PGE 338 Data Analytics and Geostatistics

Lecture 11: Spatial Modeling

Lecture outline . . .

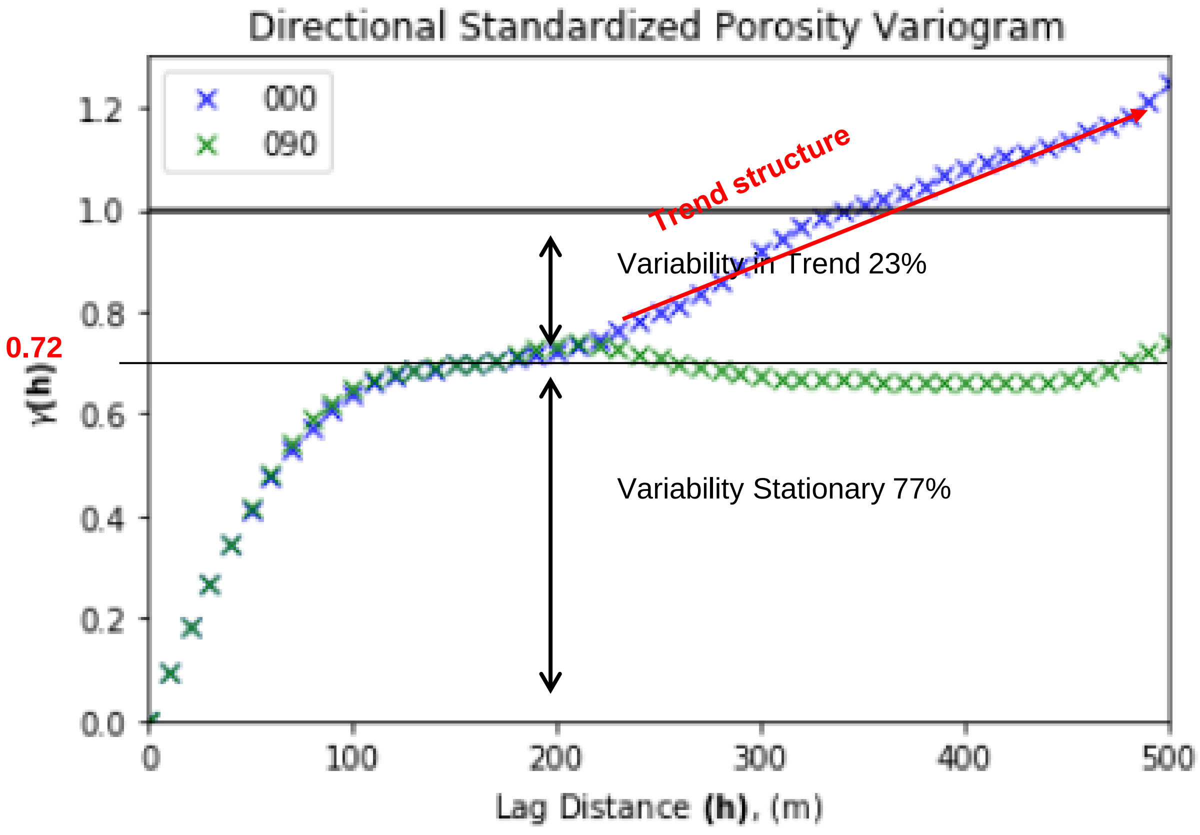
- Variogram Interpretation
- Variogram Modeling

Introduction
General Concepts
Univariate
Bivariate
Spatial
Calculation
Variogram Modeling
Kriging
Simulation
Time Series
Machine Learning
Uncertainty Analysis

Michael Pyrcz, The University of Texas at Austin

Motivation

After we calculate / quantify spatial continuity we need to model it for spatial prediction.



The proportion of trend, interpreted from the experimental variograms in the major and minor directions.

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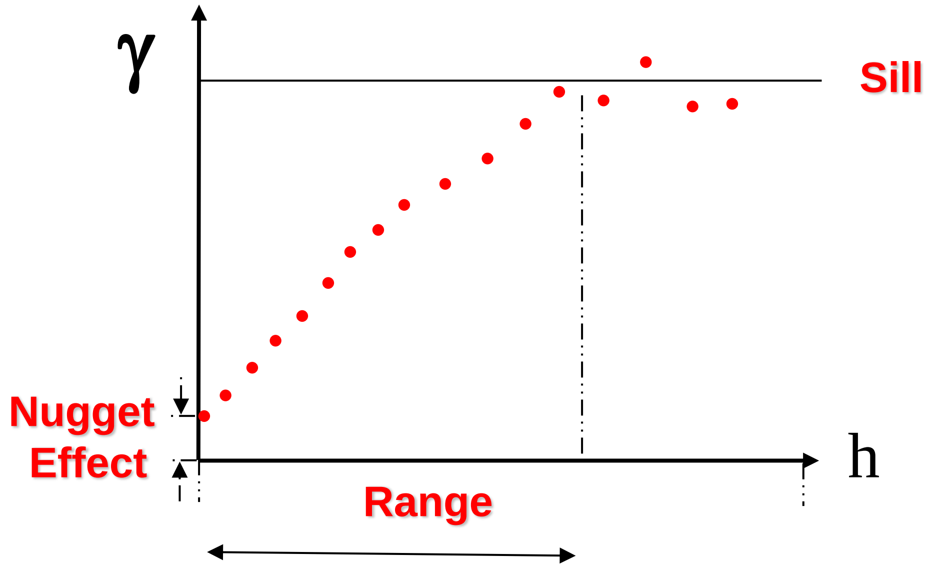
Variogram Terminology

Recall Basic Variogram Definitions:

- The ***sill*** is the variance of the data used for variogram calculation (1.0 if the data are normal scores)
- The ***range*** is the distance at which the variogram reaches the sill
- The ***nugget effect*** is the behavior at distances less than the smallest experimental lag:

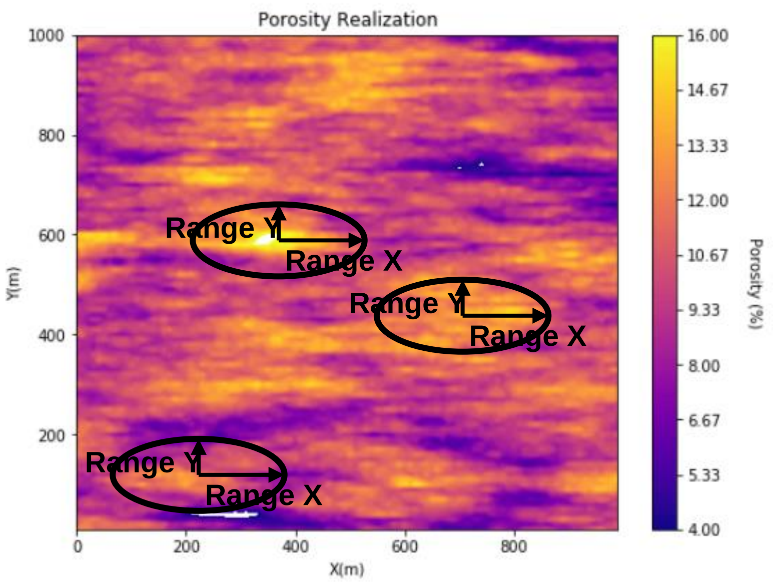


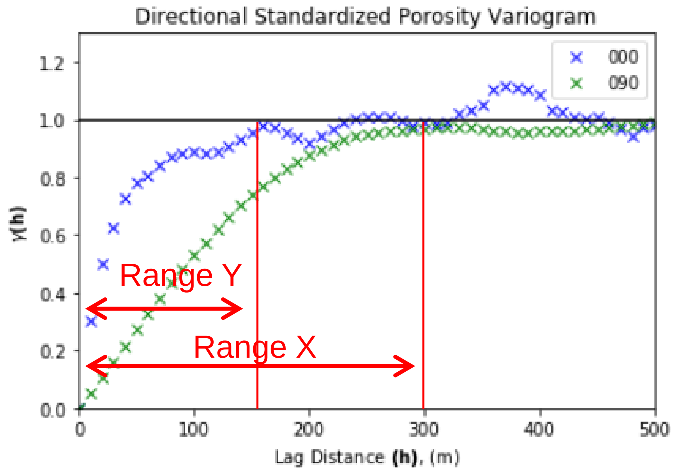
Example experimental variogram with nugget effect, sill and range indicated.

Geometric Anisotropy

Geometric Anisotropy Description

- The same structures are observed, but the range **depends on the direction**
- Commonly, the vertical range of correlation is much less than the horizontal range due to the formation of 'layering' due to sedimentary processes.
- The ratio of the **horizontal:vertical range** is commonly known as the horizontal to vertical anisotropy ratio
- Geometric anisotropy is common for the horizontal directions also
- The ratio of horizontal **major direction: horizontal minor direction range** is commonly known as the horizontal major to minor anisotropy ratio





Exhaustive data (above) and experimental variograms (below).

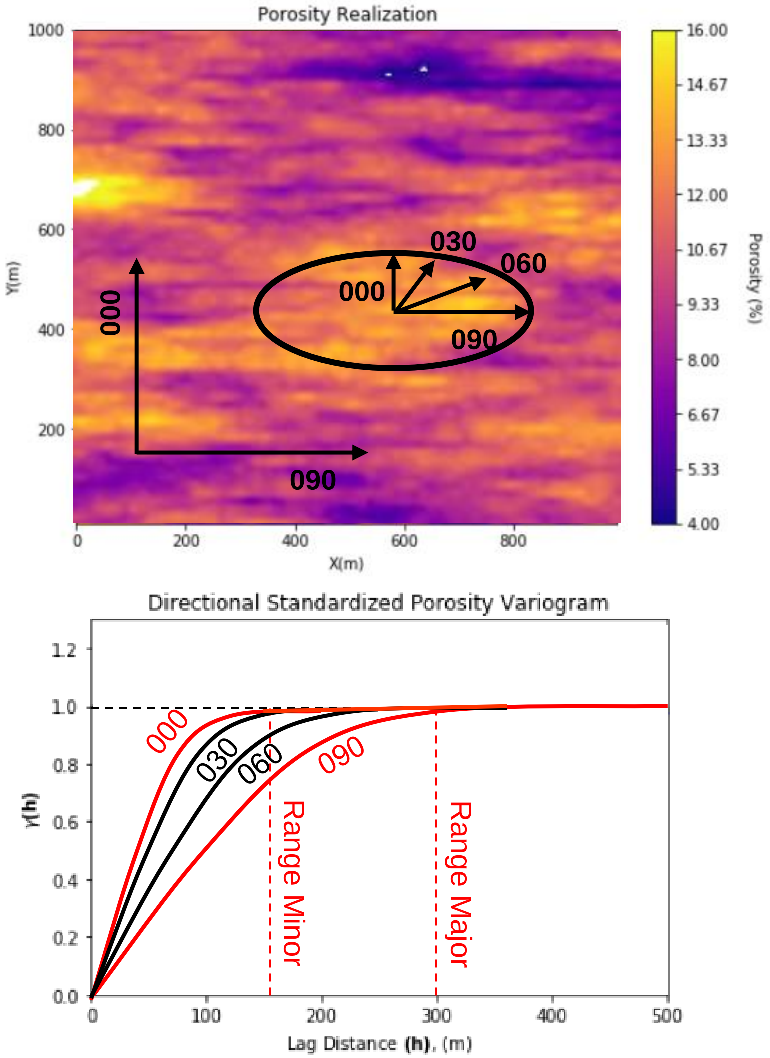
Geometric Anisotropy

Geometric Anisotropy Comments

- We assume geometric anisotropy to model 2D and 3D variogram from experimental variograms calculated in primary directions.

- This model provides a valid interpolation of the variogram between the primary directions.

- This is assumed to build nested variogram models with structures that:
  - Describe components of the variance
  - Act over all directions.



Exhaustive data (above) and experimental variograms (below).

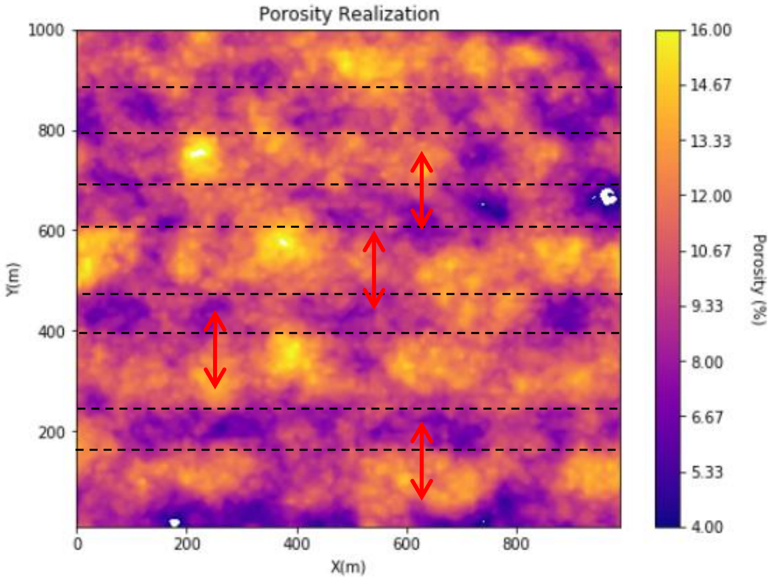
Cyclicity

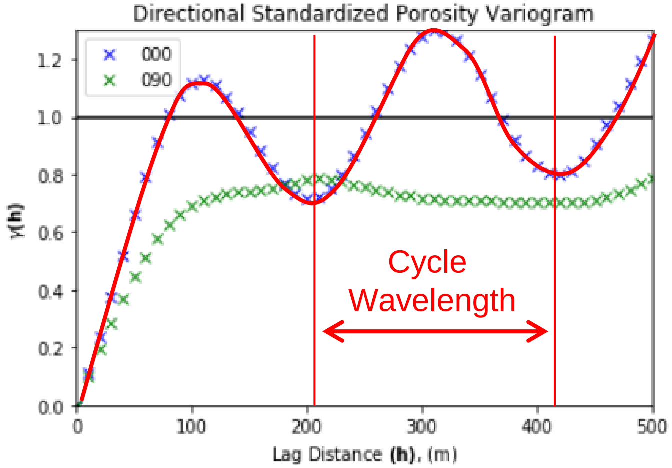
Cyclicity Description

- Cyclicity may be linked to underlying geological periodicity, cycles in the deposition

- Sometimes noise in the experimental variogram due to too few data is mistaken as cyclicity

- The wavelength of the cycles in the experimental variogram is the wavelength of the spatial cycles.



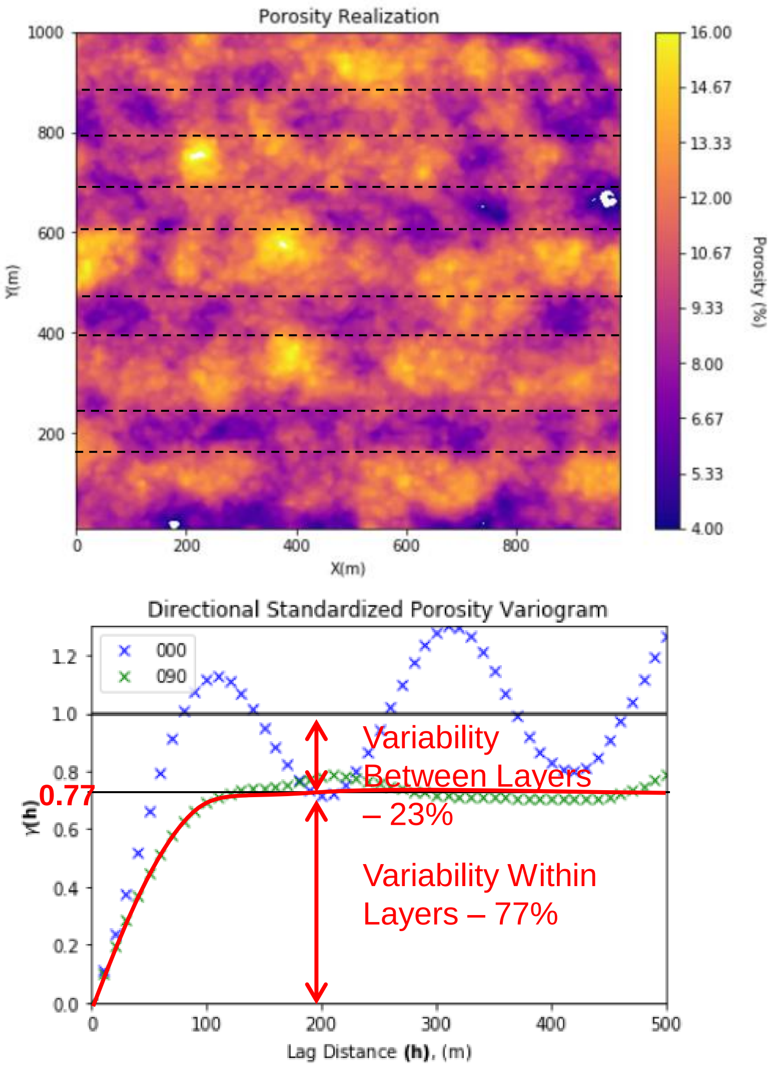


Exhaustive data (above) and experimental variograms (below).

Zonal Anisotropy

Zonal Anisotropy Description

- When the experimental variogram does not reach the sill in a direction
- Often paired with cyclicity or trend in the other (orthogonal) direction.
- The variance at which the variogram levels off is called an apparent sill.

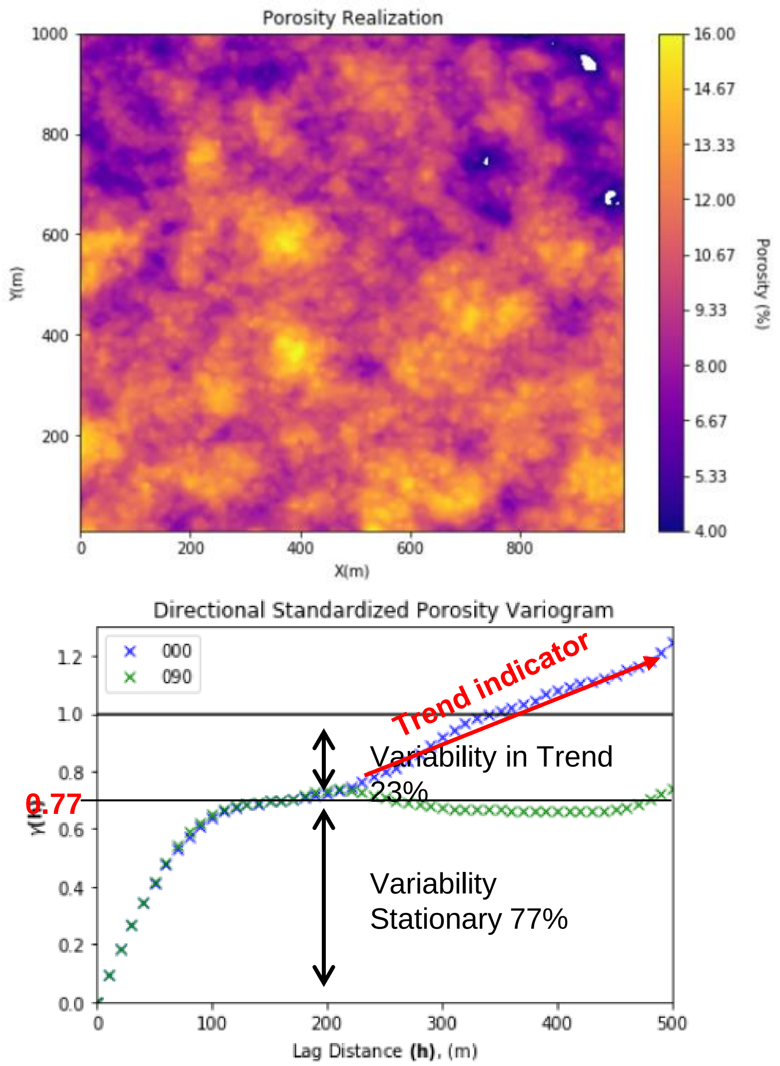


Exhaustive data (above) and experimental variograms (below).

Variogram Trend

Variogram Trend Description

- Experimental variogram points rise approximately linearly above the sill.
- Indicates a trend (fining upward, compacting with depth, etc.)
- Could be interpreted as a fractal, fit a power law function
- May have to explicitly account for the trend in later simulation/modeling
  - Model, remove the trend, work with the residual
  - If the trend is removed the residual variogram will plateau at the sill



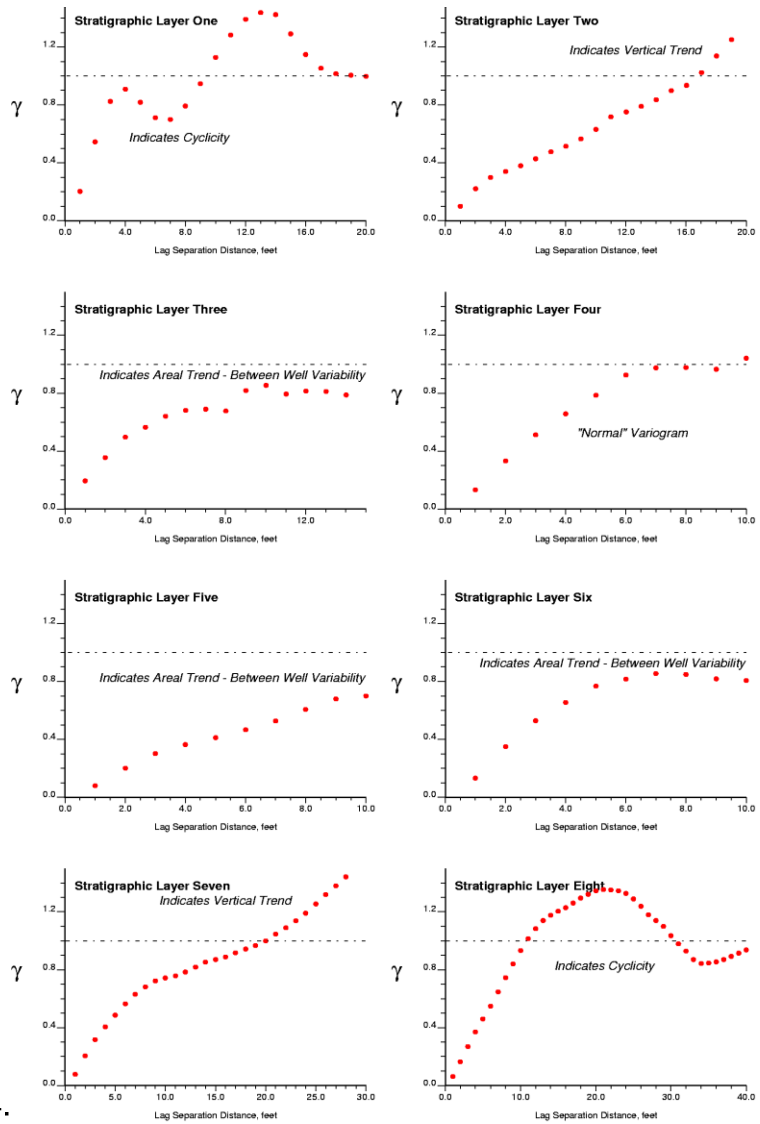
Exhaustive data (above) and experimental variograms (below).

Some Experimental Variograms

Experimental Variograms from Real Data

- Superposition of the four interpretation principles
  - Trend
  - Cyclicity
  - Zonal anisotropy
  - Geometric anisotropy

- Very few variograms are ‘textbook variograms’, natural settings are often more complicated and noisier!

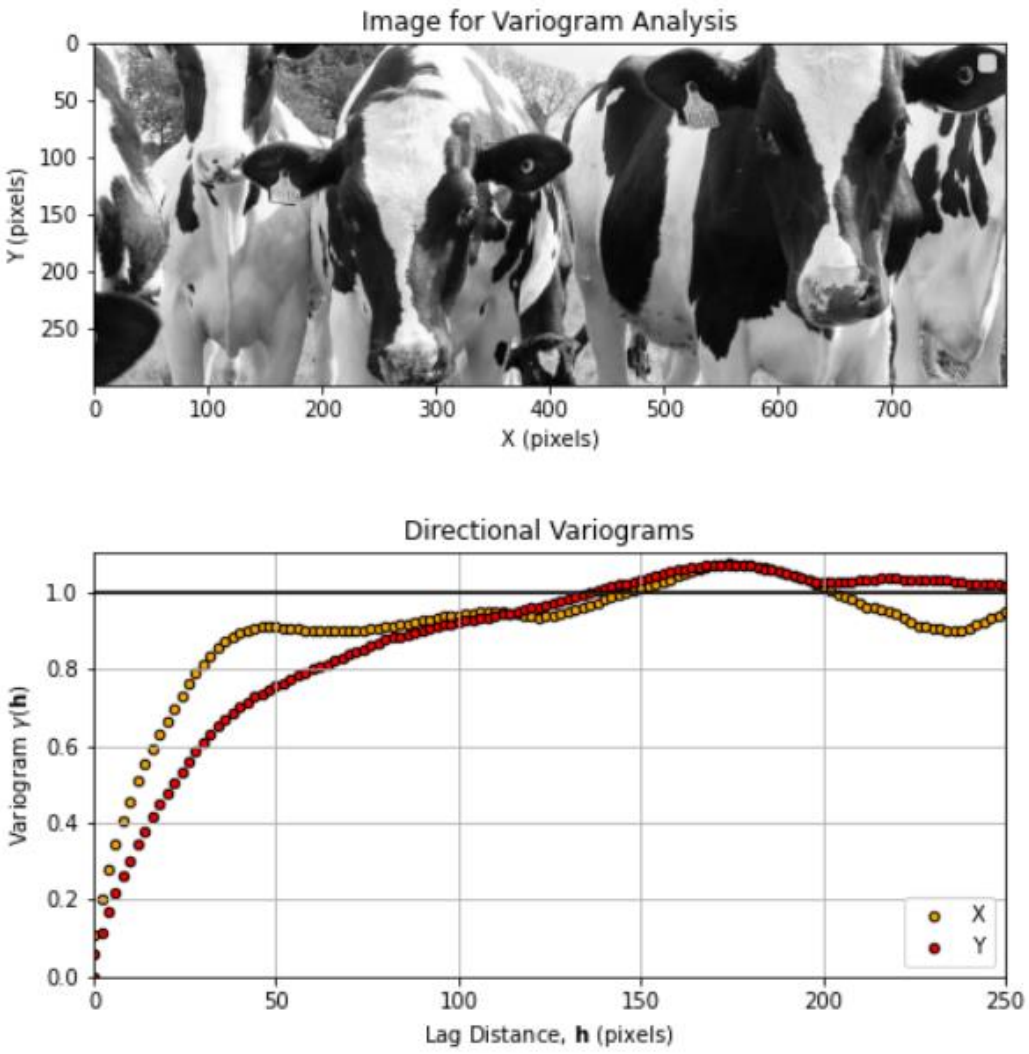


Variogram examples from Pyrcz and Deutsch, 2014.

Variogram Interpretation

Spatial Continuity of Holstein Cows

- Variogram interpretation consists of explaining the variability over different distance scales.
- Here’s a cow image converted to greyscale and cropped to improve stationarity.
- If you want to try your own image, my workflow is GeostatsPy_variogram_from_image.ipynb.



The spatial continuity of cows.

Review of Variogram Interpretation

Variogram Concepts

- Variogram is very important in spatial data analytics / geostatistical study, measure of geological (or other spatial feature) distance vs. spatial
- Initial coordinate and data transformation may be required.
- Interpretation Principles: Trend, Cyclicity, Geometric Anisotropy, Zonal Anisotropy
- Short scale structure is most important
  - nugget due to measurement error should not be modeled
  - size of geological modeling cells
- Vertical direction is typically well informed
  - can have artifacts due to spacing of core data
  - handle vertical trends and areal variations
- Horizontal direction is often not well informed
  - take from an analog field or outcrop
  - use vertical and apply a horizontal:vertical anisotropy ratios

Slide modified from Clayton Deutsch’s MinE 612 Class

Spoiler Alert

We need to practically calculate and model spatial continuity. From the available and often sparse subsurface data.

1. Calculate variogram with irregularly spaced data
   – Search templates with parameters

Complete

2. Valid spatial model
   – Fit with a couple of different, nest (additive) spatial continuity models e.g. nugget, spherical, exponential and Gaussian

3. Full 3D spatial continuity model
   – Model primary directions, i.e. major horizontal, minor horizontal and vertical and combine together with assumption of geometric anisotropy

Up Next

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General Concepts

Univariate

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- Variogram Modeling
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- Simulation

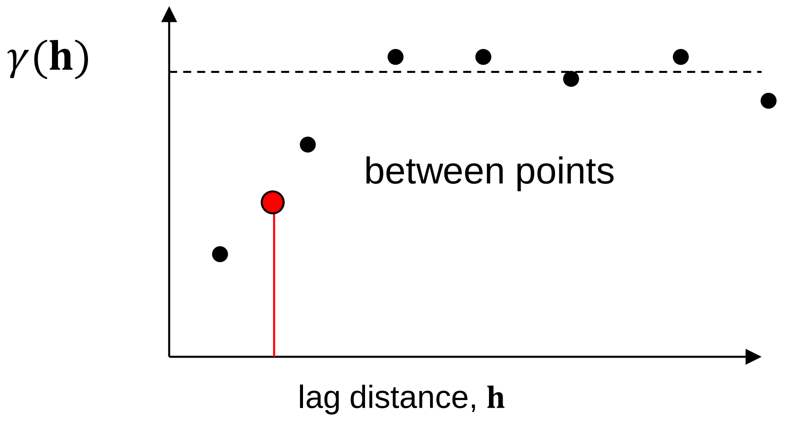
Time Series

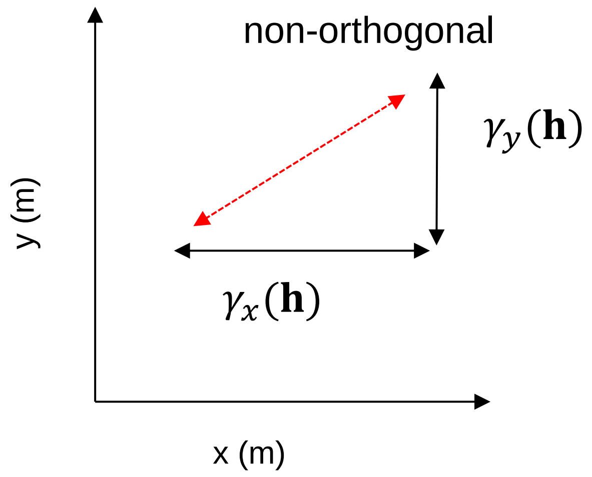
Machine Learning

Uncertainty Analysis

Reasons for Variogram Modeling

1. Need to know the variogram for *all* possible **h** lags, distances and directions – not just the ones calculated





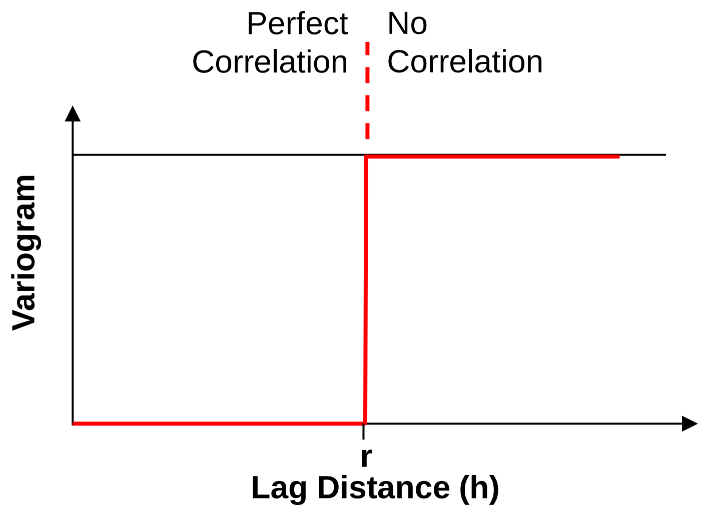
We need the variogram for all distances (left) and directions (right).

2. Incorporate additional geological knowledge (such as analogue information or information on directions of continuity …)
3. The variogram model must be **positive definite** (a legitimate measure of distance), that is, the variance of any linear combination must be positive

Slide modified from Clayton Deutsch's MinE 612 Class

Reasons for Variogram Modeling

Extreme Example to Demonstrate the Need for Using Positive Definite Variogram

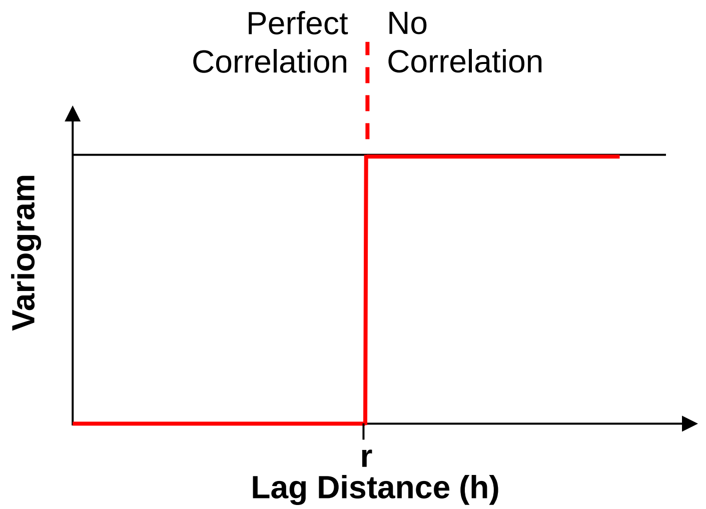


The proposed 'Pyrcz Variogram' Structure

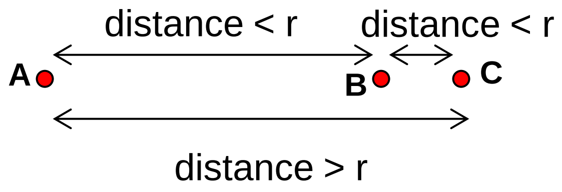
Positive-definite variogram models ensure for all possible spatial configurations that there are NO paradoxes.

Reasons for Variogram Modeling

Extreme Example to Demonstrate the Need for Using Positive Definite Variogram



The proposed ‘Pyrcz Variogram’ Structure



A and B and B and C are perfectly correlated,
but A and C are not correlated!

-Spatial paradox!

Positive-definite variogram models ensure for all possible spatial configurations that there are NO paradoxes.

Applications for Variogram Models

What can we do with variogram models? The variogram model is used:

1. in **Kriging** (next section) for spatial estimation

2. in **Kriging** (next section) to calculate spatial uncertainty

$$\text{estimation variance} = \quad \sigma_{x^*}^2 = \sigma_x^2 - \sum_{\alpha=1}^{n} \lambda_\alpha C_x(\mathbf{u}_\alpha - \mathbf{u}_0) \geq 0$$

3. in **Sequential Gaussian Simulation** (topic 13) to build heterogeneity realizations.

Now let's introduce the common set of permissible variogram models, to model spatial continuity for all distances and directions.

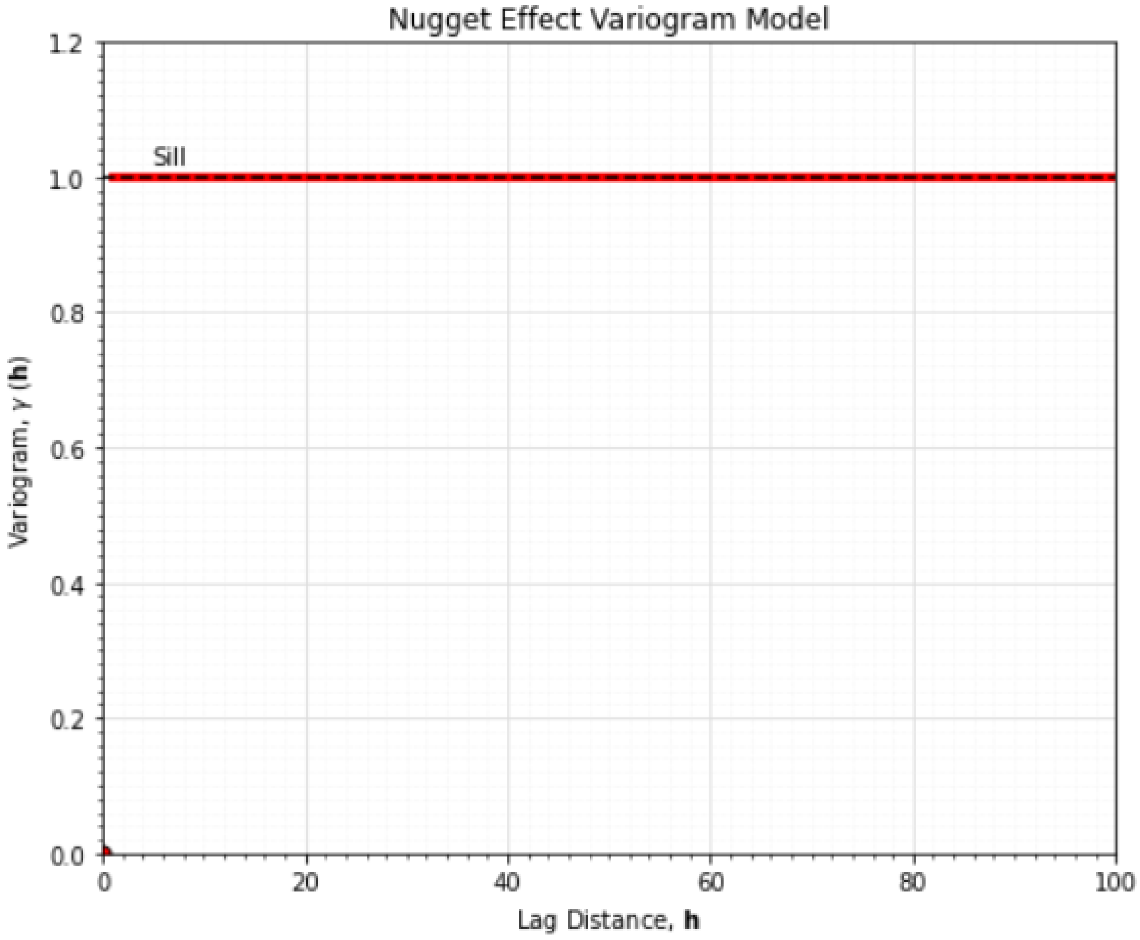
Common Variogram Models

Nugget Effect Variogram Model

- No spatial correlation
- Does not have a range, nor directionality, i.e., acts over all distances and directions.
- Should be a small component of the overall variance
  - Very uncommon in oil and gas
  - More common for mineral grades in mining
- The equation:

$$\gamma(\mathbf{h}) = C_1 \cdot Nugget = \begin{cases} 0 & ,if\ h = 0 \\ C_1 & ,if\ h > 0 \end{cases}$$

- where h is the lag distance, and there is no range parameter



Nugget effect variogram model (red line and point), file is GeostatsPy_variogram_models.ipynb.

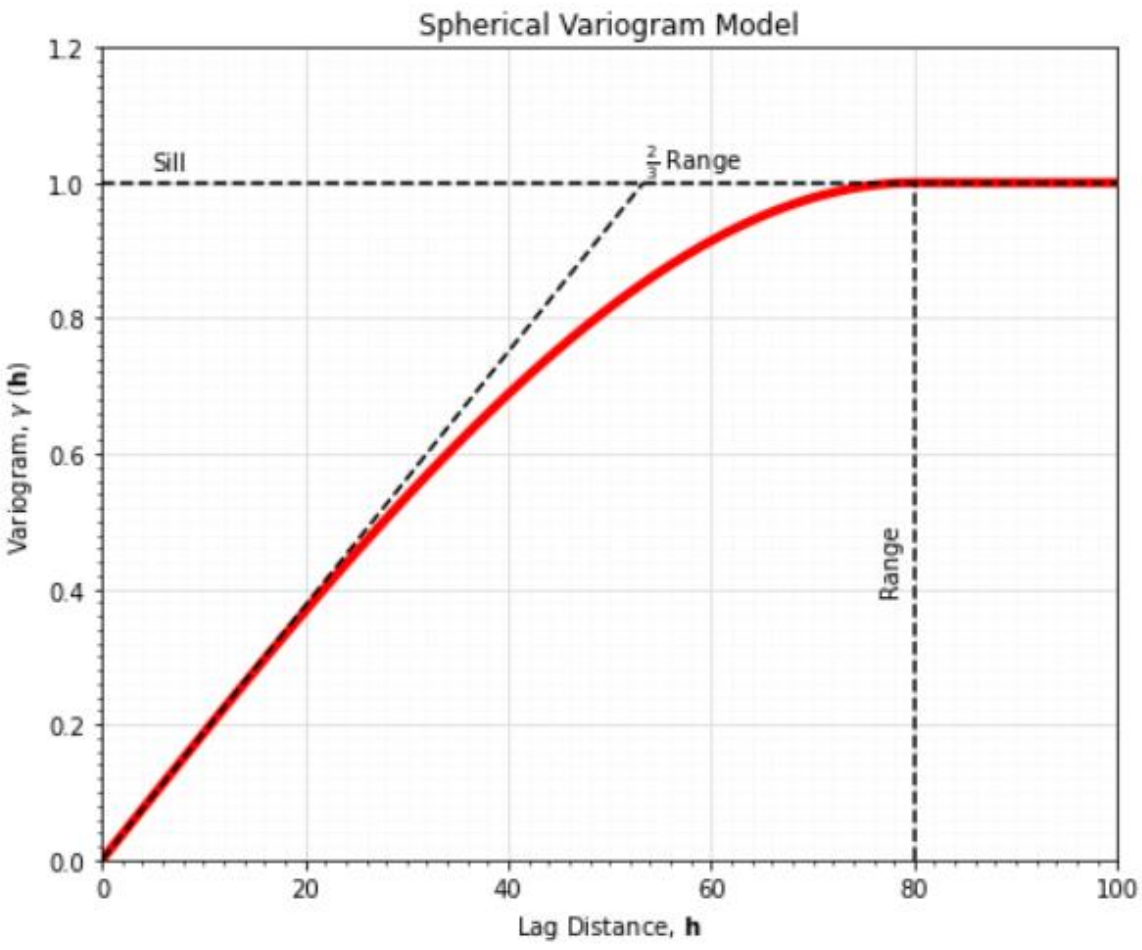
Common Variogram Models

Spherical Variogram Model

- A very commonly observed variogram / spatial continuity form
- Peicewise, beyond the range is equal to the sill
- The equation:

$$\gamma(\mathbf{h}) = C_1 \cdot Sph\left(\frac{\mathbf{h}}{a}\right) = \begin{cases} C_1 \cdot \left[1.5\left(\frac{\mathbf{h}}{a}\right) - 0.5\left(\frac{\mathbf{h}}{a}\right)^3\right] & , if\ h < a \\ C_1 & , if\ h \geq a \end{cases},$$

- where c_1 is the contribution, a is the range and $\mathbf{h}$ is the lag distance



Spherical variogram model (red line and point), file is GeostatsPy_variogram_models.ipynb.

Common Variogram Models

Exponential Variogram Model

- Also very commonly observed variogram / spatial continuity form

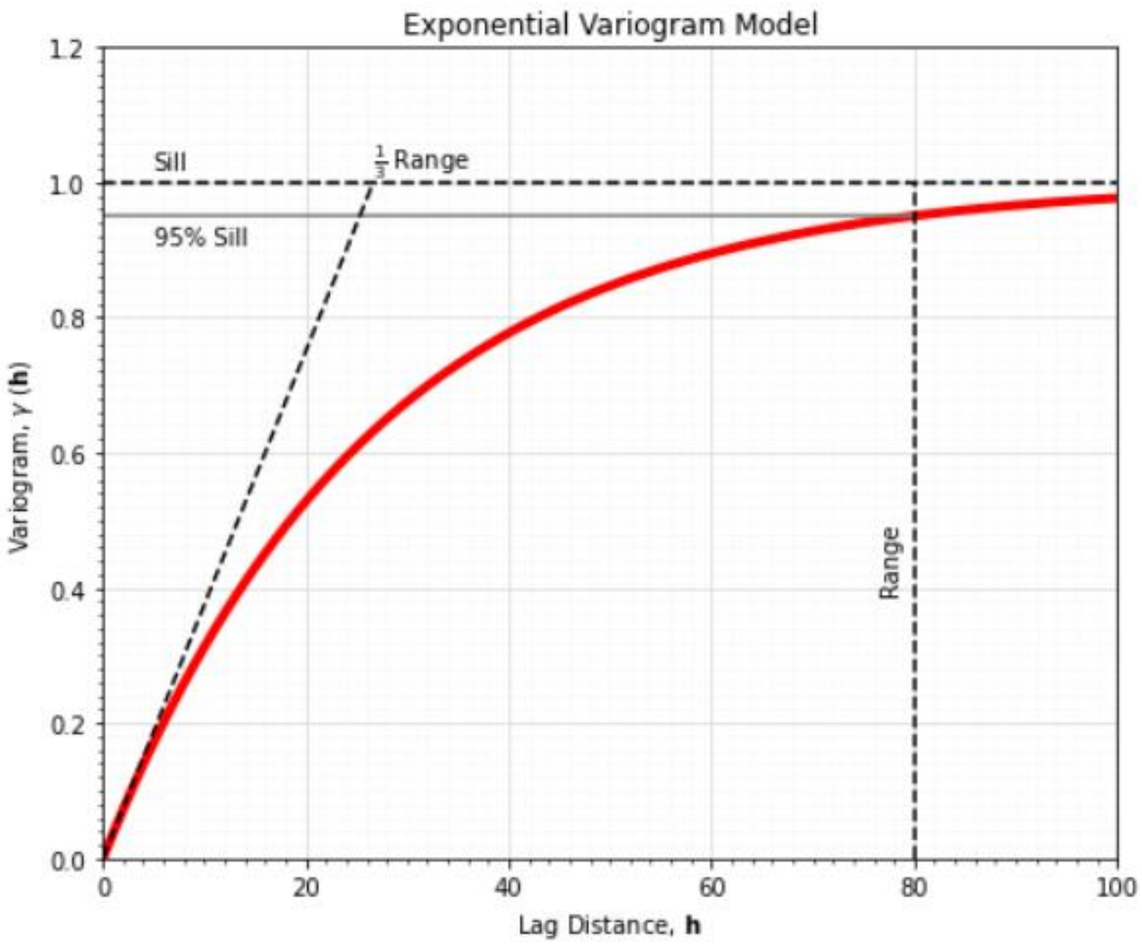
- Less short-scale continuity than spherical, and reaches sill asymptotically, range is at 95% of the sill

- The equation:

$$\gamma(\mathbf{h}) = C_1 \cdot Exp\left(\frac{\mathbf{h}}{a}\right) = C_1 \cdot \left[1.0 - exp\left(-3\left(\frac{\mathbf{h}}{a}\right)\right)\right]$$

,

- where c_1 is the contribution, a is the range and $\mathbf{h}$ is the lag distance



Exponential variogram model (red line and point), file is GeostatsPy_variogram_mmodels.ipynb.

Common Variogram Models

Gaussian Variogram Model

- Less commonly observed variogram / spatial continuity form, e.g., for thickness and elevation

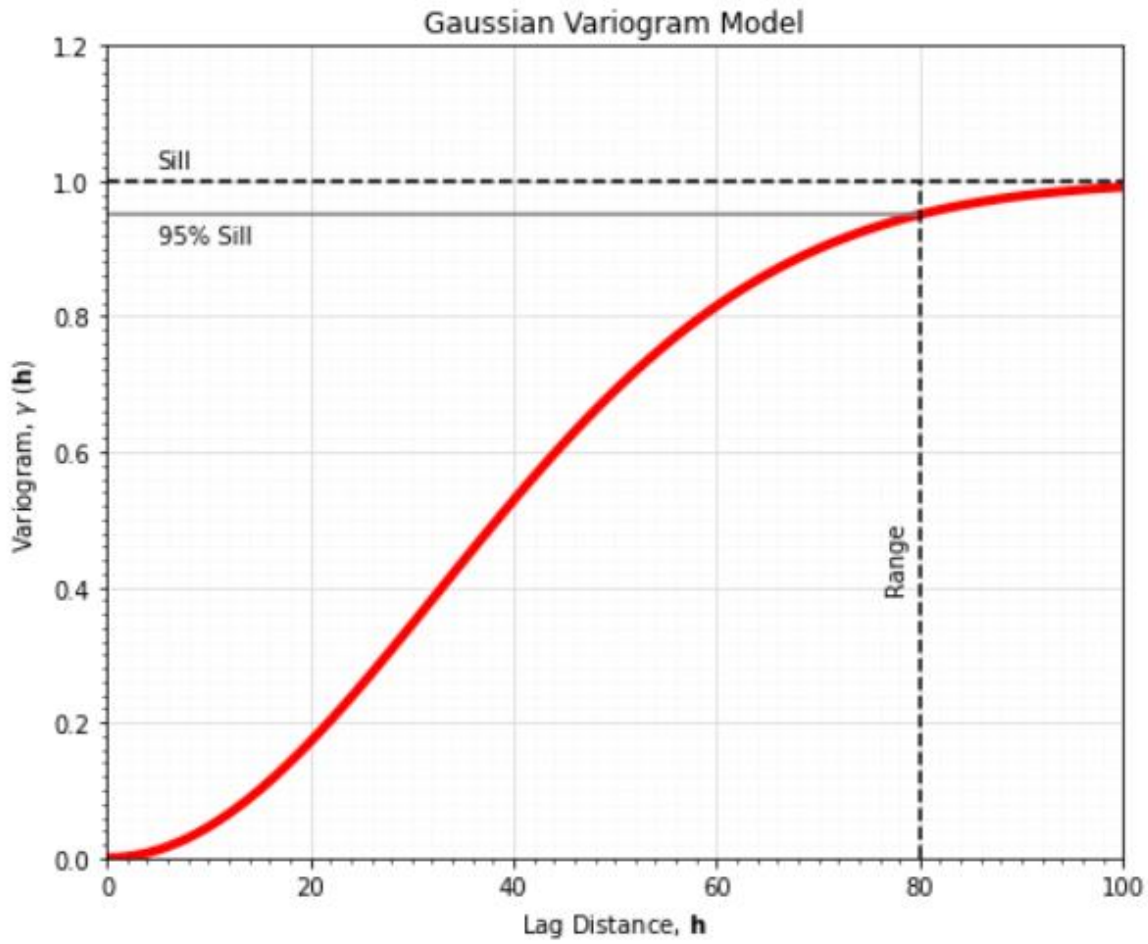
- More short-scale continuity than spherical, and reaches sill asymptotically, range is at 95% of the sill

- The equation:

$$\gamma(\mathbf{h}) = C_1 \cdot Gaus\left(\frac{\mathbf{h}}{a}\right) = C_1 \cdot \left[1.0 - \exp\left(-3\left(\frac{\mathbf{h}}{a}\right)^2\right)\right]$$

,

- where c_1 is the contribution, a is the range and $\mathbf{h}$ is the lag distance



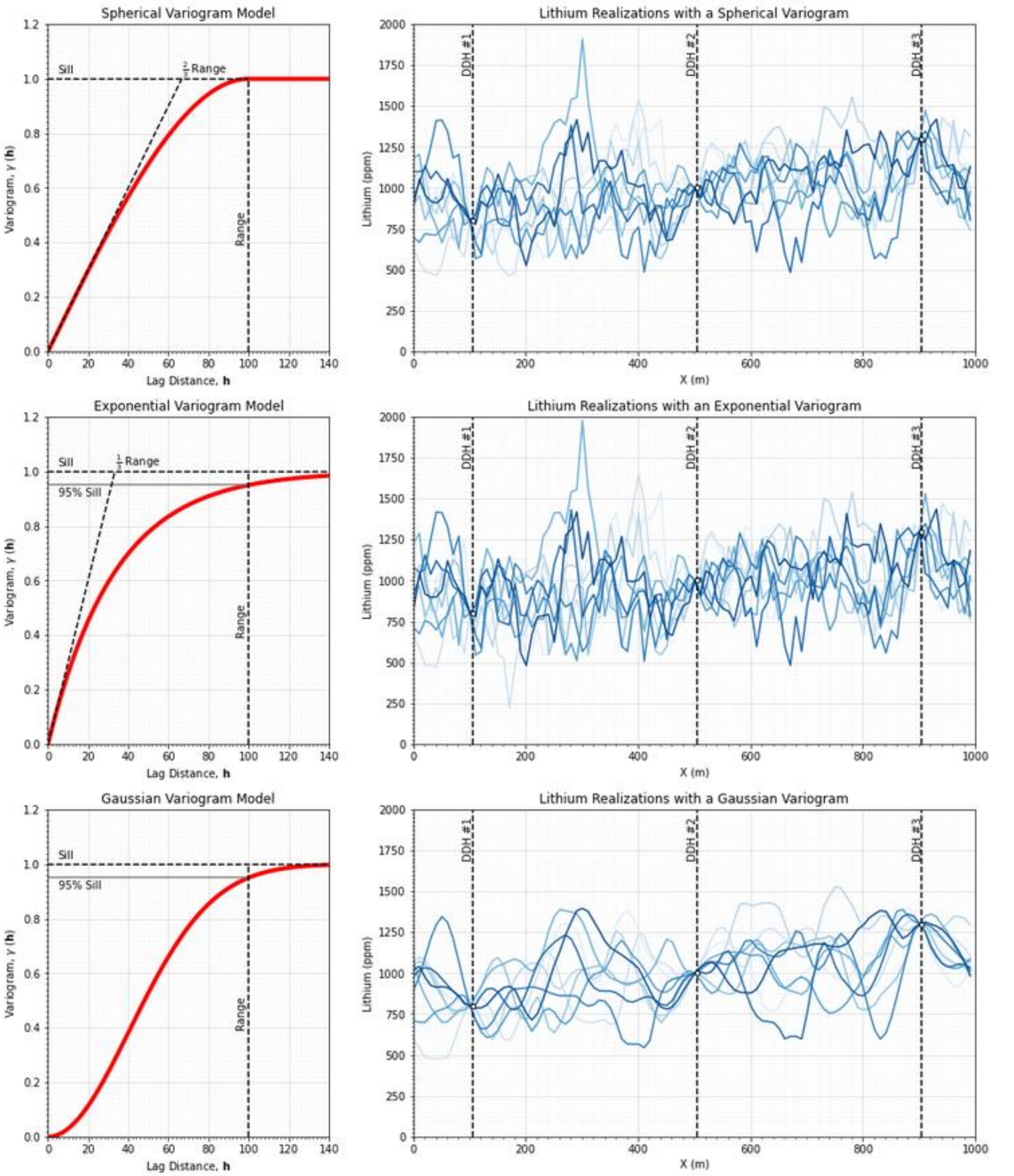
Exponential variogram model (red line and point), file is GeostatsPy_variogram_mmodels.ipynb.

Common Variogram Models

Here's an example of lithium grade simulation:

- spherical
- exponential
- Gaussian

variogram models.



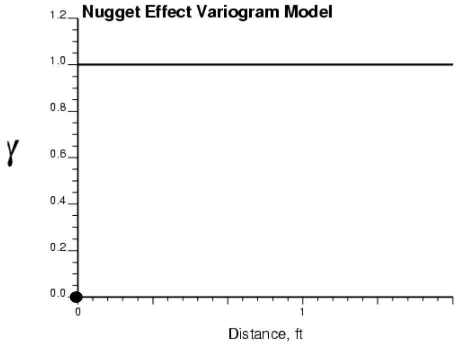
1D realizations of lithium with spherical (upper), exponential (center) and Gaussian (lower) variogram models.

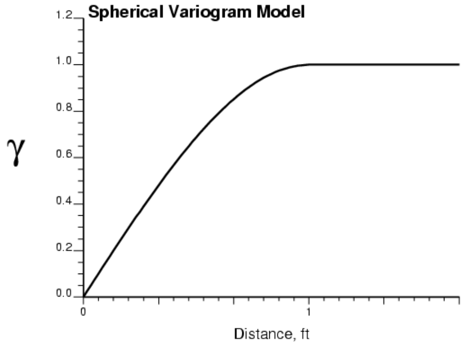
File is GeostatsPy_variogram_models_simulation.ipynb.

Common Variogram Models

No spatial correlation

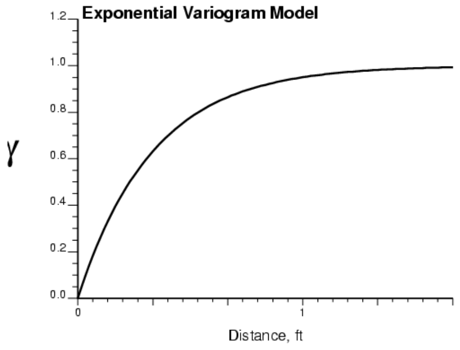
Should be a small component of the overall variance

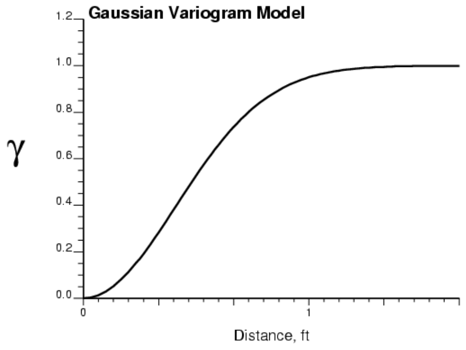




Commonly encountered variogram shape

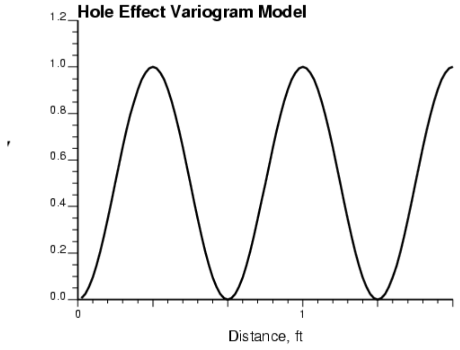
Similar to spherical but rises more steeply and reaches the sill asymptotically

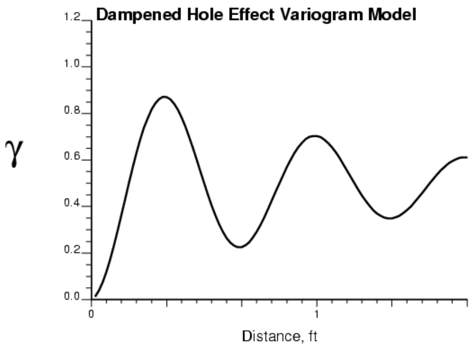




Implies short scale continuity; parabolic behavior at the origin, instead of linear

For periodic variables





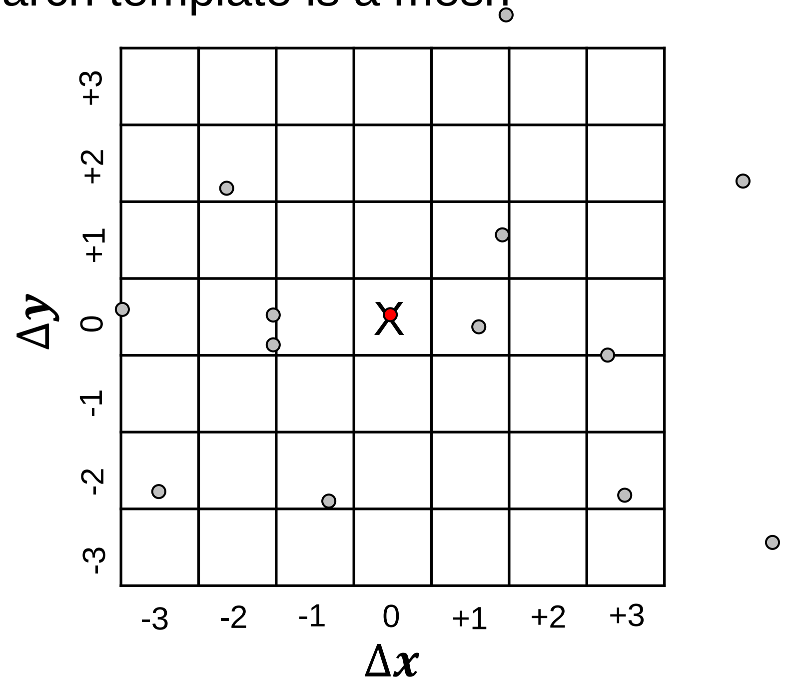
For periodic variables, when the period is not regular

Variogram examples from Pyrcz and Deutsch, 2014.

Recall: Variogram Map

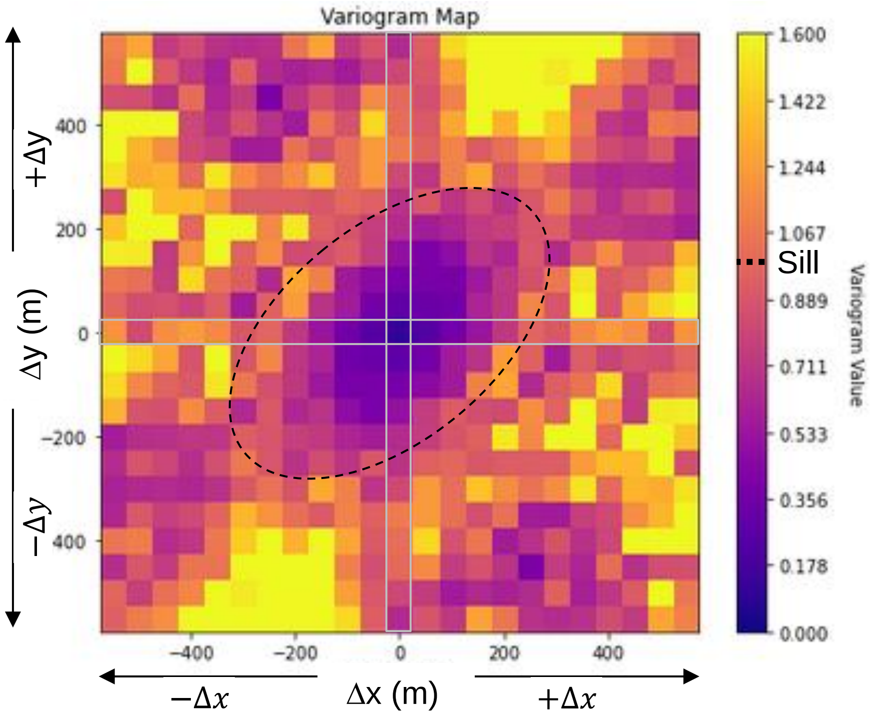
Calculate the variogram for all possible distances and directions.

Search template is a mesh



Variogram map search template.

Cell size is the search tolerance. Number of cells in each direction is number of lags.



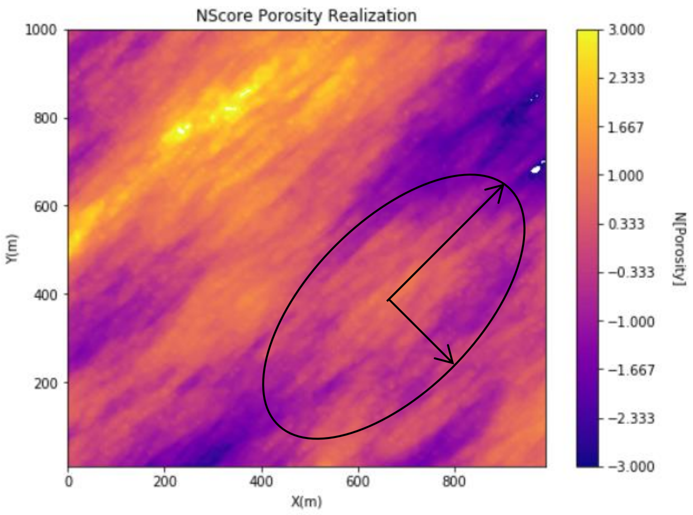
Variogram map.

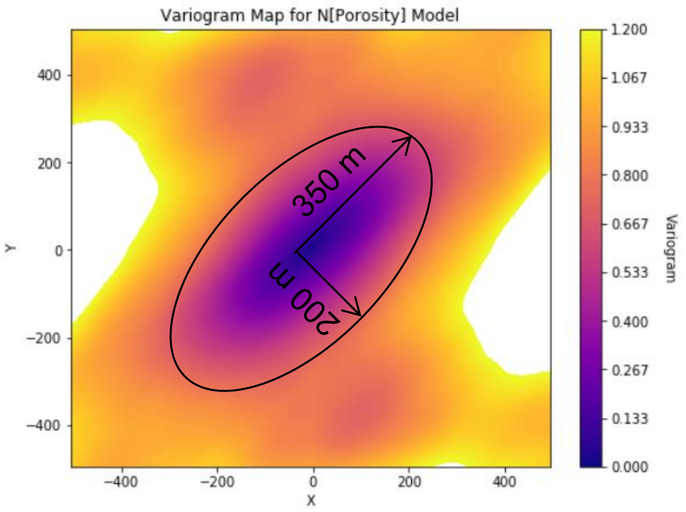
If there is enough data, can often visualize the major and minor directions and the anisotropy range, assists with interpretation and modeling.

But, we still model the variogram in the major and minor directions.

2D Variogram Models

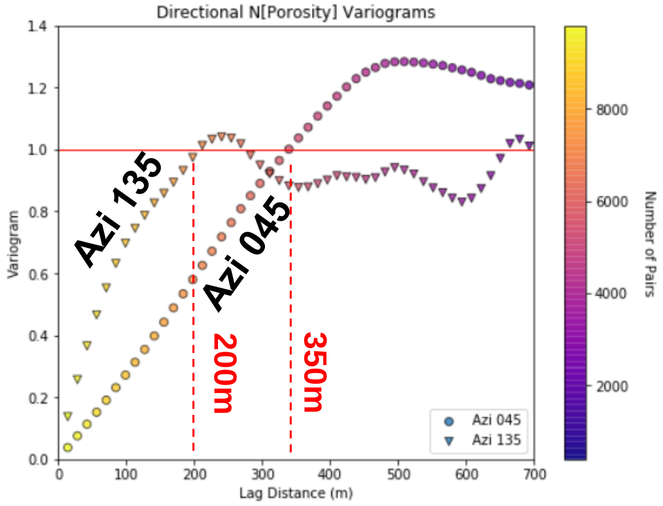
For 2D and 3D variograms, we model major and minor directions (orthogonal).



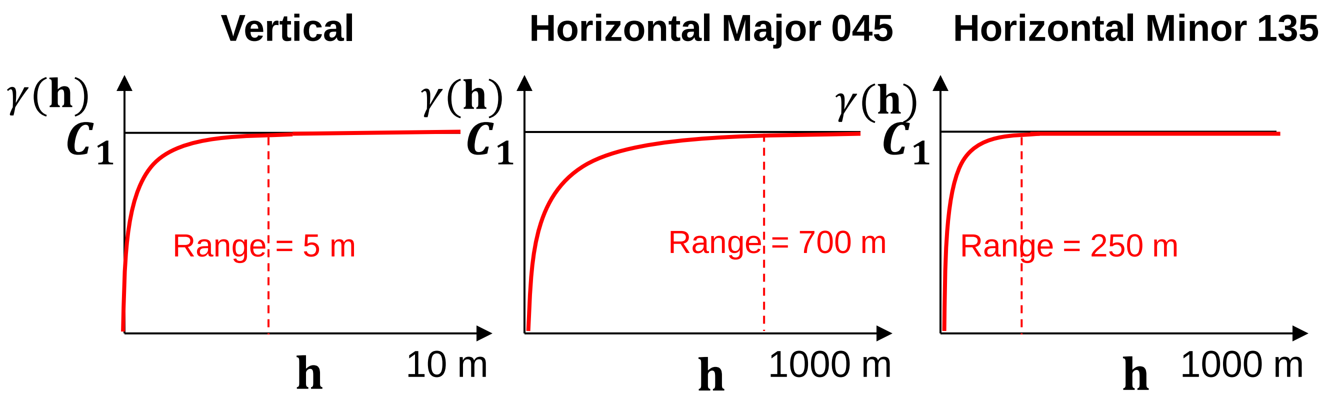


We then assume an ellipsoidal variation in continuity (geometric anisotropy):

- Parameters for a 2D variogram model:
    - direction, major and minor range, type of variogram structure (e.g., nugget, spherical, etc.)



Nested Variograms



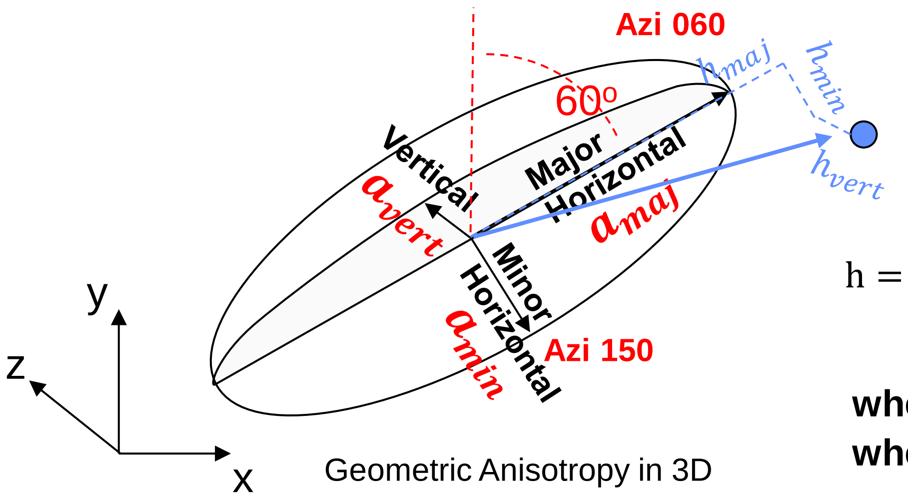
Variable range in each principal direction of a single variogram structure.

For each variogram structure you determine the:

- Contribution, amount of variance described
- Orientation, azimuth, and also dip and plunge if 3D
- Type of Structure (nugget, spherical, etc.)
- Range in the primary directions, major, minor, and vertical (geometric anisotropy).
- Note: a variogram structure is represented by a single model:
  - nugget, spherical, exponential, or Gaussian

2D / 3D Variogram Models

The variation of range along different directions is modeled using an ellipse in 2D and an ellipsoid in 3D



Geometric Anisotropy in 3D

Geometric Distance

$$h = \sqrt{\left(\frac{h_{maj}}{a_{maj}}\right)^2 + \left(\frac{h_{min}}{a_{min}}\right)^2 + \left(\frac{h_{vert}}{a_{vert}}\right)^2}$$

where a_{maj}, a_{min} and a_{vert} are range parameters
where h_{maj}, h_{min} and h_{vert} are lag components

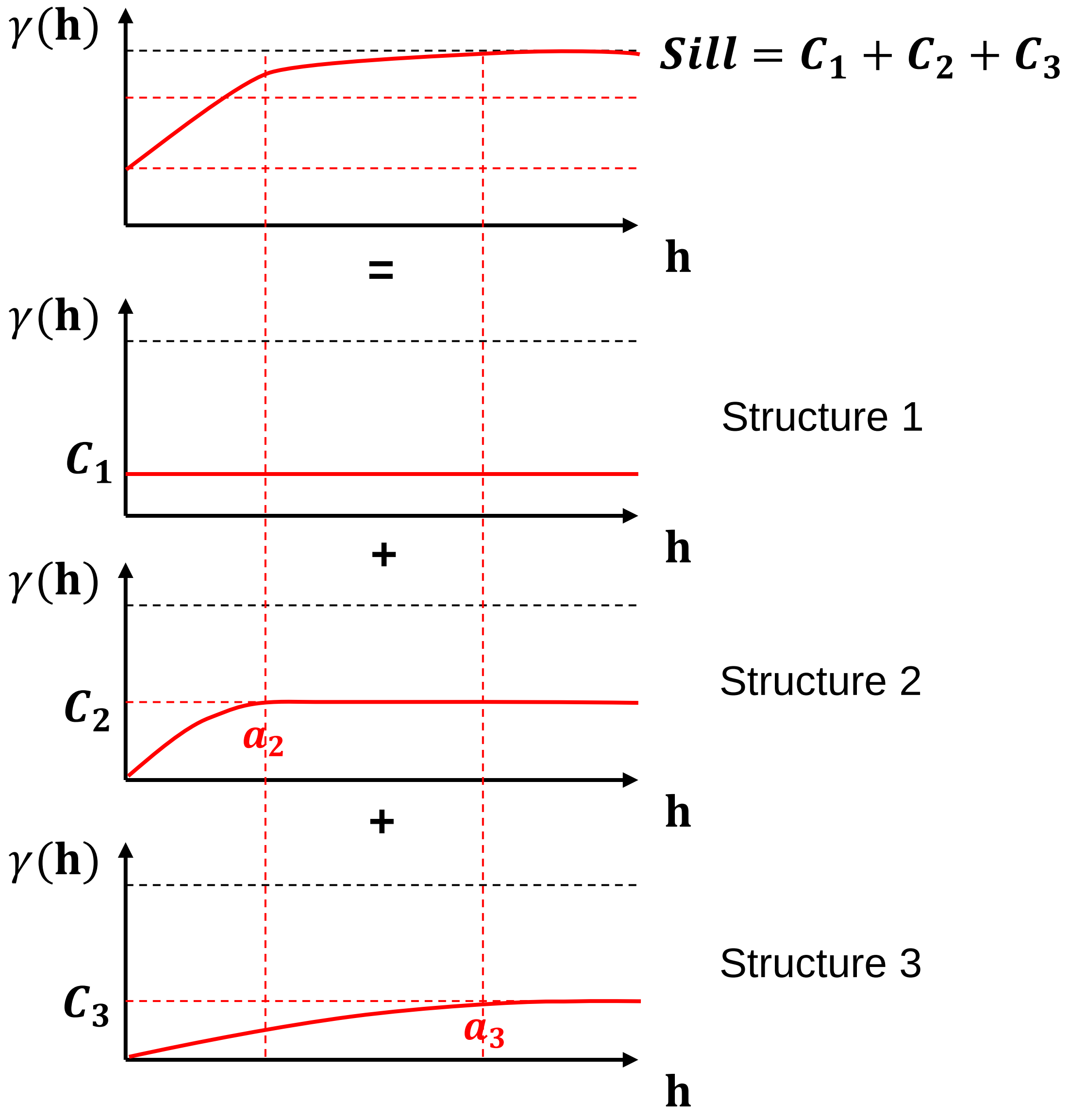
There is an ellipsoidal variation in continuity (geometric anisotropy):

- Parameters for a 2D variogram model:
  - direction, major, minor, contribution and type of variogram
- Parameters for a 3D variogram model:
  - direction, dip, major, minor and vertical range, contribution and type of variogram

Variograms Models with Nested Structures

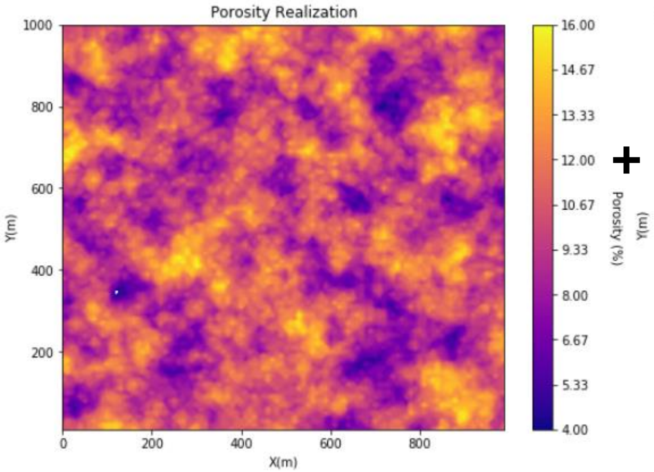
Nested Variogram Models

- The addition of positive definite variogram structures is positive definite.
- Each structure covers a proportion of the sill.
- For each structure we can change the:
  - orientation
  - range in major and minor (and vertical)
- We are spatially explaining components of the variance!



Nested Spatial Frequencies

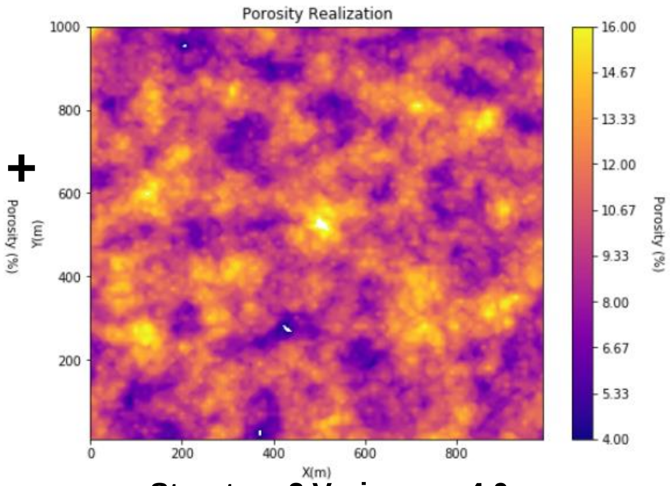
Isotropic 100 m Range, Sill = 4



Structure 1 Variance = 4.0

+

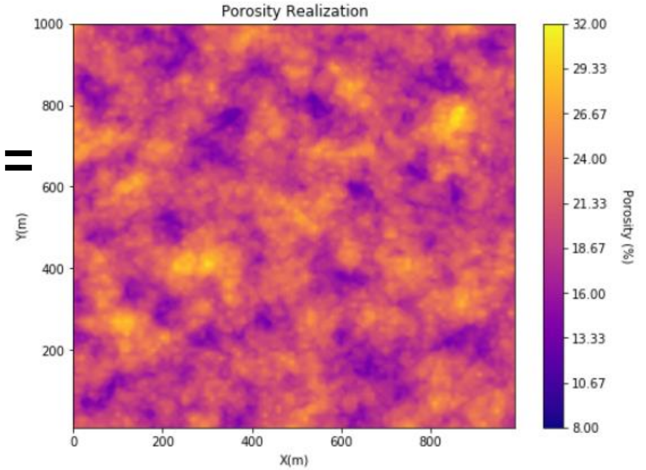
Isotropic 100 m Range, Sill = 4



Structure 2 Variance = 4.0

=

Isotropic 100m Range, Sill = 8



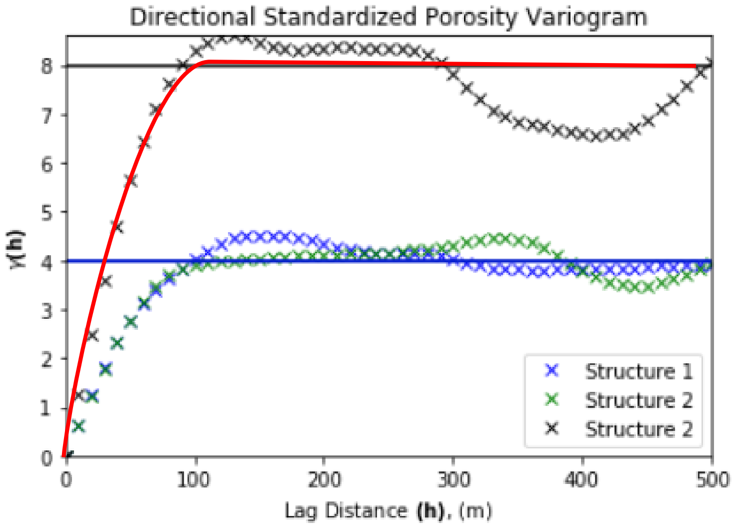
Structure 1+2 Variance = 8.0

Superposition of the same structure only changes the sill.

- The range stays the same:

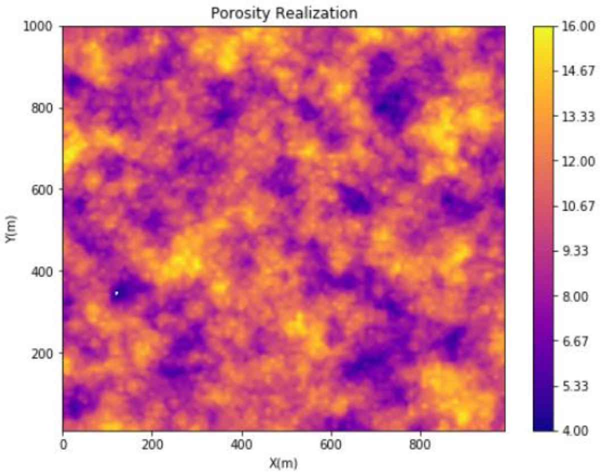
$$(C_0+C_1)\,\gamma_0(h) = C_1\gamma_0(h) + C_2\gamma_0(h)$$

- If you have a single structure in a direction just use the same range for all contributions.



Nested Spatial Frequencies

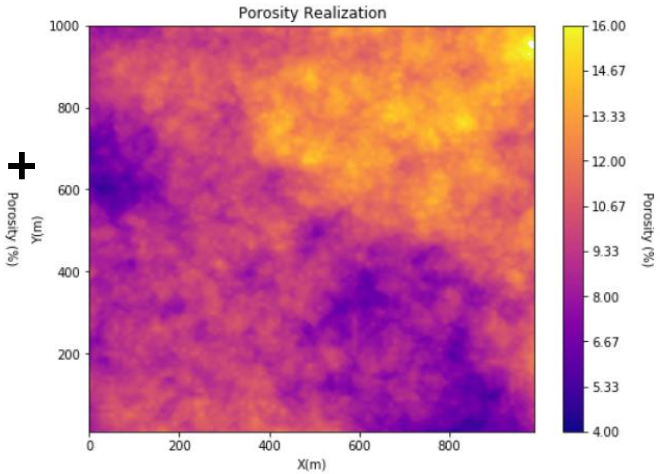
Isotropic 100 m Range, Sill = 4



Structure 1 Variance = 4.0

+

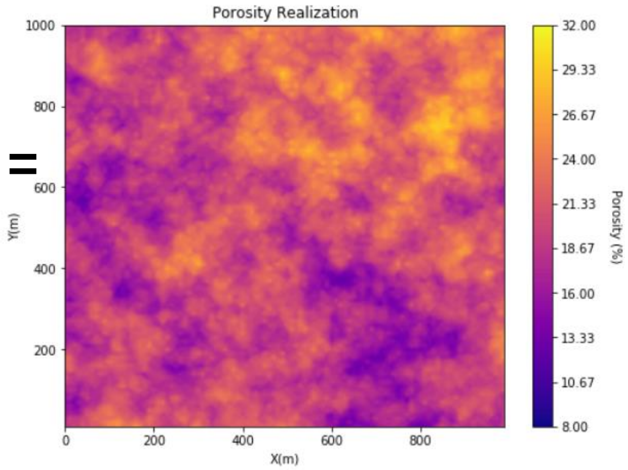
Isotropic 400 m Range, Sill = 4



Structure 2 Variance = 4.0

=

Isotropic Nested 400m Range, Sill = 8



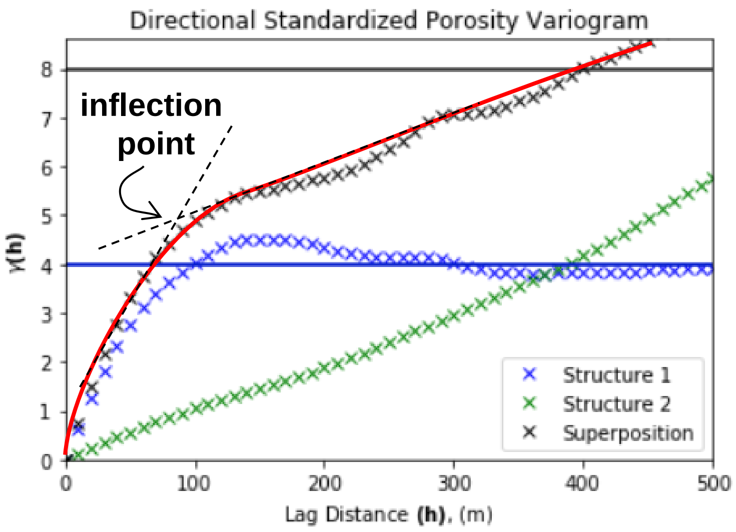
Structure 1+2 Variance = 8.0

Superposition with different ranges

- New nested model:

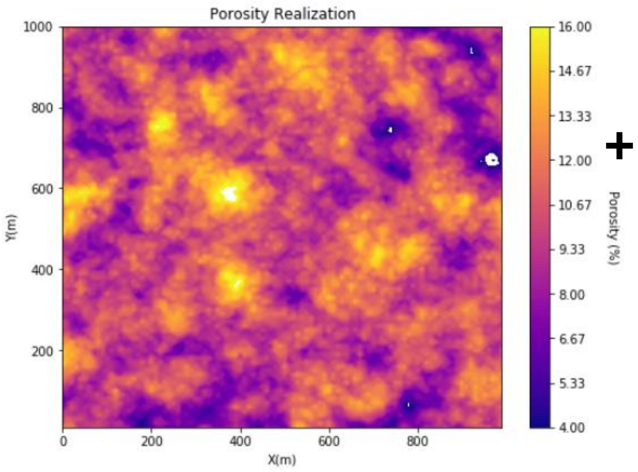
$$\gamma_{tot}(h) = C_1\gamma_1(h) + C_2\gamma_2(h)$$

- Forms an **inflection point** due to the combination of dissimilar ranges.



Nested Spatial Frequencies

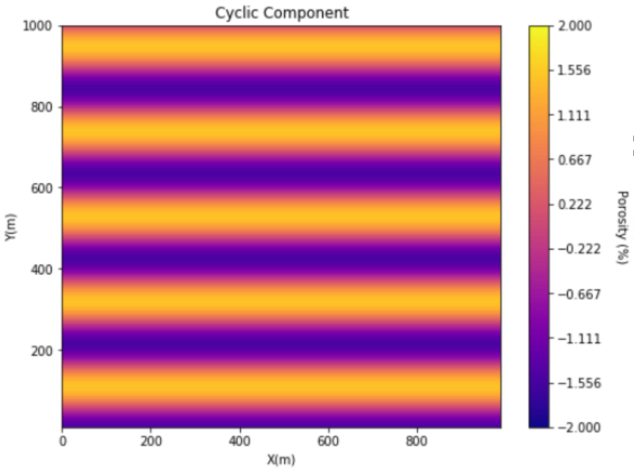
Isotropic 100 m Range



77% Variance

+

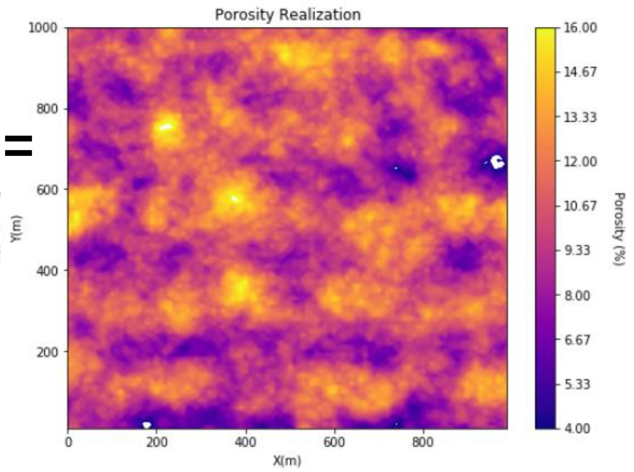
Cyclic Trend in Y 200 m Wavelength



23% Variance

=

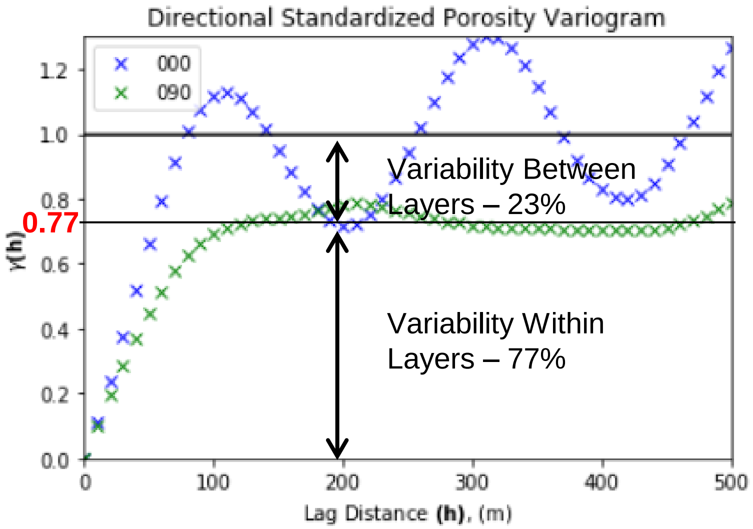
Isotropic 100m Range + Cyclic 200m



100% Variance

Variance within each spatial component is the contribution of each variogram structure.

- Superposition of multiple spatial structures each describing a proportion of the total variability

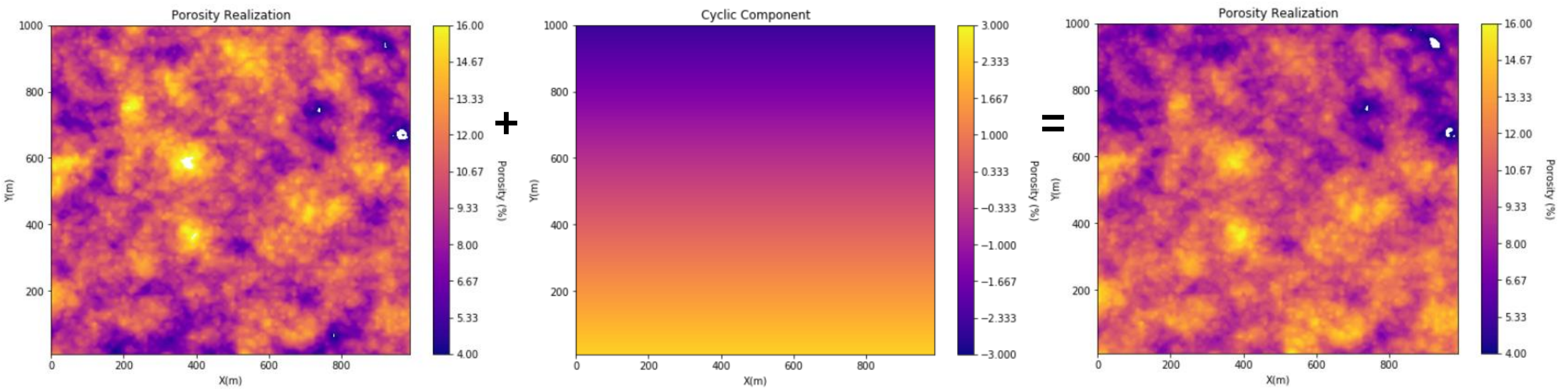


Nested Spatial Frequencies

Isotropic 100 m Range

Trend in Y

Isotropic 100m Range + Trend in Y



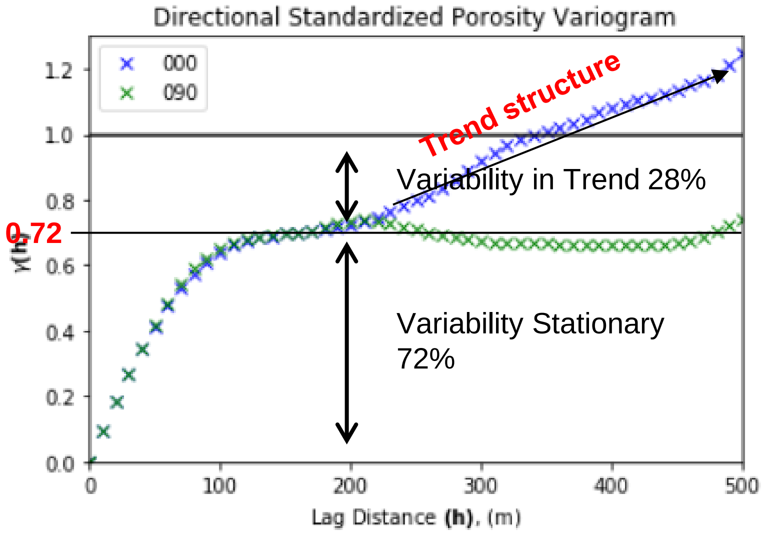
72% Variance

28% Variance

100% Variance

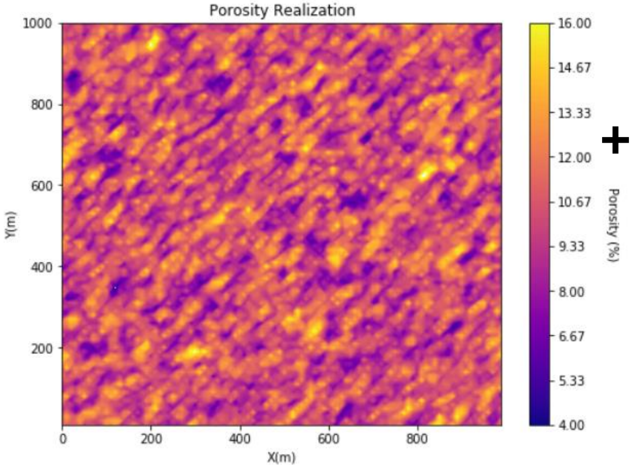
Variance within each spatial component is the contribution of each variogram structure.

- This illustrates the partitioning of spatial variance between trend and stationary, stochastic residual.



Nested Spatial Frequencies

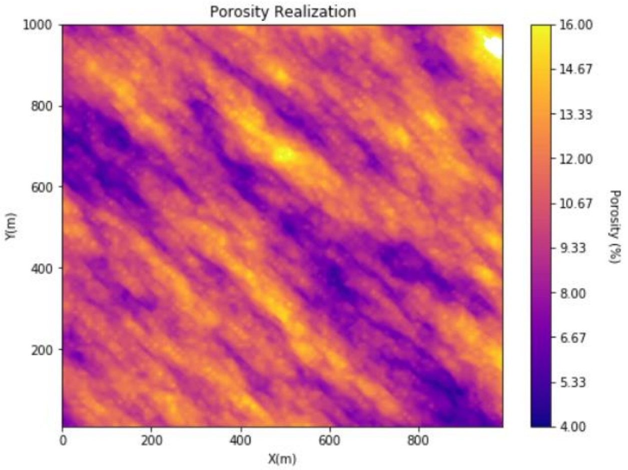
Anisotropic 045 50/20 m Range



50% Variance

+

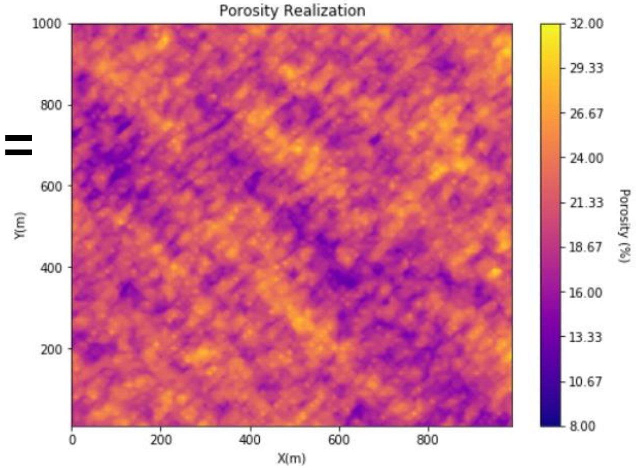
Anisotropic 135 300/100 m Range



50% Variance

=

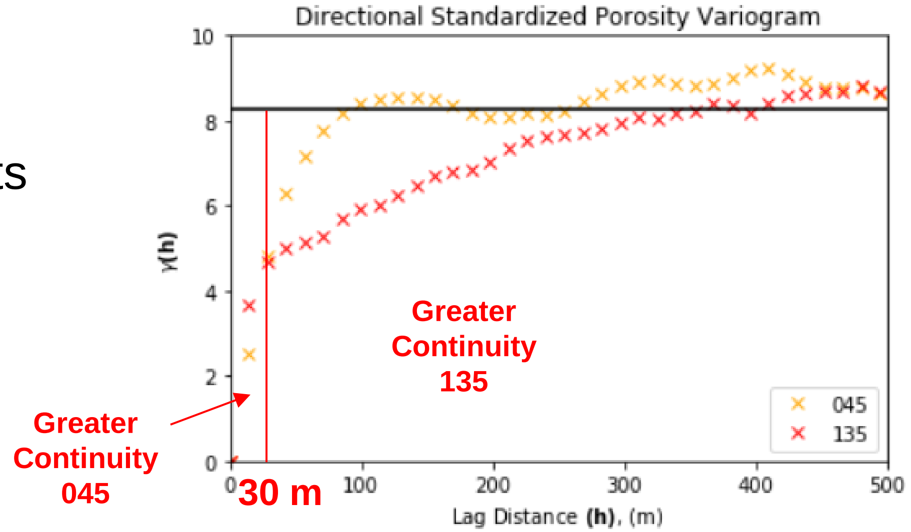
Multiscale, Orientation Variant



100% Variance

Variance within each spatial component is the contribution of each variogram structure.

- In this example the primary direction of continuity shifts over distance!
- < 30 m 045 has greater continuity
- > 30 m 135 has greater continuity



Variogram Modeling

- The following procedure is used to ensure a legitimate model:
  - Pick a single (lowest) isotropic nugget effect
  - Choose the same number of variogram structures for all directions based on the most complex direction
  - Ensure that the same contribution parameter is used for each variogram structure in all directions
  - Allow a **different range parameter** in each direction
  - Model a zonal anisotropy by setting a very large range parameter in one or more of the principal directions

- The responsibility is yours, but most software helps a little
  - force the same structure and contributions in all directions, let you modify the range and observe the result in the 3 primary directions.

Variogram Modeling Demonstration

Michael J. Pyrcz, Associate Professor
The University of Texas at Austin

Steps to modeling a 3D variogram from directional experimental variograms:

- Calculate the experimental variogram in the orthogonal principal directions, in 3D:

Directional Experimental Variograms

Vertical | Horizontal Major | Horizontal Minor

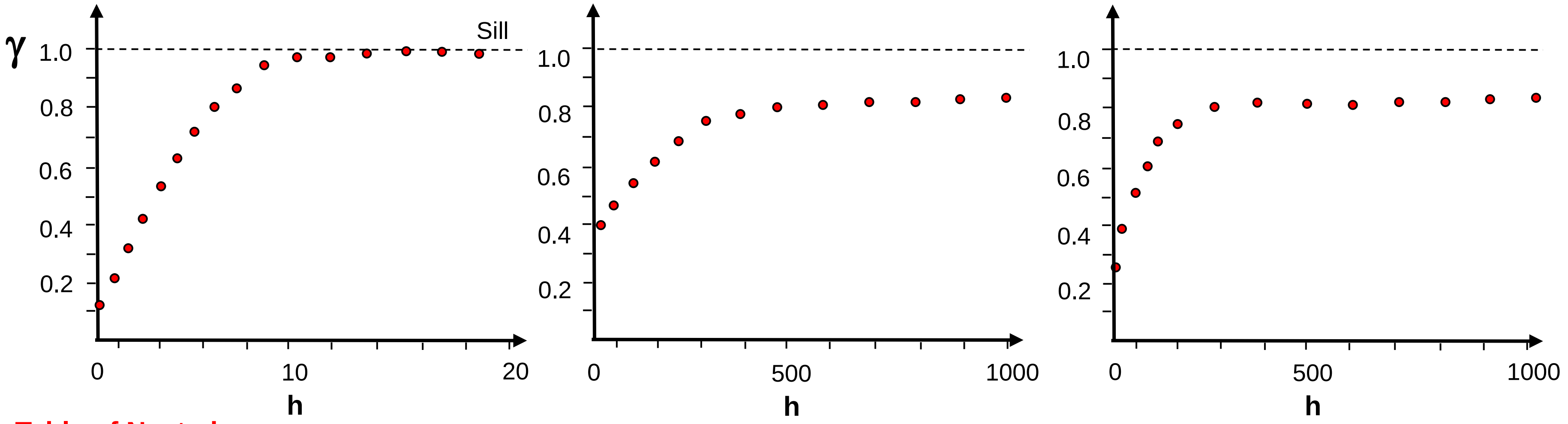


Table of Nested Variogram Model Parameters

Nested, additive variogram structures

Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1					
2	Nugget, Spherical, Exponential, or Gaussian	Variance explained	Range in Horizontal Major Direction	Range in Horizontal Minor Direction	Range in Vertical Direction
3					
4		Must sum to sill			
5					

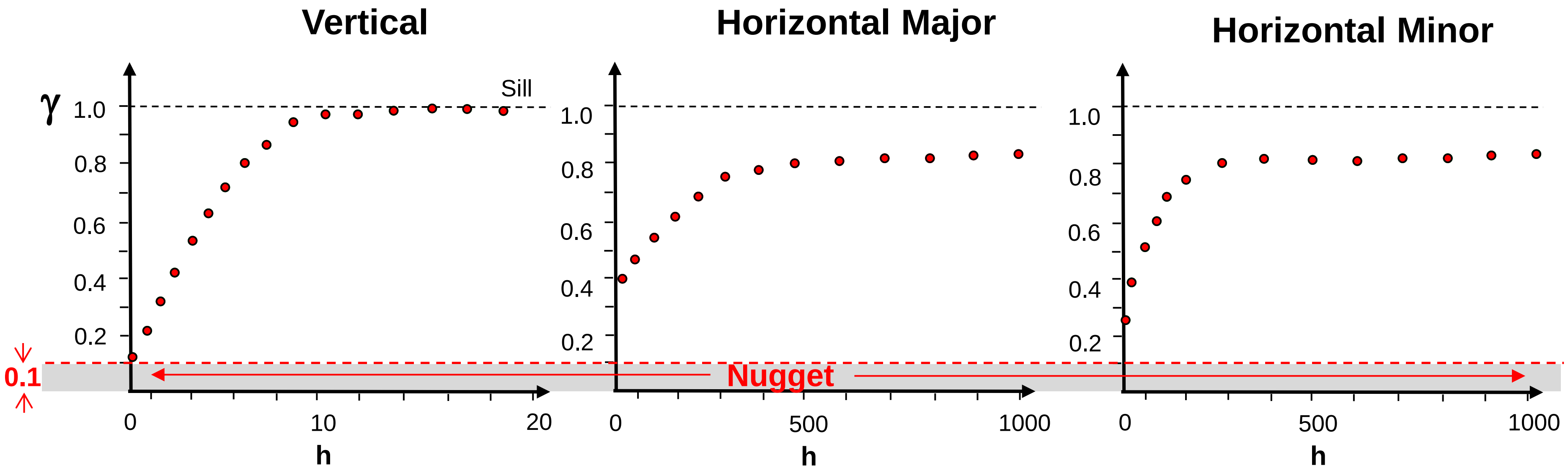
Each nested variogram structure exists in all directions, constant type and contribution.
We can only change the range and azimuths, here we assume azimuths are constant for all structures (common practice). Same major, minor and vertical!

Variogram Modeling Demonstration

Michael J. Pyrcz, Associate Professor
The University of Texas at Austin

Steps to modeling a 3D variogram from directional experimental variograms:

- Assign the isotropic nugget effect, the lowest observed nugget effect



Relative Nugget Effect of 10%

Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	**Nugget**	**0.1**	**-**	**-**	**-**
2					
3					
4					
5					

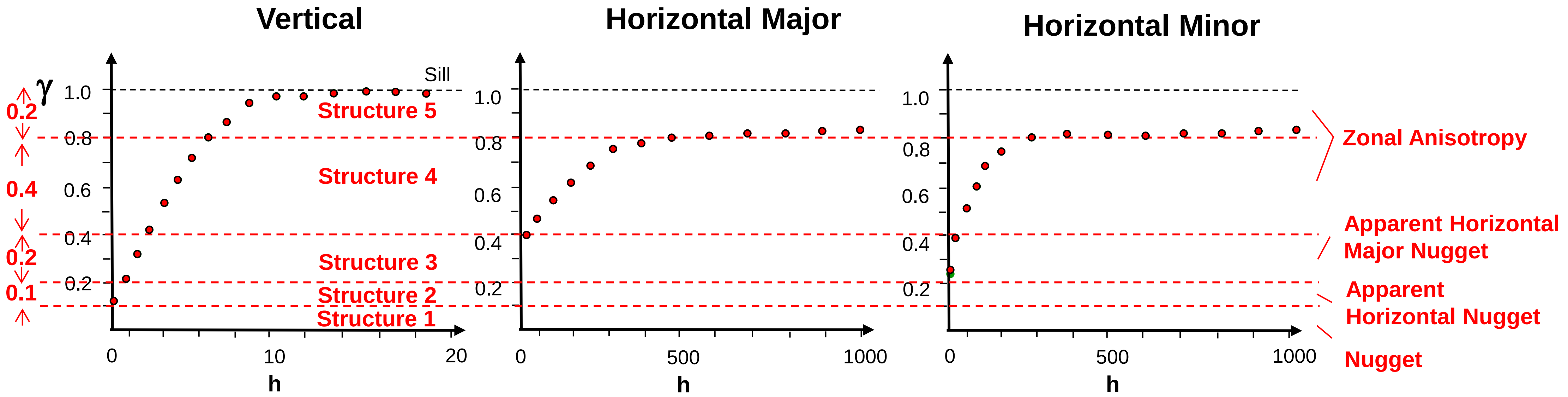
Note, nugget effect is present over all directions and distances, no azimuth nor ranges.

Variogram Modeling Demonstration

Michael J. Pyrcz, Associate Professor
The University of Texas at Austin

Steps to modeling a 3D variogram from directional experimental variograms:

- Choose the same number of nested variogram structures for all directions based number of observed stuctures



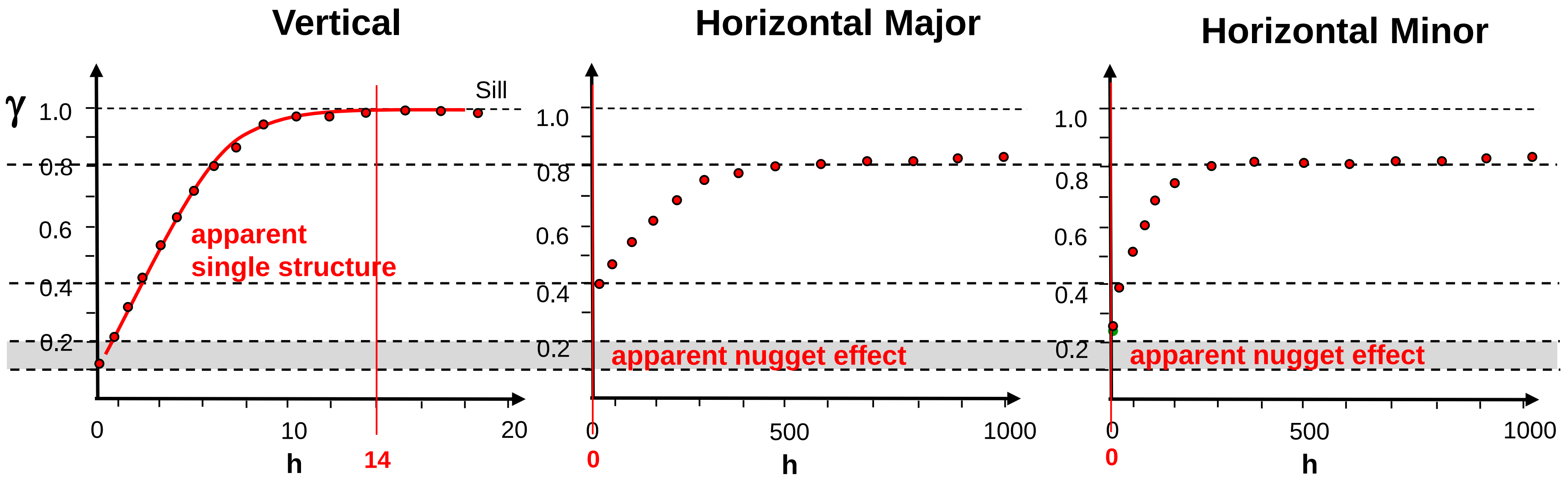
Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.1	-	-	-
2		0.1			
3		0.2			
4		0.4			
5		0.2			

Variogram Modeling Demonstration

Michael J. Pyrcz, Associate Professor
The University of Texas at Austin

Steps to modeling a 3D variogram from directional experimental variograms:

- Model the apparent nugget effect with 0 in the observed directions



Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.1	-	-	-
2	**Spherical**	0.1	**0**	**0**	**14**
3	**Spherical**	0.2			**14**
4	**Spherical**	0.4			**14**
5	**Spherical**	0.2			**14**

To make one apparent structure, all structures must be the same type of variogram model.

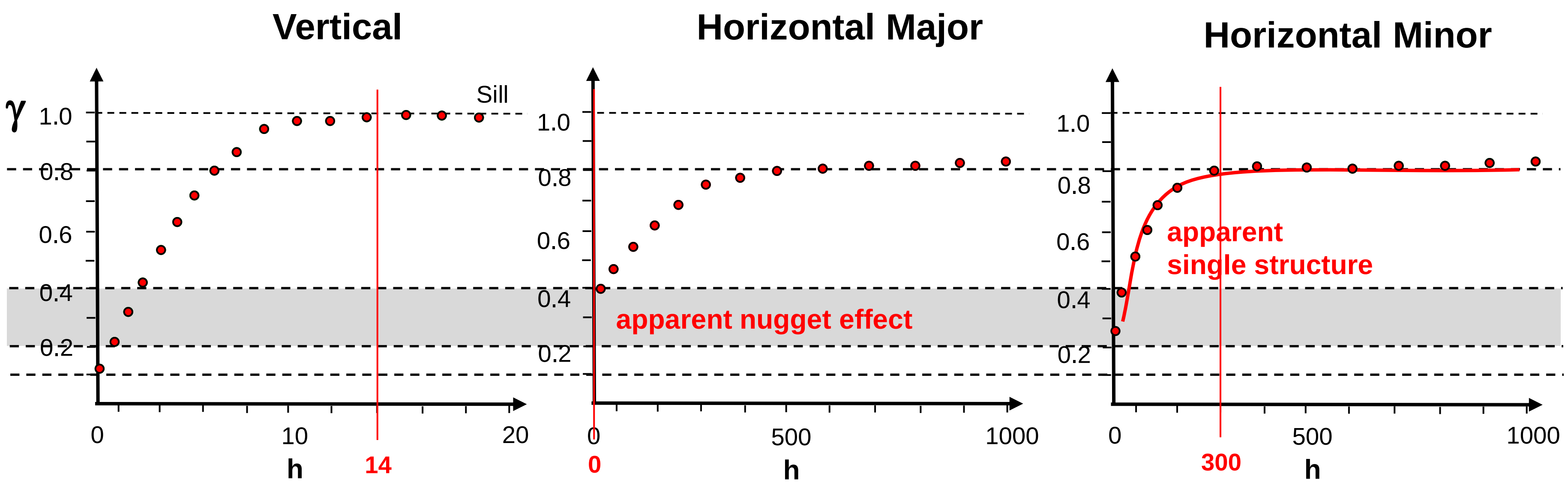
Note, in general we can change the type of variogram model for each nested structure. E.g., combine spherical and Gaussian structures.

Variogram Modeling Demonstration

Michael J. Pyrcz, Associate Professor
The University of Texas at Austin

Steps to modeling a 3D variogram from directional experimental variograms:

- Model the apparent nugget effect with 0 in the observed directions



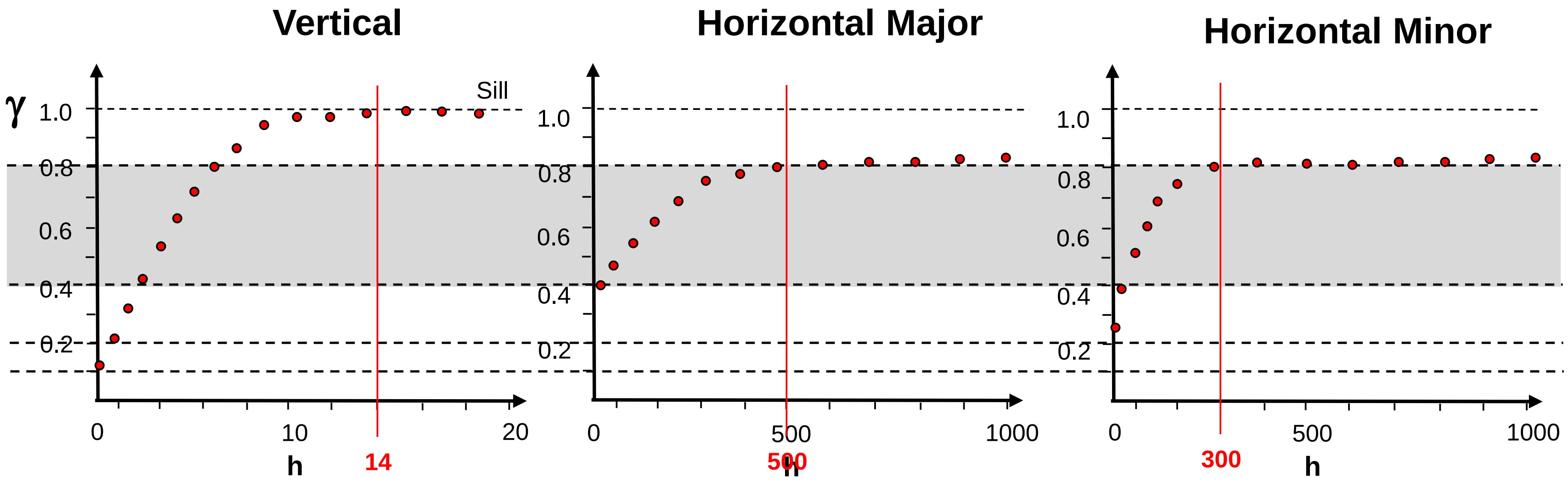
Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.1	-	-	-
2	Spherical	0.1	0	0	14
3	Spherical	0.2	0	300	14
4	Spherical	0.4		300	14
5	Spherical	0.2			14

Variogram Modeling Demonstration

Michael J. Pyrcz, Associate Professor
The University of Texas at Austin

Steps to modeling a 3D variogram from directional experimental variograms:

- model zonal with the observed range at the pseudo sill



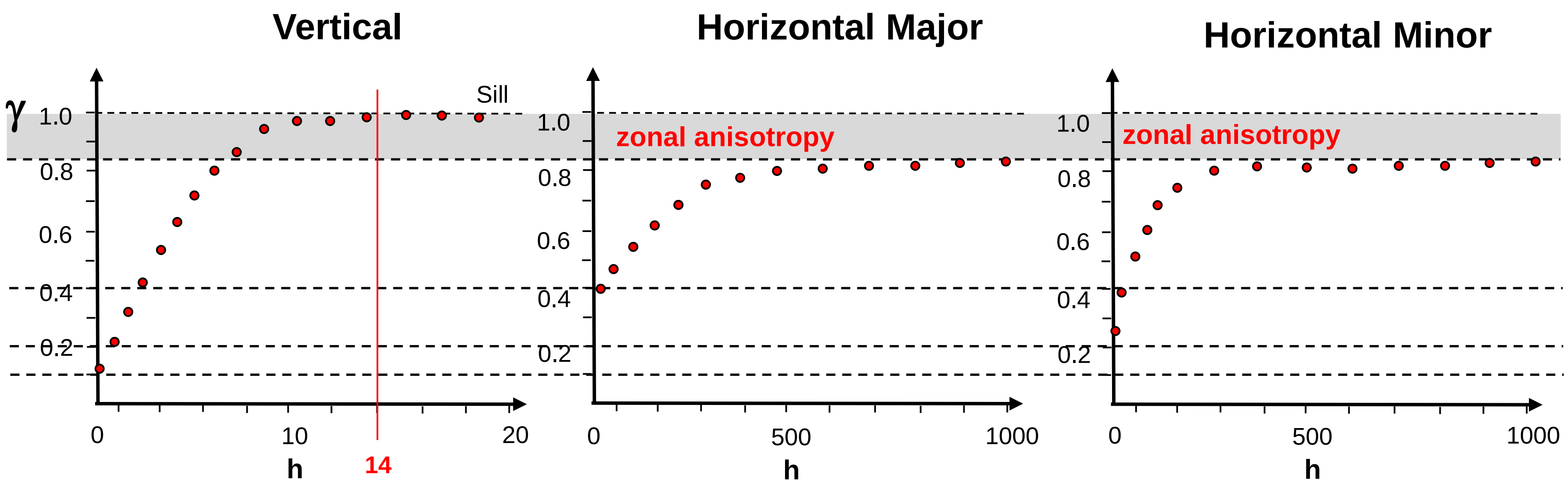
Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.1	-	-	-
2	Spherical	0.1	0	0	14
3	Spherical	0.2	0	300	14
4	Spherical	0.4	**500**	300	14
5	Spherical	0.2			14

Variogram Modeling Demonstration

Michael J. Pyrcz, Associate Professor
The University of Texas at Austin

Steps to modeling a 3D variogram from directional experimental variograms:

- model zonal anisotropy above the pseudo sill with a large value to flatten the variogram



Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.1	-	-	-
2	Spherical	0.1	0	0	14
3	Spherical	0.2	0	300	14
4	Spherical	0.4	500	300	14
5	Spherical	0.2	9999	9999	14

Variogram Modeling
Another Example

no spatial correlation in
those directions
over those structures

nugget is isotropic
over all distances

Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.1	-	-	-
2	Spherical	0.1	0	0	14
3	Spherical	0.2	0	300	14
4	Spherical	0.4	500	300	14
5	Spherical	0.2	9999	9999	14

1.0

all contributions sum to sill
we are explaining each portion
of the variance

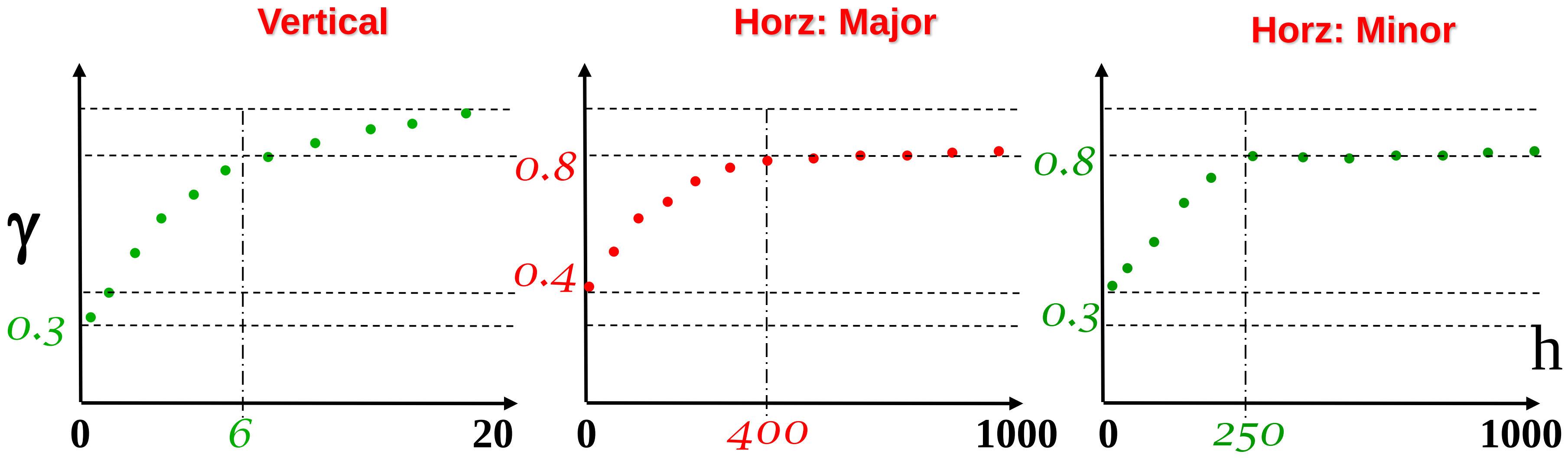
same ranges, behaves
as one structure

Representing the Variogram Model:

$$\gamma(h) = 0.1 + 0.1 \cdot sph_{\substack{av=14 \\ ah1=0 \\ ah2=0}} + 0.2 \cdot sph_{\substack{av=14 \\ ah1=0 \\ ah2=300}} + 0.4 \cdot sph_{\substack{av=14 \\ ah1=500 \\ ah2=300}} + 0.2 \cdot sph_{\substack{av=14 \\ ah1=\infty \\ ah2=\infty}}$$

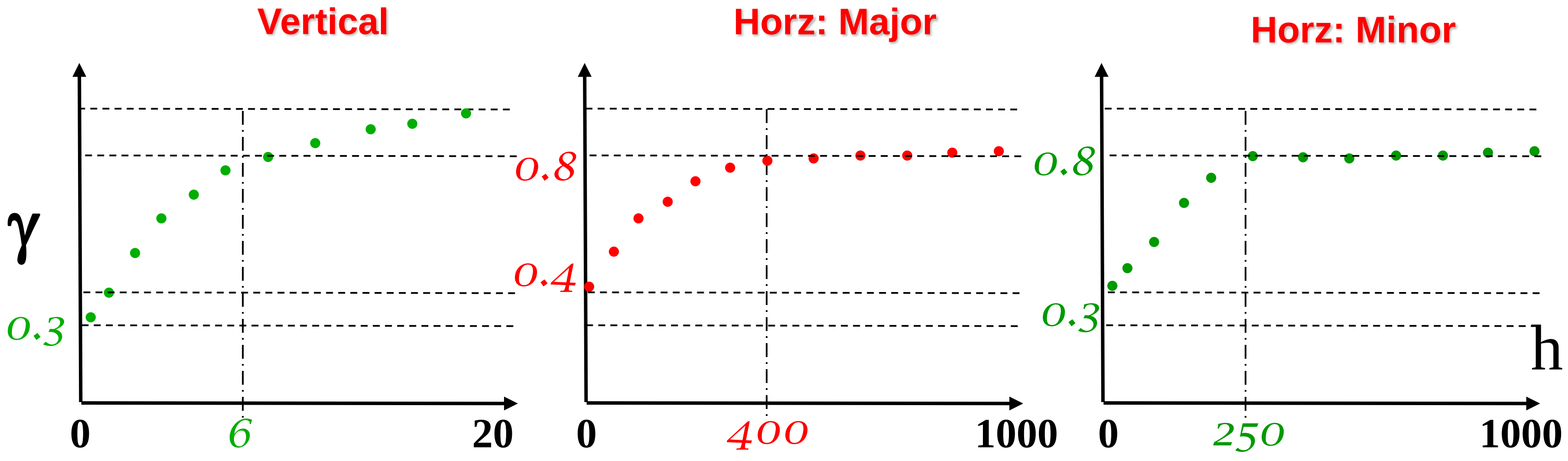
Note: $a_{maj} = a_{h1}, a_{min} = a_{h2}, a_{vert} = a_v$

Variogram Modeling Exercise #1



Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1					
2					
3					
4					
5					

Variogram Modeling Exercise #1

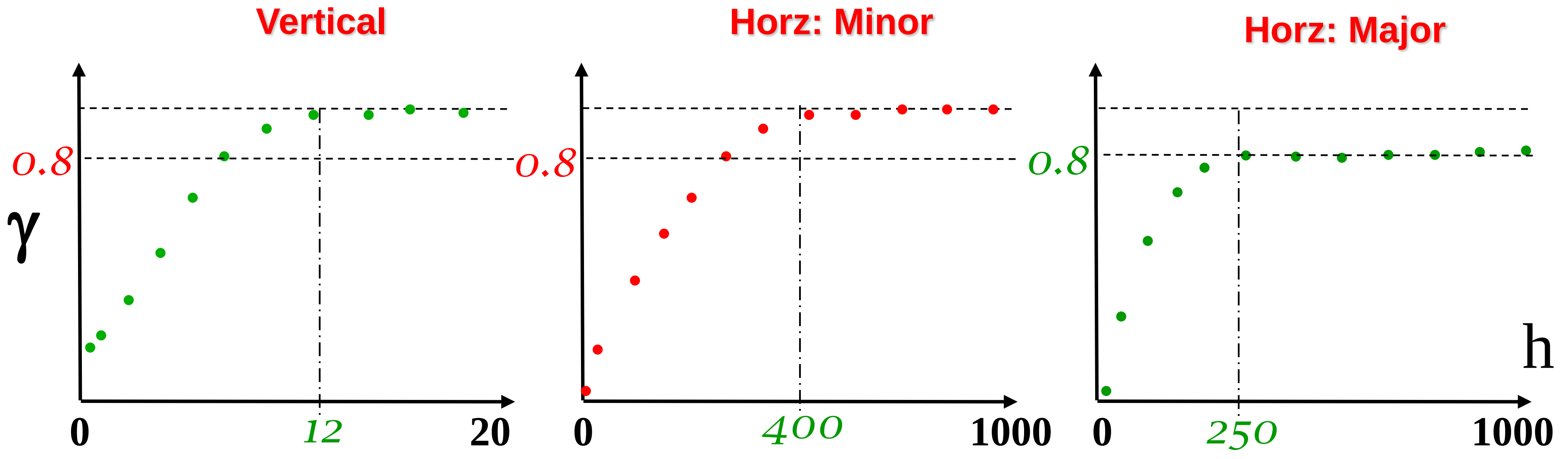


Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.3	-	-	-
2	Sph	0.1	0	0	6
3	Sph	0.4	400	250	6
4	Sph	0.2	9999	9999	20
5					

$$\gamma(h) = \mathbf{0.3} + 0.\mathbf{1} \cdot sph_{\substack{av=6 \\ a_{maj}=0 \\ a_{min}=0}} + 0.\mathbf{4} \cdot sph_{\substack{av=6 \\ a_{maj}=400 \\ a_{min}=250}} + 0.\mathbf{2} \cdot sph_{\substack{av=20 \\ a_{maj}=9999 \\ a_{min}=9999}}$$

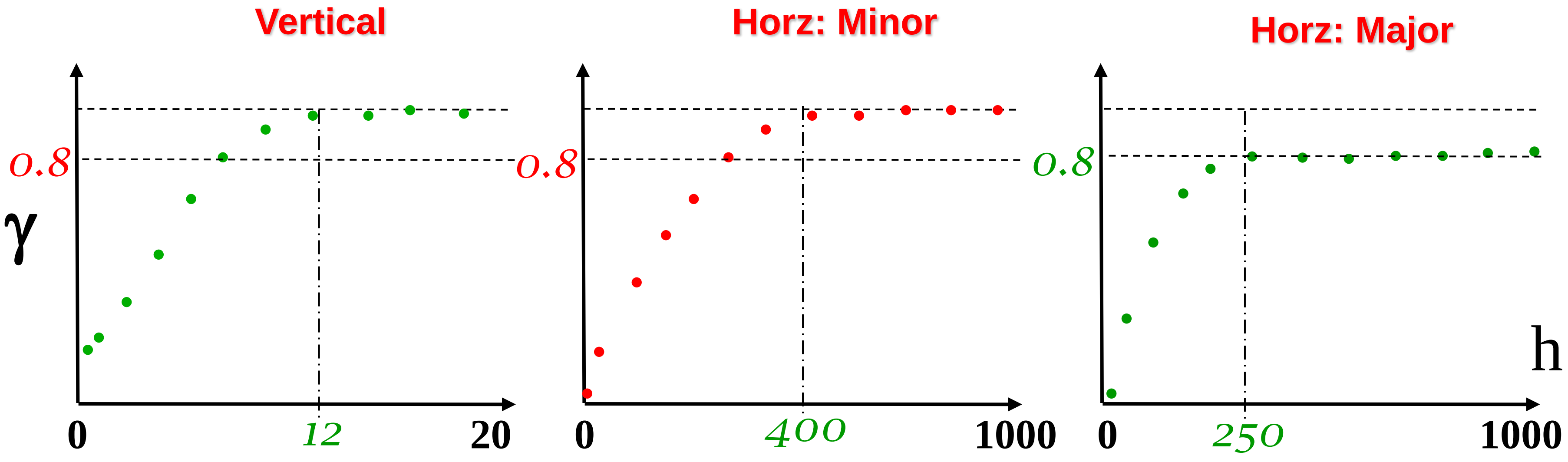
Note: $a_{vert} = a_v$

Variogram Modeling Exercise #2



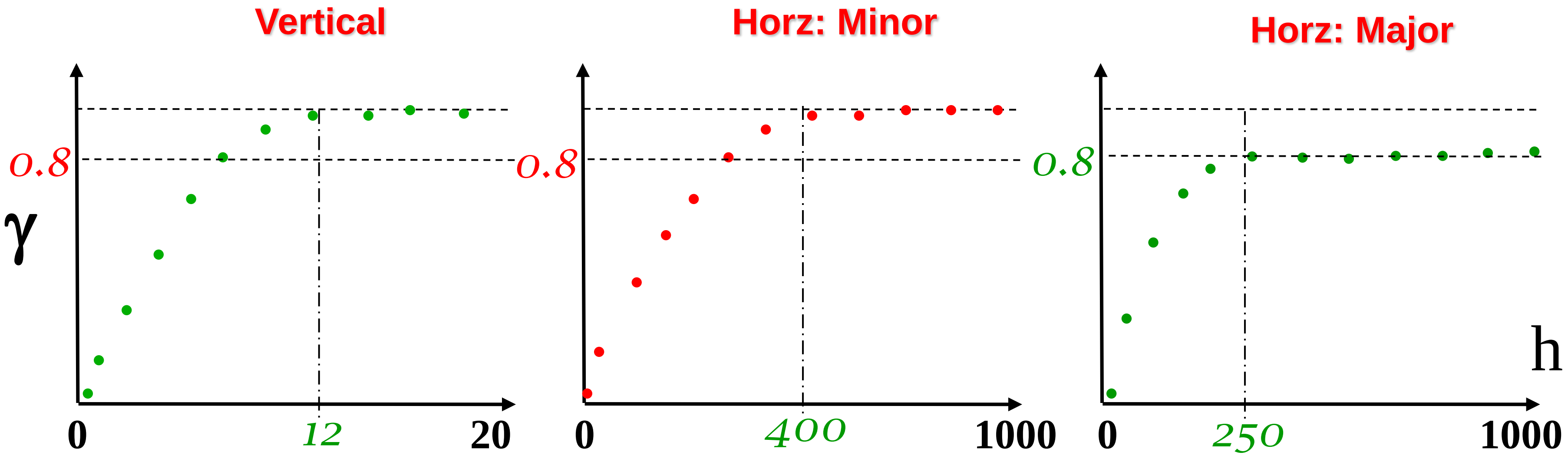
Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Sph	0.8	250	400	12
2	Sph	0.2	999999999	400	12
3					
4					
5					

Variogram Modeling Exercise #2



Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Sph	0.8	250	400	12
2	Sph	0.2	9999	400	12
3					
4					
5					

Variogram Modeling Exercise #2

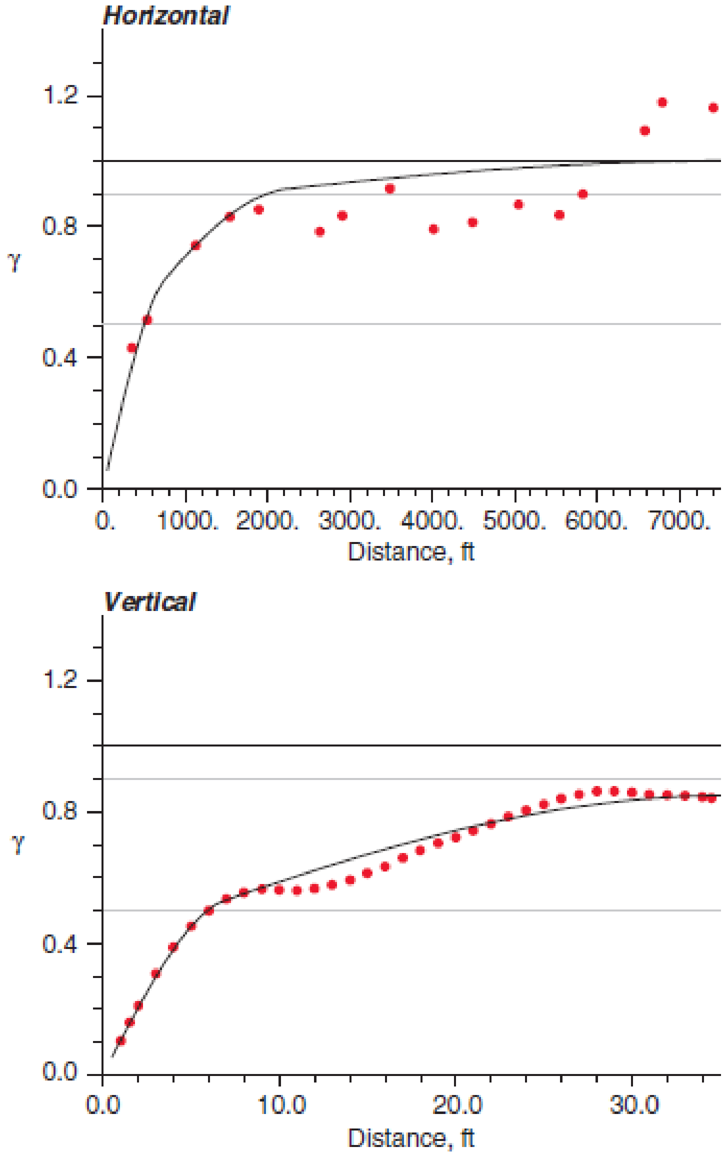


Structure	Type	c	a_{maj}	a_{min}	a_{vert}
1	Nugget	0.0	-	-	-
2	Sph	0.8	250	400	12
3	Sph	0.2	9999	400	12
4					
5					

$$\gamma(h) = 0.\mathbf{8} \cdot sph\ {}_{\substack{\mathbf{av=12} \\ \mathbf{a_{maj}=250} \\ \mathbf{a_{min}=400}}} + 0.\mathbf{2} \cdot sph\ {}_{\substack{\mathbf{av=12} \\ \mathbf{a_{maj}=9999} \\ \mathbf{a_{min}=400}}}$$

Note: $a_{vert} = a_v$

Another Variogram Model Example



Example and images from GSLIB:

- 3 variogram structures
- Isotropic horizontal
- Vertical zonal anisotropy

Variance Contribution	Type of Variogram	Horizontal Range, m	Vertical Range, m
0.50	Spherical	750.0	6.0
0.40	Spherical	2000.0	50.0
0.10	Spherical	7000.0	∞

Example from Pyrcz and Deutsch, 2014.

Inference in Presence of Sparse Data

- Most often there are inadequate data to infer a reliable horizontal variogram.

- Horizontal wells have not significantly helped with horizontal variogram inference:
  - horizontal wells have limited data, often no cores and very limited petrophysical
  - horizontal wells rarely track the stratigraphic layers, and underestimate variogram ranges!

- Vertical direction is often much better informed / aligned along wells
  - model the variogram structures from vertical and use an anisotropy ratio for horizontal

- Also use analog data deemed relevant to the site being considered such as:
  - other, more extensively sampled reservoirs
  - geological process simulation
  - outcrop measurements

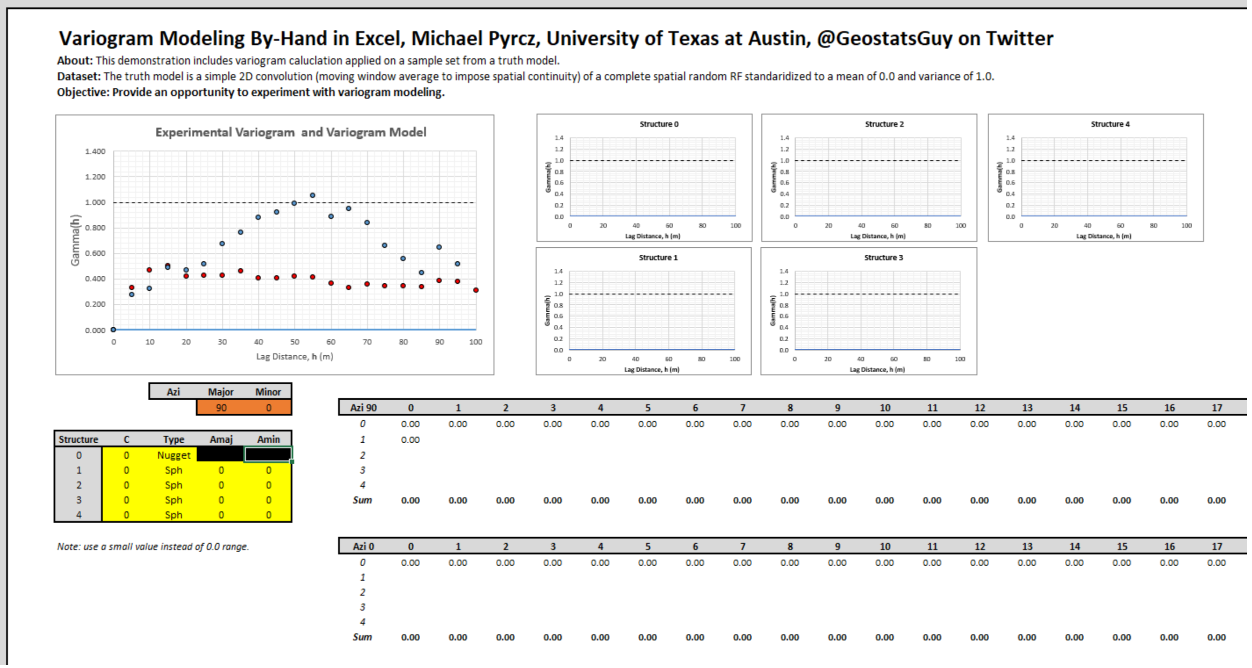
Variogram Modeling in Excel Demonstration

Variogram Calculation:

Things to try:

1. Model your experimental isotropic variogram.

2. Model your experimental anisotropic variogram.



The file is at: https://git.io/fxhxr.   The file is Variogram_Calc_Model_Demo_v2.0.xlsx

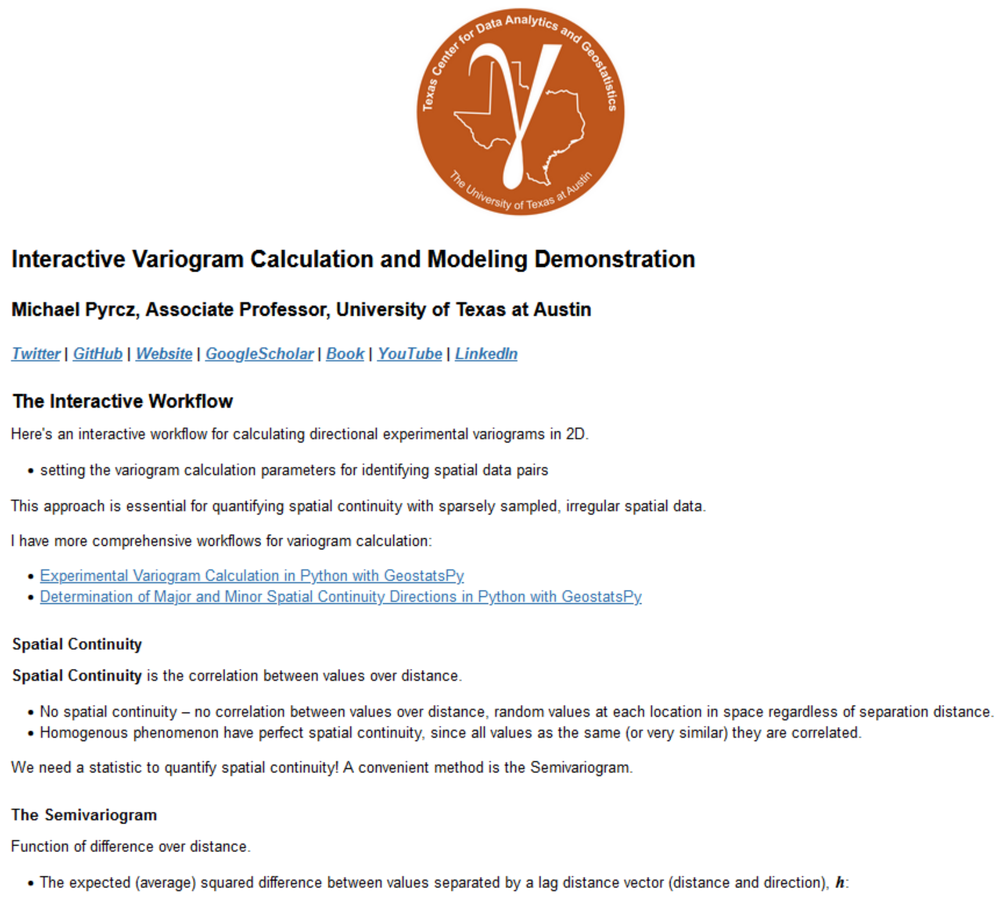
Variogram Calculation in Python Hands On

Variogram Modeling Workflow in Python

Walkthrough and to:

- Fit a nested valid, variogram model to a directional experimental variogram in 2D.

- File is: Interactive_Variogram_Calculation_Modeling.ipynb



Interactive Variogram Calculation and Modeling Demonstration

Michael Pyrcz, Associate Professor, University of Texas at Austin

Twitter | *GitHub* | *Website* | *GoogleScholar* | *Book* | *YouTube* | *LinkedIn*

The Interactive Workflow

Here's an interactive workflow for calculating directional experimental variograms in 2D.

- setting the variogram calculation parameters for identifying spatial data pairs

This approach is essential for quantifying spatial continuity with sparsely sampled, irregular spatial data.

I have more comprehensive workflows for variogram calculation:

- Experimental Variogram Calculation in Python with GeostatsPy
- Determination of Major and Minor Spatial Continuity Directions in Python with GeostatsPy

Spatial Continuity

Spatial Continuity is the correlation between values over distance.

- No spatial continuity – no correlation between values over distance, random values at each location in space regardless of separation distance.
- Homogenous phenomenon have perfect spatial continuity, since all values as the same (or very similar) they are correlated.

We need a statistic to quantify spatial continuity! A convenient method is the Semivariogram.

The Semivariogram

Function of difference over distance.

- The expected (average) squared difference between values separated by a lag distance vector (distance and direction), $\boldsymbol{h}$:

Interactive Python dashboards for variogram calculation and modeling in Python.

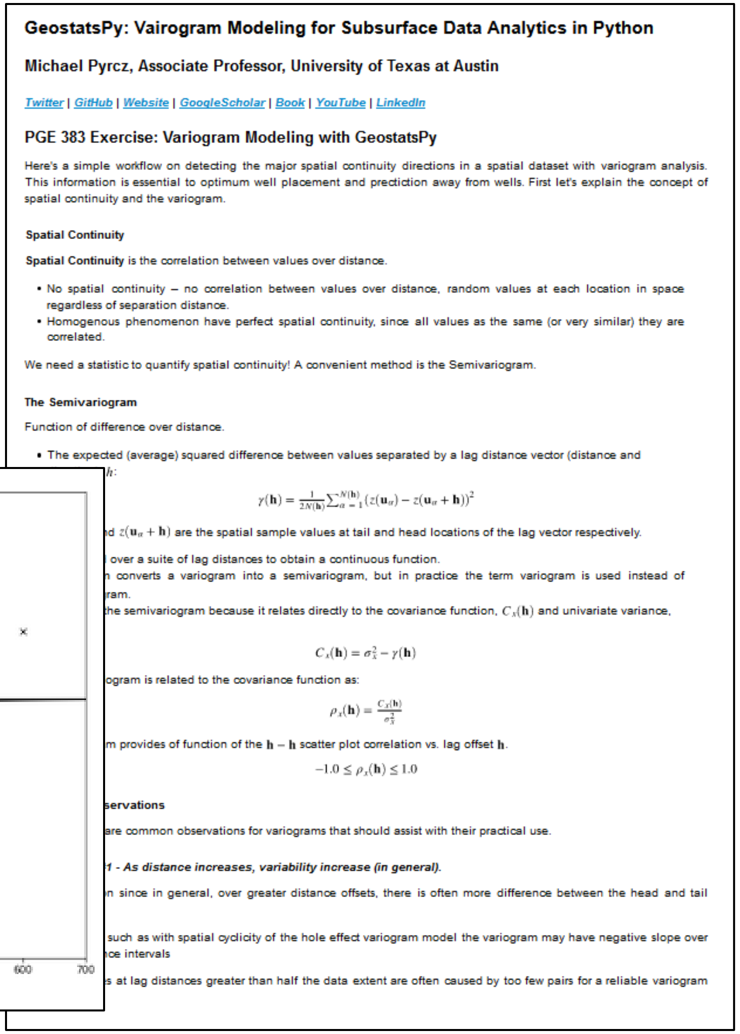
Variogram Calculation in Python Demonstration

Variogram Modeling Workflow in Python

Walkthrough and try to:

- Fit the anisotropic experimental variogram.

- File is: GeostatsPy_variogram_modeling.ipynb



GeostatsPy: Vairogram Modeling for Subsurface Data Analytics in Python

Michael Pyrcz, Associate Professor, University of Texas at Austin

Twitter | GitHub | Website | GoogleScholar | Book | YouTube | LinkedIn

PGE 383 Exercise: Variogram Modeling with GeostatsPy

Here's a simple workflow on detecting the major spatial continuity directions in a spatial dataset with variogram analysis. This information is essential to optimum well placement and prediction away from wells. First let's explain the concept of spatial continuity and the variogram.

Spatial Continuity

Spatial Continuity is the correlation between values over distance.

- No spatial continuity – no correlation between values over distance, random values at each location in space regardless of separation distance.
- Homogenous phenomenon have perfect spatial continuity, since all values as the same (or very similar) they are correlated.

We need a statistic to quantify spatial continuity! A convenient method is the Semivariogram.

The Semivariogram

Function of difference over distance.

- The expected (average) squared difference between values separated by a lag distance vector (distance and

h:

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})}\sum_{\alpha=1}^{N(\mathbf{h})}(z(\mathbf{u}_\alpha) - z(\mathbf{u}_\alpha + \mathbf{h}))^2$$

d $z(\mathbf{u}_\alpha + \mathbf{h})$ are the spatial sample values at tail and head locations of the lag vector respectively.

over a suite of lag distances to obtain a continuous function.

converts a variogram into a semivariogram, but in practice the term variogram is used instead of

ram.

he semivariogram because it relates directly to the covariance function, $C_x(\mathbf{h})$ and univariate variance,

$$C_x(\mathbf{h}) = \sigma_x^2 - \gamma(\mathbf{h})$$

ogram is related to the covariance function as:

$$\rho_x(\mathbf{h}) = \frac{C_x(\mathbf{h})}{\sigma_x^2}$$

m provides of function of the $\mathbf{h}-\mathbf{h}$ scatter plot correlation vs. lag offset $\mathbf{h}$.

$$-1.0 \leq \rho_x(\mathbf{h}) \leq 1.0$$

servations

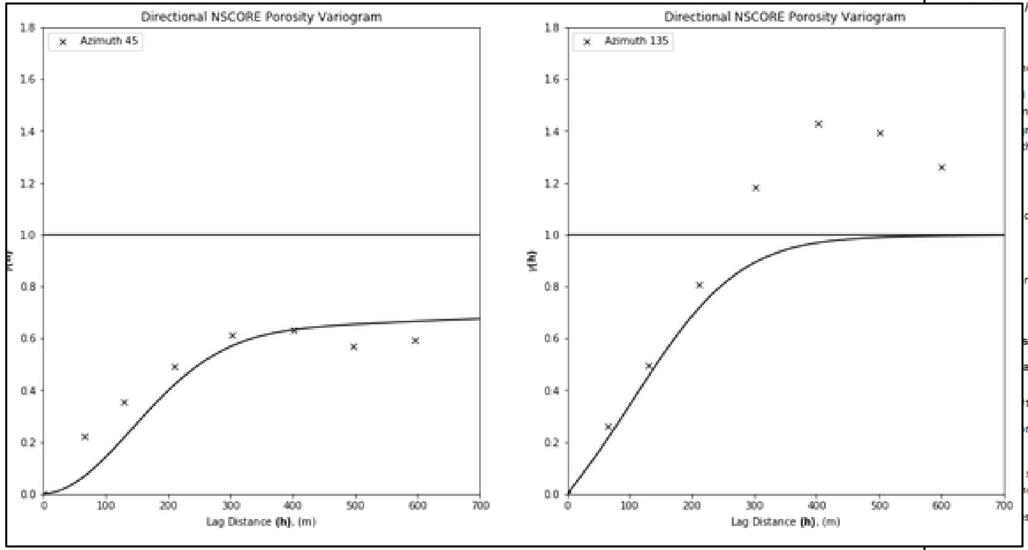
are common observations for variograms that should assist with their practical use.

1 - As distance increases, variability increase (in general).

n since in general, over greater distance offsets, there is often more difference between the head and tail

such as with spatial cyclicity of the hole effect variogram model the variogram may have negative slope over ce intervals

s at lag distances greater than half the data extent are often caused by too few pairs for a reliable variogram

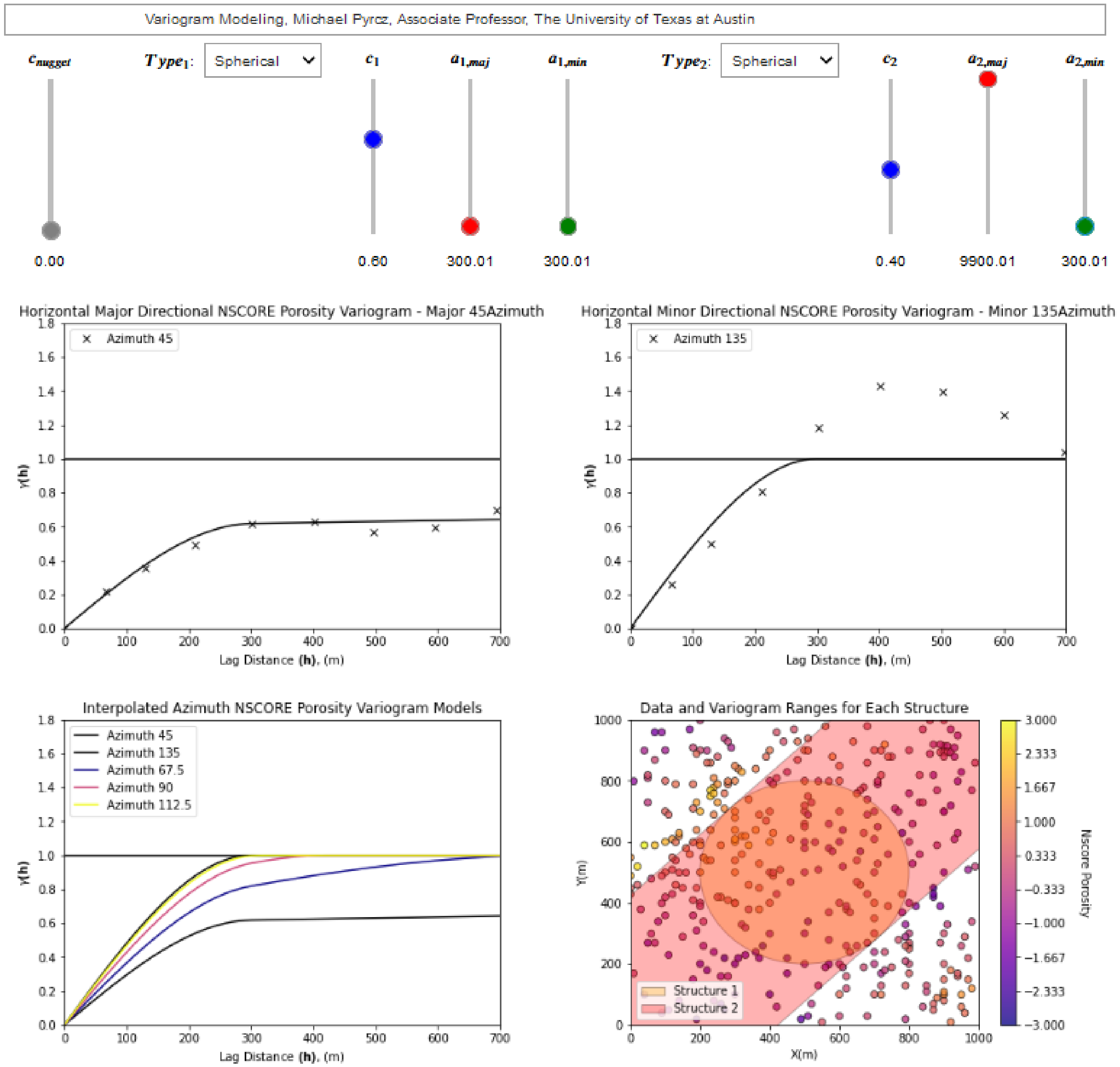


Interactive Variogram Modeling in Python Demonstration

Walkthrough and try to:

- Fit the 2D experimental variogram.

- Workflow includes only variogram modeling in the major and minor directions for a single dataset.

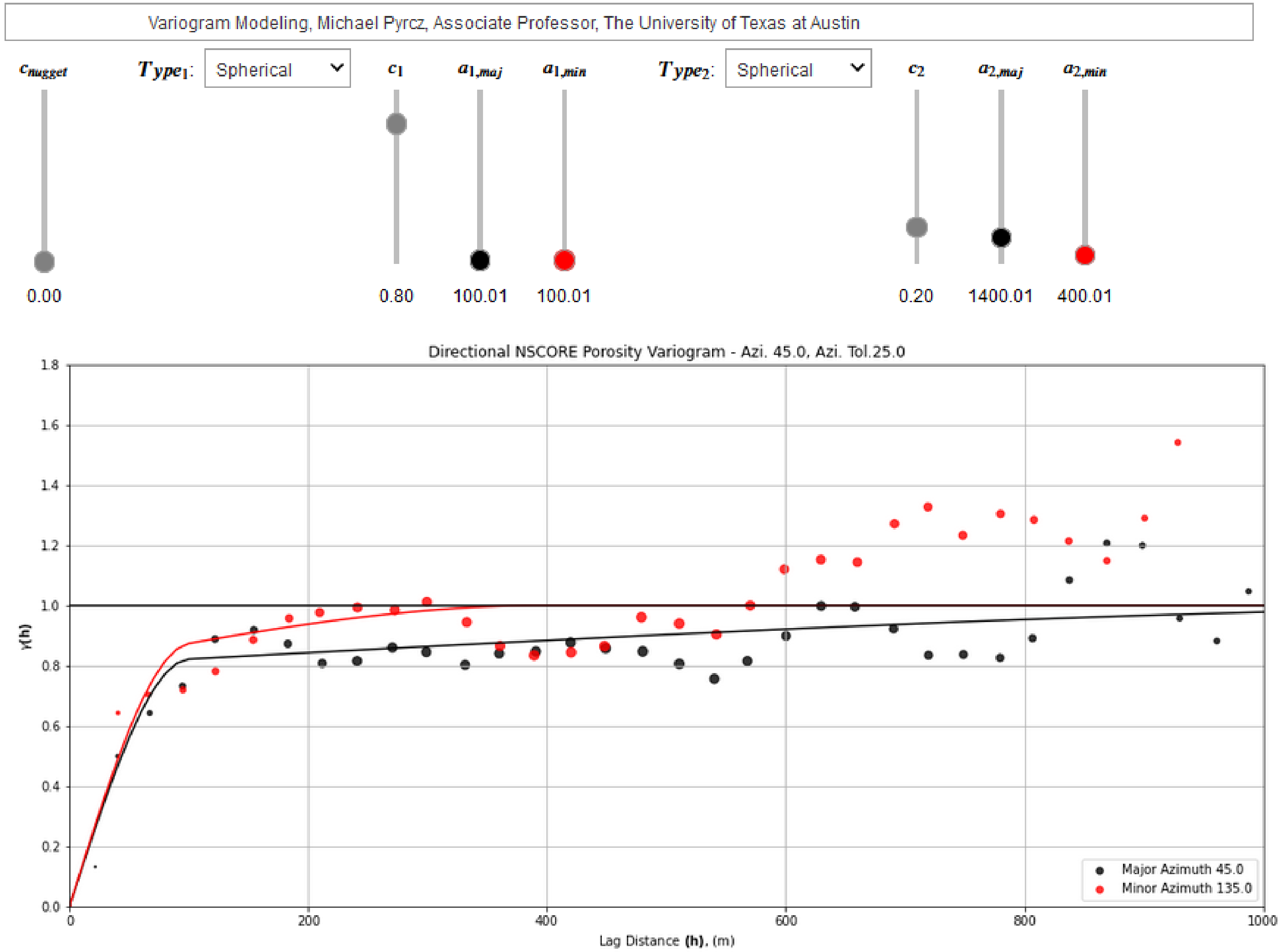


Calculate and model 2D variograms with file Interactive_Variogram_Modeling.ipynb.

Interactive Variogram Modeling in Python Demonstration

Walkthrough and try to:

- Fit the 2D experimental variogram.
- Workflow includes variogram calculation in the major and minor directions.
- Major and minor variograms on the same plot.
- Ability to select from multiple datasets.



Calculate and model 2D variograms with file Interactive_Variogram_Calculation_Modeling.ipynb.

Variogram Modeling in R Demonstration

Also made a Rmarkdown tutorial for the following workflow example in R with a lot more explanation.

- File name: variogram_demo.html
- Available at GitHub/GeostatsGuy/geostatr

Variogram Analysis in R for Engineers and Geoscientists

Michael Pyrcz, Associate Professor, University of Texas at Austin,

Contacts: Twitter/@GeostatsGuy | GitHub/GeostatsGuy | www.michaelpyrcz.com | GoogleScholar | Book

A tutorial/demonstration of variogram analysis workflow based on the gstat package by Pedesma, E. The docs are at https://cran.r-project.org/web/packages/gstat/index.html. I found Pedesma's Meuse tutorial very helpful (https://cran.r-project.org/web/packages/gstat/vignettes/gstat.pdf). Also, appreciation to Pebesma for assistance through answering questions. For this demonstration we use a 200 well 2D porosity dataset (file: 2D_MV_200Wells.csv) that may be found at https://github.com/GeostatsGuy/GeoDataSets. I used this in my Introduction to Geostatistics undergraduate class (PGE337 at UT Austin) as part of a first introduction to R for the engineering undergraduate students. It is assumed that students have no previous R experience; therefore, all steps of the code are explored and described.

Load the required libraries

```
library(gstat)                                # geostatistical methods by Edzer Pebesma
```

```
## Warning: package 'gstat' was built under R version 3.4.3
```

```
library(sp)                                   # spatial points addition to regular data frames
```

```
## Warning: package 'sp' was built under R version 3.4.3
```

```
library(plyr)                                 # manipulating data by Hadley Wickham
```

```
## Warning: package 'plyr' was built under R version 3.4.3
```

If you get an error, you may have to first go to "Tools/Install Packages…" to install these packages. Just type in the names one at a time into the package field and install. The package names should autocomplete (helping you make sure you got the right package name), and the install process is automatic, with the possibility of installing other required dependency packages. Previously I had an issue with packages not being found after install that was resolved with a reboot.

Declare functions

I was surprised that there isn't a built in method to transform a dataframe column or data vector to standard normal, Gaussian with a mean of zero, $\overline{x} = 0.0$ and a standard deviation $\sigma = 1.0$. I found this function by Ashton Shortridge (2008) and included it here. Just apply with the raw data as a vector, x, and it returns an object with the normal score values as a member vector, '[my_transform_object]$nscore'.

```
nscore <- function(x) {                        # by Ashton Shortridge, 2008
  # Takes a vector of values x and calculates their normal scores. Returns
  # a list with the scores and an ordered table of original values and
  # scores, which is useful as a back-transform table. See backtr().
  nscore <- qqnorm(x, plot.it = FALSE)$x  # normal score
  trn.table <- data.frame(x=sort(x),nscore=sort(nscore))
  return (list(nscore=nscore, trn.table=trn.table))
}
```

Variogram Modeling Review

- Variogram is very important in the geostatistical study; Measure of geological distance
- Initial coordinate and data transformation may be required.
- Interpretation Principles:
  - Trend
  - Cyclicity
  - Geometric Anisotropy
  - Zonal Anisotropy
- Short-scale structure is the most important
  - nugget due to measurement error should not be modeled
  - size of geological modeling cells
- Vertical direction is typically well informed
  - can have artifacts due to spacing of core data
  - handle vertical trends and areal variations
- Horizontal direction is not well informed
  - take from analog field or outcrop
  - typical horizontal vertical anisotropy ratios, use contributions and shape from vertical

PGE 338 Data Analytics and Geostatistics

Lecture 11: Spatial Modeling

Lecture outline . . .

- Variogram Interpretation
- Variogram Modeling

Introduction
General Concepts
Univariate
Bivariate
Spatial
Calculation
Variogram Modeling
Kriging
Simulation
Time Series
Machine Learning
Uncertainty Analysis

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