.3
5.6	107.5
8.4	160.5
3.4	64.5
7.0	132.1
6.5	120.3

5. Calculate Fraction of Total

Frac Phi h	Frac k h
1.4%	3.4%
0.8%	1.6%
0.9%	1.8%
0.7%	1.4%
1.4%	2.2%
0.7%	1.2%
1.1%	1.6%
1.2%	1.7%
0.7%	1.0%
0.9%	1.2%
1.1%	1.5%
0.9%	1.3%
1.4%	1.9%
0.6%	0.8%
1.1%	1.6%
1.1%	1.4%

6. Calculate Cumulative Fraction

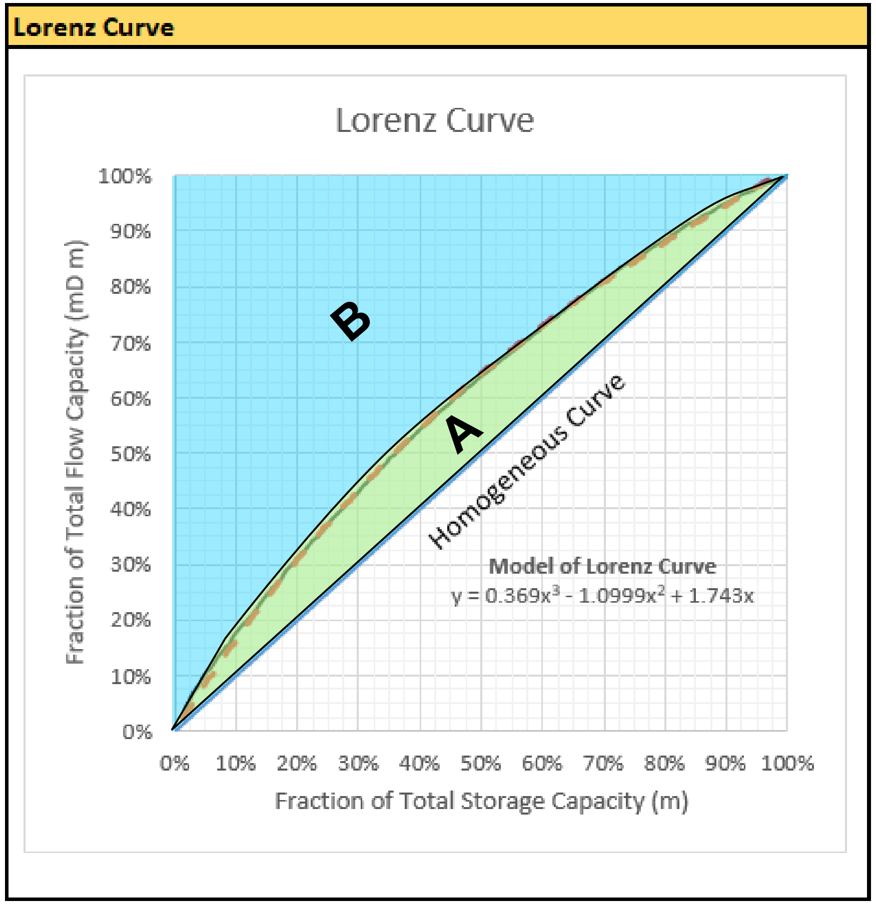
Σ Phi h	Σ k h
1.4%	3.4%
2.2%	5.0%
3.1%	6.8%
3.8%	8.2%
5.2%	10.4%
5.9%	11.6%
7.0%	13.2%
8.1%	14.9%
8.8%	15.9%
9.7%	17.1%
10.8%	18.6%
11.7%	19.9%
13.1%	21.8%
13.6%	22.6%
14.8%	24.1%
15.8%	25.6%

truncated

Lorenz coefficient in Excel file is Lorenz_Coefficient_Demo.xlsx.

Lorenz Coefficient Example

There is an example of Lorenz coefficient calculated from regularly sampled porosity and permeability on a well.



$$\text{Lorenz Coefficient} = \frac{A}{A + B}$$

Did this by fitting a function interpolating and numerical integration.

$$A = \sum_{k=1}^{K} (L(s) - H(s))\Delta s$$

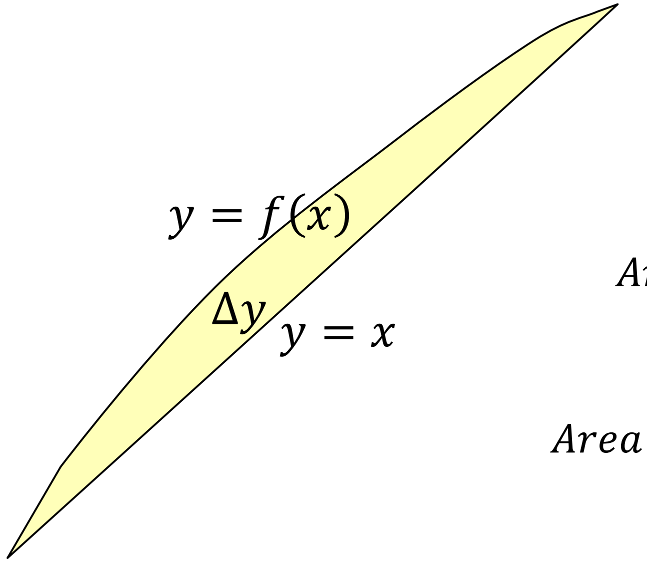
$$A + B = 0.5$$

Lorenz Coefficient Example

There is an example of Lorenz coefficient calculated from regularly sampled porosity and permeability on a well.

Calculating the area through integration of the y = f(x) polynomial – y = x line.

- Assuming a 3rd order polynomial is fit to the Lorenz curve.



$$\Delta y = f(x) - x$$

$$\Delta y = dx^3 + cx^2 + bx + a - x$$

$$Area\ (A) = \int_{0.0}^{1.0} \Delta y \, dx = \frac{dx^4}{4} + \frac{cx^3}{3} + \frac{bx^2}{2} + ax - \frac{x^2}{2}$$

$$Area(A) = \int_{0.0}^{1.0} \Delta y \, dx = \left. \frac{dx^4}{4} + \frac{cx^3}{3} + \frac{x^2(b-1)}{2} + ax + C \right|_{0.0}^{1.0}$$

$$Area(A) = \frac{d}{4} + \frac{c}{3} + \frac{(b-1)}{2}, since\ a = 0$$

Measures of Heterogeneity
Lorenz Coefficient Example

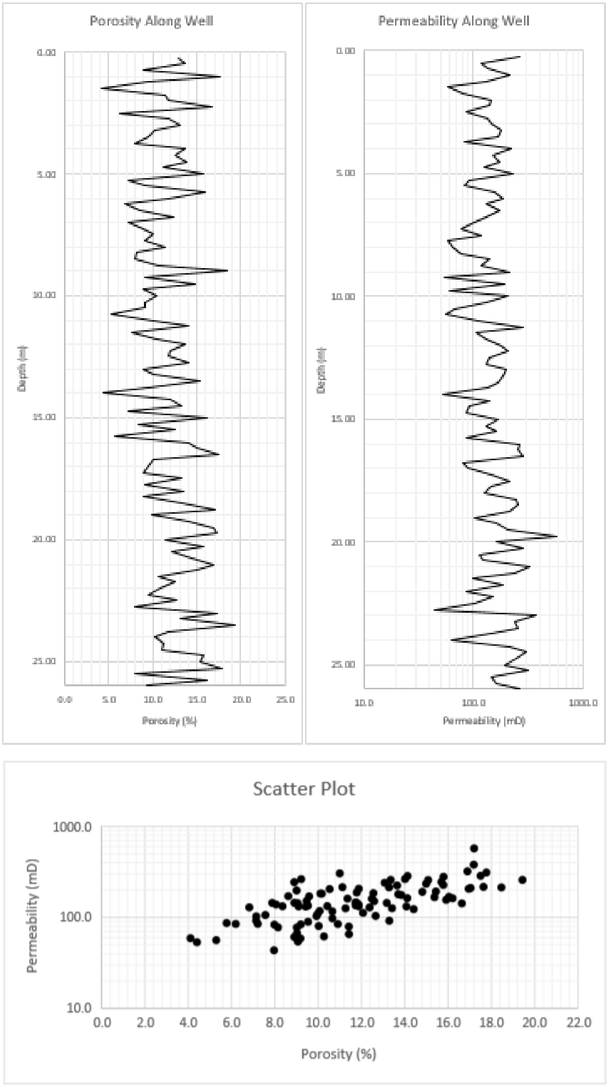
Your project team would like further information to help characterize the heterogeneity for a recent well. Previous experience has shown that Lorenz is a good predictor of flow rates.

Use small dataset available on canvas and the github example as a guide to calculate Lorenz coefficient. PorPermSample1.xlsx again.

What is the Lorenz coefficient?

What will you report for heterogeneity? High, medium or low?

Should we expect simple displacement, high recovery factor?



Porosity and permeability data.

Measures of Heterogeneity
Lorenz Coefficient Example

Your project team would like further information to help characterize the heterogeneity for a recent well. Previous experience has shown that Lorenz is a good predictor of flow rates.

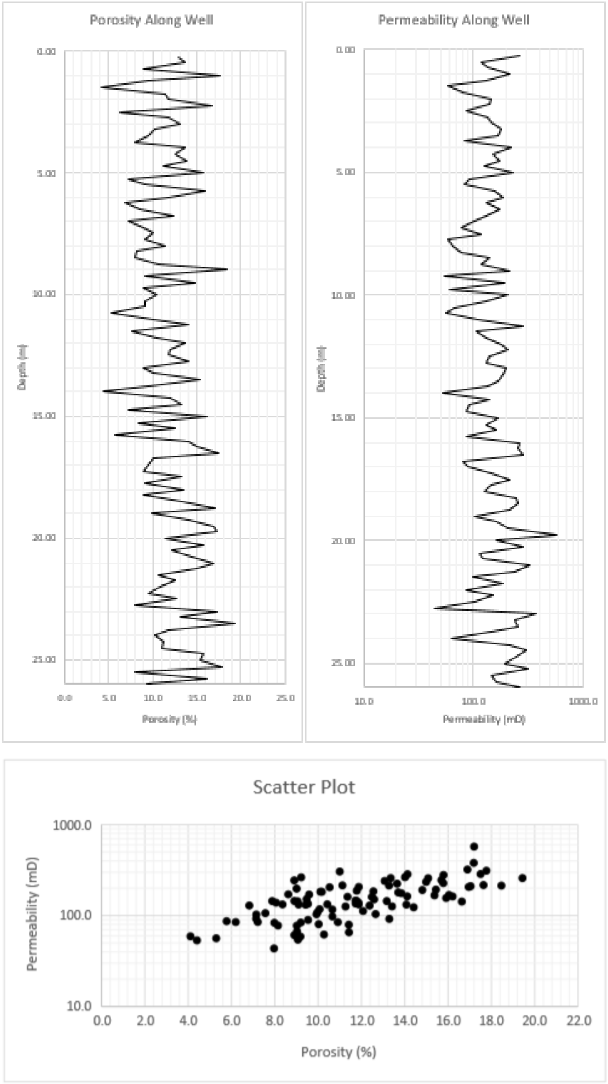
Use small dataset available on canvas and the github example as a guide to calculate Lorenz coefficient. PorPermSample1.xlsx again.

What is the Lorenz coefficient?   0.19

What will you report for heterogeneity? High, medium or low? Low

Should we expect simple displacement, high recovery factor?

Simple, piston-like displacement with high recovery factor.



Porosity and permeability data.

Measures of Heterogeneity
Lorenz Coefficient Example

Your project team would like further information to help characterize the heterogeneity for a recent well. Previous experience has shown that Lorenz is a good predictor of flow rates.

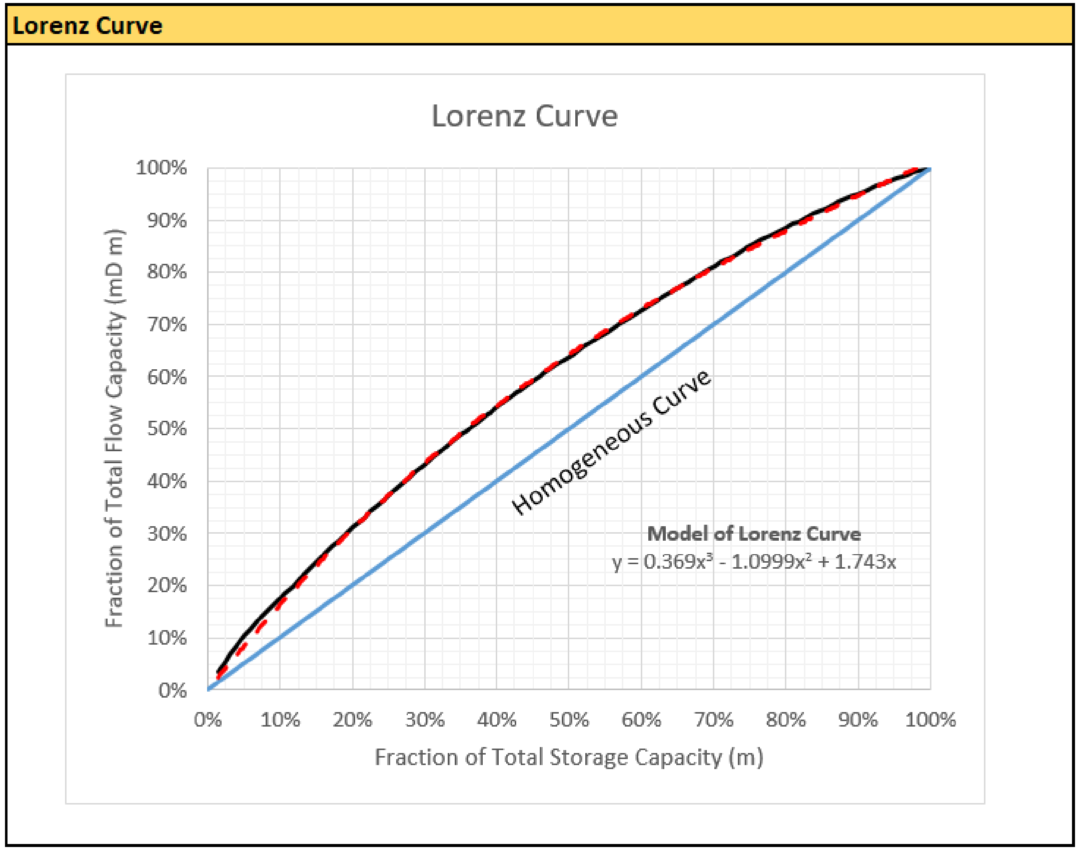
3. Sorted by Descending Ratio of Perm. / Por			4. Calculate kh and phi h		5. Calculate Fraction of Total		6. Calculate Cumulative	
k/φ	Porosity	Permeability	Phi h	k h	Frac Phi h	Frac k h	Σ Phi h	Σ k h
33.3	17.2	573.5	4.3	143.4	1.4%	3.4%	1.4%	3.4%
28.6	9.2	263.5	2.3	65.9	0.8%	1.6%	2.2%	5.0%
27.7	11.0	305.1	2.7	76.3	0.9%	1.8%	3.1%	6.8%
27.4	8.9	244.1	2.2	61.0	0.7%	1.4%	3.8%	8.2%
22.1	17.2	380.1	4.3	95.0	1.4%	2.2%	5.2%	10.4%
22.0	9.0	197.8	2.3	49.4	0.7%	1.2%	5.9%	11.6%
20.4	13.0	265.5	3.2	66.4	1.1%	1.6%	7.0%	13.2%
20.2	14.1	285.3	3.5	71.3	1.2%	1.7%	8.1%	14.9%
19.9	8.6	171.4	2.2	42.8	0.7%	1.0%	8.8%	15.9%
19.5	10.5	205.1	2.6	51.3	0.9%	1.2%	9.7%	17.1%
19.4	13.4	258.6	3.3	64.6	1.1%	1.5%	10.8%	18.6%
19.3	11.1	214.9	2.8	53.7	0.9%	1.3%	11.7%	19.9%
19.0	16.9	321.1	4.2	80.3	1.4%	1.9%	13.1%	21.8%
18.9	6.8	129.1	1.7	32.3	0.6%	0.8%	13.6%	22.6%
18.9	14.0	264.2	3.5	66.0	1.1%	1.6%	14.8%	24.1%
18.4	13.1	240.6	3.3	60.1	1.1%	1.4%	15.8%	25.6%
18.3	7.9	144.3	2.0	36.1	0.6%	0.9%	16.5%	26.4%
18.1	10.1	183.1	2.5	45.8	0.8%	1.1%	17.3%	27.5%
18.0	10.2	182.6	2.5	45.7	0.8%	1.1%	18.1%	28.6%
17.8	9.6	170.5	2.4	42.6	0.8%	1.0%	18.9%	29.6%
17.7	15.8	279.8	3.9	70.0	1.3%	1.7%	20.2%	31.2%
17.5	17.8	311.6	4.4	77.9	1.4%	1.8%	21.6%	33.1%
17.4	11.9	206.7	3.0	51.7	1.0%	1.2%	22.6%	34.3%
17.2	8.1	138.8	2.0	34.7	0.7%	0.8%	23.3%	35.1%
17.1	15.1	257.4	3.8	64.4	1.2%	1.5%	24.5%	36.6%
16.4	13.6	224.4	3.4	56.1	1.1%	1.3%	25.6%	38.0%
16.4	17.5	286.8	4.4	71.7	1.4%	1.7%	27.0%	39.7%
16.3	8.9	144.7	2.2	36.2	0.7%	0.9%	27.8%	40.5%
16.1	13.3	214.1	3.3	53.5	1.1%	1.3%	28.8%	41.8%
16.0	9.5	151.5	2.4	37.9	0.8%	0.9%	29.6%	42.7%
15.9	11.8	187.9	2.9	47.0	1.0%	1.1%	30.6%	43.8%
15.9	8.4	132.7	2.1	33.2	0.7%	0.8%	31.3%	44.6%
15.9	9.1	143.7	2.3	35.9	0.7%	0.8%	32.0%	45.4%
15.7	15.7	245.7	3.9	61.4	1.3%	1.5%	33.3%	46.9%
15.6	15.0	234.5	3.7	58.6	1.2%	1.4%	34.5%	48.3%
15.3	9.0	136.9	2.2	34.2	0.7%	0.8%	35.2%	49.1%

truncated

Lorenz coefficient in Excel demonstration.

Measures of Heterogeneity
Lorenz Coefficient Example

Your project team would like further information to help characterize the heterogeneity for a recent well. Previous experience has shown that Lorenz is a good predictor of flow rates.



$$Area(A) = \frac{d}{4} + \frac{c}{3} + \frac{(b-1)}{2}$$

$$= \frac{(0.369)}{4} + \frac{(-1.100)}{3} + \frac{(1.743 - 1.0)}{2}$$

$$Lorenz = \frac{Area(A)}{0.5} = \frac{0.097}{0.5} = 0.194$$

Lorenz curve and integration to calculate the Lorenz coefficient (area under the curve above the homogeneous line / area of upper triangle (0.5).

Lorenz Coefficient Example in Excel

Here’s a complete workflow in Excel to:

- perform each step of the Lorenz coefficient calculation
- calculate the Lorenz coefficient and produce the associated plot

Lorenz Coefficient Demonstration, Michael Pyrcz, The University of Texas at Austin

Demonstration of calculating Lorenz coefficient in Excel.

Result: Lorenz coefficient, a concept taken from economics to describe inequality in income used to quantify rock heterogeneity 0 = homogeneous, 1 heterogeneous.

Metholodolgy: calculate ratio perm. / por., sort in descending order. Calculate Lorenz coefficient cumulative fraction kh and phih and plot. Calculate fraction of area between Lorenz curve and homogeneous curve to area above homogeneous curve.

1. Raw Well-based Data

Depth	Porosity (ϕ)	Permeability (k)
0.25	13.0	265.5
0.50	13.6	116.9
0.75	9.0	136.9
1.00	17.6	216.7
1.25	9.4	131.6
1.50	4.1	59.1
1.75	11.4	79.4
2.00	11.7	146.0
2.25	16.6	142.7
2.50	6.2	84.9
2.75	11.7	135.8
3.00	13.2	144.4
3.25	10.2	182.6
3.50	9.6	170.5
3.75	8.0	83.3
4.00	13.6	224.4
4.25	12.6	151.0
4.50	13.8	176.4
4.75	11.3	125.9
5.00	15.8	227.4
5.25	7.1	92.7

2. Ratio Perm. /

k/ϕ	k/phi	Por	Perm
20.4	33.3	17.2	573.5
8.6	28.6	9.2	263.5
15.3	27.7	11.0	305.1
12.3	27.4	8.9	244.1
14.0	22.1	17.2	380.1
14.4	22.0	9.0	197.8
7.0	20.4	13.0	265.5
12.5	20.2	14.1	285.3
8.6	19.9	8.6	171.4
13.7	19.5	10.5	205.1
11.6	19.4	13.4	258.6
11.0	19.3	11.1	214.9
18.0	19.0	16.9	321.1
17.8	18.9	6.8	129.1
10.4	18.9	14.0	264.2
16.4	18.4	13.1	240.6
12.0	18.3	7.9	144.3
12.7	18.1	10.1	183.1
11.2	18.0	10.2	182.6
14.4	17.8	9.6	170.5
13.0	17.7	15.8	279.8

3. Sorted by Descending Ratio of Perm. / Por

k/ϕ	Porosity	Permeability
33.3	17.2	573.5
28.6	9.2	263.5
27.7	11.0	305.1
27.4	8.9	244.1
22.1	17.2	380.1
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19.0	16.9	321.1
18.9	6.8	129.1
18.9	14.0	264.2
18.4	13.1	240.6
18.3	7.9	144.3
18.1	10.1	183.1
18.0	10.2	182.6
17.8	9.6	170.5
17.7	15.8	279.8

4. Calculate kh and phi h

Phi h	k h
4.3	143.4
2.3	65.9
2.7	76.3
2.2	61.0
4.3	95.0
2.3	49.4
3.2	66.4
3.5	71.3
2.2	42.8
2.6	51.3
3.3	64.6
2.8	53.7
4.2	80.3
1.7	32.3
3.5	66.0
3.3	60.1
2.0	36.1
2.5	45.8
2.5	45.7
2.4	42.6
3.9	70.0

5. Calculate Fraction of Total

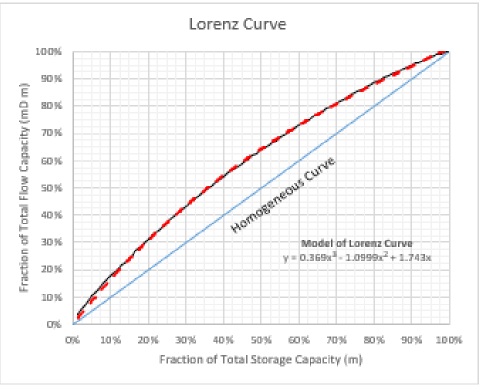
Frac Phi h	Frac k h
1.4%	3.4%
0.8%	1.6%
0.9%	1.8%
0.7%	1.4%
1.4%	2.2%
0.7%	1.2%
1.1%	1.6%
1.2%	1.7%
0.7%	1.0%
0.9%	1.2%
1.1%	1.5%
0.9%	1.3%
1.4%	1.9%
0.6%	0.8%
1.1%	1.6%
1.1%	1.4%
0.6%	0.9%
0.8%	1.1%
0.8%	1.1%
0.8%	1.0%
1.3%	1.7%

6. Calculate Cumulative

Σ Phi h	Σ k h
1.4%	3.4%
2.2%	5.0%
3.1%	6.8%
3.8%	8.2%
5.2%	10.4%
5.9%	11.6%
7.0%	13.2%
8.1%	14.9%
8.8%	15.9%
9.7%	17.1%
10.8%	18.6%
11.7%	19.9%
13.1%	21.8%
13.6%	22.6%
14.8%	24.1%
15.8%	25.6%
16.5%	26.4%
17.3%	27.5%
18.1%	28.6%
18.9%	29.6%
20.2%	31.2%

7. Plot Lorenz Curve

Lorenz Curve



Lorenz coefficient demonstration in Excel, file is Lorenz_Coefficient_Demo.ipynb.

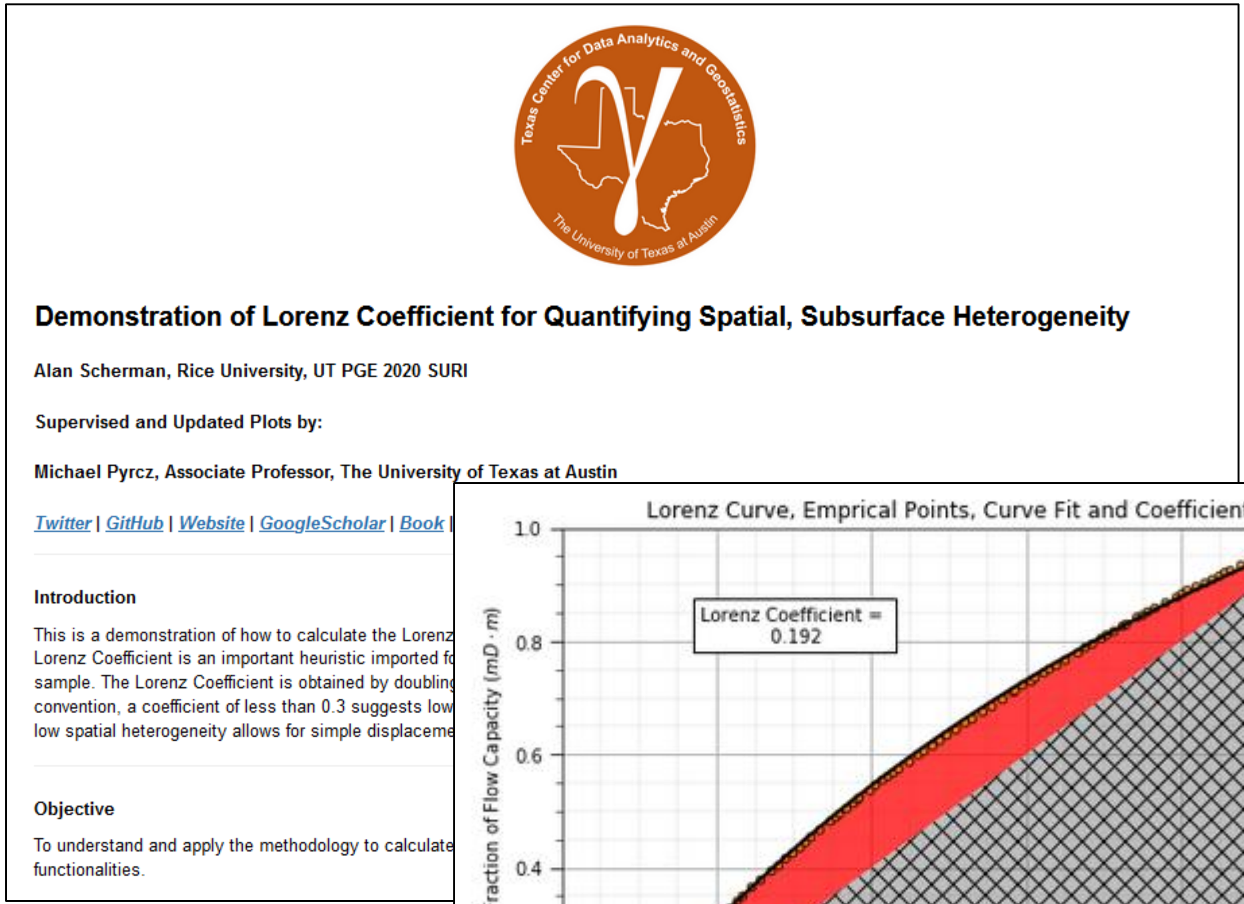
Lorenz Coefficient Example in Python

Here's a complete workflow in Python to:

- load a data table
- perform each step of the Lorenz coefficient calculation
- calculate the Lorenz coefficient and produce the associated plot

Credit to my former SURI student, Alan Scherman from Rice University for Python:

- functions for Lorenz
- well-documented workflow



Demonstration of Lorenz Coefficient for Quantifying Spatial, Subsurface Heterogeneity

Alan Scherman, Rice University, UT PGE 2020 SURI

Supervised and Updated Plots by:

Michael Pyrcz, Associate Professor, The University of Texas at Austin

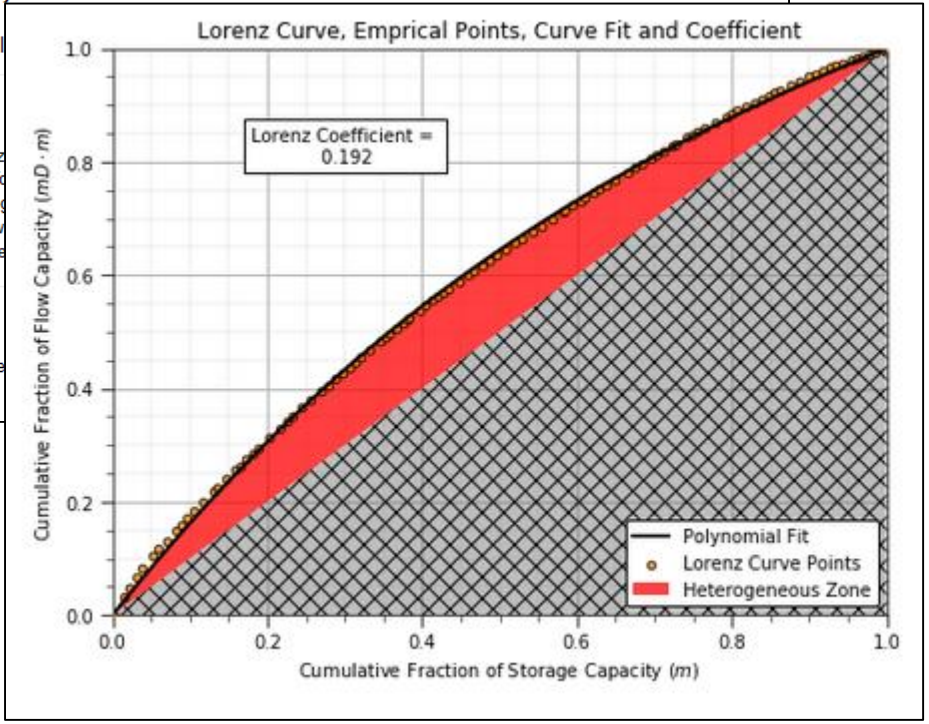
Twitter | GitHub | Website | GoogleScholar | Book |

Introduction

This is a demonstration of how to calculate the Lorenz
Lorenz Coefficient is an important heuristic imported fo
sample. The Lorenz Coefficient is obtained by doubling
convention, a coefficient of less than 0.3 suggests low
low spatial heterogeneity allows for simple displaceme

Objective

To understand and apply the methodology to calculate
functionalities.



Lorenz coefficient demonstration in Python, file is Lorenz_coefficient_demo.ipynb.

PGE 338 Data Analytics and Geostatistics

Lecture 6: Heterogeneity

Lecture outline . . .

- Static Measures of Heterogeneity
- Lorenz Coefficient

Michael Pyrcz, The University of Texas at Austin