Decision Making

Lecture outline . . .

- Decision Making
- Loss Functions
- Decision Making Example
- Decision Making Hands-on

MOTIVATION

- We have built uncertainty models…

…now we need to make decisions in the presence of uncertainty.

DECISION MAKING

BUILDING UNCERTAINTY MODELS

How Have We Calculated Uncertainty Models? Some examples:

1. Bayesian Updating
   - prior plus likelihood to calculate the posterior
   - e.g., probability that a coin is fair, uncertainty in reservoir OIP

2. Bootstrap + Monte Carlo Simulation and Transfer Function
   - uncertainty mean porosity by bootstrap, then MCS for uncertainty in OIP

3. Kriging
   - kriging estimate and variance with Gaussian assumption for uncertainty at a location
   - indicator kriging of uncertainty at a location

4. Simulation and Transfer Function
   - reservoir realizations applied to flow simulation to calculate pre-drill production uncertainty at a new well

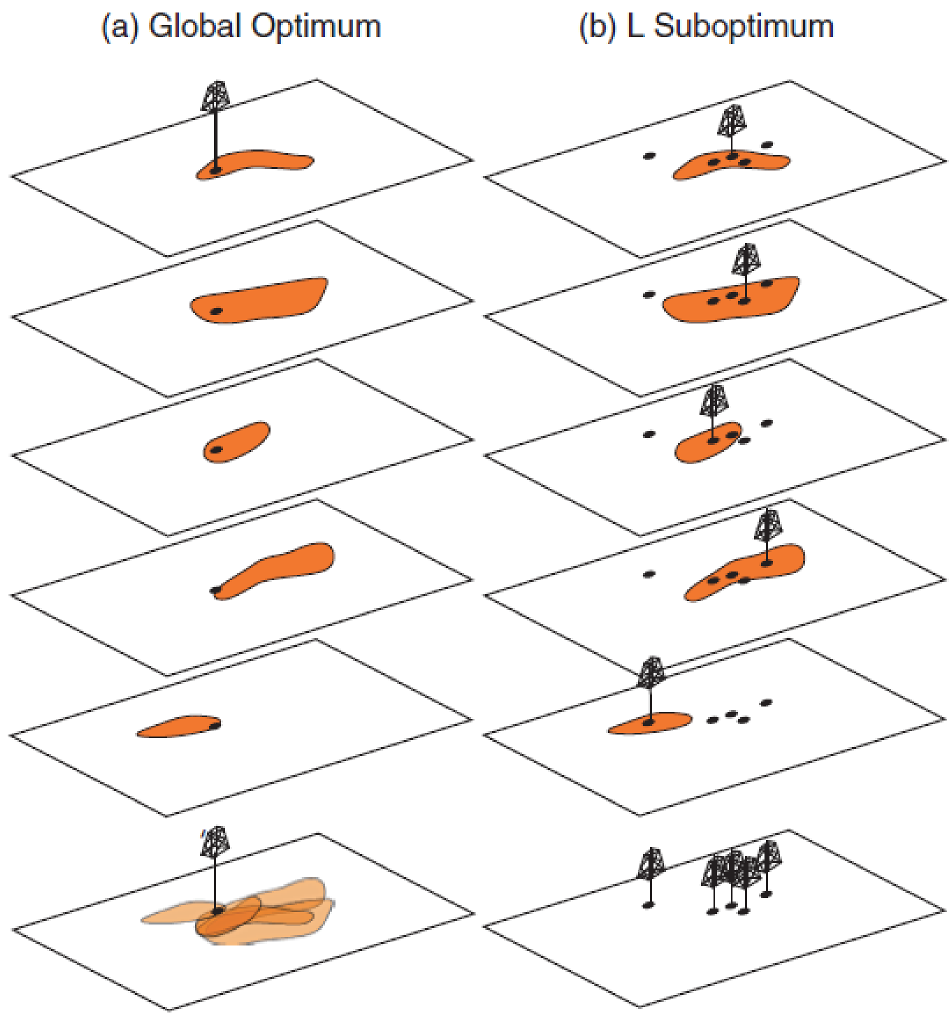
DECISION MAKING WITH UNCERTAINTY

- We have calculated a reasonable uncertainty model:
  - Connected volume of water or hydrocarbon to a location
  - Mineral or hydrocarbon resource in place
  - Recover factor mineral or hydrocarbon extraction
- We must make a decision with that uncertainty model
  - Number and locations of wells
  - Time and volume injected for water of chemical flood, pump and treat remediation
  - Dig limits and mine plan
- Going from Uncertainty to a Discrete Decision
  - There is a best decision for each possible subsurface model
  - We need the best single decision (can't make probabilistic decisions!)

DECISION MAKING WITH UNCERTAINTY

- Well site selection methods:
  - Integer programing problem, output must be an integer, not a fraction
  - Optimization combined with simulation one potential well at a time
  - Experimental design and response surface methodology

- Optimize the profitability of reservoir production
  - High dimensional: complicated by large number of parameters and timing
  - Interactions: locations of injector and producers,
  - Multivariate: injection, well completions



Global optimum vs. L suboptimum (Pyrcz and Deutsch, 2014).

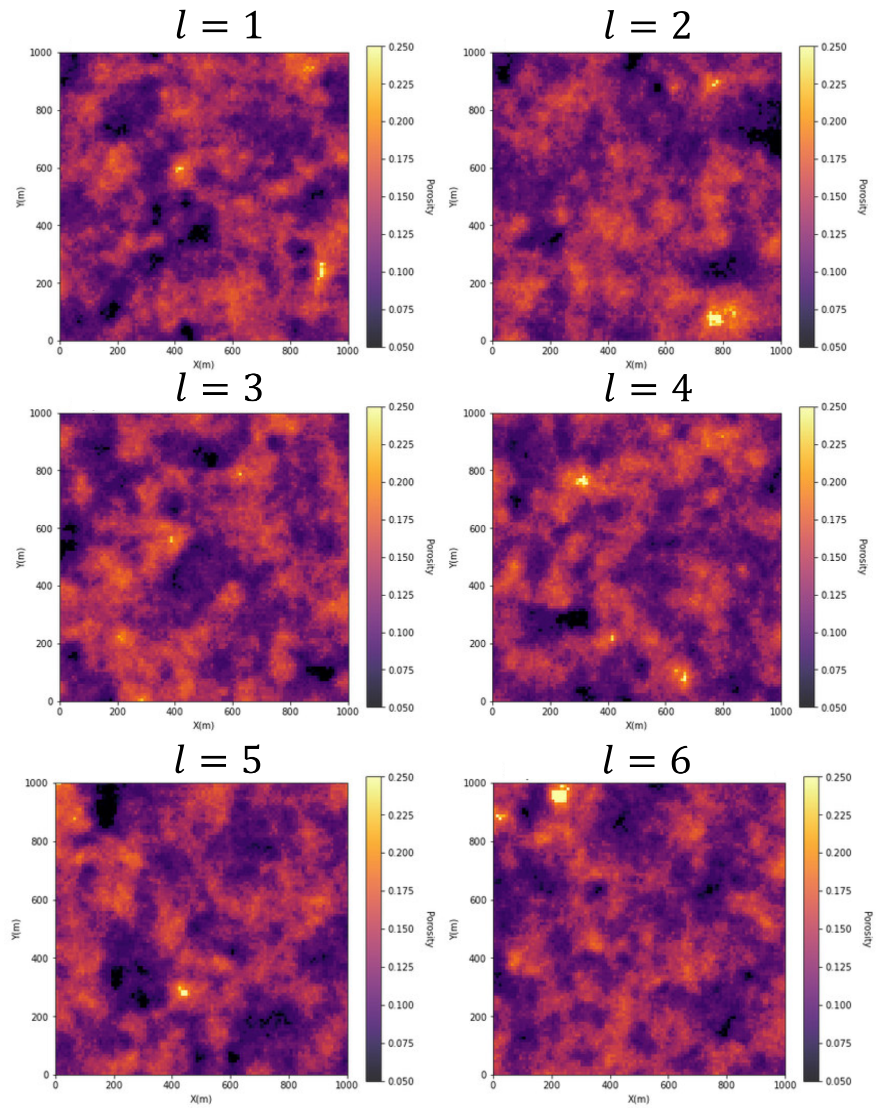
DECISION MAKING WITH UNCERTAINTY

- Expected Profit Workflow:

1. Calculate the uncertainty model, ***l*=1,…,*L*** subsurface realizations

The Uncertainty Model

- Scenarios change the model inputs, and assumptions to capture uncertainty in the modeling decisions.
- Realizations change the random number seed to capture spatial uncertainty
- Multivariate, includes all features needed to simulate extraction.
- The uncertainty model is represented by multiple models of the subsurface.



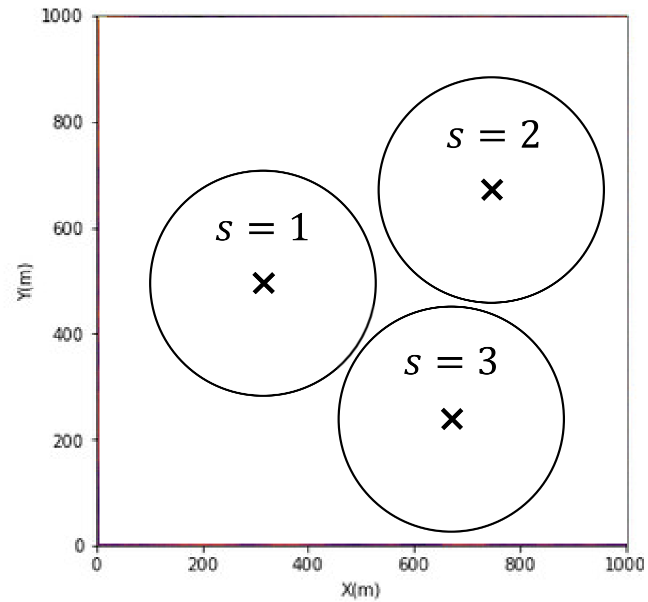
$l = 1, \dots, 6$ subsurface models, realizations and scenarios of the subsurface.

DECISION MAKING WITH UNCERTAINTY

- Expected Profit Workflow:

1. Calculate the uncertainty model, $\boldsymbol{l}=\mathbf{1},\ldots,\boldsymbol{L}$ subsurface realizations
2. Establish $\boldsymbol{s} = \mathbf{1}, \ldots, \boldsymbol{S}$ development scenarios

- The Development Scenarios
  - Discrete alternatives for developing the spatial, subsurface resource
  - Includes all details needed to simulate extraction, e.g., well locations, completion types, dig limits, mining block sequence
  - For example, 3 potential well locations with drainage radius.



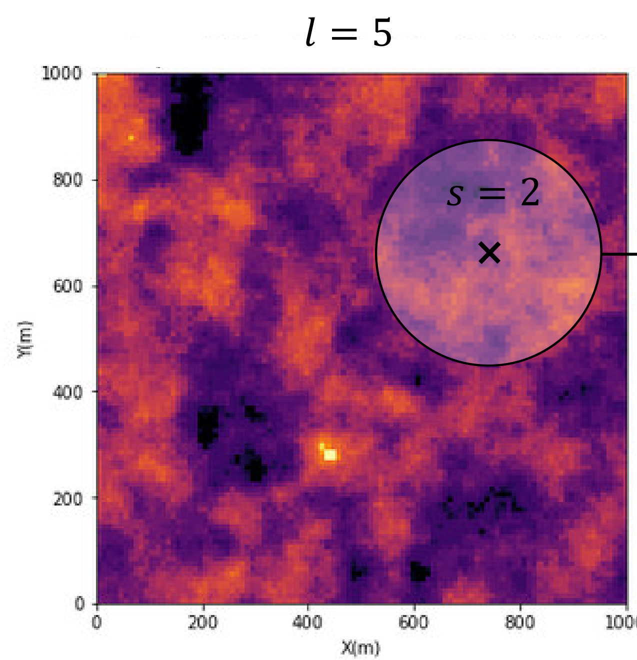
$s = 1, \ldots, 3$ development scenarios.

DECISION MAKING WITH UNCERTAINTY

- Expected Profit Workflow:

1. Calculate the uncertainty model, $\boldsymbol{l}$=$\mathbf{1}$,...,$\boldsymbol{L}$ subsurface realizations
2. Establish $\boldsymbol{s} = \mathbf{1}, \dots, \boldsymbol{S}$ development scenarios
3. Establish $\boldsymbol{P}$ profit metric

- The Profit Metric
  - Transfer function to go from spatial features to value, $\boldsymbol{P}(\boldsymbol{l},\boldsymbol{s})$
  - Input is a subsurface realization, $\boldsymbol{l}$, and development scenario, $\boldsymbol{s}$.
  - In the thing are we trying to maximize? E.g., net present value, contaminant removed from soil, resource recovered.



$l = 5$, realization, $s = 2$ development scenario.

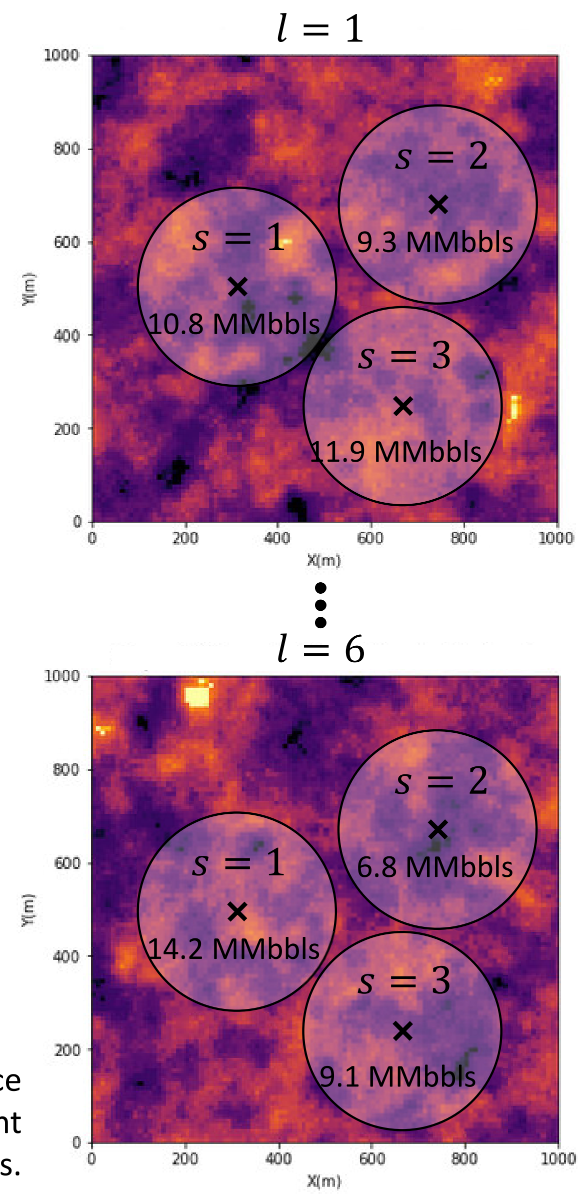
$\boldsymbol{P}(\boldsymbol{l} = \mathbf{5}, \boldsymbol{s} = \mathbf{2})$ = 13 MMbbls

DECISION MAKING WITH UNCERTAINTY

- Expected Profit Workflow:

1. Calculate the uncertainty model, $\boldsymbol{l}$=**1**,...,$\boldsymbol{L}$ subsurface realizations
2. Establish $\boldsymbol{s} = \mathbf{1}, \dots, \boldsymbol{S}$ development scenarios
3. Establish $\boldsymbol{P}$ profit metric
4. Calculate profit, $\boldsymbol{P}$, over all $\boldsymbol{L}$ and $\boldsymbol{S}$

- Repeat the calculation over all $\boldsymbol{L}$ and $\boldsymbol{S}$
  - Full combinatorial of L subsurface realizations and S development scenarios, $\boldsymbol{P}(\boldsymbol{l}=\mathbf{1},\dots,\boldsymbol{L},\boldsymbol{s}=\mathbf{1},\dots,\boldsymbol{S})$



Profit calculated over all subsurface realizations and development scenarios.

DECISION MAKING WITH UNCERTAINTY

- Expected Profit Workflow:

1. Calculate the uncertainty model, $\boldsymbol{l}=\mathbf{1},\ldots,\boldsymbol{L}$ subsurface realizations
2. Establish $\boldsymbol{s} = \mathbf{1}, \ldots, \boldsymbol{S}$ development scenarios
3. Establish $\boldsymbol{P}$ profit metric
4. Calculate profit, $\boldsymbol{P}$, over all $\boldsymbol{L}$ and $\boldsymbol{S}$
5. Calculate the expected profit for each development scenario, $\boldsymbol{E}\{\boldsymbol{P}(\boldsymbol{S})\}$

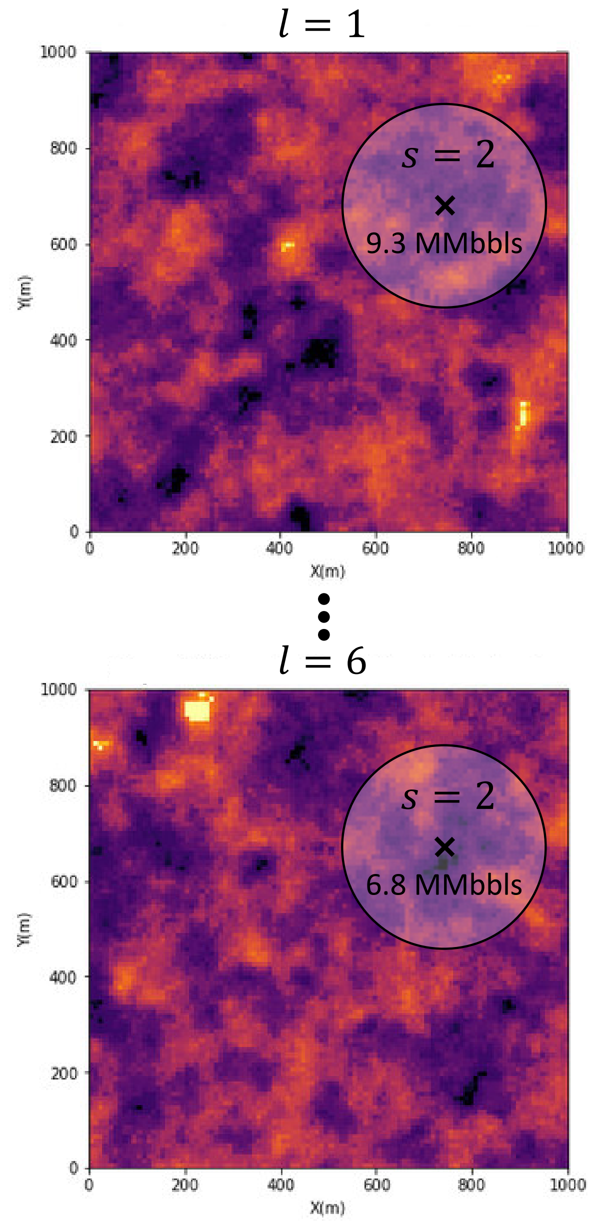
- Calculation expected profit $\boldsymbol{E}\{\boldsymbol{P}(\boldsymbol{S})\}$
  - Expected profit is a probability, λ_l, weighted average of profit over subsurface uncertainty

$$\boldsymbol{E}\{\boldsymbol{P}(\boldsymbol{s}=\mathbf{2})\} = \frac{1}{\sum_{l=1}^{L} \lambda_l} \sum_{l=1}^{L} \lambda_l \cdot \boldsymbol{P}(\boldsymbol{l}, \boldsymbol{s}=\mathbf{2})$$

  - if all models are equiprobable:

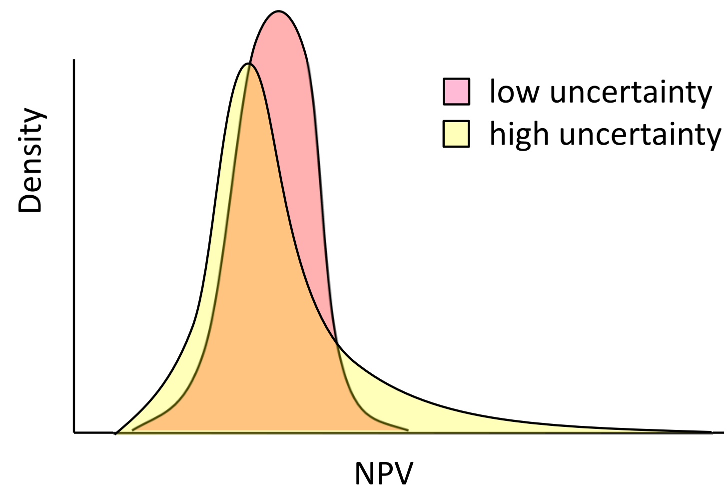
$$\lambda_l = \frac{1}{L}, l = 1, \ldots, L$$

Profit calculated over all subsurface realizations and development scenarios.



IMPACT OF UNCERTAINTY

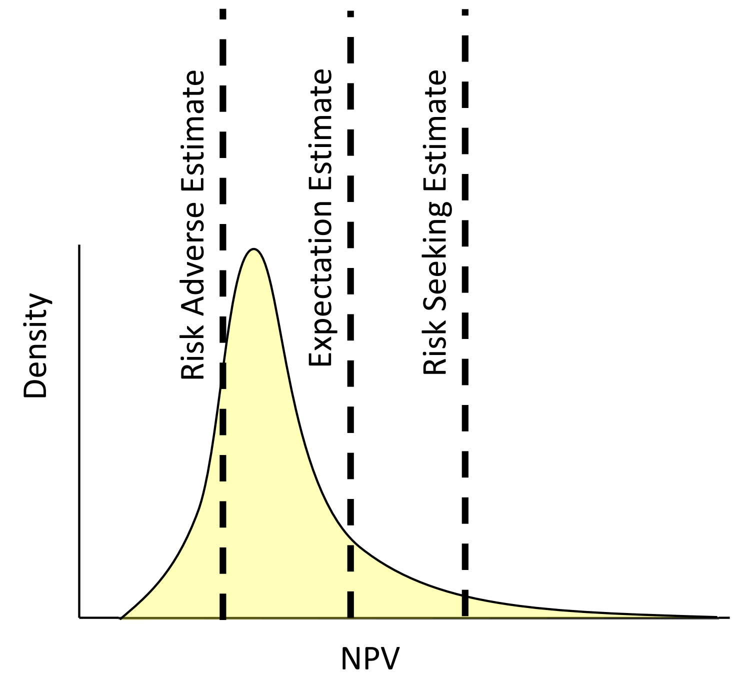
- What Uncertainty Distribution is Best?
  - Is narrower variance better?
  - Higher variance may have more downside risk but also more upside potential.
  - e.g., in shale play well product has a high variance, lognormal distribution and there are large well counts
  - a few high production wells can pay for the operation
  - metal grades may also be approximately lognormally distributed
  - a small proportion of selective mining units impact the entire operation



2 uncertainty models

RISK AND DECISION MAKING

- How do we make optimum estimates in the presence of uncertainty?
- We cannot provide an uncertainty distribution to operations for decision making.
- We must provide a single estimate. We must choose a single estimate in the presence of uncertainty
- The expectation estimate (arithmetic average) assumes the cost of under- and over-estimation is symmetric and squared (L2 norm)



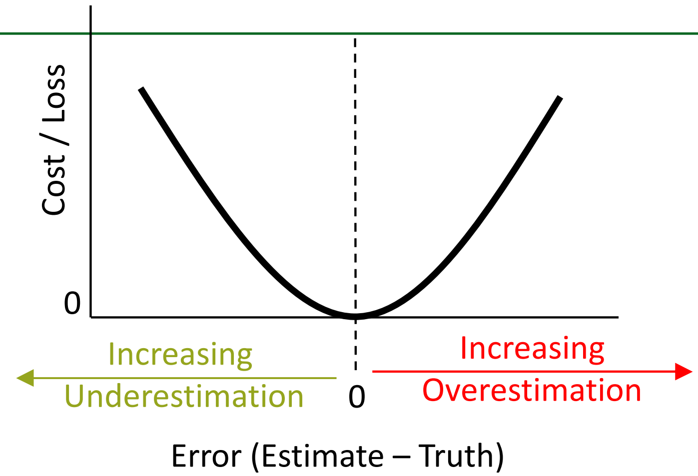
2 uncertainty models

- Risk Adverse – cost of overestimation > cost of underestimation
- Risk Seeking – cost of overestimation < cost of underestimation
- Let's formalize this with the concept of Loss Functions.

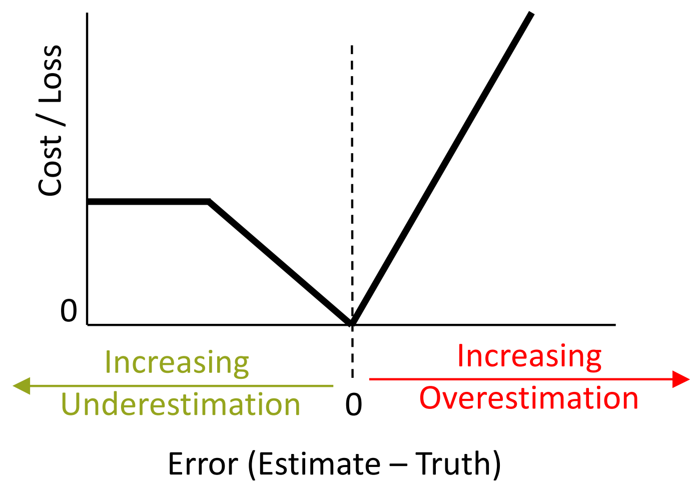
LOSS FUNCTIONS

LOSS FUNCTIONS

- Loss Functions
  - To make a decision in the presence of uncertainty we need to quantify the loss function
  - Loss due to over- and underestimation of the true value.
  - No loss if the estimate is correct, Estimate – Truth = 0
  - Note: the estimating with the mean minimizes the quadratic loss function
  - For a more complicated example, overestimation is more costly than underestimation and at some threshold underestimation any further has no more cost.



Simple loss function, the cost of estimation error represented with a symmetric quadratic function.



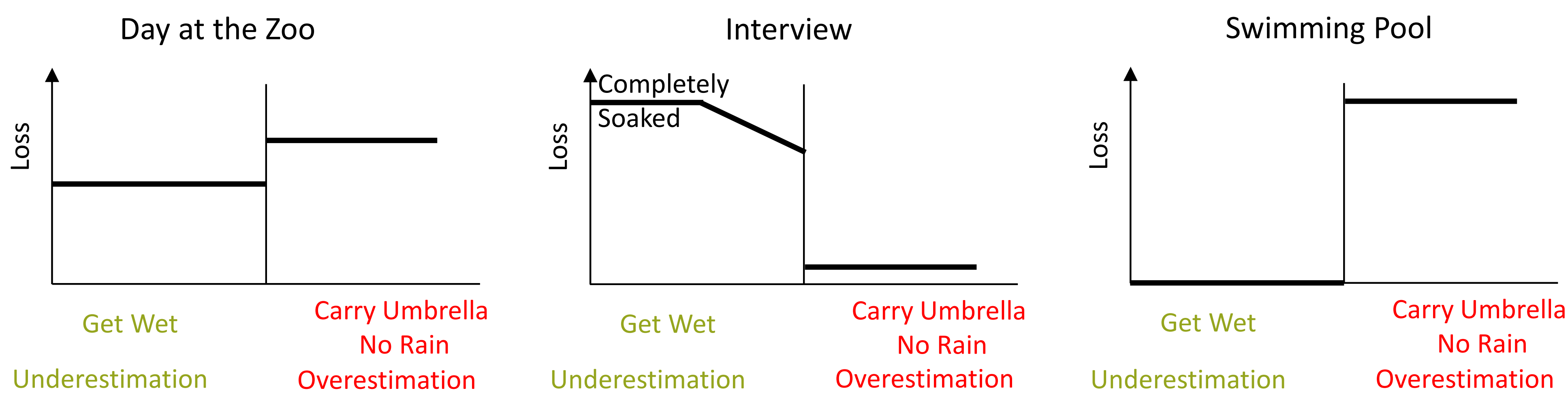
Complicated loss function, the cost of estimation error with asymmetry and thresholds.

LOSS FUNCTIONS

- Loss Function Example
  - Loss function example to support the decision to carry an umbrella

Overestimation = you estimate rain, but no rain happens
Underestimation = you estimate no rain, but rain happens

  - Should you carry an umbrella? The loss function depend on where you are going?

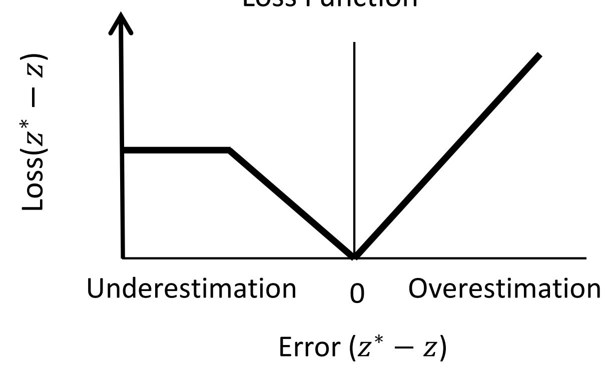


Loss functions for estimating rain for going to the zoo, an interview and the swimming pool.

LOSS FUNCTIONS

- Decision Making in the Presence of Uncertainty :
  - quantify cost of over and underestimation in a loss function
  - apply the loss function to the random variable of interest for a range of estimates
  - calculate the expected loss for each estimate
  - make decision that minimizes loss

Loss Function



Loss function.

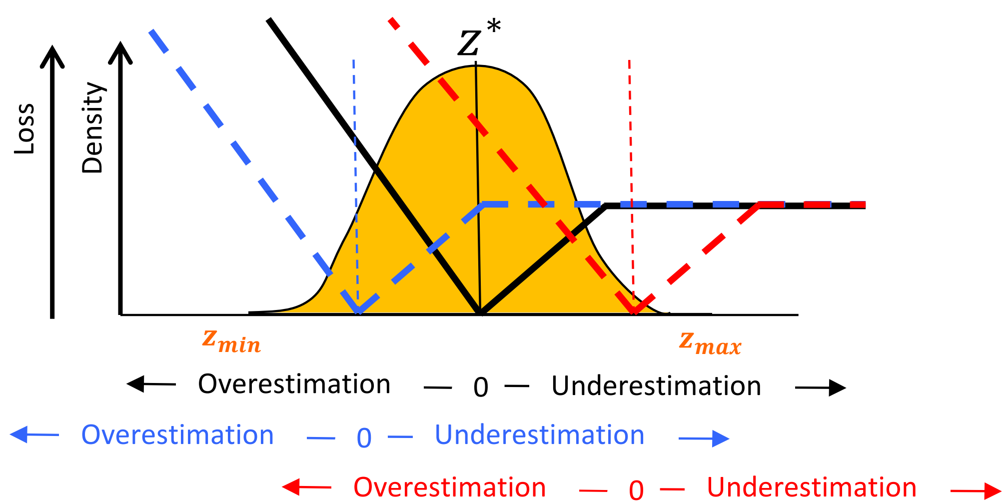
flipped

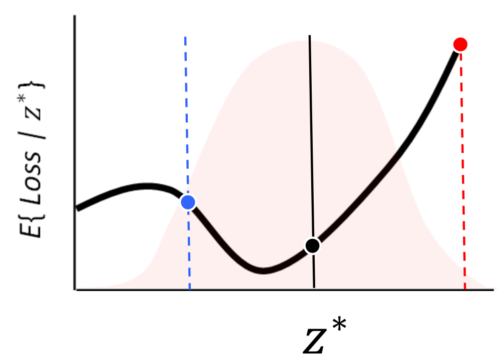
$$E\{Loss(z^*)\} = \int_{-\infty}^{\infty} Loss(z - z^*) \cdot f_z(z)\, dz$$

Loss for a specific amount of error

Probability of that error

Uncertainty PDF and Loss for 3 Estimate Cases





Schematic of expected loss calculation for 3 estimate cases.

LOSS FUNCTIONS

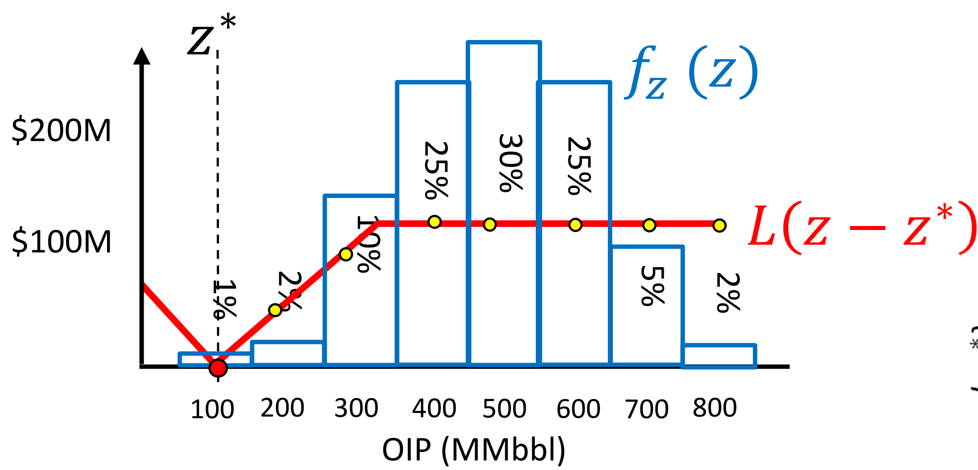
- Expected Loss Calculation:
  - Recall, expectation is a probability weighted average
  - Calculation of the expected loss for a single estimate: $E\{Loss(z^\wedge *\}$

$$E\{Loss(z^*\} = \int_{-\infty}^{\infty} L(z-z^*) \cdot f_z(z)\, dz = \sum L(z-z^*) \cdot P(z)$$

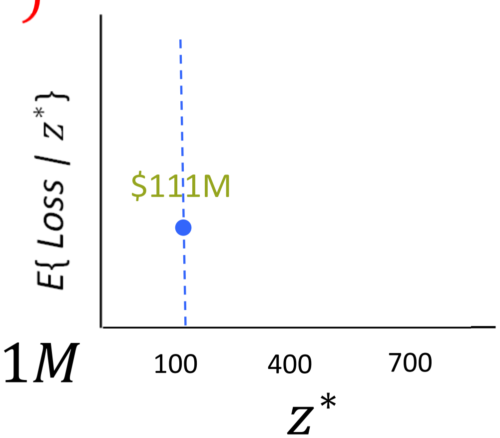
discrete approximation

Recall expectation is a probability weighted average.

$$E\{z\} = \int_{-\infty}^{\infty} z \cdot f_z(z)\, dz$$



$$E\{Loss(z^* = 100\ MMbbl)\} \sum \text{1\% x \$0M} \quad \text{2\% x \$50M} \quad \text{10\% x \$100M} \quad \text{25\% x \$115M} \quad \text{25\% x \$115M} \quad \text{5\% x \$115M} \quad \text{2\% x \$115M} = \$111M$$

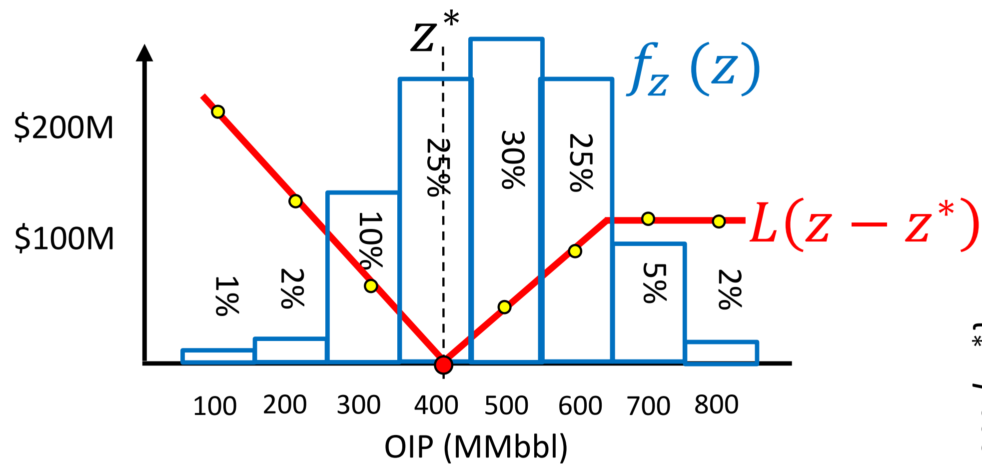


Expected Loss vs. estimate.

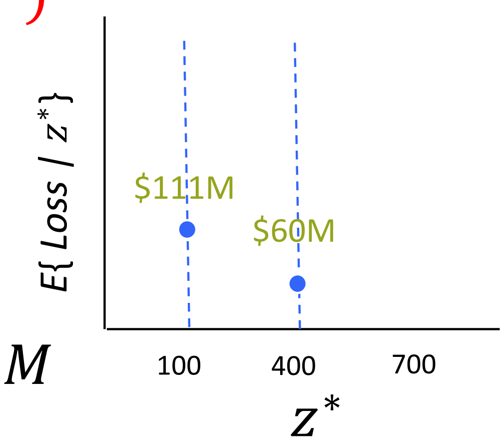
LOSS FUNCTIONS

- Expected Loss Calculation:
  - Recall, expectation is a probability weighted average
  - Calculation of the expected loss for a single estimate: $E\{Loss(z^*\}$

$$E\{Loss(z^*\} = \int_{-\infty}^{\infty} L(z - z^*) \cdot f_z(z)\, dz = \underbrace{\sum L(z - z^*) \cdot P(z)}_{\text{discrete approximation}}$$



$$E\{Loss(z^* = 400\ MMbbl)\} \sum \begin{matrix} 1\%\ \text{x}\ \$210M \\ 2\%\ \text{x}\ \$140M \\ 10\%\ \text{x}\ \$70M \\ 25\%\ \text{x}\ \$0M \\ 30\%\ \text{x}\ \$50M \\ 25\%\ \text{x}\ \$100M \\ 5\%\ \text{x}\ \$115M \\ 2\%\ \text{x}\ \$115M \end{matrix} = \$60M$$



Expected Loss vs. estimate.

LOSS FUNCTIONS

- Expected Loss Calculation:
  - Recall, expectation is a probability weighted average
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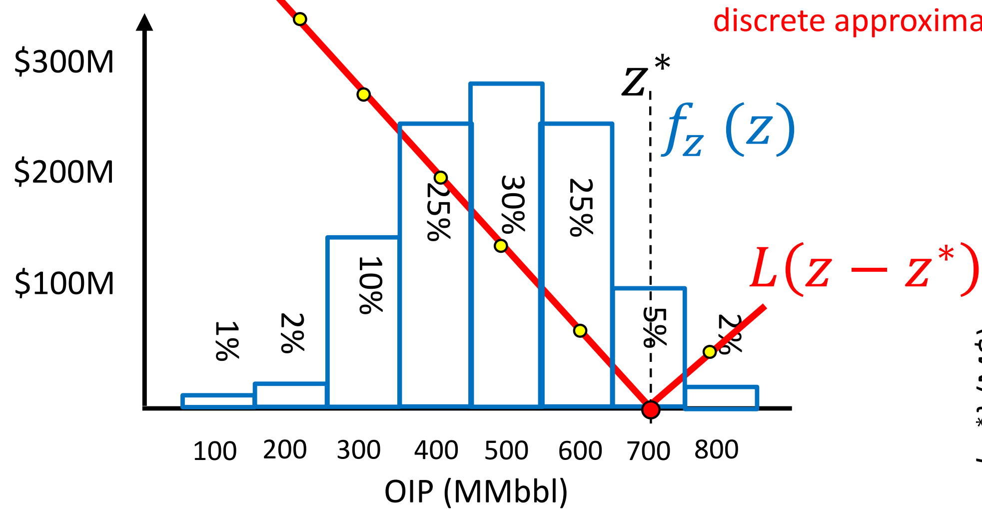
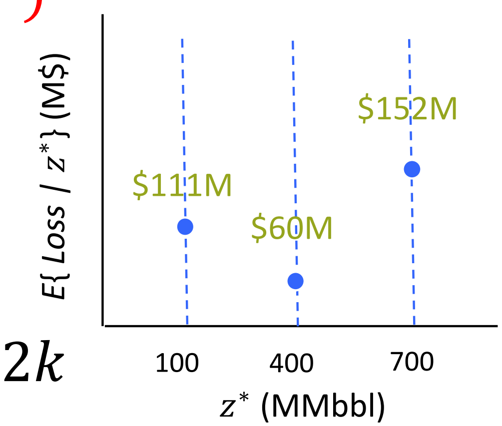
$$E\{Loss(z^*\} = \int_{-\infty}^{\infty} L(z - z^*) \cdot f_z(z)\, dz = \sum L(z - z^*) \cdot P(z)$$

discrete approximation



$$E\{Loss(z^* = 700\ MMbbl)\} \sum \begin{matrix} 1\% \times \$420k & 2\% \times \$350k & 10\% \times \$280k & 25\% \times \$210k & 30\% \times \$140k & 25\% \times \$70k & 5\% \times \$0k & 2\% \times \$50k \end{matrix} = \$152k$$



Expected Loss vs. estimate.

DECISION MAKING EXAMPLE

DECISION MAKING EXAMPLE

- Srivastava (1990) provided a great intuitive example:
  - How much solvent should we inject to assist sweeping oil?
  - Depends on connected volume, Mohan built 500 indicator realizations of sand / shale.
  - Calculated the connected pore volume for each.

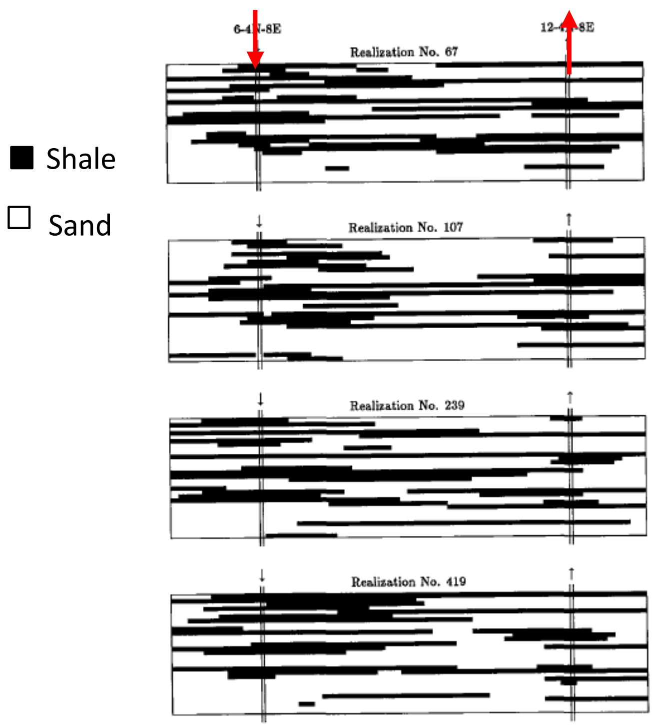


Fig. 3—Local detail for four of the 500 realizations.

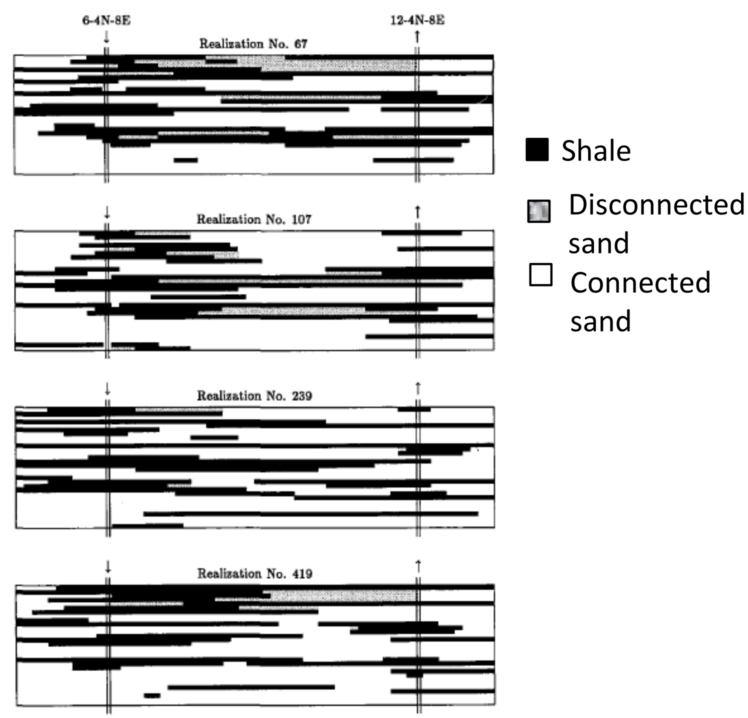


Fig. 4—Connected pore volume for the realizations shown in Fig. 3.

Example and figure from Srivastava, 1990, An Application of Geostatistical Methods for Risk Analysis in Reservoir Management, Society of Petroleum Engineers, SPE# 20608.

DECISION MAKING EXAMPLE

- Srivastava (1990) provided a great intuitive example:
  - From the 500 realizations from sequential indicator simulation, we get this distribution of connected pore volume.
  - But we have to decide on a specific volume of solvent treatment.
  - We need a single estimate of pore volume!

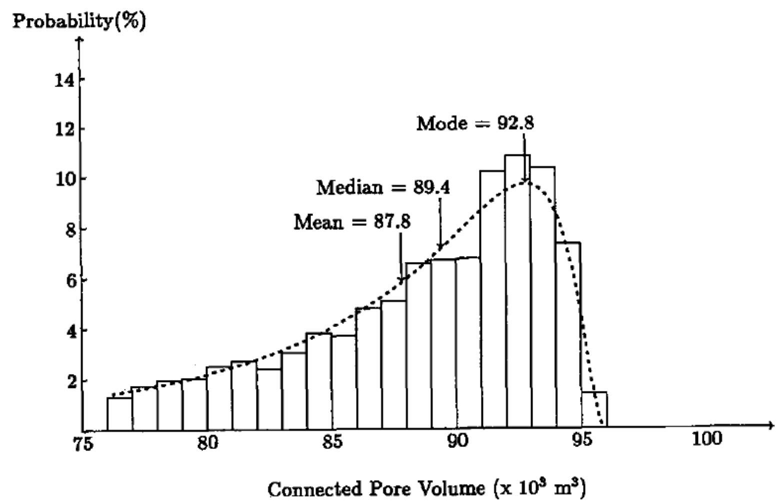


Fig. 6—Traditional *best* estimates from the probability distribution shown in Fig. 5.

  - This would depend on the risk associated with under- and overestimation of pore volume – we use a loss function to model this

Example and figure from Srivastava, 1990, An Application of Geostatistical Methods for Risk Analysis in Reservoir Management, Society of Petroleum Engineers, SPE# 20608.

DECISION MAKING EXAMPLE

- Srivastava (1990) provided a great, intuitive example:
  - The loss function is asymmetric as a function of error in connected pore volume ‘X’ m3 :
  - Overestimate by ‘X’ m3, waste that solvent, cost of solvent
  - ‘X’ m3 x solvent cost$ / m3
  - Underestimate by ‘X’ m3, leave ‘X’ m3 Oil behind, but save on solvent

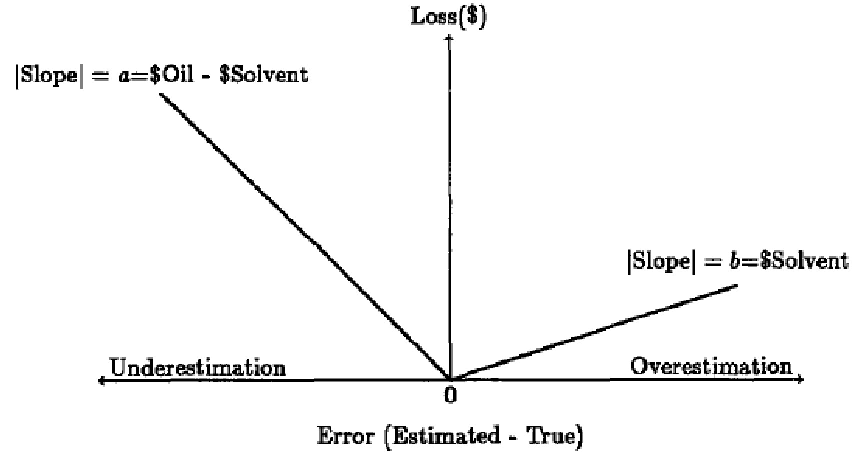


Fig. 7—An asymmetric linear loss function.

‘X’ m^3 x price of oil$ / m^3 - ‘X’ m^3 x solvent cost$ / m^3

Loss function depends on cost of solvent and price of oil.

Example and figure from Srivastava, 1990, An Application of Geostatistical Methods for Risk Analysis in Reservoir Management, Society of Petroleum Engineers, SPE# 20608.

DECISION MAKING EXAMPLE

- Srivastava (1990) provided a great, intuitive example:
  - Applying the loss function to the uncertainty distribution and select the pore volume estimate that minimizes the expected loss.
  - The best estimate of the pore volume depends on the ratio of the price of oil to the cost of solvent.

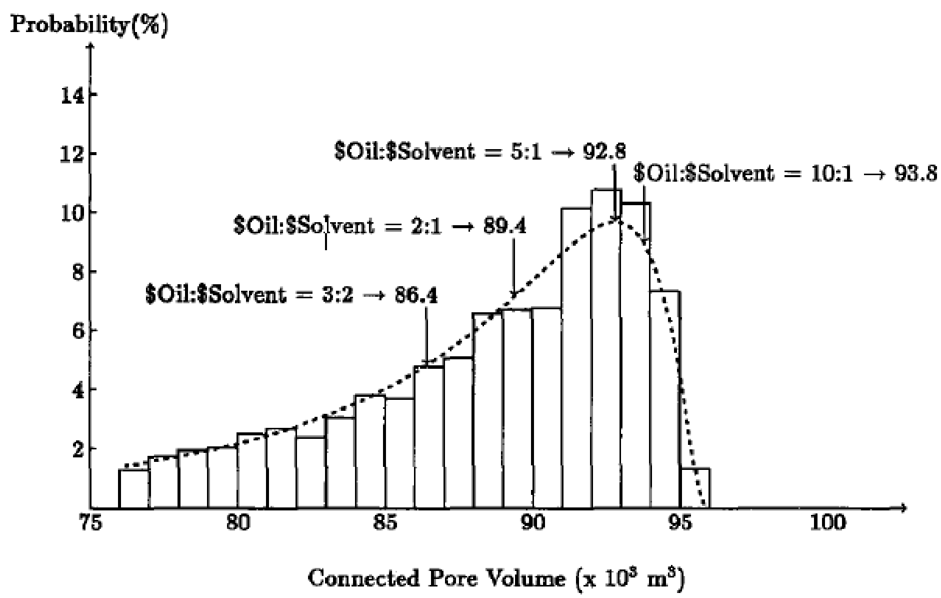


Fig. 8—*Best* estimates using the loss function shown in Fig. 7.

Example and figure from Srivastava, 1990, An Application of Geostatistical Methods for Risk Analysis in Reservoir Management, Society of Petroleum Engineers, SPE# 20608.

DECISION MAKING HANDS ON

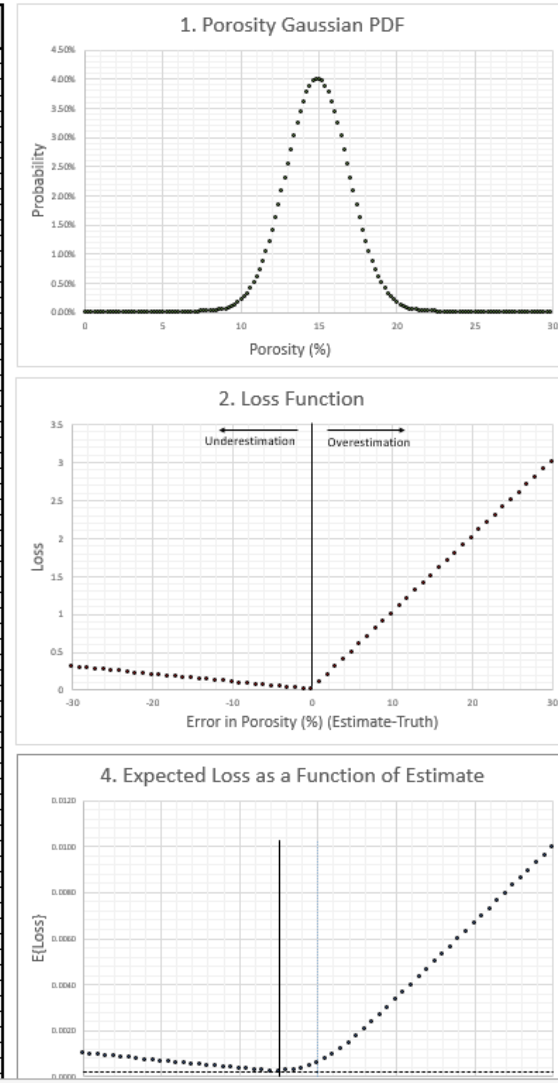
DECISION MAKING HANDS ON

- Things to Try Out:
  - Try increasing and decreasing the cost of overestimation and observe the estimate.
  - Try increasing and decreasing the standard deviation of the uncertainty model.
  - Try asymmetric loss functions.

The File is Decision_Making_with_Loss_Function.xlsx.

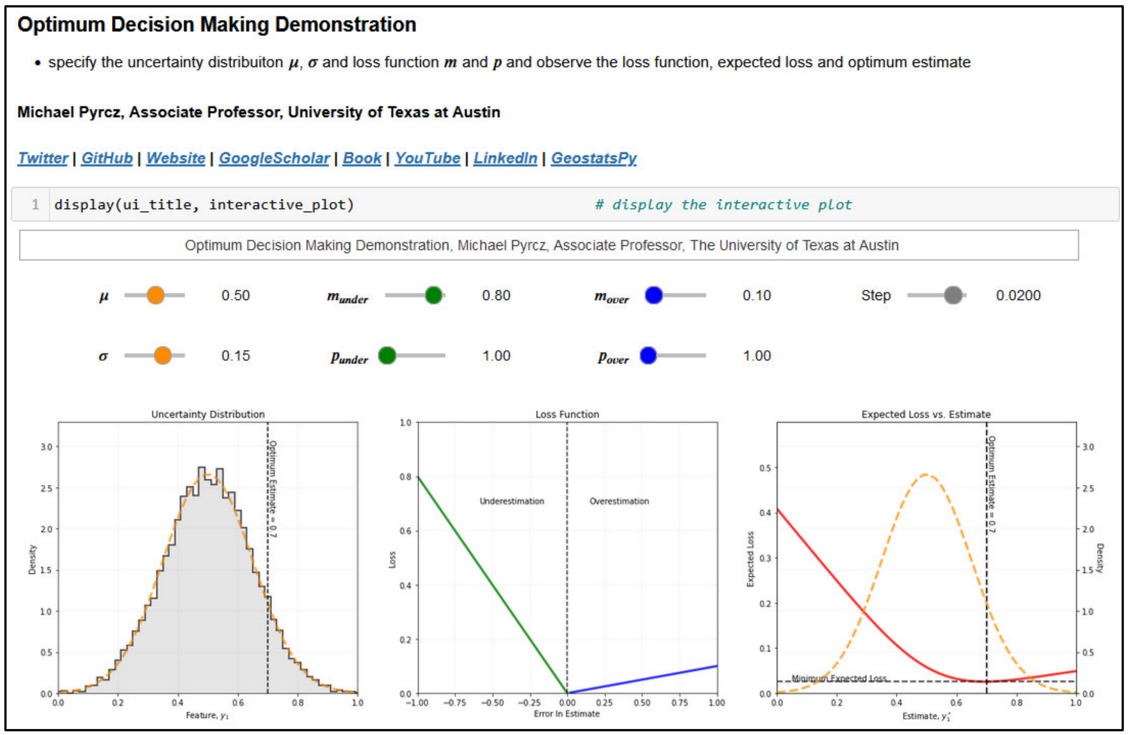
1. Gaussian Distribution Parameters		
Mean	15	
Standard Deviation	2	

Porosity (%)	Probability	Probability Normalized
0.1	1.8E-13	0.00%
0.3	3.7E-13	0.00%
0.5	7.7E-13	0.00%
0.7	1.6E-12	0.00%
0.9	3.2E-12	0.00%
1.1	6.5E-12	0.00%
1.3	1.3E-11	0.00%
1.5	2.5E-11	0.00%
1.7	5.0E-11	0.00%
1.9	9.6E-11	0.00%
2.1	1.8E-10	0.00%
2.3	3.5E-10	0.00%
2.5	6.6E-10	0.00%
2.7	1.2E-09	0.00%
2.9	2.2E-09	0.00%
3.1	4.1E-09	0.00%
3.3	7.4E-09	0.00%
3.5	1.3E-08	0.00%
3.7	2.3E-08	0.00%
3.9	4.1E-08	0.00%
4.1	7.1E-08	0.00%
4.3	1.2E-07	0.00%
4.5	2.1E-07	0.00%
4.7	3.5E-07	0.00%
4.9	5.8E-07	0.00%
5.1	9.5E-07	0.00%
5.3	1.6E-06	0.00%
5.5	2.5E-06	0.00%
5.7	4.0E-06	0.00%
5.9	6.4E-06	0.00%
6.1	1.0E-05	0.00%
6.3	1.6E-05	0.00%
6.5	2.4E-05	0.00%
6.7	3.6E-05	0.00%
6.9	5.5E-05	0.00%
7.1	8.2E-05	0.00%
7.3	1.2E-04	0.00%
7.5	1.8E-04	0.00%
7.7	2.6E-04	0.01%
7.9	3.7E-04	0.01%
8.1	5.2E-04	0.01%
8.3	7.3E-04	0.01%
8.5	1.0E-03	0.02%
8.7	1.4E-03	0.03%
8.9	1.9E-03	0.04%
9.1	2.6E-03	0.05%
9.3	3.4E-03	0.07%
9.5	4.5E-03	0.09%
9.7	6.0E-03	0.12%
9.9	7.7E-03	0.15%
10.1	9.9E-03	0.20%
10.3	1.3E-02	0.25%
10.5	1.6E-02	0.32%
10.7	2.0E-02	0.40%
10.9	2.4E-02	0.49%
11.1	3.0E-02	0.60%
11.3	3.6E-02	0.72%
11.5	4.3E-02	0.86%
11.7	5.1E-02	1.02%
11.9	6.0E-02	1.20%
12.1	7.0E-02	1.39%
12.3	8.0E-02	1.60%
12.5	9.1E-02	1.83%
12.7	1.0E-01	2.06%
12.9	1.1E-01	2.30%
13.1	1.3E-01	2.54%



DECISION MAKING HANDS ON

- Things to Try Out:
  - Try increasing and decreasing the cost of overestimation and observe the estimate.
  - Try increasing and decreasing the standard deviation of the uncertainty model.
  - Try asymmetric and nonlinear loss functions.



The File is Interactive_DecisionMaking.ipynb.

DECISION MAKING

New Tools

Topic	Application to Subsurface Modeling
Loss Functions	Quantify cost of over and underestimation. *Use loss functions to support optimum decision making.*
Decision Making in Presence of Uncertainty	Decision making with loss functions over uncertainty model and development choice scenarios. Apply loss functions and statistical expectation to find decision to maximizes expected net present values.

Decision Making

Lecture outline . . .

- Decision Making
- Loss Functions
- Decision Making Example
- Decision Making Hands-on